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# PREFACE

This is Number 2 of Volume 48 of the A. I. E. E. Quarterly TRANSACTIONS. In it appears the majority of the papers and discussions presented at the Winter Convention held in New York from January 28 to February 1, 1929. The remaining papers, together with those from the Regional Meeting held in Cincinnati during March, will be published in the July Quarterly.

# Anomalous Conduction as a Cause of Dielectric Absorption*

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*Synopsis.—The paper describes a series of experiments on the fundamental electrical properties of a number of waxes and oils, such as are used in composite high-voltage insulation. Particular attention has been directed to dielectric absorption as the origin of dielectric loss and the probable cause of deterioration and short life. The relation of the properties of the constituent parts to the properties of mixtures has been examined, with special reference to purification methods and the influence of small amounts of impurities. The studies have also afforded opportunity to test the validity of certain existing theories of absorption and loss. The method of experiment is the recording of short time absorption curves by means of the string galvanometer. Charge and discharge curves have been traced from a small fraction of a second following the beginning of charge or discharge, these curves having been brought much nearer to their beginnings than any heretofore recorded. It is this region near the beginning of these phenomena, which is most important in its bearing on dielectric loss.*

*The principal results are summarized at the end of the paper, among the most striking being: (a) The extensive evidence that dielectric absorption is purely a conduction phenomenon; (b) The conductivity of dielectrics is highly anomalous in character, sometimes increasing, sometimes decreasing with increases in voltage and temperature. Absorption always follows the conductivity in these changes. (c) No existing theory, including that of Maxwell, can account for the variations in the absorption of mixtures of dielectrics here studied, and (d) Temperature elevation increases conduction and is a most serious factor in causing deterioration. The control of the processes involved in changes of anomalous conduction offers the best opportunity for improvement of composite insulation.*

* * * * *

## I. INTRODUCTION

THE two most important properties of an insulating material are high dielectric strength and long life. The best insulators from this point of view are the crystalline, rigid, and refractory materials, such as porcelain, mica, quartz, and the like. These materials do not lend themselves to the insulation of wires and the various forms of conductor necessary in electric machinery. The composite and fibrous materials necessary in these cases have in general lower dielectric strength and shorter life. The short life is associated with the relatively high dielectric loss, always inherent in this class of materials. Under the combined influence of electric stress and temperature, progressive chemical and physical changes occur in the structure of the material. These changes are almost invariably in the direction of a deterioration of the important insulating properties: phase difference, resistivity, and electric strength.

The nature of these changes is not understood. The close correlation with changes in dielectric loss indicates clearly that an understanding and control of the loss will point the way to longer life. The evidence of recent years is that the greater part of dielectric loss is due to the phenomenon known as dielectric absorption, the nature of the origin of which is itself very obscure. The close relationship between dielectric absorption and dielectric loss has been shown experimentally,[37] and in fact the familiar decaying charging current curve under continuous voltage, typical of absorption, has been shown by Tank, Bouasse[37] and others to be sufficient to account for the essential characteristics of dielectric loss.

Unfortunately this does not help us very much, since in spite of the early recognition of the phenomenon of absorption (1843) and the constant study given it over many years by both physicists and engineers, it remains almost as obscure in its origin as during the early active period of its study. Only a single rational theory, that of Maxwell, has been proposed. Other suggestions of purely hypothetical character, and broadly qualitative, in relation to recent theories of atomic structure and ionic movement, have been proposed from time to time. None of these in the light of present knowledge is subject to experimental test. In fact, the same may be said of the proposal of Maxwell. Maxwell's theory, in short, has remained alive in spite of the absence of experimental proof, largely because of its simple rational character, because of its obvious sufficiency, if its premises be granted, and principally because no better theory has yet been proposed. As an indication of this condition, K. W. Wagner[40] has recently extended Maxwell's analysis to the alternating case and shown that many of the most striking experimental observations may thereby be satisfactorily explained.

The probable explanation of this vague position of Maxwell's proposal lies in the vast amount of experimental data accumulated in the general field of dielectrc research, which contains many suggestions of other causes which might account for the phenomenon of dielectric absorption. Recently there has been a particularly striking increase in interest in the problem of dielectric conductivity and polarization. All this work brings prominently forward the evidence of the motion of ions through the body of the dielectric,

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37. For numbered references see Bibliography.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-31, 1929.*

whether liquid or solid, inequalities of potential gradient, and various suggestions of an accumulation of space charge. We comment in further detail on this work later on. In general, it may be said that almost any single one of these various types of conductivity may be invoked with reasonable success as a qualitative explanation of the more conspicuous manifestations of dielectric absorption.

## II. Purpose of the Work

It therefore appeared desirable to investigate more closely some of the more common dielectrics utilized in the manufacture of commercial insulation, with special reference to (1) A study of the charge and discharge absorption current curves at shorter intervals than heretofore observed, following the application and removal of voltage. This is the region of greatest influence on the value of dielectric loss. (2) To examine how closely these materials may be made by ordinary methods to approach the simple character postulated by Maxwell. (3) To study the dielectric properties of several substances, singly and in combination. (4) To study the mutual relationship of absorption and conduction as related to variations in temperature, electric gradient, and admixed impurities.

Fig. 1—General View of Measuring Equipment

Left: string galvanometer and auxiliaries. Center: condenser, cover removed. Right: charge and discharge switch.

As materials for study we have selected some of the better known waxes and oils. These are, for the most part, good dielectrics, are used commonly in the manufacture of insulation, and lend themselves readily to convenient and reliable assembly for test. The several samples have been measured in a parallel plate condenser. The quantities measured are the charge and discharge current curves, the dielectric constant, the final conductivity, over a range of values of temperature and of electric gradient.

## III. Apparatus and Methods

The condenser consists essentially of two circular brass plates. The lower of these is 60.96 cm. in diameter, and serves also as the bottom of a pan to contain the dielectric. This pan is mounted on insulators and is connected to the high side of the potential. The upper plate is 52.7 cm. in diameter and is surrounded by a guard ring 2.54 cm. wide, with air gap 0.32 between. See Fig. 2.

The back of the condenser plate is screened by metal cover, forming a part of the guard system. All metal surfaces of the electrodes, guard rings, and pan were nickel plated and polished. The computed capacity of the condenser at 0.249 cm., separation, is 0.000,784 $\mu$ f. (air).

The condenser as described is placed in an outer electrically heated oil bath, which in its turn rests in a deep bed of sand, the whole being contained in a stout wooden box, reinforced with steel bars and covered with galvanized iron with carefully soldered joints to render it air tight. This box is provided with a removable top,

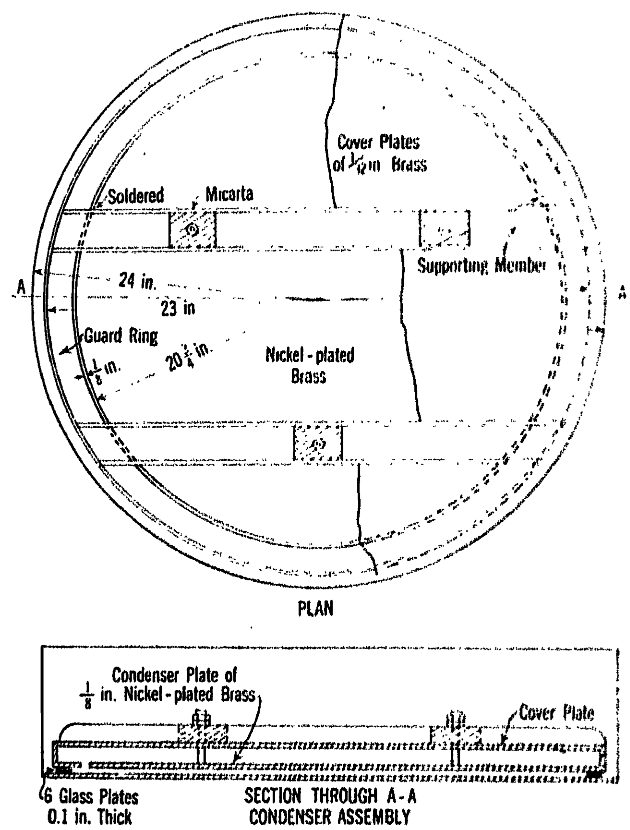


Fig. 2—Condenser Used in Tests

the joint between consisting of polished brass bars, sealed with wax. The whole provides means for temperature control and evacuation of the air during the preparation of the specimens.

After considering various methods and some preliminary experiments for measuring the very small absorption current of good insulators, we adopted the Einthoven[7] string galvanometer, for which see also the series of articles by Williams.[38]

The galvanometer itself consists of a powerful electro-

magnet with a narrow air gap, across which is stretched a coated quartz fiber. By a system of a condensing lens and microscope, light from an arc is passed through the galvanometer and the shadow of a portion of the string is focused on a revolving photographic film. The essential elements will be seen in Fig. 1. The film revolves at a known uniform speed and the shadow of the string gives a trace of the variation of the current passing through the string. Simple methods permit also the projection on the film of uniform vertical and horizontal scales. As the speed is slow, one revolution in 2 to 3 sec., bromide paper is sufficiently rapid for taking the exposure. It has the advantage of giving immediate and legible records without printing, and at great reduction of cost. The complete record is about 10 in. in length.

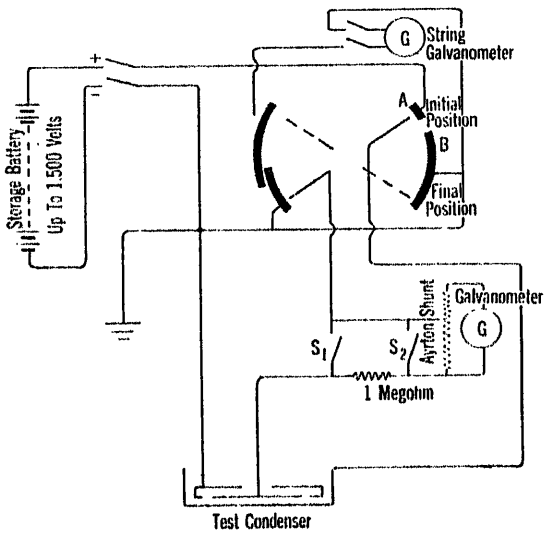


Fig. 3—Circuit for Measuring Absorption and Conductivity

$S_1$ is closed during absorption test.
$S_2$ is to stop swing
Connections are shown for discharge. For charge reverse connections to $A$ and $B$

The sensitivity of the galvanometer can be varied over a wide range by altering the tension of the string. In our work the sensitivity usually ranged from $10^{-9}$ to $10^{-7}$ amperes per mm. The instrument is extremely sensitive to induced electrical disturbances, as well as mechanical vibration. Progressive elimination of all such disturbances, not always easy to find, is essential for satisfactory records.

An automatic switch operated by gravity was used for connecting the galvanometer into the condenser circuit. This switch was arranged to close the condenser circuit on charge and insert the galvanometer into the circuit after a small fraction of a second. Similarly, for discharge the condenser was short circuited for a brief interval, the galvanometer then being cut in. In the case of discharge the period of open circuit following charge was 0.005 sec., the period of short circuit 0.0022 sec., the galvanometer then being cut in. Similar intervals obtained for the period of charge, these figures being obtained by oscillograms. An approximate computation indicated that the initial transient of the geometric charge was negligible before the galvanometer was cut in the circuit. See Fig. 3.

The conductivity was measured on a D'Arsonval galvanometer of sensitivity $4 \times 10^{-10}$ amperes per cm. The measurement was taken following the taking of absorption current on charge. The readings were taken shortly after the absorption record and at intervals during an hour thereafter, this being ample time to give a practically constant value.

The values of dielectric constant on the materials studied lie within a very narrow range as compared with the variations in conductivity, and it was not felt that a high degree of accuracy in its measurement was necessary. We therefore adopted the rapid and convenient method of measuring the current under alternating voltage as indicated in Fig. 4. The dielectric constant so taken is the ratio of the currents in the condenser, when filled with the dielectric in question, and when empty. The introduction of two auxiliary fixed condensers and of a Grondahl rectifier reduced the time required in these measurements to a short interval.

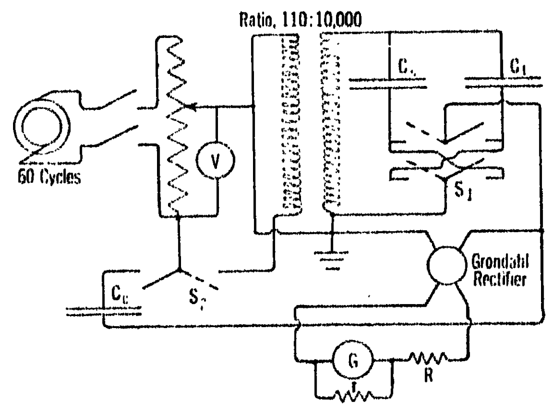


Fig. 4—Circuit for Measuring Dielectric Constant

$C_t$ test condenser
$C_s$ and $C_c$, standard condensers. Value not necessarily known

The absorption and conduction of dielectrics are extremely sensitive to temperature variation so that the accurate determination of the temperature received special attention. Two methods were used: A resistance thermometer was placed in the oil below the condenser. This was used in melting the material and as a means of holding the temperature at a definite value. A second method was by the use of copper-Advance thermocouples, three placed on the central plate of the condenser and one in the oil. The couples were mounted in brass blocks placed immediately on the upper side of the condenser plate, and one below the condenser on top of the heater. When the condenser was filled all of these blocks were covered by the dielectric. The cold junctions were kept at 0 deg. cent.

The true temperature was assumed to be the mean of the values of all four couples. The resistance thermometer and the thermocouples were calibrated by reference to an accurate mercury thermometer, graduated to 0.2 deg.

For calibrating the string galvanometer and several of the instruments, and for measuring temperatures, the potentiometer as indicated in Fig. 5 was used.

As a source of potential a lead storage battery of 768 cells was available. Values of voltage used were 1500, 1000, and 500 volts.

## IV. Description of Materials

*Paraffin.* One of the best of the better known insulators. Ours was purchased as "Refined Wax," melting point 128–130 deg. fahr. (53.4–54.5 deg. cent.). It was a hard white paraffin, being the purest and most refined grade obtainable from one of the large manufacturers. Thorpe[33] gives the following definition of paraffin:

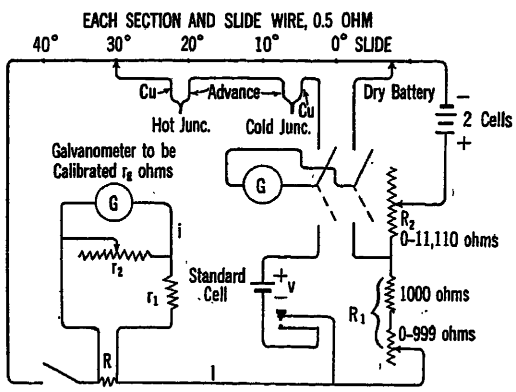


Fig. 5—Potentiometer for Galvanometer Calibration and Temperature Measurements

"A solid wax-like substance composed of saturated hydrocarbons of the series $C_nH_{2n+2}$. Melting point 38–57 deg. cent." Values of $n$ from 12 to 60. The variety of constituents explains why commercial paraffin, although presumably it has a crystalline structure, softens over a wide range of temperatures. The paraffin used by us softened slightly at 35 deg. cent. and at 45 deg. cent. was plastic like putty.

Paraffin when melted and allowed to cool in air, absorbs a considerable quantity of air which is set free on solidification, causing the usual opaque appearance. Melting and cooling in even a moderate vacuum reduces this and makes it fairly transparent; Möller.[17]

Commercial paraffin may also contain noticeable quantities of water. Sometimes on first melting globules of water would collect in the bottom of the vessel. In one instance about half a teaspoonful was obtained from five pounds of paraffin, but this was exceptional. This excess of water is presumably mechanically entrapped in manufacture, since it at once falls down on melting. As in the case of transformer oils one would expect a certain amount of water to be actually dissolved. Since at 65 deg. cent., the temperature generally used for melting, the vapor pressure of water is 18.8 cm., which is much above the vacuum of around 5 cm., the water should be largely eliminated. Whether, however, sufficient water is taken up to affect the properties we were measuring is open to question, since with perhaps one or two exceptions, paraffin merely melted and cooled in air had as low absorption and conductivity as when vacuum treated.

*Ceresine.* This is a natural or fossil paraffin. The composition and structure are probably much the same, although some oxidation may have occurred. It is obtained by purifying ozokerite or mineral wax, which is found principally in Poland. As an insulating material it ranks with paraffin.

The particular material used resembled a hard white paraffin. Its melting point was 52-53 deg. cent. It absorbed air in much the same way as paraffin, and like paraffin became clear when given a vacuum treatment.

*Spermaceti.* This is a wax-like substance obtained from a cavity in the head of the sperm whale. The material used by us was white, rather opaque, and had a slight odor resembling tallow. Thorpe[33] gives the composition as mainly cetyl palmitate $(C_{16}H_{33})$ $C_{16}H_{31}O_2$. He gives the melting point as 41-49 deg. cent., but by repeated purification rising to 53.5 deg. cent. Our material showed a melting point of 45-48 deg. cent., depending upon the method used.

Unlike paraffin, a vacuum treatment produces no visible change. The structure is in all cases rather coarse and crystalline, and does not suggest good dielectric strength, which was also indicated by its failing during one of the tests. It did not show characteristics justifying extensive study.

*Carnauba Wax.* This is a vegetable wax. Thorpe[33] describes it as follows: "Obtained from the leaves of the carnauba palm, Brazil. Color: dirty yellow or greenish, melting point 83-86 deg. cent. specific gravity 0.990-0.999 at 15 deg. cent. The principal constituent is a myricyl ester of ceratic acid, $C_{27}H_{53}O_2$. $C_{30}H_{61}$, together with some minor constituents." The melting point was 85 deg. cent. specific gravity at 20 deg. cent. being 0.979. While hard, it is very brittle, has little adhesion to metal, and contracts greatly on cooling. As a consequence it cracks badly, and is very unsatisfactory as a dielectric.

*Stearic Acid.* This is obtained from animal fats. The composition is $C_{18}H_{36}O_2$. The melting point when pure is 69.3 deg. cent. It is crystalline in structure. Palmitic acid $C_{16}H_{32}O_2$, another member of the same series, is usually associated with it, and is very difficult to eliminate. Pure stearic acid is odorless. The grade used by us was that purified by alcohol, and presumably of good quality. The melting point was only 55 deg. cent., indicating the presence of palmitic acid, and possibly lower members of the same series. The

odor was quite strong, resembling that of tallow. It had a light cream color.

Stearic acid has high absorption and conductivity, making it an interesting material for our work. While the adhesion to metal is poor, which would tend to promote cracking, no trouble was ever experienced with the dielectric strength.

*Refined Lubricating Oil.* This is a high grade oil of excellent insulating qualities and was selected in order to study the effect of mixing it with paraffin. It was supplied by the Standard Oil Company of New Jersey, as their purest and highest grade insulating oil. It is clear oil with a bright yellow color, somewhat viscous, but flows quite freely at room temperature. The specific gravity at 26 deg. cent. is 0.877.

*Black Oil.* Furnished by request as a petroleum oil of poor insulating qualities for use in the study in mixtures. It is evidently in an intermediate stage of refining. The color is a dense black, probably indicating the presence of an asphalt compound. At 20 deg. cent. it flows with difficulty, and approaches the consistency of a grease. Even at 65 deg. cent. it was so viscous that it was difficult to remove from the condenser plates.

## V. THE EXPERIMENTS

The chief observations were the charge and discharge absorption current curves as taken with the string galvanometer. These afforded data as to absorption and conduction. In each case they were taken over a range of temperature and of voltage, and the dielectric constant measured as well. The influence of combined temperature and reduced air pressure on the electric properties was also studied. Paraffin showing itself practically free of absorption, was used as a basic material for the study of its combination with other materials, and with "impurities."

A large number of observations has been taken. Complete presentation is unnecessary. We give, therefore, in Figs. 6 to 11, a series of typical charge and discharge current curves as observed on various specimens. In all we have taken over 200 such curves. They constitute the principal record on which our conclusions are based. The numerical values presented in Tables I to IV are measured directly from the photographic records. These tables are a condensed compilation from the more extended individual tables pertaining to the respective specimens. The materials and details of treatment corresponding to the individual specimens as numbered are given in Table V. Three standard temperatures were used, 25, 35, and 45 deg. cent., and three standard voltages, 500, 1000, 1500 volts; nine sets of tests on each sample. Occasionally higher temperatures were included, but as these usually approach the melting point of the solids, we have selected the three low temperatures mentioned.

From the charging current records the matter of chief interest is the excess of the current over the final conduction current, measured after one hour. The comparison of this excess of the current with the values of the discharge current indicates the possible presence of irreversible absorption current. In the tables are recorded the values of absorption current at 0.2, 0.4, and 0.8 sec., and also these values less the final conduction current denoted by $i''$.

From the records of discharge currents we have found it best to record the values taken at time intervals in geometric progression 0.1, 0.2, 0.4, 0.8, 1.6, 3.2 sec. The rate at which the current dies out is of significance, and is indicated by the ratio of the current at 0.2 and 0.4 sec., and also at 0.4 and 0.8 sec.

The maximum deflection is clearly indicated on many records. It is, however, reported in only a few instances in which the deflection was so small that the maximum was the only readable feature. We have not used it in our deductions. We have analyzed elsewhere the possible error in the maximum deflection and in general have shown that throughout our work the deflection of the string follows closely the variation of the current, except in a few instances of the higher sensitivity, at the beginning of the record. This conclusion has been proved by photographic records.

## VI. CHARACTERISTICS OF ABSORPTION CURVES

Figs. 6 and 7 are records of charging current, Figs. 8 and 9 of discharge.

Many attempts have been made to express the absorption current curve by a simple algebraic relation. Our accurate curves beginning at brief intervals following short circuit furnish us with excellent material to test the validity of some of these expressions. Prominent among them are those involving the time to a constant negative power and that based on a negative exponential function of the time.

In Fig. 12 is shown a comparison between an actual curve and that given by the formula $i = I\,t^{-n}$, where $I$ and $n$ are constants. The constants have been so chosen that the curves intersect at 0.1 sec. and 2 sec. The agreement is seen to be fair. Such a curve is a straight line when plotted to logarithmic coordinates. A large number of our curves was so plotted and almost invariably gave a line slightly concave towards the time axis.

The negative exponential $i = I\,E^{-at}$ has also been plotted on Fig. 12, in the same manner. Its simplicity and definite value at zero time make it attractive, but the deviation from the actual curve is so great that it is scarcely even a rough approximation. This curve has particular interest from the fact that it is that indicated by Maxwell's analysis. This departure of the results of experiment from the indication of this simple theory has often been noted and is one reason for the discredit now commonly attached to the validity of Maxwell's proposal.

In view of the foregoing discrepancies we have preferred to describe the current curves directly in terms

of the ordinates at the intervals already mentioned, and to compare the variation among different samples in terms of the changes in the ratio of these values.

## VII. Influence of Electrical Gradient

Early theoretical analysis of dielectric absorption, including Maxwell's, attributes to it a linear increase

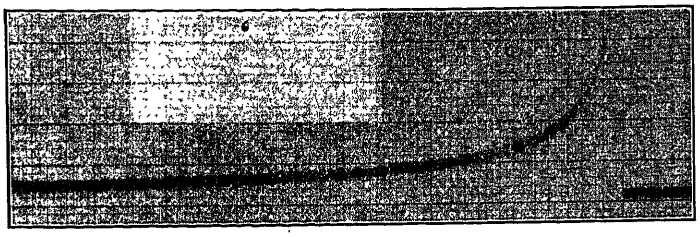

Fig. 6—Specimen No. 24—Carnauba Wax

Dec. 21, 1927—Charge—1500 volts
Temperature 46.1 deg. cent.
Sensitivity 6.28 $\times 10^{-8}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

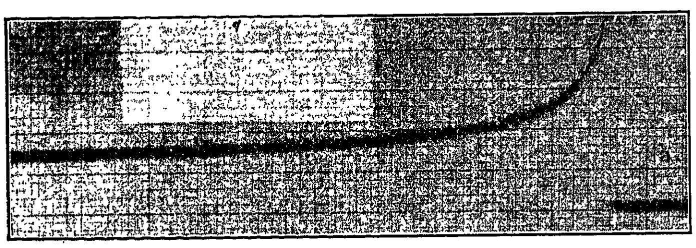

Fig. 7—Specimen No. 28—Paraffin, Equal Parts Good and Bad

Jan. 24, 1928—Charge—1500 volts
Temperature 45.2 deg. cent.
Sensitivity 6.24 $\times 10^{-8}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

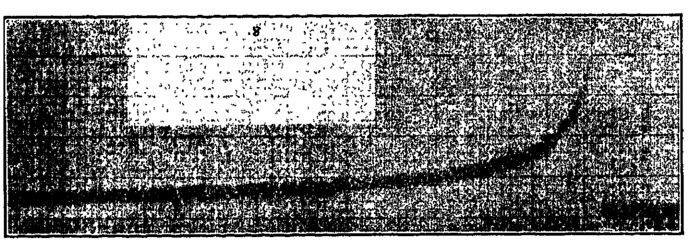

Fig. 8—Specimen No. 11—Stearic Acid

Oct. 6, 1927—Discharge—1500 volts
Temperature 44.8 deg. cent
Sensitivity 1.84 $\times 10^{-7}$ampere/mm.
Time, 1 small sq. = 1/25 sec.

with electric gradient. Curie,[3] from experiments on a number of crystals, announces this as the first of his three well known empirical laws. We have found noticeable variations from this simple relation.

As regards the relation of conduction to electric gradient much confusion exists. Curtis[4] using samples about 1 cm. thick and measuring the conduction at 50 and 500 volts, found no difference with fair insulators, and only a slight increase at the higher voltage for poor and porous insulators. He states that with the best insulators, as paraffin and ceresine, the effect, if any, was masked by absorption. On the other hand Poole[19, 20, 21] investigating mica, glass, and celluloid at high potential gradients found a noticeable effect which could be approximately expressed by the formula $\lambda = A + B X$, where $\lambda$ is the conductivity, $X$ the potential gradient, and $A$ and $B$ constants. This has

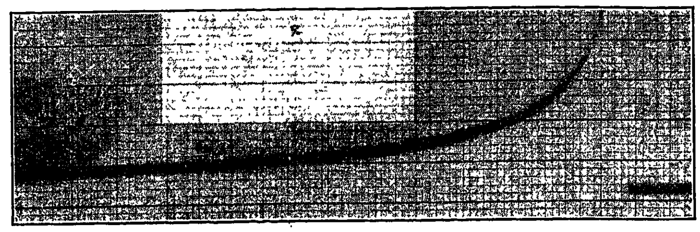

Fig. 9—Specimen No. 24—Carnauba Wax

Dec. 21, 1927—Discharge—1500 volts
Temperature 46.8 deg. cent.
Sensitivity 5.22 $\times 10^{-8}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

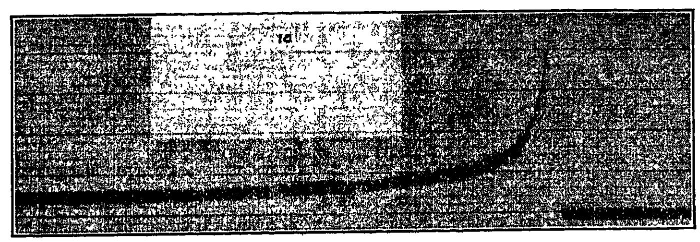

Fig. 10—Specimen No. 34—"Black Oil"

March 2, 1928—Discharge—1500 volts
Temperature 45 deg. cent.
Sensitivity 3.92 $\times 10^{-9}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

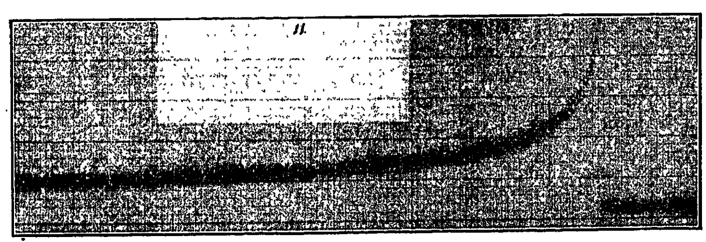

Fig. 11—Specimen No. 38—Paraffin 99 Per Cent, "Black Oil" 1 Per Cent

March 28, 1928—Discharge—500 volts
Temperature 25.1 deg. cent.
Sensitivity 2.09 $\times 10^{-9}$ ampere/mm.
Time, 1 small sq. = 1/25 sec.

been confirmed by Schiller[28] also working on glass, and using values of $X$ up to 500 kv./cm. Mündel[18] likewise experimented with glass plates at all voltages up to breakdown, and concluded that while at low gradients the current is proportional to the gradient, that is constant conduction, at high values the current is nearly proportional to the square of the gradient. Tadeschi[32] studied various paper products at high potential gradients, and found a marked increase in conduction with increasing gradient, although no definite relation

was observed. Further instances could be cited, but these are sufficient to indicate that at gradients which are fairly high the conduction increases with the gradient. Our own results in general indicate the same relation.

The influence of electric gradient in our work may be

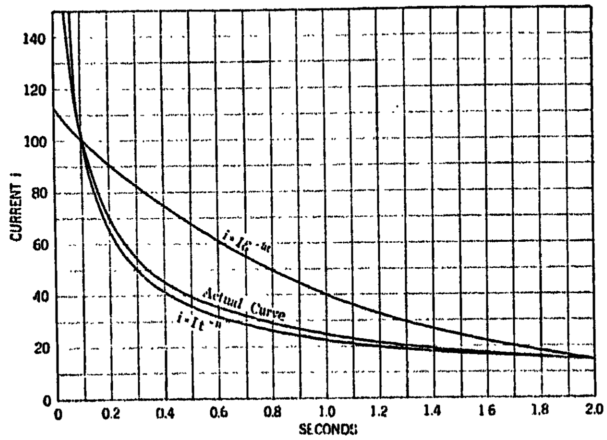


FIG. 12—COMPARISON BETWEEN ACTUAL ABSORPTION CURVE ON DISCHARGE AND EMPIRICAL EXPRESSIONS

Actual curve, paraffin, specimen No. 28, at 45.3 deg. cent. and 1500 volts. Plotted to arbitrary scale of 100 at 0.1 sec.

Empirical curves drawn to intersect actual at 0.1 and 2 sec.

studied from Tables I to IV. The influence on absorption is shown in the ratio of the current at 1500 volts to that at 500 volts, for two time intervals, 0.4 and 1.6 secs. after discharge. The influence on conduction is shown in the ratio of the final currents at 1500 volts and 500 volts. If the absorption were proportional to the electric gradient and the conduction constant, these ratios would all be "3."

In the case of absorption, we notice that there is a general tendency to a value less than "3" for these ratios and this effect is most noticeable at the higher temperatures. Perhaps the most noticeable feature is that the ratio of the currents is almost invariably less at 1.6 sec. than at 0.4 sec. The meaning of this is that the current is falling off more rapidly at the higher voltage. If this is correct then the ratio of all consecutive ordinates should be greater at 1500 volts than at 500 volts. We have examined the results on a number of our specimens from this point of view, and out of 23 cases in which the comparison is possible we have found the ratio is greater at 1500 volts, in 13 cases, in 10 others approximately equal to that at 500, and in no case definitely lower. The evidence, therefore, is clear that the absorbed charge falls off more rapidly the higher the voltage, thus showing a noticeable departure from Curie's empirical law. As regards the conduction a study of the table plainly shows large variations of the ratio of conduction current at 1500 volts to that at 500 volts from the value "3" in both directions. At 25 deg. there are two values out of 15, at 45 deg. there are seven out of 25 which are less than "3." When the ratios for the two temperatures are compared, in nearly every case it is greater at the lower temperature. Since, as will be seen, the conduction usually increases with the temperature, it thus appears that as the conduction increases the relative influence of increasing electric gradient becomes less.

In addition to the data here given for paraffin a large number of check tests was made on the values of conduction. All these agree in showing a marked increase with voltage. However, attempts relating them to a definite law such as that of Poole, for example, were not successful.

Instances in which the conduction has decreased with increase of voltage are of special interest. In the case of No. 30 refined oil and No. 33, equal parts of this

TABLE I

RESULTS OF TESTS AT 500 VOLTS AND 25 DEG. CENT.

UNIT OF CURRENT $10^{-8}$ AMPERE

| Specimen No. 1 | Conduction current 2 | Charge $i$ (0.2) 3 | $i$ (0.8) 4 | $i$ (3.2) 5 | $i''$ (0.2) 6 | $i''$ (0.8) 7 | $i''$ (3 2) 8 | Discharge $i$ (0.2) 9 | $i$ (0.4) 10 | $i$ (0.8) 11 | $i$ (1.6) 12 | $i$ (3.2) 13 | $\frac{i(0.2)}{i(0.4)}$ 14 | $\frac{i(0.4)}{i(0.8)}$ 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 0.154 | .. | .. | .. | .. | .. | .. | Trace at | peak | | | .. | | |
| 8 | 0.242 | .. | .. | .. | .. | .. | .. | Traco at | peak | | | | | |
| 9 | 0.236 | .. | .. | .. | .. | .. | .. | 7.53 | 5.94 | 4.63 | 3.24 | .. | 1.27 | 1.28 |
| 10 | 0.0176 | .. | .. | .. | .. | .. | .. | 0.54 | at peak | | | | | |
| 11 | 18.4 | .. | .. | .. | .. | .. | .. | 207. | 129. | 79. | 43. | 27. | 1.68 | 1.64 |
| 24 | 0.007 | 15.0 | 7.55 | 3.17 | 15.0 | 7.54 | 3.16 | .. | 10.9 | 7.85 | 5.34 | 3.44 | .. | 1.39 |
| 25 | 0.065 | 46.6 | 22.8 | 9.45 | 46.5 | 22.7 | 9.39 | .. | 31.1 | 21.4 | 14.5 | 9.45 | .. | 1.45 |
| 28 | 10.96 | .. | 42.8 | 26.9 | .. | 33.8 | 15.9 | 58.2 | 41.4 | 28.6 | 19.2 | 12.3 | 1.40 | 1.45 |
| 29 | 1.116 | .. | 33.6 | 11.9 | .. | 32.5 | 10.8 | .. | 55.4 | 30.3 | 16.3 | 9.2 | .. | 1.83 |
| 32 | 0.091 | 1.69 | 1.54 | 1.43 | 0.78 | 0.63 | 0.52 | 1.5 | at peak | | | | | |
| 34 | 216. | .. | .. | .. | .. | .. | .. | .. | .. | .. | 18.6 | 11.8 | .. | .. |
| 35 | 194. | .. | .. | .. | .. | .. | .. | 35.4 | 31.5 | 26.7 | 21.9 | 16.4 | 1.12 | 1.18 |
| 36 | 19.8 | 57.4 | 37.1 | 25.3 | 37.6 | 17.3 | 5.5 | 43.7 | 31.3 | 21.8 | 14.7 | 9.9 | 1.39 | 1.44 |
| 37 | 6.66 | .. | 21.5 | 13.6 | .. | 14.8 | 6.9 | .. | .. | 15.6 | 11.6 | 10.4 | .. | .. |
| 38 | 0.409 | 10.14 | 6.09 | 3.57 | 9.73 | 5.68 | 3.16 | 8.44 | 6.74 | 4.92 | 3.87 | 2.89 | 1.25 | 1.37 |
| 39 | 0.276 | 18.1 | 11.5 | 7.2 | 17.8 | 11.2 | 6.9 | 16.1 | 13.1 | 10.4 | 8.2 | 6.1 | 1.23 | 1.26 |

oil and paraffin, it might reasonably be explained as due to the approach to a saturation value of the current, as has often been noted in liquid dielectrics. The remaining instances are all paraffin. In the direction of possible explanation we report the following examples of a kind of slow moving polarization found with

TABLE II

RESULTS OF TESTS AT 1500 VOLTS AND 25 DEG. CENT.

UNIT OF CURRENT $10^{-8}$ AMPERE

| | | Charge | | | | | | Discharge | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Specimen No. 1 | Conduction current 2 | $i$ (0.2) 3 | $i$ (0.8) 4 | $i$ (3.2) 5 | $i''$ (0.2) 6 | $i''$ (0.8) 7 | $i''$ (3.2) 8 | $i$ (0.2) 9 | $i$ (0.4) 10 | $i$ (0.8) 11 | $i$ (1.6) 12 | $i$ (3.2) 13 | $\frac{i(0.2)}{i(0.4)}$ 14 | $\frac{i(0.4)}{i(0.8)}$ 15 |
| 6 | 0.425 | .. | .. | .. | .. | .. | .. | 0.76 | at peak | | | | | |
| 8 | 1.92 | .. | .. | .. | .. | .. | .. | 1.40 | at peak | | | | | |
| 9 | 1.30 | .. | .. | .. | .. | .. | .. | 25.9 | 18.6 | 11.8 | 7.24 | .. | 1.39 | 1.58 |
| 10 | 0.100 | .. | .. | .. | .. | .. | .. | 1.62 | at peak | | | | | |
| 11 | 138. | .. | .. | .. | .. | .. | .. | 798. | 486. | 260. | 159. | 91. | 1.64 | 1.87 |
| 21 | 0.885 | .. | 4.60 | 2.88 | .. | 3.71 | 1.99 | .. | 18.5 | 13.4 | 9.56 | 6.53 | .. | 1.37 |
| 22* | 0.282 | 5.40 | 3.72 | 2.40 | 5.12 | 3.44 | 2.12 | 10.3 | 7.86 | 5.69 | 4.07 | 2.76 | 1.32 | 1.38 |
| 23† | 0.294 | 10.6 | 7.56 | 4.47 | 10.3 | 7.27 | 4.18 | 14.7 | 10.8 | 7.98 | 5.48 | 3.74 | 1.36 | 1.36 |
| 24 | 0.031 | .. | 23.0 | 15.2 | .. | 23.0 | 15.2 | .. | 33.5 | 23.7 | 16.0 | 10.3 | .. | 1.41 |
| 25 | 0.343 | .. | 96.4 | 30.9 | .. | 96.1 | 30.6 | .. | .. | 69.5 | 47.0 | 30.3 | .. | .. |
| 28 | 57.2 | .. | .. | 98.5 | .. | .. | 41.3 | .. | .. | 90.6 | 60.3 | 37.0 | .. | .. |
| 29 | 14.6 | .. | .. | 61.3 | .. | .. | 46.7 | .. | .. | .. | 74.6 | 41.3 | .. | .. |
| 32 | 0.023 | 0.432 | 0.308 | 0.308 | 0.409 | 0. 285 | 0.285 | 0.6 | 0.4 | .. | .. | .. | .. | .. |
| 34 | 845. | .. | .. | .. | .. | .. | .. | 85.4 | 72.7 | 57.0 | 40.3 | 23.7 | 1.17 | 1.27 |
| 35 | 660. | .. | .. | .. | .. | .. | .. | .. | .. | 54.0 | 40.9 | 29.3 | .. | .. |
| 36 | 64.5 | 191. | 129. | 91.9 | 127. | 64.5 | 27.4 | 129. | 96.5 | 67.4 | 46.4 | 30.2 | 1.34 | 1.43 |
| 37 | 20.4 | .. | 72.4 | 47.6 | .. | 52.0 | 27.2 | .. | 60.2 | 44.1 | 31.1 | 21.4 | .. | 1.37 |
| 38 | 1.56 | 23.9 | 14.3 | 9.58 | 22.3 | 12.7 | 8.02 | 21.8 | 16.7 | 12.7 | 9.7 | 7.5 | 1.30 | 1.31 |
| 39 | 0.690 | .. | 22.9 | 12.7 | .. | 22.2 | 12.0 | 37.2 | 27.1 | 19.4 | 13.4 | 9.4 | 1.51 | 1.40 |

*No. 22 is at 35 deg. cent.
†No. 23 is at 35 deg. cent.

TABLE III

RESULTS OF TESTS AT 500 VOLTS AND 45 DEG. CENT

UNIT OF CURRENT $10^{-8}$ AMPERE

| | | Charge | | | | | | Discharge | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Specimen No. 1 | Conduction current 2 | $i$ (0.2) 3 | $i$ (0.8) 4 | $i$ (3.2) 5 | $i''$ (0.2) 6 | $i''$ (0.8) 7 | $i''$ (3.2) 8 | $i$ (0.2) 9 | $i$ (0.4) 10 | $i$ (0.8) 11 | $i$ (1.6) 12 | $i$ (3.2) 13 | $\frac{i(0.2)}{i(0.4)}$ 14 | $\frac{i(0.4)}{i(0.8)}$ 15 |
| 6 | 0..125 | .. | .. | .. | .. | .. | .. | 2.43 | 1.90 | 1.35 | 0.86 | .. | 1.29 | 1.40 |
| 8 | 0.137 | .. | .. | .. | .. | .. | .. | 1.62 | at peak | | | | | |
| 9 | 1.32 | .. | .. | .. | .. | .. | .. | 7.69 | 6.37 | 4.80 | 3.40 | .. | 1.20 | 1.33 |
| 10 | 0.048 | .. | .. | .. | .. | .. | .. | 1.78 | 1.30 | 0.98 | 0.70 | .. | 1.38 | 1.33 |
| 11 | 88.1 | .. | .. | .. | .. | .. | .. | 186. | 135. | 100. | 67. | 41. | 1.38 | 1.34 |
| 13 | 0.013 | .. | .. | .. | .. | .. | .. | 4.84 | 3.57 | 2.51 | 1.83 | 1.25 | 1.35 | 1.42 |
| 14 | 0.037 | .. | .. | .. | .. | .. | .. | 5.84 | 4.32 | 3.19 | 2.26 | 1.56 | 1.35 | 1.34 |
| 15 | 0.004 | .. | .. | .. | .. | .. | .. | 2.86 | 1.68 | 1.35 | 0.94 | 0.54 | 1.70 | 1.25 |
| 16 | 3.94 | .. | .. | .. | .. | .. | .. | 22.6 | 16.3 | 10.8 | 7.5 | 4.8 | 1.38 | 1.52 |
| 17 | 6.84 | .. | .. | .. | .. | .. | .. | 31.9 | 20.3 | 13.2 | 8.2 | 5.1 | 1.56 | 1.54 |
| 18 | 13.5 | .. | .. | .. | .. | .. | .. | 49.3 | 33.9 | 23.6 | 14.4 | 8.7 | 1.45 | 1.44 |
| 19 | 16.9 | .. | .. | .. | .. | .. | .. | 47.2 | 30.8 | 20.0 | 12.6 | 7.4 | 1.53 | 1.54 |
| 20 | 272. | .. | .. | .. | .. | .. | .. | 98.8 | 56.7 | 30.6 | 14.7 | 6.5 | 1.74 | 1.85 |
| 21* | 0.472 | 15.6 | 10.5 | 6.54 | 15.1 | 10.0 | 6.07 | 8.78 | 6.58 | 4.88 | 3.48 | 2.25 | 1.34 | 1.35 |
| 28 | 36.8 | 145. | 86.2 | 56.6 | 108. | 49.4 | 19.8 | 95.0 | 62.8 | 41.8 | 26.6 | 17.1 | 1.51 | 1.50 |
| 29† | 10.4 | .. | 70.2 | 33.9 | .. | 59.7 | 23.4 | .. | 100. | 55.8 | 31.2 | 17.9 | .. | 1.80 |
| 32 | 0.065 | 0.315 | 0.197 | 0.197 | 0.250 | 0.132 | 0.132 | 1.76 | 1.45 | 1.19 | 0.94 | 0.54 | 1.22 | 1.28 |
| 34 | 1500. | .. | .. | .. | .. | .. | .. | 10.8 | 6.60 | 4.60 | 3.40 | 2.60 | 1.64 | 1.44 |
| 35 | 1405. | .. | .. | .. | .. | .. | .. | 15.7 | 11.7 | 7.85 | 4.80 | 3.33 | 1.34 | 1.49 |
| 36 | 957. | 1455. | 1332. | 1260. | 498, | 375. | 303. | .. | 78.1 | 44.8 | 24.6 | 11.0 | .. | 1.74 |
| 37 | 334. | 864. | 734. | 660. | 530. | 400. | 326. | 186. | 121. | 76.1 | 47.2 | 25.4 | 1.54 | 1.59 |
| 38 | 46.3 | .. | 153. | 126. | .. | 107. | 80. | 82.5 | 54.1 | 35.6 | 22.0 | 12.8 | 1.53 | 1.52 |
| 39 | 1.47 | 49.8 | 32.1 | 20.5 | 48.3 | 30.6 | 19.0 | 71.8 | 48.4 | 32.4 | 20.5 | 12.4 | 1.48 | 1.49 |

*No. 21 at 35 deg. cent.
†No. 29 at 35 deg. cent.

TABLE IV
RESULTS OF TESTS AT 1500 VOLTS AND 45 DEG. CENT.
UNIT OF CURRENT $10^{-8}$ AMPERE

| Specimen No. 1 | Conduction current 2 | Charge $i$ (0.2) 3 | $i$ (0.8) 4 | $i$ (3.2) 5 | $i''$ (0.2) 6 | $i''$ (0.8) 7 | $i''$ (3.2) 8 | Discharge $i$ (0.2) 9 | $i$ (0.4) 10 | $i$ (0.8) 11 | $i$ (1.6) 12 | $i$ (3.2) 13 | $\frac{i(0.2)}{i(0.4)}$ 14 | $\frac{i(0.4)}{i(0.8)}$ 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 0.488 | .. | .. | .. | .. | .. | .. | 8.40 | 5.93 | 3.40 | 1.94 | .. | 1.42 | 1.74 |
| 8 | 0.350 | .. | .. | .. | .. | .. | .. | 3.56 | 1.36 | 1.40 | .. | .. | 1.64 | 1.63 |
| 9 | 2.54 | .. | .. | .. | .. | .. | .. | 26.0 | 18.6 | 11.9 | 9.70 | .. | 1.39 | 1.57 |
| 10 | 0.209 | .. | .. | .. | .. | .. | .. | 4.38 | 3.14 | 2.33 | 1.62 | .. | 1.40 | 1.35 |
| 11 | 350. | .. | .. | .. | .. | .. | .. | 645. | 432. | 287. | 180. | 105. | 1.49 | 1.50 |
| 13 | 0.119 | .. | .. | .. | .. | .. | .. | 11.2 | 8.32 | 5.98 | 4.08 | 2.67 | 1.34 | 1.39 |
| 14 | 0.189 | .. | .. | .. | .. | .. | .. | 19.0 | 14.5 | 10.3 | 6.78 | 4.52 | 1.31 | 1.40 |
| 15 | 0.032 | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. | .. |
| 16 | 11.8 | .. | .. | .. | .. | .. | .. | 51.8 | 38.3 | 25.9 | 16.5 | 10.0 | 1.35 | 1.48 |
| 17 | 73.3 | .. | .. | .. | .. | .. | .. | 117. | 74.8 | 45.4 | 26.7 | 14.7 | 1.57 | 1.65 |
| 18 | 61.4 | .. | .. | .. | .. | .. | .. | 130. | 88.5 | 57.8 | 35.8 | 21.4 | 1.47 | 1.53 |
| 19 | 114. | .. | .. | .. | .. | .. | .. | 131. | 83.0 | 50.8 | 29.5 | 14.7 | 1.58 | 1.63 |
| 20 | 1068. | .. | .. | .. | .. | .. | .. | 245. | 130. | 64.1 | 31.3 | 12.5 | 1.90 | 2.03 |
| 21* | 3.35 | 21.9 | 14.2 | 8.66 | 18.5 | 10.8 | 5.31 | 29.6 | 20.5 | 13.4 | 8.61 | 5.61 | 1.44 | 1.53 |
| 22 | 0.075 | 3.36 | 2.18 | 1.36 | 3.28 | 2.10 | 1.28 | 3.39 | 2.67 | 1.96 | 1.58 | 1.21 | 1.27 | 1.37 |
| 23 | 0.133 | 12.0 | 8.82 | 5.10 | 11.87 | 8.68 | 4.97 | 14.0 | 11.3 | 8.97 | 6.58 | 4.47 | 1.24 | 1.26 |
| 24 | 0.583 | 269. | 100. | 31.4 | 269. | 99.4 | 30.8 | 326. | 209. | 133. | 81.0 | 51.2 | 1.56 | 1.56 |
| 28 | 261. | 465. | 303. | 218. | 204. | 42. | −43. | 233. | 145. | 91.0 | 54.5 | 34.2 | 1.60 | 1.60 |
| 29† | 73.6 | .. | .. | 169. | .. | .. | 95. | .. | .. | 198. | 111. | 62.7 | .. | .. |
| 32 | 0.225 | 2.84 | 1.55 | 0.64 | 2.61 | 1.32 | 0.41 | 4.66 | 3.52 | 2.52 | 1.62 | 0.97 | 1.32 | 1.40 |
| 34 | 6180. | .. | .. | .. | .. | .. | .. | 16.4 | 10.0 | 6.88 | 4.72 | 3.72 | 1.65 | 1.46 |
| 35 | 4730. | .. | .. | .. | .. | .. | .. | 24.5 | 17.1 | 11.2 | 7.14 | 4.28 | 1.43 | 1.53 |
| 36 | 3070. | 4690. | 4380. | 4180. | 1620. | 1310. | 1110. | .. | 277. | 148. | 72.0 | 30.5 | .. | 1.87 |
| 37 | 1114. | 2170. | 1876. | 1747. | 1056. | 762. | 633. | 353. | 221. | 128. | 71.2 | 34.8 | 1.60 | 1.72 |
| 38 | 132. | .. | 320. | 200. | .. | 188. | 128. | 175. | 113. | 70.9 | 42.2 | 23.6 | 1.55 | 1.59 |
| 39 | 2.00 | 92.0 | 47.6 | 16.0 | 90.0 | 45.6 | 14.0 | 71.8 | 48.4 | 32.4 | 20.5 | 12.4 | 1.48 | 1.49 |

*No. 21 at 35 deg. cent.
†No. 29 at 35 deg. cent

several of the better grade of specimens. Our tests have usually been made in the order 500, 1000, 1500 volts, the condenser being short circuited between tests. In a number of instances it was found that at 500 volts the current at 10 min. is large compared to that at 60 min. In applying the next higher voltage the current at 10 min. would differ but little from that at 60 min. A series of check measurements confirmed these observations. It will be noted that there is a suggestion of a sweeping out or removal of some conducting element, which on reversal of polarity is brought back and must again be swept out. The known magnitude of the absorption current makes it certain that this is a much greater effect than that due to a possible residual absorption from a previous charge.

## VIII. Influence of Temperature

We find convenient reviews presenting knowledge of the nature of conduction in solid dielectrics by Curtis[5] and by Kraus.[12] The tendency is to attribute the conduction of insulating solids to ions of widely varying character. Some of these are of electrolytic type, others of greater mass, and occasionally the electron is invoked. Much uncertainty surrounds the question of the nature of these carriers.

There is a common tendency to consider an increase in conductivity with temperature as a requisite of electrolytic conducting, but as pointed out by Kraus, this is not necessarily so, since conductivity is the result of both mobility and dissociation and the temperature affects these in opposite ways, so that it is quite possible for conductivity to fall with rising temperature.

The majority of the investigations on the influence of temperature on dielectric conduction have been made on glass, various crystals, and impregnated paper. For materials such as we are now considering, Curtis[4] gives the variation in resistivity between 20 and 30 deg. cent. Paraffin (Parawax) is stated to have double the resistance, or half the conduction, at 20 deg. that it has at 30 deg. cent.

Considering the influence of temperature on absorption we find a number of investigations. Von Schweidler[29] gives results on glass at 18 to 47 deg. cent. He found that they were well expressed by the relation $t = I\,t^{-n}$. He found that $n$ was approximately constant, while $I$ increased with rising temperature. From this he concluded that the effect of temperature was to increase the magnitude of the absorption current, while not altering the form of the curve. This would be equivalent to an increase in the total absorbed charge with temperature.

Zeleny[39] prepared condensers of paper impregnated with paraffin, and also with ozokerite, in such a

manner as to contain various amounts of moisture. With temperatures ranging from − 70 to + 30 deg. cent., the throw on the ballistic galvanometer of 5 sec. period was measured, the condenser having previously been short-circuited for 0.3 sec. In all cases a marked increase in throw was observed as the temperature increased. These results are open to criticism because of the short period of the galvanometer. The observed values could be explained either as due to a greater absorbed charge, or else that this charge came out more readily at the higher temperature.

Wagner,[34] working on gutta-percha, balata, rubber, hard rubber, paraffin, and ceresine, over a considerable range of temperatures, using an a-c. method, indicates an increasing magnitude in the absorption current as the temperature increases, followed finally by a decrease as the melting point is approached. He also shows that the effect found with alternating stress would be equivalent with constant stress to a change in the rate at which the absorbed charge fell off, but to only a slight variation in the total absorbed charge, differing in this respect from von Schweidler.

Our principal results as regards temperature are contained in Tables I to IV. The influence on conduction is found in the ratio of the final current at approximately 35 deg. and 45 deg. to that at 25 deg. cent. at 500 volts. The voltage being the same at both temperatures, this is also the ratio of the conductivities.

The influence on absorption is brought out in the ratios of the currents on discharge at 35 deg. and 45 deg. to that at 25 deg. cent. After 0.4 sec., and 1.6 sec., and in a few cases for the peak values, all at 1500 volts.

In the majority of cases the conduction increases with the temperature. The large range of variation is noticeable. It is suggestive that the mixtures of distinctly different substances, Nos. 29, 33, 36, 37, and 38, all show large values. This is probably due to some dissociation among the constituents.

The number of instances in which conduction has fallen with a temperature increase is noteworthy. Examining each specimen we see that this occurs with the better quality insulators; paraffin in fair to good condition, ceresine, and refined oil. This is significant, and leads to the inference that it is related to the abnormalities found with these high insulators which were noted in considering the influence of voltage.

As to absorption, we find a general tendency to an increase with rise in temperature,—quite large in some cases. But just as with conduction we notice exceptions to this rule. That of ceresine, Nos. 22 and 23, is conspicuous. A duplicate test was made on No. 22 and is included to show the same tendencies in both tests. The interesting feature is that ceresine was also exceptional in its conduction, so that here is a substance which contrary to most materials has both decreasing conduction and absorption current with rising temperature. The other exception is "Black Oil," Nos. 34 and 35. This being a liquid at all temperatures, the decrease is in line with that always noted in passing from the solid to the liquid state.

Comparing the results we note all through a definite tendency for the conduction and absorption current to vary more or less proportionately. That is, the greater the increase in conduction, the greater the absorption current.

In comparing the magnitude of the absorption currents at a single time interval we must keep in mind that the curves from which they are taken may be falling at different rates. If we compare the ratios at different time intervals, we have an indication as to whether the total absorbed charge has changed, or merely that the rate of discharge has altered. We find 11 cases where we can compare the ratio of the currents at 0.4 and 1.6 sec. with the change in conductivity. In 6 instances the ratio of the absorption currents is less at 1.6 than at 0.4 sec., while the conduction has increased with the temperature. This agreement is striking, and in line with Wagner's conclusion that the chief influence of temperature is on the rate of discharge, and only slightly on the total absorbed charge. In the other 5 the ratio of the absorption currents is greater at 1.6 than at 0.4 sec. In 3 of these, all ceresine, the conduction decreases with the temperature. Thus these also fall in line. The other two, paraffin No. 9 and stearic acid No. 11, show an increase in conduction, and the relation is not so definite.

As is well known the effect of melting a dielectric is in most cases practically to remove the absorption. This was found to be true of paraffin. The mixture of paraffin and stearic acid No. 29, is interesting in this connection. Although both constituents have melting points above 50 deg. cent., yet the mixture even at 45 deg. cent. had the consistency of soft butter. Under this condition only a trace of absorption on discharge was found. At 54.4 deg. cent. the mixture was a clear liquid, and here also only a trace of absorption could be detected.

The "Black Oil," Nos. 34 and 35, is an instance of a liquid with a large reversible absorption. It is commonly assumed that liquids do not show this phenomenon. The high viscosity of the oil suggests itself in explanation. Even at about 70 deg. cent. it was difficult to wipe off from the condenser plates, whereas paraffin at this temperature would wipe off as readily as water.

Reviewing briefly, we note that in the case of solid dielectrics, in general both conduction and absorption current increase with rising temperature. As the melting point is reached the absorption falls off, usually almost disappearing on complete fusion. In some instances a decrease has been found in both conduction and absorption with temperature rise. In the case of both increase and decrease the proportionate changes in absorption and conduction are almost the same. There are indications that the rate at which the absorption current dies out is greater the greater the conduction.

TABLE V
DESCRIPTION OF SPECIMENS

| Specimen No. | Material | Treatment |
|---|---|---|
| 6 | Paraffin | 100 deg. cent. for 9 hr. in vacuum |
| 8 | Paraffin | No. 6 heater to 110 deg. cent. for 6 hr. in vacuum |
| 9 | Paraffin | No. 8 heater to 110 deg. cent. for 3 hr. in vacuum |
| 10 | Paraffin | 65 deg. cent. for 6 hr. in vacuum |
| 11 | Stearic acid | Melted in air at 72 deg. cent. |
| 13 | Paraffin | Melted in air at 69 deg. cent. |
| 14 | Paraffin | No. 13 heated to 70 deg. cent. for 9 hr. in vacuum |
| 15 | Paraffin | No. 14 heated to 70 deg. cent. for 8 hr. in air |
| 16 | Paraffin | No. 15 heated to 85 deg. cent. for 8 hr. in vacuum |
| 17 | Paraffin | No. 16 heated to 85 deg. cent. for 8 hr. in air |
| 18 | Paraffin | No. 17 heated to 100 deg. cent. for 8 hr. in vacuum |
| 19 | Paraffin | No. 18 heated to 100 deg. cent. for 8 hr. in air |
| 20 | Paraffin | No. 19 heated to 145 deg. cent. for 8 hr. in air |
| 21 | Ceresine | Melted in air at 64 deg. cent. |
| 22 | Ceresine | No. 21 heated to 66 deg. cent. for 3 hr. in vacuum |
| 23 | Ceresine | No. 22 heated to 70 deg. cent. for 5 hr. in vacuum |
| 24 | Carnauba wax | Heated to 96 deg. cent. for 5 hr. in vacuum |
| 25 | Spermaceti | Melted in air at 64 deg. cent. |
| 28 | Paraffin. Equal parts of new paraffin and No. 20 | Heated to 67 deg. cent. for 30 min. in vacuum |
| 29 | Equal parts by volume of paraffin and stearic acid | Heated to 67 deg. cent. for 2 hr. in vacuum |
| 32 | Paraffin | Heated to 65 deg. cent. for 4 ½ hr. in vacuum |
| 34 | "Black oil" | Oil poured in with condenser at 45 deg. cent. |
| 35 | "Black oil" | No. 34 heated to 69 deg. cent. for 3 hr. in vacuum |
| 36 | Paraffin, 91.7 %: "Black Oil," 8.3 % | Heated to 68 deg. cent. for 1 ½ hr. in vacuum |
| 37 | Paraffin, 96 %: "Black Oil," 4 % | Heated to 71 deg. cent. for 2 hr. in vacuum |
| 38 | Paraffin, 99 %: "Black Oil," 1 % | Heated to 68 deg. cent. for 3 hr. in vacuum |
| 39 | Paraffin. No. 20 filtered through Fuller's earth | Heated to 71 deg. cent. for hr in vacuum |

We have noticed a number of cases of reversible absorption in liquids.

## IX. Influence of Treatment

As far back as 1883 Hertz[10] showed that the conduction and absorption of benzine could be largely reduced by careful distillation. The injurious effect of even a trace of moisture in transformer oil has been long recognized.

We have heated all our materials at reduced air pressure for the removal of air and water, but in only a few cases was there definite evidence of improvement. One such instance is ceresine, where No. 22, vacuum treated, shows distinctly lower conduction and absorption current than No. 21, merely melted in air. On the other hand "Black Oil," although a poor insulator, was substantially unaltered by heating in a vacuum.

Paraffin when heated for a long time, shows increased conduction and absorption current. The same sample of material was heated for 8-hr. periods both in air and under vacuum, and with a gradually increasing temperature. All tests were made at 45 deg. cent. As a basis of comparison we take the conductivity at 1000 volts, and the absorption current after 0.2 sec. at the same voltage, as these are available for all specimens. These values are given in Table VI. The unit of current is $10^{-8}$ ampere. We note that so long as the temperature does not exceed 70 deg. cent. the injury to the paraffin is slight, if any. Above this temperature deterioration is more and more rapid. There is evidently a rough parallelism between the increase in conduction, and that in absorption current.

The general nature of the deterioration was indicated by the appearance of a greenish color, evidently due to solution of nickel from the condenser plates by some acid in the paraffin. This action is well known, although the exact composition of the acids had not been determined.

This deterioration of paraffin has been already noted by Mikola.[16] He found that after heating to 200 deg. cent. its conduction was much increased, and its electrical properties modified in other ways.

TABLE VI
INFLUENCE OF PROLONGED HEATING

| Specimen | Treatment | Temperature | Conduction current at 1000 volts 45 deg. cent. | Absorption current at 1000 volts, 0.2 sec. after discharge, 45 deg. cent. |
|---|---|---|---|---|
| 13 | Air | 69 | 0.062 | 7.02 |
| 14 | Vac. | 70 | 0.142 | 13.4 |
| 15 | Air | 70 | 0.051 | 6.24 |
| 16 | Vac. | 85 | 8.64 | 39.8 |
| 17 | Air | 85 | 25.6 | 68.4 |
| 18 | Vac. | 100 | 40.0 | 101. |
| 19 | Air | 100 | 65.3 | 97.5 |
| 20 | Air | 145 | 706. | 164. |

A similar effect occurs in petroleum oils, such as transformer oil, and for these the subject has been much studied; see the series of papers by Haslam, Frölich, and Mead.[9,15]

We filtered the deteriorated paraffin through Fuller's earth, a clay having high absorbing properties, and commonly used in the refining of paraffin. The paraffin after filtration had largely lost its green color, and its electrical properties were greatly improved. (Specimen No. 39.) This study was not carried further, but it appears evident that the temperature deterioration of paraffin consists largely in the formation of other compounds sufficiently definite and different in properties as to permit separation by filtration.

## X. Conduction and Absorption in Mixtures

We have already mentioned the apparent impossibility of finding a solid insulating substance meeting

Maxwell's requirement of conduction without absorption. Rowland and Nichols[25] found no absorption in calcite (Iceland spar). But later investigations on calcite show absorption, (Richardson[22] and Joffe[11]). In this connection we may call attention to the extensive tests by Lee and Lowry[13] on materials of the class we are considering. They give data on 31 different waxes, resins, and bitumens. They determined the d-c. resistance and the a-c. conduction at 1000 cycles. Comparing the reciprocal of the conduction with the d-c. resistance, it is in all cases lower, showing the influence of absorption. Also, in general, the lower the d-c. resistance the more pronounced was the effect of absorption.

Since no materials could be found exactly meeting Maxwell's requirements, it was necessary to study mixtures of materials of which one at least had noticeable absorption. Two methods suggest themselves. The first is to derive a mathematical relation, following the general method laid down by Maxwell, for a two layer dielectric, either or both of the layers having absorption. From such an expression the value of the absorption current could be calculated, and compared with the measured values. However, the absorption currents in the constituents are functions of the time, and the introduction of these functions into the relations leads to apparently insuperable mathematical difficulties. The other, and more feasible method is to analyze the measured values, and notice any possible relations between them which would be suggested by a combination of the characteristics of the ingredients.

To form an idea of the absorption which would occur if the mixture followed Maxwell's theory we may treat it as a two-layer dielectric, each layer having the capacity and conductance determined by the tests for that material. Maxwell has shown that the order in which the layers are arranged is immaterial. A mixture could be considered as an infinite number of layers of each constituent, and if all of those of the same kind were arranged together it would reduce to two layers. Evidently in arranging the materials in layers we have taken the case most favorable to absorption, since if the substances were arranged in parallel columns between the condenser plates there would be merely two dielectrics in parallel, and consequently no absorption. Of necessity we must leave to one side the question of the solution of one substance in the other,[31] and our discussion is thus only broadly qualitative as a test of Maxwell's theory for good insulating materials in their best available condition.

Considering Maxwell's expression for the current in the two-layer case we find that when the dielectric constants are nearly the same, and one constituent has much higher conduction than the other, the low conduction determines the magnitude of the conduction current, and the high conduction determines the magnitude of the absorption current.

Among others, mixture No. 28 was composed on equal parts by volume of new paraffin and the deteriorated paraffin, No. 20. This mixture was studied at 500 and at 1500 volts, and its properties compared with those of the constituents and with those computed from Maxwell's two-layer expression. The general results are as follows:

While the conduction of the mixture is lower than that of the poor paraffin, No. 20, its absorption currents are greater in all cases. The computed absorption currents at 0.4 sec. are fairly near the measured values, while at 1.6 sec. they are decidedly lower. If we assume that the computed value should be added to that of the constituents, we may do this approximately by adding the computed value to half that of No. 20, since the absorption of the fresh paraffin is low. Making this comparison, we see that it would much exceed the measured value at 0.4 sec., and would be somewhat low at 1.6 sec. While there is a hint of Maxwell's effect, yet it is not conspicuous, and the results are better accounted for by a change in the distribution or freedom of the ions present in the deteriorated dielectric.

Following this a mixture, No. 29, of equal parts by volume of paraffin and stearic acid was studied. The conduction of the mixture was only about one-fifth that of the stearic acid. The absorption current of the mixture was on the average 0.45 of that of the stearic acid, this being fairly constant. That is, as regards absorption, the effect is nearly the same as if, instead of being mixed, one-half of the condenser had been filled with paraffin, the other half with stearic acid. There is here no indication of any increase in absorption through mixture. The conduction of the mixture is much greater than if the materials were in layers, but less than if they were side by side. As regards the computed values, since the mixture had less absorption than if the stearic acid it contained had been put by itself in the condenser, that is less than half the value for the acid alone, it is evident that no increase in absorption was produced by mixing, and the Maxwell effect did not exist.

The next specimen was No. 33, a mixture of equal parts by volume of paraffin and refined lubricating oil. The oil had a low conduction, only a little higher than the paraffin and only a trace of reversible absorption. The mixture closely resembled the constituents, having a conductivity of the same magnitude, and only a trace of absorption. Here again, there is no evidence of any increase in absorption due to the mixture.

We finally tried what may be considered the addition of a conducting impurity to paraffin by mixing with it varying proportions of the highly conducting "Black Oil." The properties of the paraffin, of the oil, and of 8.33, 0.4, and 1 per cent mixtures respectively of the oil with the paraffin were studied separately. The values of conduction are best compared by expressing them as percentages of the value for "Black Oil," see Table VII.

TABLE VII
PER CENT CONDUCTION OF MIXTURES OF PARAFFIN AND "BLACK OIL"

| | No. 35 | No. 36 | No. 37 | No. 38 |
|---|---|---|---|---|
| Per cent "Black Oil" | 100 | 8.33 | 4.0 | 1.0 |
| 25 deg. cent. 500 volts | 100 | 10.2 | 3.46 | 0.21 |
| 25 deg. cent. 1500 volts | 100 | 9.8 | 3.10 | 0.24 |
| 45 deg. cent. 500 volts | 100 | 68.1 | 23.8 | 3.30 |
| 45 deg. cent. 1500 volts | 100 | 65.0 | 23.6 | 2.79 |

We notice that at 25 deg. cent., the conductivity is nearly proportional to the amount of oil, that is, the oil behaves as if the paraffin were not present. At 45 deg. cent., however, the conduction at each percentage is from 3 to 8 times greater than for the oil by such a proportional relation, indicating that at the higher temperature there is more reaction between the oil and the paraffin. The computed current for the two-layer case is omitted since, as already explained, it would be nearly the same as for paraffin alone, that is far less than any of the measured values of the mixtures.

Examining the absorption current we find that it is greater for all the mixtures than for the oil alone. There is evidently a maximum in the absorption in the vicinity of Nos. 36 and 37. Thus there is an approximate accord with Maxwell's theory. But the oil is a liquid and its absorption can scarcely be expected to follow the same relations as for a solid. As an instance, we notice that with the oil the absorption current falls off in going from 25 deg. cent. to 45 deg. cent., while with the mixture it increases. Examining the computed values for the two-layer case we find two noticeable peculiarities. The first is the small magnitude of the absorption currents, particularly at 45 deg. cent. The other is the rapid rate at which even these small values fall off as compared to the actual decrease. These figures definitely show the inability of Maxwell's theory to explain the absorption.

From the experiments described it may be concluded that a mixture of two solid dielectrics acts much the same as if the two were separate. In some cases there may be a slight increase in absorption, due probably to a change in the number and distribution of free ions. There is little or no evidence to show that Maxwell's theory applies in any way. The addition of a conducting liquid to a solid dielectric with which it is miscible causes a large increase in the absorption. The absorption current is much greater and falls off more slowly than can possibly be explained by Maxwell's theory.

## XI. Irreversible Absorption

The presence of an irreversible absorption in certain cases has long been recognized. It occurs commonly in composite dielectrics made up of one or more originally liquid ingredients, as for example, impregnated paper. It usually appears when the final conduction has high value.

We have examined all our records for the presence of irreversible absorption, by studying the difference between the discharge current and the charging current reduced by the amount of the final steady conduction current. For the most part we have found evidence of only traces of irreversible absorption. There is, however, one notable group in which it occurs, *viz.*, mixtures of paraffin and relatively small amounts of "Black Oil" at 45 deg. cent. (Nos. 36, 37, and 38). In these cases the irreversible component of the charging current was from 2 to 7 times the corresponding value of the discharge current, increasing with the amount of admixed oil. This irreversible current undoubtedly contributes to the loss in the alternating case. That it can assume such large values is striking, and indicates the importance of its further study.

## XII. Results and Discussion

The main conclusions of the work are as follows:

1. The shape of the current-time curve of the absorption current on discharge is essentially the same with every substance studied. Its striking peculiarity is a very rapid decrease at first, followed by a very gradual dying out. No simple mathematical formula has been found to express this relation satisfactorily. There is a fair approximation to the relation $i = A\,t^{-n}$. The variation from the negative exponential relation, as proposed by Maxwell, is very wide.

2. All the solid dielectrics showed appreciable absorption. Heating at reduced air pressure in no case reduced the absorption. The absorption was approximately proportional to the final conduction in all cases. There was no indication that the simple dielectric having conductivity but no absorption, as postulated by Maxwell, can exist.

3. Noticeable departures were found from Curie's law of proportionality between the ordinates of the current time curve on discharge and the charging voltage.

4. The absorption current on discharge falls off more rapidly the higher the charging voltage.

5. The conduction in general increases with the voltage. In a few cases there is evidence of the approach to a saturation current.

6. In general both conduction and absorption currents in solids increase with rise in temperature. As the melting point is approached the absorption current falls off, and usually almost disappears on complete fusion, as noted by other observers.

7. In some instances a decrease was found in both conduction and absorption as the temperature rose. While an exception to 6 they agree with it in showing a proportionate change in both conduction and absorption currents.

8. Reversible absorption in liquids, while not as conspicuous as in solids, is very noticeable in some cases. This shows that whatever the nature of absorption it is not fundamentally related to the solid state.

9. Any treatment of a dielectric, such as prolonged heating, which increases or diminishes the conduction, increases or diminishes the absorption current in much the same proportion.

10. There is evidence that a mixture of two solid dielectrics acts much the same as though the two were separate.

11. The addition to a solid dielectric of a conducting liquid which can mix with it causes a large increase in the absorption. The absorption current is much greater, and falls off far more slowly than can be explained by Maxwell's theory.

12. The solids tested gave no definite evidence of irreversible absorption current. However, mixtures of paraffin with a small percentage of conducting oil, showed at 45 deg. cent. a noticeable irreversible absorption current.

The most striking feature throughout these studies is the intimate connection between the absorption and the conduction. Increase or decrease in either, under the influence of other variables, is accompanied always by a similar change in the other. Other observers have noted a similar correlation. We have shown that it extends over such a wide range of materials and conditions as to make it certain that the ultimate seat of the phenomenon of absorption is to be found in the laws of ionic conduction.

Throughout the course of this work we have used Maxwell's theory as a sort of criterion for our results. We have done this partly because of the distinction of its originator and of the place which it apparently still holds in the minds of many physicists, but principally because it is the only available rational theory offering reasonable opportunity for experimental check. We have emphasized the failure of the theory to account for the observations in either magnitude or general qualitative relation. We therefore dismiss the theory as, if not positively disproved, at least useless for application to commonly available dielectrics.

The anomalous conduction found in dielectrics both liquid and solid offers a far more promising prospect for the explanation of dielectric absorption. We use the word "anomalous" since conduction in dielectrics rarely follows Ohm's law. If electrons are involved in this conduction, their contribution is in general overshadowed by that of much larger, heavier, and slower ions. This is true for both liquids and solids. All the work of recent years points to this type of conductivity. It may be seen at once how readily the conception lends itself to an explanation of absorption. The ions being large and slow, move towards the opposite electrodes; in doing so they upset the potential gradient; or, if by any cause their motion is interrupted, they may accumulate and by the resulting space charge cause the counter or polarization e. m. fs. often reported. There are, however, obvious difficulties in this simple picture, some of which we discuss below.

Curie[3] describes experiments on porous porcelain showing pronounced absorption, which decreased as the porcelain was dried. From this he suggests polarization as the cause of absorption, but makes no attempt to develop a complete theory.

Hartshorn[8] suggests that absorption is due to a non-conducting air film between the dielectric and the terminals, so that the actual contact exists at only a few points. The motion of the ions through the dielectric is stopped by the air layer, resulting in a counter electromotive force. The conduction current is determined by the occasional points of actual contact. As one of the proofs of this theory, he points to the absence of absorption in liquids, but from what we have already shown this is not a valid objection. It is also evident that this is nothing but an application of Maxwell's theory, the dielectric being one layer, the air film the other.

Richardson[22,23,24] gives the results of extensive tests on conduction and absorption. The materials studied were: quartz, Iceland spar, rock salt, glass, ebonite, and paraffin. He takes the view that absorption is a manifestation of polarization, and states as his conclusion[24] that the charging current in a dielectric with constant impressed voltage is expressed by the sum of two terms, one the rate of change of the charge in the dielectric, and the second the difference between the impressed e. m. f. and a polarization e. m. f. divided by a constant resistance. The second term gives the current at infinite time. The polarization is assumed as the cause of absorption, but no explanation of the polarization is offered.

Somerville and Buckley[30] show that in porcelain at high temperatures, as much as 1100 deg. cent., the current $I$ due to an electromotive force $E$ can be represented as $I = (E - e)/R$, where $e$ is a counter electromotive force dependent on applied voltage, time of charge, and temperature, and $R$ is the resistance. This is introduced as showing the possibility of a counter electromotive force of polarization where water is very definitely absent.

Joffe[11] has extensively investigated the absorption in crystals, and considers it due to the motion of both ions and electrons, resulting in a polarization in the crystal. He finds two distinct types of polarization; one represented by Iceland spar, the other by quartz. With Iceland spar, if the crystal is charged, and a thin layer, as little as 0.01 mm., removed from the side next to the cathode, the residual charge disappears. If the layer is removed from the side adjoining the anode, no change is noticed. With quartz the removal of layers from either side has only a small effect. The potential changes gradually and symmetrically through the crystal.

The fundamental difficulty in a proposal of polarization, due to space charge, as an explanation of dielectric absorption is the inconsistency of the idea of a free flow of charge through the mass of the dielectric, this charge being more or less abruptly halted at the surface of the

dielectric, where the latter is in contact with a metallic electrode. There is ample evidence of the motion of the ions, even in a solid dielectric. It is important to note, however, that Joffe reports only one case of a sharply defined surface layer of charge, with consequent high potential gradient to the electrode, and that even in so good an insulator as quartz, it was apparent that the internal accumulation of charge was spread over a much greater volume and the variation of potential gradient, although not uniform, was gradual rather than abrupt.

From the point of view of free motion of ions, and internal accumulation of charge with consequent reaction on the electric gradient, the work of Smekal[44] and of Schmidt,[45] as discussed by Rogowski,[46] is much more suggestive and acceptable. Smekal has tried to explain conduction in solids as due to the motion of free ions in sub-microscopic cleavages or surfaces of crystal separation. The ions are liberated from the crystal more readily in these spaces and follow easily the direction of the electrical intensity, probably in accordance with the laws of motion of gaseous ions. Schmidt and Rogowski have extended this suggestion to studies of the influence of voltage and temperature and show that there is marked agreement in the behavior of the conduction, so predicted, with the observations of experiment. Rogowski has also used these sub-microscopic spaces in the development of an interesting theory of dielectric breakdown.

The presence of such open spaces or cleavages in crystals even when prepared under most careful conditions, has been shown beyond question for many crystals. They offer a more satisfactory explanation of dielectric absorption than the proposal of an elastic and retarded polarization, such as suggested by Joffe, for we have the picture of a motion of free ions over limited distances, presumably in a gaseous atmosphere. The analysis of Rogowski explains the proportionality of current and potential gradient for low values of the latter, and also an increasing conduction with increasing gradient as proposed by Poole.[21] Looking further we may see that it leads without too violent assumptions to an explanation of dielectric absorption. It is easy to picture the accumulation of charges at the ends of the sub-microscopic spaces. Moreover, beyond this, simple mathematical analysis shows that there results a steadily decreasing current-time relation, in fact, the negative exponential relation, characteristic of absorption as suggested by many experiments. This interesting suggestion obviously depends on the presence of the sub-microscopic cleavages as described. The presence of these has been recognized only in certain crystals. A somewhat similar suggestion for noncrystalline solids, involving minute canals with electrolytic conduction, has recently been made by Böning.[47] We are, however, far from the proposal of a general theory covering all solid dielectrics.

We have already called attention to the variation of density of distribution of the ions in a conducting insulating liquid, when subject to electric stress. In many cases this results in a marked variation in potential gradient near the electrodes, as long ago shown by Mie,[41] Warburg,[35] and others. This leads to the question of the influence of this accumulated space charge on the external field as a possible explanation of the typical current-time curves, often noticed in viscous liquids and similar to those observed in solids. We have reported above one especially conspicuous case. The difficulty here, as in the case of polarization in solids, is to explain the failure of this accumulated charge to pass to the electrodes. We venture to suggest that this difficulty in the case of the liquids may be explained by assuming that an ion, owing to its intense electric field, surrounds itself with an atmosphere of neutral molecules which move with it, and which when an obstacle, such as an electrode, is encountered, prevents the charge ion from passing to it. Extending this conception we can easily conceive the accumulation of layers of space charge in the proximity of the electrodes. Simple mathematical analysis of the influence of this space charge on the applied electric field of a parallel plate condenser, leads to the negative exponential current-time relation approximating the charging current time curves, as often observed in experiment.

The increasing evidence that absorption and conduction always go hand in hand, and that the latter in solids frequently partakes of the character of that of gaseous and liquid ions, emphasizes clearly that further progress towards a better understanding of the phenomena in dielectrics can best be obtained through a study of this type of ion. Moreover, as it is the slow moving ions which are of the greatest interest and importance, we propose, as a continuation of this work, further studies of ionic mobilities and conductivities as found in liquid insulators of increasing viscosity, merging into the type of solid compounds such as studied in this paper and those commonly used for electrical insulation.

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## Discussion

**E. R. Thomas:** Clerk Maxwell demonstrated that if two ideal condensers having leakage resistances such that the product of their respective capacitances and resistances are not equal, are connected in series, they produce the external phenomenon of causing voltage to build up at their terminals after an initial discharge, which is now called dielectric absorption, dielectric hysteresis, residual charge, etc.

This was demonstrated recently by oscillograph tests on a 1-$\mu$ f laboratory standard condenser. The two 0.5-$\mu$ f sections were connected in series and leakage resistances of approximately 1 and 2 megohms were connected across the respective sections. Such an arrangement gives an unequal product of $r$ and $c$ for the respective condenser sections.

This equivalent circuit after being charged at 50 volts and momentarily short-circuited showed a building up of a residual voltage to a maximum value of about 3.2 volts in 0.6 sec.

If this equivalent circuit produces the same phenomenon as is met with in commercial insulation then it would seem advantageous to use it as a unit of measure on commercial insulation to evaluate the degree to which it deviates from ideal insulation.

Suppose we pass a plane parallel and midway through a sheet of insulation between two plate electrodes, the degree of uniformity in conductivity of the two halves of the insulation will have a ratio to each other which is indicated by the ratio of the maximum residual voltage occurring after discharge to the original voltage charge. Thus on the assumption of the Maxwell equivalent circuit the degree of uniformity of the insulation resistance is a function of the percentage peak residual voltage compared to the original charge.

The time at which the peak residual voltage occurs after the initial discharge is a function of the insulation resistance and capacitance as well as the ratio of resistivity.

The discharge characteristic of the residual voltage is by this reasoning the difference of two negative exponentials with respect to time that is $e_t = A\ (\epsilon^{-xt} - \epsilon^{-yt})$ where $A$ is a function of the original voltage charge and the degree of uniformity of insulation resistance and $x$ and $y$ are functions of the insulation resistance, the degree of uniformity, and the capacitance.

Mr. Marbury pointed out in his paper on dielectric absorption in 1925 similar data which he obtained in a point-by-point method.

I regret that Messrs. Whitehead and Marvin conclude from the test data presented that the Maxwell equivalent circuit fails in application to the commonly available dielectrics. When the magnitudes of the physical constants of the dielectrics investigated and described in their paper are considered, the period of time between discharge and the starting of the graphic record, even though it is only 0.005 sec., may be a relatively long time period as far as recording the true picture of the phenomenon. A method of investigation which starts from zero time would tend to be more conclusive.

**G. M. J. Mackay:** Dr. Whitehead has developed a very effective method for studying d-c. conductivity and absorption in a dielectric but he has investigated materials which are very complex from both the chemical and physical standpoints.

It should help very greatly in the interpretation of the relationship between d-c. and a-c. effects if these characteristics could be studied with relatively simple geometric assemblies or with an exaggeration of the variations which occur in commercial materials. For instance, in commercial insulation we have materials which consist of conducting portions in series with very good dielectric material; we have emulsions of water in oils, and conducting and non-conducting particles suspended in liquids. By making artificial systems of this nature and determining the characteristics, both according to Dr. Whitehead's method and by a-c. measurements, a good deal of light might be thrown upon the mechanism of dielectric losses. As an example, suppose two quartz plates with liquid between. On direct current there would be practically no conductivity because of the high insulating value of quartz. I am not sure what the absorption effects would show, but on alternating current the losses would be relatively high and dependent on the resistance of the liquid. Again, with an emulsion of water and an oil, the d-c. measurements would show a different relationship to

the alternating characteristics. If Dr. Whitehead would show us typical characteristics representative of different arrangements of this nature, it should be very helpful in the interpretation of both d-c. and a-c. phenomena.

**Herman Halperin:** During the past year and a half in Chicago, we have had in experimental operation six 1000-ft. lengths of cable operating at 132 kv. Five of the cables have "solid" insulation while the other one is of the oil-filled type. Periodically with the high voltage removed, measurements have been made of direct current versus time, using a 200-volt battery. Generally, the curve of charging current has been higher than the curve of the discharging current. The curves as plotted on logarithmic paper are not straight lines, but curves upward and downward and change from time to time indicating a disagreement with the Maxwell theory. To date we have not been able to learn much from the curves.

The impression that I get from the first paragraph of the article, is that, under the combination of influence under electric stress and temperature, progressive physical and chemical changes occur in the structure of the material with the result that radical changes in the insulating properties are bound to occur. This sweeping statement does not seem to be justified. Laboratory and operating data, which are not nearly as complete as they should be, indicate for most cable insulations, however, that there is a critical stress for a given operating condition below which the given insulation should operate without any material change in characteristics.

**R. W. Atkinson:** There has, in the past, been a great deal of theoretical discussion as to the validity of Maxwell's theory of absorption but very little quantitative data bearing on the matter. We have at last been presented with a practical amount of test data on the subject.

I have been very much interested to look over Dr. Whitehead's work showing the actual physical characteristics of dielectric absorption and showing that Maxwell's theory does not apply to the physical conditions that we get there. I think there have been many partial data showing things of that sort, but it is worth while to see the systematic work and know that we need to use another physical theory to account for the particular phenomenon.

Up until the time when dielectric losses became of large importance in cable insulation the particular factor of dielectric loss that was important undoubtedly was conduction; the dielectric losses were important at the higher temperatures of 60 deg. cent and above, that is, where the insulation consisted of paper saturated with a fluid oil having a considerable conductivity. Also it was possible to determine a pretty direct relation between the conductivity of the oil itself and the dielectric power factor of the cable saturated with that oil. In view of these facts, since the cable engineer was at that time primarily interested in power factor within that range, he was principally concerned with the conductivity of the oil which could be measured by simple means, as with direct current. Having developed facilities devoted to that particular phase of the subject, it has been possible to reduce dielectric losses within that range until it is now possible to bring those losses commercially to values that were considered impossible even in the laboratory a few years ago. With the reduction of losses of that type and with the possibility of largely increased stresses brought about by reduction in dielectric losses and by other means, we now find that the loss which is left over, which was formerly practically negligible, may become in the future a matter of larger relative importance. The matter of absorption, which we have been neglecting for a few years, may become vastly more important commercially than the conductivity. Thus only from the standpoint of interest in physics and of interest in helping us to understand the laws of the dielectric, this question of absorption in dielectrics is something that is very likely to become of increasing commercial importance.

**A. Nyman:** I should first like to ask Dr. Whitehead whether the absorption really determines or affects in any definite way the life of the dielectric.

**H. H. Race:** I agree with Dr. Whitehead that in 60-cycle phenomena the short-time point of view is certainly of interest. At the same time, in getting at the ultimate nature of absorption, if we can, the long-time absorption curves will have to be considered. A paper on some of this work was published in the A. I. E. E. Quarterly TRANS., October 1928, p. 1044.

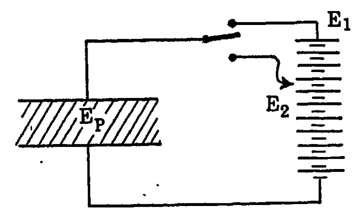


FIG. 1

I should like to present some data obtained this fall on hard rubber, which is as near a heterogeneous solid as we could possibly get. The experiment deals with the so called polarization potential which is built up within the body of the dielectric during a charge at constant potential. Suppose a potential $E_1$ is applied at the instant $t_0$ of dielectric and maintained constant until the charging current approaches a constant value.

In our previous work we found that this takes several days. There is evidence that during this time an electric potential difference called the polarization potential ($E_p$), has been built up within the body of the dielectric. The current which has been flowing during this time will be considered positive and is shown by the curve between the ordinates $t_0$ and $t_1$.

Now suppose the switch is thrown at the instant $t_1$ so that a constant potential $E_2$ less than $E_1$ is applied to the sample. If $E_2$ is less than $E_p$, a current proportional to $(E_p - E_2)$ will flow in a direction opposite to that in which it had previously been flowing. The current will continue to flow in this direction until $E_p$ has dropped to a value equal to $E_2$ at which time the

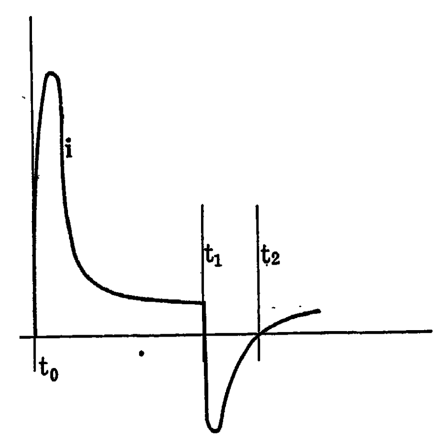


FIG. 2

current will be zero as shown by the point $t_2$. After this time the current will be positive and will approach the conduction current due to the positive potential $E_2$.

If several runs are made with exactly similar conditions of charge but with increasing values of $E_2$, the negative portions of the current curve will get smaller and the several points $t_2$ will come nearer to $t_1$ since the value of $(E_p - E_2)$ is getting smaller. If, as $E_2$ is increased, the first value could be found at which no reversal of current occurred, this would be the value of $E_p$ for the particular charging conditions being studied. The data for five runs all having a charging potential of 750 volts are shown in the following table.

| $E_1$ Volts | $E_2$ Volts | $t_2 - t_1$ Minutes |
|---|---|---|
| 750 | 703 | 59 |
| 750 | 712 | 36 |
| 750 | 725 | 16 |
| 750 | 736 | 11 |
| 750 | 748.5 | 1 |

The values of $t_2 - t_1$ depend upon the final conduction currents of the specimen which are not definite because the apparatus errors are of the same order of magnitude.

The last run shows that $E_p$ was greater than 99.8 per cent of $E_1$ and the first run shows that when $E_2$ was 94 per cent of $E_1$, $E_p$ remained greater than $E_2$ for an hour. Therefore the important points are that after a long charge at a potential $E_1$, $E_p$ is very nearly equal to $E_1$ and the relaxation of the material is relatively slow.

**C. F. Hill:** (communicated after adjournment) In this paper the authors have pointed out several conclusions of importance. Of these, the conclusions that Maxwell's theory of dielectrics does not fit experimental evidence and that a reversible absorption current has been measured for liquids, are the most important. It has been evident for some time that a model built after Maxwell's theory is difficult to make conform to our ideas of ionization, electric conduction, atomic and molecular structure and the like which we associate with electrical phenomena in matter. It would be very important and certainly worth some effort to try to tie up the action of polar molecules of these liquids, for which Professor Whitehead and Mr. Marvin have found reversible absorption, with the dielectric absorption. The application of Debye's theory to dielectrics to explain the anomalous properties is very promising.

One or two questions concerning the conclusions of this paper have arisen, the first of which concerns the reversible absorption of liquids. Do the results distinguish in any way if the reversible absorption is a space charge effect or if it might be a polarization on the electrodes? Also, it is not evident what the irreversible absorption current might be, and how detected or measured. Is there a plausible explanation of this lost component?

The extension of the current-time curve towards the zero time axis is much needed data and the authors have developed a method which has carried the curves to a small fraction of a second which is a worth-while step as it aids in a study of what is happening over the time when losses take place under a-c. voltage. Apparently to get much nearer the zero time axis we must resort to devices for measurement with time lags comparable with the periods of the electric-charge carriers themselves, which suggests that cathode-ray devices must be used before we get farther with this problem.

**R. H. Marvin:** Mr. Thomas' discussion of the possibility of applying Maxwell's theory and his illustration of tests on circuits made up with actual condensers and leakages develop very interesting relations and certainly give a qualitative idea of the phenomena, but, as we have pointed out in our paper, there are many ways in which the actual dielectric differs from any such simple representation. Perhaps the most striking is the great difference between the discharge curve as shown from actual samples and the exponential curve which would follow from any simple representation in accordance with Maxwell's theory.

Replying to Dr. Mackay on the connection between a-c. absorption and d-c. losses, that has not been attempted in this paper but we have other investigations under way at the University which are attempting to correlate these phenomena.

In connection with Mr. Halperin's discussion of the curves, we have pointed out ourselves that the absorption curves do not agree exactly with the relation $i = I\, t^{-n}$ but show a slight curve, and although in general this curve is in one direction, we also found cases where it curved in the other direction, so that it is quite certain that this curve is only an approximation, but it does seem to be a very fair approximation.

Replying to Mr. Nyman's inquiry as to the relation between absorption losses, all we referred to there, was that the absorption increased as the heating did, and the heating in turn with the deterioration.

# Corona Ellipses

BY VLADIMIR KARAPETOFF[1]

Fellow, A. I. E. E.

*Synopsis.—The purpose of this investigation is to give a mathematical theory of the cyclograms of corona obtained by a cathode-ray oscillograph. In the case investigated a long wire of small diameter is connected to one terminal of an a-c. source; the other terminal of the source is connected to a concentric cylinder of considerable diameter or to a metal plate at some distance from the wire. A cathode-ray oscillograph with two pairs of deflecting plates, at right angles to each other, is so connected that one pair of plates causes deflections of the cathode beam proportional to the values of instantaneous voltage of the source, and the other pair of plates causes deflections proportional to the instantaneous values of the charging and loss current flowing into the wire.*

*As long as the sinusoidal amplitude of the applied voltage is below the visual corona point, the charging current is also sinusoidal, in time quadrature with the voltage. The oscillograph record is therefore an ellipse, with the amplitudes of the voltage and the current as the principal semi-axes. When, however, the minimum ionization voltage is exceeded during a part of each alternation, the cyclogram ceases to be an ellipse, but consists of four portions per cycle, two of which correspond to the intervals of time during which the corona is extinct, and the other two when corona is present, with quite short transients in between.*

*F. W. Peek (A. I. E. E. TRANS., Vol. XLVI, 1927, p. 1009) published a number of such oscillograph records, with voltage amplitudes both below and above the visual critical point. In order to explain the mechanism of corona formation and the influence of the space charge upon the instantaneous critical voltage, he also produced "artificial corona," by using two condensers in series, one of which was shunted by a sphere gap.*

*The purpose of the present investigation is to give a mathematical theory of the observed cyclograms, on the basis of two condensers in series, with the space charge as a fictitious dividing line. The condenser nearest to the wire is assumed to be shunted by a conductance, and to have a resistance in series, to account for the actual motion of ions and the power loss. Approximate equations are derived for the current and the voltage as functions of time.*

*For the artificial corona it is shown that the composite curve consists of arcs of two ellipses, with their principal axes along those of the cyclograms. Assuming the visual critical voltage to be known, an expression is derived for the instant of the cycle at which the corona is re-established.*

*For the actual corona, it is shown that the cyclogram also consists of portions of two ellipses, only their principal axes are at some angles with the principal axes of the cyclogram. The theory is applied to one of Mr. Peek's records, and it is shown that both the shape of the experimental curves and the instants at which the corona is re-established check fairly well with the theory.*

* * * * *

## I. Introduction

NUMEROUS laboratory tests and theoretical researches on corona formation about long cylindrical conductors of comparatively small cross-section have been made by various investigators; many noteworthy results have been recorded in Institute papers for over twenty years. The modern cathode-ray oscillograph, which is a further development of the original Braun tube, has made it possible to study the a-c. corona in a much greater detail quantitatively, from instant to instant, and thus has brought us nearer a rational explanation of the ionic mechanism involved.

A number of a-c. corona records (cyclograms), obtained by means of a cathode-ray oscillograph, was published by Lloyd and Starr[2] and by Peek.[3] Peek's Fig. 9A is reproduced in Fig. 1 below, using a somewhat different ratio of the scales for abscissas and ordinates. The beam was slightly diffuse (not focused), and rather than to draw an arbitrary average line, the boundaries of the actual trace on the photograph are shown by the cross-hatched strip. The theoretical ellipses drawn in the same sketch do not concern us yet.

The horizontal distances from the vertical axis are proportional to instantaneous values of the sinusoidal applied voltage between the wire and a metal plate. The vertical ordinates represent the corresponding instantaneous values of the current flowing into the wire. The cyclogram refers to the established conditions, and is different from one for the first few cycles. The cathode beam traced the figure counter-clockwise.

Beginning at point $p$, where the applied voltage is zero and the corona is maintained by the previously accumulated space charge (see Peek's paper for a physical explanation of this seeming paradox), we arrive at point $b$ at which the applied voltage reaches its maximum. Shortly afterwards the flow of current stops and the corona is extinguished somewhere between the points $b$ and $s$. On the decreasing voltage the corona is re-established at point $h$ and continues until slightly beyond point $d$; $d\,k$ is again that portion of the next alternation during which the corona is extinct. It is re-established at point $k$. The two parts of the actual cyclogram, $p\,n\,b$ and $r\,q\,d$, are not quite alike, because of the difference in the mass and mobility of positive and negative ions.

*The purpose of the paper is to outline a mathematical theory which explains the general shape of the observed corona cyclograms.* The narrower of the two theoretical ellipses shown in Fig. 1 represents the current-voltage relations during those portions of the cycle when the corona is extinct. The broader ellipse gives a similar

1. Professor of Electrical Engineering, Cornell University, Ithaca, N. Y.

2. W. L. Lloyd, Jr., *Methods Used in Investigating Corona Loss by Means of the Cathode-Ray Oscillograph*, A. I. E. E. TRANS., 1927, Vol. 46, p. 997.

3. F. W. Peek, *The Law of Corona and the Dielectric Strength*; *ibid.*, p. 1009.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

relationship with the corona present. The theoretical transition points are indicated by the small circles at $h$ and $k$. It will be seen that the agreement with the observed cyclogram is quite satisfactory considering the complicated nature of the phenomenon.

## II. Two Condensers in Series, One of Which is Shunted by a Spark Gap

### A. *The Circuit*

We shall first consider the circuit (Fig. 2) which, according to Peek, roughly imitates the conditions in an actual a-c. corona. This circuit consists of two condensers, $C_1$ and $C_2$, in series, the first one being shunted by a "glow gap" $G_1$. In this ideal glow gap, as distinct from an actual "spark gap," a copious corona promptly takes place between the "hair brush" electrodes when the voltage between them reaches a value $e_1$. When the applied voltage is raised further, the discharge remains corona-like at the same voltage $e_1$ and does not change to streamers or spark-over. The excess voltage is supposed to be consumed in the condenser $C_2$. The corona stops instantly when the voltage $p_1$ drops below $e_1$. We shall not discuss here the question as to whether such a gap is realizable in practise or not, because the circuit shown in Fig. 2 is intended merely as an analog, to simplify the mathematical treatment of an actual a-c. corona around a conductor.

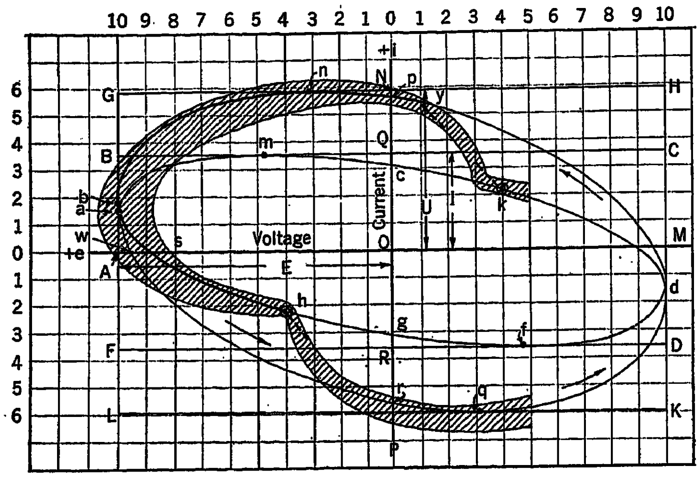


FIG. 1—AN ACTUAL OSCILLOGRAM OF A-C. CORONA (PEEK) WITH THE CORRESPONDING ELLIPSES

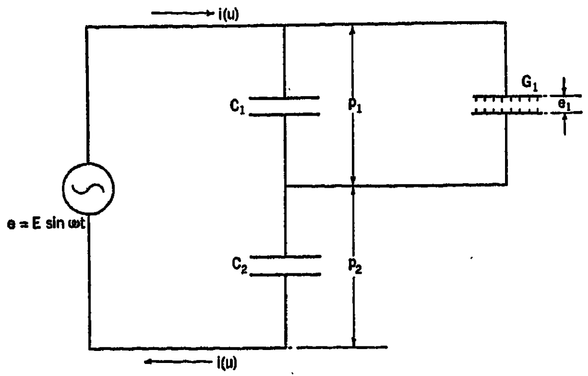


FIG. 2—TWO CONDENSERS IN SERIES ONE OF WHICH IS SHUNTED BY A GLOW-GAP

We shall disregard the non-periodic transient conditions of the first few cycles, and assume the phenomenon to be periodic, as shown in Fig. 3. In this cyclogram, the cathode beam undergoes two harmonic motions simultaneously. One is along the axis of abscissas, and the deflections are proportional to the total applied voltage $e = E \sin \omega t$. (See notation at the end of the paper.) The other motion is along the axis of ordinates, the deflections being proportional to the instantaneous values, $i$, of the current through the condenser $C_2$. The cyclogram is described by the cathode beam counter-clockwise. The points of discontinuity correspond to the instants at which the current suddenly increases when the corona is re-established. The corona is extinguished at the extreme right and left points where the voltage reaches its maxima, because at these instants the current becomes zero and is reversed.

Because of these discontinuities, the cathode beam

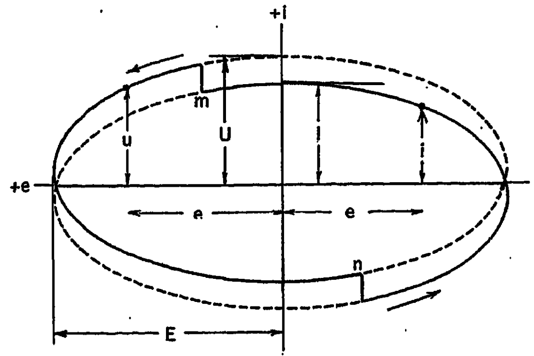


FIG. 3—AN ACTUAL CYCLOGRAM (PEEK) TAKEN ON THE ARRANGEMENT SHOWN IN FIG. 2, WITH THE CORRESPONDING ELLIPSES

alternately follows two distinct curves, and it is necessary to consider the equation of each separately. It is shown below that both curves are arcs of ellipses.

### B. *Corona Discharge is Out*

Let an instantaneous voltage across $C_1$ (Fig. 2) be $p_1$ and that across $C_2$ be $p_2$. Since by assumption we are considering that portion of an alternation when no current is flowing through the gap $G_1$, the charging currents of the two condensers are equal, and we have:

$$i = C_1 \, d\, p_1 / d\, t = C_2 \, d\, p_2 / d\, t \tag{1}$$

The sum of $p_1$ and $p_2$ is equal to the instantaneous value of the applied voltage, so that

$$p_1 + p_2 = E \sin \omega t \tag{2}$$

These are simultaneous differential equations for $p_1$ and $p_2$. Integrating Equation (1), gives

$$C_1 \, p_1 = C_2 \, p_2 + \Gamma \tag{3}$$

where $\Gamma$ is a constant of integration. Experiment

shows that the corona goes out, approximately, at the instant when the current is reversed, that is, at $\omega t = \pi/2$; at this instant $p_1 = e_1$, and $p_2 = E - e_1$. Substituting these values in Equation (3), gives

$$C_1 e_1 = C_2 (E - e_1) + \Gamma \tag{4}$$

Eliminating $\Gamma$ from Equations (3) and (4) and solving the resulting equation with Equation (2) for $p_1$ and $p_2$, we obtain:

$$p_1 = (e_1 - P_1) + P_1 \sin \omega t \tag{5}$$

$$p_2 = -(e_1 - P_1) + P_2 \sin \omega t \tag{6}$$

where

$$P_1 = E\, C_2/(C_1 + C_2) \tag{7}$$

$$P_2 = E\, C_1/(C_1 + C_2)\,. \tag{8}$$

$P_1$ and $P_2$ are the voltages across the condensers $C_1$ and $C_2$ respectively when the total applied voltage reaches its maximum value, $E$. They are inversely as the corresponding capacitances and their sum is equal to $E$.

From Equations (1) and (6) we have:

$$i = \omega\, C_2\, P_2 \cos \omega t = I \cos \omega t \tag{9}$$

where the amplitude of the current is

$$I = \omega\, C_2\, P_2 \tag{10}$$

Let the instantaneous value of the applied voltage be $e$, so that

$$e = E \sin \omega t \tag{11}$$

From Equations (9) and (11) we obtain

$$(i/I)^2 + (e/E)^2 = 1 \tag{12}$$

This is the equation of an ellipse (Fig. 3) whose horizontal and vertical semi-axes are $E$ and $I$ respectively. Only the full-drawn portions of this curve appear on an actual experimental cyclogram.

*C. Corona Discharge is Present*

By assumption, while the corona is going, the glow gap, $G_1$, maintains a constant voltage, $e_1$, across the condenser $C_1$. Hence, in place of Equation (1), we now write

$$u = C_2\, d\, p_2/d\, t \tag{13}$$

where $u$ is the charging current of condenser $C_2$ while the glow gap is operating. By using two distinct letters, $i$ and $u$, for the current, confusion is avoided as to which part of the cycle is meant. Equation (2) becomes

$$e_1 + p_2 = E \sin \omega t \tag{14}$$

Substituting the value of $p_2$ from Equation (14) in Equation (13), we get

$$u = U \cos \omega t \tag{15}$$

where the amplitude of the current is

$$U = \omega\, C_2\, E \tag{16}$$

Comparing this amplitude with that given by Equation (10), we see that $U$ is greater than $I$ in the same ratio in which $E$ is greater than $P_2$. In other words, with reference to Equation (8),

$$U/I = E/P_2 = (C_1 + C_2)/C_1 \tag{17}$$

From Equations (11) and (15), we get

$$(u/U)^2 + (e/E)^2 = 1 \tag{18}$$

This result means that when a glow discharge is taking place, the relationship between the current and the voltage is again an ellipse. Its horizontal semi-axis, $E$, is the same as before, but the vertical semi-axis is equal to $U$, and is larger than $I$, as indicated by Equation (17). The arcs of this ellipse shown in Fig. 3 by solid lines are portions of this ellipse that would appear on an actual cyclogram.

*D. Points of Discontinuity*

To determine the instants of time at which the corona starts ($m$ and $n$ in Fig. 3), we put in Equation (5)

$$p_1 = -e_1 \tag{19}$$

The minus sign is necessary for the following reason: The constant of integration, $\Gamma$, in Equation (3) has been determined from the condition that $p_1 = +e_1$, at the points where the ellipse for $i$ crosses the axis of abscissas. At these points the glow discharge goes out, because the total applied voltage begins to diminish. So when the discharge is re-established, it is due to a voltage, $e_1$, in the opposite direction. Putting $p_1 = -e_1$ at the instant $t = t_1$, and solving Equation (5) for $\sin \omega t_1$, we get:

$$\sin \omega t_1 = 1 - 2\,(e_1/P_1) \tag{20}$$

In this expression, the angle $\omega t_1$ at which the corona is reestablished, can have only values greater than $\pi/2$, because the corona goes out at $\omega t = \pi/2$. A negative value of $\sin \omega t_1$ means that $\omega t_1$ lies between 180 deg. and 270 deg. (point $n$ in Fig. 3 and also Peek's Fig. 18B). A positive value of $\sin \omega t_1$ means that $\omega t_1$ lies between 90 deg. and 180 deg. (Peek's Fig. 18C).

Equation (20) determines the points $m$ and $n$ for the simplified corona diagram shown in Fig. 2, when the values of $e_1$ and $P_1$ are known. However, in application to an actual corona around a conductor, the voltage $e_1$ is not known. Instead, it is the visual critical voltage, $e_v$, that can be readily measured. By definition, $e_v$ is that value of the applied voltage (direct voltage or crest of alternating voltage) at which a corona barely appears or just goes out. Therefore, in Equation (20) it is convenient to express $e_1$ through $e_v$, in order to make the formula applicable to an actual corona.

Let both condensers be originally uncharged and let an increasing d-c. voltage be slowly applied to the combination of $C_1$ and $C_2$, in place of an a-c. voltage. At the instant just before a glow appears at $G_1$, we have for the total charge or electric displacement in each of the condensers:

$$e_1\, C_1 = p_2\, C_2 \tag{21}$$

Moreover,

$$e_1 + p_2 = e_v \tag{22}$$

Eliminating $p_2$ and solving for $e_1$, we get

$$e_1 = e_v \, C_2/(C_1 + C_2) \tag{23}$$

Substituting this value of $e_1$ in Equation (20) and eliminating $P_1$ by means of Equation (7), we finally obtain

$$\sin \omega \, t_1 = 1 - 2 \, (e_v/E) \tag{24}$$

This formula may also be written in the form

$$e_1 = E - 2 \, e_v \tag{25}$$

where $e_1 = E \sin \omega \, t_1$ is the instantaneous critical voltage at which the corona starts.

When $E = e_v$, $\sin \omega \, t_1 = -1$; $\omega \, t_1 = 270$ deg., and we have an ionized state for an instant only (Peek's Fig. 18A). When $E = 2 \, e_v$, $\sin \omega \, t_1 = 0$; $\omega \, t_1 = 180$ deg., the discharge starts each time when the applied voltage passes through zero, and the ionized state continues for one-half of each alternation. When $E$ has a value between $e_v$ and $2 \, e_v$, $\omega \, t_1$ lies between 180 deg. and 270 deg., and the discharge continues for less than one-half of each alternation, as shown in Fig. 3. When $E$ is greater than $2 \, e_v$, $\sin \omega \, t_1$ is positive, $\omega \, t_1$ lies between 90 deg. and 180 deg., and the corona glow lasts for more than one-half of each alternation (Peek's Fig. 18C).

*E. A Mechanical Analog*

The electrical conditions shown in Figs. 2 and 3 may

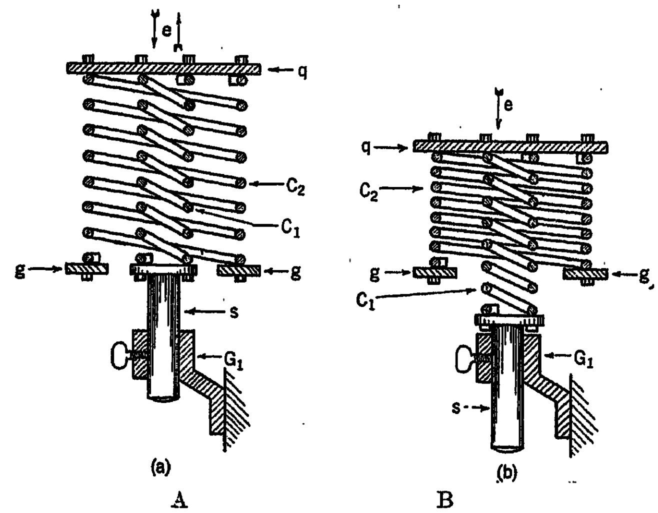


FIG. 4—A MECHANICAL ANALOG OF A-C. CORONA

be made clearer by the following mechanical analog. Fig. 4a shows a combination of two spiral springs, $C_1$ and $C_2$, which correspond in their behavior to the two condensers in the circuit in Fig. 2. The tops of the springs are fastened to the plate $q$. The bottom of the spring $C_2$ is fastened to a solid foundation $gg$. The bottom of the spring $C_1$ is attached to the plunger $s$ which can slide through an immovable sleeve $G_1$. So long as the vertical force on $C_1$ in either direction does not exceed a certain value, $e_1$, the plunger remains stationary because of friction in $G_1$. When the force exceeds $e_1$, the plunger begins to slide, so that a force greater than $\pm \, e_1$ can at no time be applied to $C_1$.

Let a mechanical force, $e$, which varies harmonically with time, be applied to the upper plate $q$, causing the springs alternately to expand and to contract. When there is no slippage at $G_1$, this force is divided between the springs $C_1$ and $C_2$ in proportion to their elastances (force per unit deformation), just as in Fig. 2, without the corona, the applied voltage $e$ is divided between $C_1$ and $C_2$ in proportion to their electrical elastances. When the plunger begins to slide, the spring $C_2$ takes more than its share of the force and becomes highly compressed (Fig. 4b), whereas $C_2$ is only compressed by the force $e_1$.

On the upward stroke, the compression in $C_1$ may be changed to tension long before $C_1$ has reached its normal state, and the plunger $G_1$ may begin to move upward before the total force $e$ has been reversed. The parts of the mechanical cycle during which the plunger is moving correspond to the parts of the electrical cycle when a glow discharge takes place. By following this analog in greater detail, the reader may make clear to himself the cyclogram, the points of discontinuity, charge, and energy storage in $C_2$, the effect of the value of $E$ as compared to $e_v$, etc. The applied mechanical force takes the place of the applied voltage; mechanical displacements are analogous to electric charges; and the velocity of the motion of the plate $q$ represents the current.

## III. ELLIPSES OF ACTUAL CORONA

The proper scientific way to treat mathematically a d-c. glow discharge about a round wire is by determining the distribution of electric space-charge density and the voltage gradient as functions of the distance from the geometric axis of the conductor. This can be done by using the divergence theorem and considering the mobilities of ions. While the mathematics of such a deduction is quite complicated and some uncertainties and simplifications are unavoidable, Otto Mayr[4] has succeeded in obtaining approximate expressions for d-c. corona in finite form. He also has shown that the relationship between the critical voltage gradient at the surface of the wire, and the values of mobilities of ions determined by separate experiments of a different kind, is in fair agreement with his theory.

The theory of the a-c. corona is much more involved, since the space charge and the velocities of ions are then functions not only of the distance from the axis of the conductor, but of time as well. Moreover, the phenomenon becomes dependent on the frequency of the supply, since an ion which at 60 cycles may travel during one alternation from the inner wire almost to the outer cylinder, at 5000 cycles may be caught and reversed near the inner conductor. Although Mayr shows by rough computations (*ibid.*, p. 276) that the conditions at 60 cycles are qualitatively the same as

4. Otto Mayr, "Raumladungsprobleme der Hochspannungstechnik", *Arch. f. Elek.*, 1927, Vol. 18, p. 270.

with direct current, yet so far no rational theory covering this case has been worked out.

Our problem here is much more narrow, namely to represent mathematically the general features of the appearance of corona cyclograms, such as shown in Fig. 1, and we shall attempt to do this in a semi-empirical way on the basis of two equivalent diagrams (Figs. 5 and 6) which are a generalization of that shown in Fig. 2.

*A. An Expression for the Current when Corona is Extinct*

Strictly speaking, the actual conditions in an a-c. corona cannot be adequately represented by an equivalent diagram of condensers, fixed glow gaps, resistances, etc., such as shown in Fig. 2, nor by any extension thereof. There is in reality no definite stationary cylindrical surface, concentric with the wire, which could serve as a boundary between some fictitious condensers, such as $C_1$ and $C_2$; nor is there anything in the mechanism of the corona to correspond to a "set" glow gap $G_1$. However, the diagram shown in Fig. 2 leads to a cyclogram in Fig. 3 which checks with the experiment. This suggests the possibility

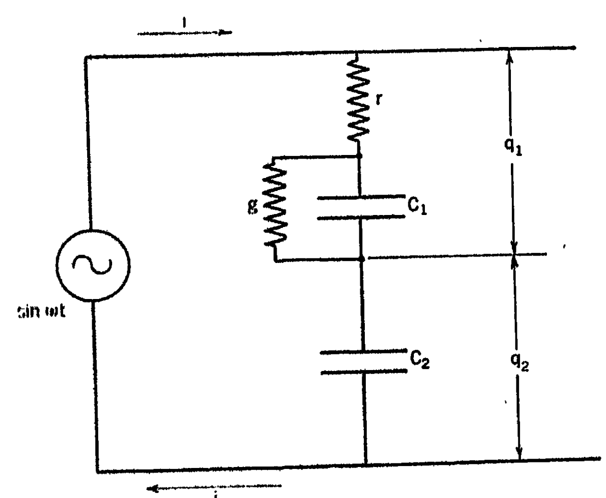


FIG. 5—AN IMPERFECT AND A PERFECT CONDENSER IN SERIES

that a more elaborate series combination (Figs. 5 and 6) of an imperfect condenser, $C_1$, representing the ionized portion of the gas adjacent to the wire, and a perfect condenser, $C_2$, representing the remote portions of the gas in which no ionization by collisions takes place, may adequately (although semi-empirically) reproduce the cyclogram in Fig. 1.

In Fig. 5, the imperfections in the condenser $C_1$ are represented by a fictitious series resistance $r$ and a shunted conductance $g$. We shall first assume that there is no corona and no glow gap, that the combination of the two condensers is connected to a source of voltage, $E \sin \omega t$, and that permanent conditions have been established. The equivalent impedances of the condensers $C_1$ and $C_2$ are

$$Z_1 = r + (j\,\omega\,C_1 + g)^{-1} \quad (26)$$

$$Z_2 = (j\,\omega\,C_2)^{-1} \quad (27)$$

Hence, the partial voltage across $C_2$ is

$$q_2' = E\,\epsilon^{j\omega t}\,\frac{(j\,\omega\,C_2)^{-1}}{r + (j\,\omega\,C_1 + g)^{-1} + (j\,\omega\,C_2)^{-1}} \quad (28)$$

In this expression, $E\,\epsilon^{j\omega t}$ is to be thought of as a rotating vector whose length is equal to the amplitude of the applied voltage. This vector is multiplied by a complex fraction which changes its length and turns it by a constant angle. Hence, $q_2'$ is also a vector in the same sense as $E\,\epsilon^{j\omega t}$. The voltage $q_2$ is marked with a prime sign to indicate that only the sinusoidal component of the complex exponential voltage is to be used later.

Multiplying the numerator and the denominator of the righthand side of Equation (28) by $j\,\omega\,C_2$, we get

$$q_2' = E\,\epsilon^{j\omega t}\,[1 + r\,j\,\omega\,C_2 + (j\,\omega\,C_2)/(j\,\omega\,C_1 + g)]^{-1} \quad (29)$$

Expanding the last fraction by long division, this equation is transformed into

$$q_2' = E\,\epsilon^{j\omega t}\,[1 + (C_2/C_1) + j\,r\,\omega\,C_2 + j\,g\,C_2/(\omega\,C_1^2) + \text{etc.}]^{-1} \quad (30)$$

Since $r$ and $g$ are small quantities (imperfections), the terms with the higher powers of $g$ may be neglected. Combining the terms with $j$ into one, we may finally write

$$q_2' = E\,\epsilon^{j\omega t}\,[1 + (C_2/C_1) + j\,n]^{-1} \quad (31)$$

where $n$ is an empirical correction term. Thus, we may generalize the meaning of expression (31) by saying that *it represents the a-c. voltage across the perfect condenser $C_2$, no matter what the nature of imperfections in $C_1$ is, provided that they are sufficiently small.* In other words, from now on it is not necessary to think of these imperfections as being due to a particular constant resistance or conductance; the loss may be caused by collisions of residual ions and other factors in gaseous conduction. Equation (31) may also be written in the form[5]

$$q_2' = E\,\epsilon^{j(\omega t - \gamma)}\,C_1 \cos\gamma/(C_1 + C_2) \quad (32)$$

where $\gamma$ may be called the imperfection angle of condenser $C_1$. For a perfect condenser, $\gamma = 0$, and Equation (32) agrees with Equation (8). The angle $\gamma$ is determined by Equation (34).

For the sake of brevity we shall introduce

$$Q = E\,C_1 \cos\gamma/(C_1 + C_2) \quad (36)$$

so that Equation (32) becomes

5. To transform expression (31) into Equation (32), rewrite the factor by which $E\,\epsilon^{j\omega t}$ is multiplied in the form

$$K = C_1/(C_1 + C_2 + j\,n\,C_1) \quad (33)$$

and put $\quad n\,C_1 = (C_1 + C_2)\tan\gamma \quad (34)$

This will give

$$K = C_1 \cos\gamma/[(C_1 + C_2)(\cos\gamma + j\sin\gamma)] \quad (35)$$

It remains to replace $(\cos\gamma + j\sin\gamma)$ by $\epsilon^{j\gamma}$, to obtain Equation (32).

$$q_2' = Q\,\epsilon^{j(\omega t - \gamma)} \qquad (37)$$

The sine component of $q_2'$, corresponding to $E \sin \omega t$, is

$$q_2' \text{ (sine)} = Q \sin(\omega t - \gamma) \qquad (38)$$

So far we have assumed our circuit to be operating continuously, under established conditions, and with the initial charges equal to zero. In reality, the arrangement shown in Fig. 5, if it applies to a corona at all, can hold true only during those portions of each alternation during which ionization by collision is not taking place, but only motion and recombination of residual ions; in other words, when the corona is "out." Just before the corona goes out, the distribution of electric stress in the layers of the gas adjacent to the wire is quite different from that when the corona is extinct, more voltage being thrown upon the fictitious condenser $C_2$. Thus, during the portion of an alternation under consideration, $C_2$ behaves as if it were "pre-charged" by a d-c. source.[6] This extra charge adds a d-c. component to the voltage $q_2'$ and causes a similar component to be subtracted from $q_1'$. Thus, Equation (38) should be generalized to

$$q_2 = Q \sin(\omega t - \gamma) + D \qquad (39)$$

where $D$ is the d-c. component of the voltage and corresponds to $\Gamma$ in part II of the paper.

Experiment shows that the corona goes out when the applied voltage has reached its maximum value, $E$, and just begins to decrease. We shall assume that at such instants the voltage across the strongly ionized portion of the dielectric is $e_1$, as in Fig. 2. At the same instants, the voltage across the condenser $C_2$ is $q_2 = E - e_1$. Substituting this value for $q_2$ in Equation (39), at $\omega t = \pi/2$, gives

$$D = E - e_1 - Q \cos\gamma \qquad (40)$$

so that Equation (39) becomes

$$q_2 = Q\,[\sin(\omega t - \gamma) - \cos\gamma] + (E - e_1) \qquad (41)$$

The charging current through $C_2$ is still determined by Equation (1), because a constant initial charge has no effect on it. Thus,

$$i = C_2\, d\, q_2/dt = \omega\, C_2\, Q \cos\ \omega t - \gamma) \qquad (42)$$

and the amplitude of the current is

$$I = \omega\, C_2\, Q \qquad (43)$$

For the case in Fig. 2, $\gamma = 0$, and Equations (42) and (43) become identical with Equations (9) and (10). Thus, for the part of an alternation when no new ions are produced, the cyclogram curve is represented by the two equations

$$e = E \sin \omega t \qquad (44)$$

$$i = I \cos(\omega t - \gamma) \qquad (45)$$

It is shown in the Appendix that eliminating $t$ from these equations gives an ellipse between $e$ and $i$, but the principal axes of this ellipse are not equal to $E$ and $I$ and are not in the direction of those of the cyclogram. The ellipse $a\ m\ c\ d\ f\ g\ h$ in Fig. 1 corresponds to the Equations (44) and (45). The values of $I = 3.56$ divisions and $\gamma = 28$ deg. have been chosen by trials to fit the portions $a\ h$ and $d\ k$ of the cyclogram as closely as possible.

Equation (39) upon which Equation (45) is based, may be called semi-empirical for the following reason: The phenomenon under consideration is a "periodic transient" (using Dr. Steinmetz's expression), so that, strictly speaking, it is not legitimate to begin with Equations (26) to (28) which apply to a steady state only. The starting point should be equations similar to Equations (1) and (2), applied to Fig. 5. These would give a sinusoidal term and a transient exponential term. The result comes out quite involved for application to cyclograms, because of the exponential term containing $t$. Moreover, such an accurate procedure is hardly justifiable in view of the uncertainties in the quantities $C_1$, $C_2$, $r$, and $g$, which are fictitious and variable quantities anyway. It was therefore felt that the shorter method used above, with the addition of a constant term at the end, is justified by the circumstances of the case and by the specific purpose in view.

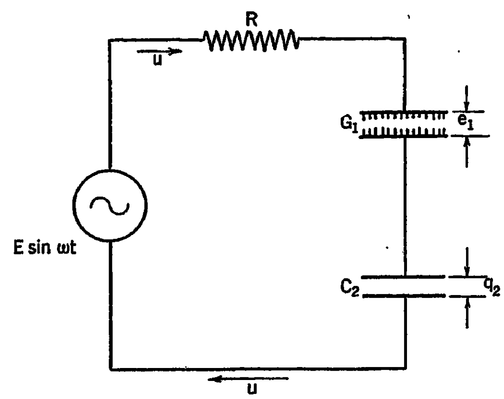


FIG. 6—A GLOW GAP IN SERIES WITH A CONDENSER

*B. An Expression for the Current when Corona is Present*

The assumptions made for this portion of the cyclogram are shown in Fig. 6. A comparison with Fig. 2 will show that a series resistance, $R$, has been added, to make the case more general and to account for the loss of power in the corona. The condenser $C_1$ has been omitted since it is shunted by a glow gap of constant voltage $e_1$, so that the charge on $C_1$ remains constant and does not enter in the equations. Or else, $C_1$ may be thought of as having been converted into the glow gap $G_1$; this mode of representation agrees with the physical facts more closely.

Equation (13) applies in this case, and Equation (14) is generalized by the addition of the ohmic drop, $R\,u$. Thus, the equations corresponding to Fig. 6 are:

6. V. Karapetoff, *Precharged Condensers in Series and in Parallel*, A. I. E. E. PROC., 1918, Vol. 37, p. 509.

$$u = C_2\, d\, q_2/dt \quad (46)$$

$$R\, u + e_1 + q_2 = E \sin \omega\, t \quad (47)$$

The correct procedure in the solution of Equations (46) and (47) is to substitute the value of $u$ from Equation (46) in Equation (47) and to solve the resulting equation for $q_2$. However, this would bring in an exponential transient term of doubtful accuracy, since the whole equivalent diagram shown in Fig. 6 is only a crude approximation to an actual corona. We therefore reason as follows:

If the constant voltage, $e_1$, did not exist, both the current $u$ and the voltage $q_2$, under steady conditions, would be sinusoidal. The presence of $e_1$ causes the condenser to become "pre-charged," that is, its voltage does not alternate between say $Q_2'$ and $-Q_2'$ but between $+Q_2' - e_1$ and $-Q_2' - e_1$. On the other hand, the current remains sinusoidal because, according to Equation (46), it depends only on the rate of change of $q_2$ with the time. Thus, disregarding the transient conditions, we may put

$$q_2 = Q_2 \sin (\omega\, t - \beta) - e_1 \quad (48)$$

where $\beta$ is a phase displacement or imperfection angle analogous to $\gamma$ introduced before. Consequently, from Equation (46),

$$u = \omega\, C_2\, Q_2 \cos (\omega\, t - \beta) = U \cos (\omega\, t - \beta) \quad (49)$$

where $$U = \omega\, C_2\, Q_2 \quad (50)$$

is the amplitude of the current $u$. Substituting the foregoing values of $q_2$ and $u$ in Equation (47), gives

$$R\, \omega\, C_2\, Q_2 \cos (\omega\, t - \beta) + Q_2 \sin (\omega\, t - \beta) = E \sin\, \omega\, t \quad (51)$$

This relationship must hold true at any value of $t$. Equating, therefore, the coefficients of $\sin \omega\, t$ and of $\cos \omega\, t$ on both sides of the equation, we obtain, after simplification:

$$\tan \beta = \omega\, C_2\, R \quad (52)$$

$$U = \omega\, C_2\, E \cos \beta \quad (53)$$

When $R = 0$, $\beta = 0$, and Equation (53) agrees with Equation (16), Equation (49) becomes identical with Equation (15).

The equations

$$e = E \sin \omega\, t \quad (54)$$

$$u = U \cos (\omega\, t - \beta) \quad (55)$$

determine an ellipse, just as Equations (44) and (45) do; see Appendix. This is the ellipse $b\, n\, p\, d\, q\, r$ in Fig. 1. The values of $U = 5.9$ divisions and $\beta = 18$ deg. have been chosen by trial to fit the portion $y\, p\, n\, b$ of the cyclogram as closely as possible. Positive corona differs somewhat from negative corona so that the two halves of the cyclogram are not quite symmetrical. A somewhat different ellipse would fit the lower corona region better.

*C. Points of Discontinuity*

As explained in Sec. II, under D, we shall assume that the corona starts when Equation (19) is satisfied. Since in Fig. 5

$$q_1 + q_2 = E \sin \omega\, t \quad (56)$$

the condition of $q_1 = -e_1$ is equivalent to

$$q_2 = E \sin \omega\, t_1 + e_1 \quad (57)$$

where $t_1$ is an instant of time when the corona starts. Substituting this value of $q_2$ in Equation (41), we obtain

$$E \sin \omega\, t_1 - Q \sin (\omega\, t_1 - \gamma) = E - Q \cos \gamma - 2\, e_1 \quad (58)$$

To solve this equation for $t_1$, we introduce an auxiliary voltage $S$ and an auxiliary phase angle $\theta$ such that

$$E \sin \omega\, t_1 - Q \sin (\omega\, t_1 - \gamma) = S \sin (\omega\, t_1 + \theta) \quad (59)$$

Equating the coefficients of $\sin \omega\, t_1$ and $\cos \omega\, t_1$ on both sides of Equation (59), we get

$$E - Q \cos \gamma = S \cos \theta \quad (60)$$

$$Q \sin \gamma = S \sin \theta \quad (61)$$

so that

$$\tan \theta = Q \sin \gamma/(E - Q \cos \gamma) \quad (62)$$

$$S = Q \sin \gamma/\sin \theta = (E - Q \cos \gamma)/\cos \theta \quad (63)$$

Therefore, $S$ and $\theta$ may be considered as known quantities. Equations (58), (59), and (60) then give

$$\sin (\omega\, t_1 + \theta) = \cos \theta - 2\, (e_1/S) \quad (64)$$

When $\gamma = 0$, $\theta = 0$, and Equation (64) becomes identical with Equation (20). The statements made there regarding the limits of values of $\omega\, t_1$ apply here to $(\omega\, t_1 + \theta)$. Having calculated the values of $\theta$ and $S$ from Equations (62) and (63), the value of sin $(\omega\, t_1 + \theta)$ may be computed from Equation (64) and then the angle $\omega\, t_1$ determined.

The relationship between $e_1$ and $e_v$ may be assumed to be represented by Equation (23), because at the visual critical voltage the imperfections represented by the resistances may be neglected.

From Equation (36)

$$C_2/(C_1 + C_2) = 1 - [Q/(E \cos \gamma)] \quad (65)$$

so that

$$e_1 = e_v \, \{ 1 - [ Q/(E \cos \gamma)] \} \quad (66)$$

Dividing Equation (43) by Equation (53), we obtain

$$I/U = Q/(E \cos \beta) \quad (67)$$

Therefore, Equation (66) may also be written in the form

$$e_1 = e_v \, \{ 1 - (I/U)\, (\cos \beta/\cos \gamma) \} \quad (68)$$

By means of either Equation (66) or (68) $e_1$ may be computed if $e_v$ has been determined experimentally and the ellipses drawn.

In Fig. 1, the points $k$ and $h$ have been determined as follows: (a) By means of Equation (68) the value of $e_1$ was computed using the value of $e_v = 18.1$ kv., given by Peek for this particular cyclogram; (b) $Q$ was computed from Equation (67) for the given voltage

$E = 65.6$ kv. (c) The angle $\theta$ was determined from Equation (**62**) and $S$ from Equation (**63**); (d) sin $(\omega\, t_1 + \theta)$ was found from Equation (**64**); (e) Having found the angle $\omega\, t_1$, the abscissas $\pm E \cos \omega\, t_1$ gave the positions of the points $k$ and $h$. It will be seen that these points agree quite well with the experimental discontinuities in the cyclogram.

## IV. Conclusion

It has been shown that the general shape of corona cyclograms, such as in Fig. 1, may be adequately represented by arcs of two ellipses, whose equations can be derived from an equivalent diagram of the corona. It has also been shown that cyclograms of a circuit consisting of two condensers and a glow gap (Fig. 2) also consist of arcs of two ellipses. In the latter case the ellipses are "straight," whereas with the actual corona they are "oblique."

While experimental and theoretical work on the detailed mechanism of corona is going on, based on a statistical consideration of actual moving ions in the glow region, the computations and the point of view presented in this paper may be of some value in that certain fictitious "bulk quantities" (capacitances and resistances) are introduced which may be calculated from experimental data. Variations in these quantities with the conditions of the experiment may give an indication as to the corresponding changes in the state of the gas, changes which otherwise it may be difficult to measure or to express by numbers.

The particular equivalent diagrams used in this paper are perhaps the simplest and the crudest possible, so that the field is open for further refinements. For example, it is natural to assume that the space charge does not retain its position during an alternation, but contracts and expands, so that the values of the two capacitances, $C_1$ and $C_2$, vary harmonically with the time. However, such refinements will be of value only when suitable experimental material is available to check theoretical conclusions and when the cathode ray in an oscillograph can be better focused, so as to produce a much sharper record.

## Appendix

*Some properties of an oblique ellipse.* By an oblique ellipse (Figs. 1 and 7) is meant an ellipse boxed in a rectangle whose bisectors ($A\,M$ and $Q\,R$) do not coincide with the principal axes of the curve. Let the ordinates of the ellipse be denoted by $i$ and the abscissas by $e$. We shall prove that the curve may be represented by two simultaneous equations of the form

$$e = E \sin \alpha \qquad (69)$$

$$i = I \cos (\alpha - \gamma) \qquad (70)$$

where $E = O\,A$, $I = O\,Q$, $\alpha = \omega\, t$ is an auxiliary variable angle, and $\gamma$ a constant angle which characterizes the ellipse. Equations (**44**) and (**45**) and Equations (**54**) and (**55**) are of this form.

A mathematical point may be thought of as performing a harmonic motion, $E \sin \omega\, t$, along the axis of abscissas and at the same time being subjected to another harmonic motion, $I \cos(\omega\, t - \gamma)$, along the axis of ordinates. It is well known in physics that a superposition of two such motions gives an elliptical path (for example, elliptically polarized light).

To prove that Equations (**69**) and (**70**) represent an ellipse, eliminate $\alpha$ between them. The result is

$$I^2 e^2 + E^2 i^2 - 2\, i\, e\, I\, E \sin \gamma - E^2 I^2 \cos^2 \gamma = 0 \qquad (71)$$

This equation is of the form

$$a\, x^2 + b\, y^2 + 2\, h\, x\, y + c = 0 \qquad (72)$$

which is the equation of a conic section referred to the origin at its center. The curve is an ellipse when

$$a\, b - h^2 > 0 \qquad (73)$$

In our case

$$a\, b - h^2 = I^2 E^2 - I^2 E^2 \sin^2 \gamma = I^2 E^2 \cos^2 \gamma \qquad (74)$$

and is intrinsically positive, so that the curve is an ellipse. The angle $\phi$ which its major axis forms with the axis of abscissas is determined by the equation

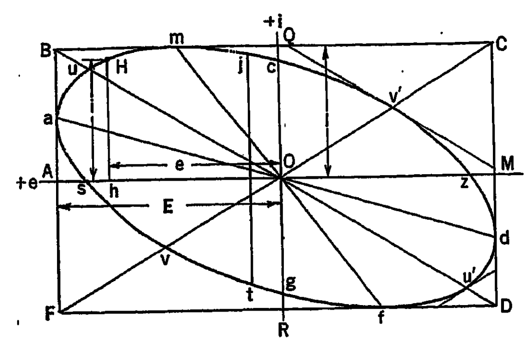


Fig. 7—Conjugate Diameters of an Ellipse

$$\tan 2\, \phi = 2\, h/(a - b) \qquad (75)$$

In our case

$$\tan 2\, \phi = 2\, I\, E \sin \gamma/(E^2 - I^2) \qquad (76)$$

On first sight the last equation may seem to be physically inhomogeneous, in that (amperes)$^2$ is subtracted from (volts)$^2$. However, in this equation both $E$ and $I$ are supposed to be expressed in some units of length, such as divisions of cross-section paper; otherwise $\phi$ has no meaning.

The lengths of the principal axes are determined from the "discriminating quadratic"

$$k^2 - (E^2 + I^2)\, k + E^2 I^2 \cos^2 \gamma = 0 \qquad (77)$$

One value of $2\, k$ gives the length of the major axis, the other that of the minor axis.

*Conjugate Diameters.* Two diameters of an ellipse are said to be conjugate when any chord parallel to one is bisected by the other. For example, in Fig. 7, $a\, d$ and $c\, g$ are conjugate diameters. All vertical chords, such as $j\, t$, are bisected by $a\, d$. The tangent to the ellipse at the extremity of one of the conjugate

diameters is parallel to the other diameter. Therefore, $a$ is the point of tangency with $B\,F$. For the same reason, the diameter $m\,f$, conjugate to $s\,z$, passes through the tangency points, $m$ and $f$, of the horizontal sides of the circumscribed rectangle.

The diagonals $B\,D$ and $F\,C$ also determine a pair of conjugate diameters of the ellipse; that is, the tangents to the ellipse at points $u$ and $u'$ are parallel to $F\,C$, while the tangents at $v$ and $v'$ are parallel to $B\,D$. This may be proved as follows: Equations **(69)** and **(70)** give

$$d\,e/d\,\alpha = E\cos\alpha;\; d\,i/d\,\alpha = -\,I\sin(\alpha - \gamma) \qquad \textbf{(78)}$$

Consequently, the slope of the curve, at a point determined by the coordinates $e$ and $i$, is

$$d\,i/d\,e = -\,(I/E)\sin(\alpha - \gamma)/\cos\alpha \qquad \textbf{(79)}$$

For the point $u$, from the geometry of the figure

$$(i/e)_u = I/E \qquad \textbf{(80)}$$

On the other hand, from Equations **(69)** and **(70)**

$$(i/e)_u = (I/E)\cos(\alpha_u - \gamma)/\sin\alpha_u \qquad \textbf{(81)}$$

Hence, for point $u$

$$\cos(\alpha_u - \gamma) = \sin\alpha_u \qquad \textbf{(82)}$$

which means that

$$\alpha_u = 45° + 0.5\,\gamma \qquad \textbf{(83)}$$

Substituting this value in Equation **(79)**, gives

$$(d\,i/d\,e)_u = -\,I/E \qquad \textbf{(84)}$$

This proves that the tangent at $u$ is parallel to $F\,C$, the slope of the latter line being $-\,I/E$. For the point $v$, the slope is equal to $+\,(I/E)$, so that Equation **(79)** is satisfied when

$$\alpha_v = 135° + 0.5\,\gamma \qquad \textbf{(85)}$$

Substituting the value of $\alpha_u$ from Equation **(83)** in Equations **(69)** and **(70)**, we get

$$(O\,u)^2 = e_u^2 + i_u^2 = (E^2 + I^2)\sin^2(45° + 0.5\,\gamma) \qquad \textbf{(86)}$$

Similarly

$$(O\,v)^2 = (E^2 + I^2)\cos^2(45° + 0.5\,\gamma) \qquad \textbf{(87)}$$

Consequently,

$$(O\,u)^2 + (O\,v)^2 = E^2 + I^2 = (O\,B)^2 \qquad \textbf{(88)}$$

Both $E$ and $I$ are of course supposed to be expressed in some units of length. One of the fundamental properties of the ellipse is that the sum of the squares of any pair of its conjugate diameters is constant. Hence, in our ellipse the sum of the squares of the principal axes, or of any other pair of conjugate diameters, is equal to the square of the diagonal of the circumscribed rectangle.

The angle $\delta$ between any two conjugate diameters is determined by the equation

$$\tan\delta = 2\,I\,E\cos\gamma/[E^2\sin 2\,\alpha - I^2\sin(2\,\alpha - 2\,\gamma)] \qquad \textbf{(89)}$$

For the sake of convenience in computations and in fitting an experimental curve to an ellipse, some values of $e$, $i$, and $d\,i/d\,e$ are given in the table below, determined from Equations **(69)**, **(70)**, and **(79)**. The ellipses shown in Fig. 1 were drawn with the aid of this table.

| $\alpha$ | $e$ | $i$ | $d\,i/d\,e$ |
|---|---|---|---|
| $0$ | $0$ | $I\cos\gamma$ | $(I/E)\sin\gamma$ |
| $\gamma$ | $E\sin\gamma$ | $I$ | $0$ |
| $45° + 0.5\,\gamma$ | $\left\{ \begin{array}{l} E\sin(45° + 0.5\,\gamma) = \\ E\cos(45° - 0.5\,\gamma) \end{array} \right.$ | $\left. \begin{array}{l} I\sin(45° + 0.5\,\gamma) = \\ I\cos(45° - 0.5\,\gamma) \end{array} \right\}$ | $-(I/E)$ |
| $90°$ | $E$ | $I\sin\gamma$ | infinity |
| $90° + \gamma$ | $E\cos\gamma$ | $0$ | $(I/E)/\sin\gamma$ |
| $135° + 0.5\,\gamma$ | $\left\{ \begin{array}{l} E\sin(45° - 0.5\,\gamma) = \\ E\cos.(45° + 0.5\,\gamma) \end{array} \right.$ | $\left. \begin{array}{l} -I\sin(45° - 0.5\,\gamma) = \\ -I\cos(45° + 0.5\,\gamma) \end{array} \right\}$ | $(I/E)$ |
| $180°$ | $0$ | $-\,I\cos\gamma$ | $(I/E)\sin\gamma$ |
| $180° + \gamma$ | $-\,E\sin\gamma$ | $-I$ | $0$ |

## Notation

| | |
|---|---|
| $a$ | parameter in Equation **(72)** |
| $b$ | parameter in Equation **(72)** |
| $C_1, C_2,$ | capacitances |
| $c$ | parameter in Equation **(72)** |
| $D$ | constant of integration in Equation **(39)** |
| $E$ | amplitude of applied a-c. voltage $e$ |
| $e$ | instantaneous a-c. voltage |
| $e_1$ | critical voltage of a glow gap |
| $e_i$ | instantaneous critical voltage |
| $e_v$ | visual critical voltage |
| $g$ | shunted conductance |
| $h$ | parameter in Equation **(72)** |
| $I$ | amplitude of current $i$ |
| $i$ | instantaneous current |
| $j$ | $\sqrt{-1}$ |
| $K$ | complex quantity defined by Equation **(33)** |
| $k$ | either of the principal axes of an ellipse; Equation **(77)** |
| $n$ | correction term in Equation **(31)** |
| $P_1, P_2$ | voltage amplitudes defined by Equations **(7)** and **(8)** |
| $p_1, p_2$ | instantaneous component voltages |
| $Q$ | amplitude of voltage $q_2'$, defined by Equation **(36)** |
| $Q_2$ | voltage amplitude defined by Equation **(48)** |
| $q_1, q_2$ | instantaneous voltages across condensers $C_1$ and $C_2$ |
| $q_2'$ | sinusoidal component of voltage $q_2$ |
| $R, r$ | resistances |
| $S$ | auxiliary voltage defined by Equation **(59)** |
| $t$ | time |
| $t_1$ | instant at which corona is re-established |
| $U$ | amplitude of current $u$ |
| $u$ | instantaneous current |
| $u, v$ | subscripts referring to points $u$ and $v$ in Fig. 7 |
| $x, y$ | coordinates in Equation **(72)** |
| $Z_1, Z_2$ | impedances defined by Equations **(26)** and **(27)** |
| $\alpha = \omega\,t$ | variable time angle |
| $\beta$ | imperfection angle, Equation **(48)** |
| $\Gamma$ | constant of integration in Equation **(3)** |
| $\gamma$ | imperfection angle of condenser $C_1$ |

$\delta$ angle between conjugate diameters of an ellipse

$\epsilon$ base of natural logarithms

$\theta$ auxiliary phase angle defined by Equation (59)

$\phi$ angle between the major axis of an ellipse and the axis of abscissas

$\omega = 2\,\pi\,f$ angular frequency

## Discussion

**E. B. Payne:** In a cooperative research with the Utilities Research Commission we have conducted corona experiments at the University of Illinois. By means of tube amplifiers we have obtained oscillograms which record discontinuities in current-time curves. When such discharges occur in lead-covered cables we have found them to be associated with high-frequency disturbances which travel along the cable and may by proper apparatus be picked up at the cable ends.

**J. B. Whitehead:** It is of very great interest to see an effort to represent so highly complicated a phenomenon as the alternating-voltage corona by means, first, of a mechanical model and then of a simple electrical analogy.

I confess to some surprise that Professor Katapetoff has been able to advance so far in this type of analysis. He has himself called attention to the complexity of this phenomenon, and I hope that his remarks indicate that he is going to offer us something further in the way of a deeper and more fundamental analysis. Perhap that is the reason why he has not mentioned the recent work of one or two others on the fundamental character of corona and on the underlying laws of the travel of space charge in its reaction on the corona curves. I have particularly in mind the work of Holm, who has analyzed the travel of the space charge in a state of ionization, and under the influence of the sustained voltage corresponding to one-half of the alternating cycle. Holm has computed the limit of the travel of space charge, and used it to explain the shapes of the alternating-voltage curves as we see them.

The changes in the position of the critical voltage, on the alternating wave in the direction of the zero value of voltage, as described by Mr. Peek, are to be explained by the return of the space charge, which has gone out during one-half wave, under the influence of the succeeding wave of opposite polarity. This results in a very much higher potential gradient around the wire than would be due to the voltage itself, and so break down at an *apparently* lower value of voltage. It is this out-going travel and return of space charge that has engaged the attention of Holm.

In our own laboratory at Johns Hopkins, Dr. Willis two years ago made some very interesting extensions of Holm's theory, and since then one of our men, Dr. Waldorf, has been making some further studies in the hope of checking up the theories of Holm.

# Flux Linkages and Electromagnetic Induction in Closed Circuits

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***Synopsis.***—*It is shown that the flux linkages of a circuit may be changed in two very different ways—either the flux may be varied causing a voltage to be induced according to Faraday's Law of Electromagnetic Induction, or the turns may be varied by a substitution of circuit without inducing a voltage. In the Appendix it is mathematically shown that the flux may be changed either by transformer or cutting action, but that the presence of one or the other of these actions is dependent on the choice of reference axes. Thus any argument to the effect that one of them in particular is a necessary part of all induction phenomena is futile. It is possible to identify in every d-c. machine the building up of flux linkages so as to generate a voltage, and the reduction of flux linkages by a substitution of circuit so as not to generate a voltage. The alternate working of these two methods for changing the flux linkages of a circuit is an essential and necessary feature of every d-c. dynamo-electric machine. General criteria are introduced for ascertaining in any given case the nature of the changes in interlinkages which occur, and whether voltages are induced thereby. By way of application, a new restriction on the use of coefficients of inductance is pointed out, the sliding contact and homopolar machine are discussed, and finally a table has been prepared illustrating the various types of flux linkages found in familiar apparatus.*

## INTERLINKAGES

THE purpose of this paper is to examine the various changes in flux linkages that occur in electrical circuits, and to classify them with respect to the type of voltage generated. With this end in view a general equation is derived for calculating the voltage induced in a circuit of any shape moving or changing its configuration in a variable field of magnetic flux. Therefrom criteria are developed, and their application demonstrated, for determining in any given case the nature of the changes in interlinkages that take place and whether voltages are induced thereby.

There are many examples of changing flux linkages which are exceedingly difficult to analyze, and uncertainty may exist about the way in which the voltage is generated. Nor is it always easy to transpose the change in interlinkages that seem to occur to an equivalent switching operation and substitution of circuits, or to a rate of change of flux or turns. Oftentimes these phenomena are masked by each other, or seem to admit of a double interpretation. It is therefore convenient, if not essential, to classify the different methods for bringing about a change in flux linkages, in such a way and subject to such interpretation, that this uncertainty is reduced to a minimum.

In order that no ambiguity shall exist as to the meaning of the term *circuit*, as used in this paper, the following definition shall apply—

*Any closed contour in space, whether in conducting media or not, and regardless if parts thereof are common to any other selected contours, constitutes a closed circuit.*

On the basis of this definition, a network of linear conductors having two or more branches in parallel will consist of just as many separate and distinct circuits as it is possible to trace closed paths in that network, even though some of the conductors are common to two or more of those circuits. It is thus possible to construct a network of $n\ (n-1)/2$ complete circuits with $n$ flexible linear conductors if only two junctions are used. Each circuit of a network will in general link a different amount of flux, and if the network is not constrained to move as a rigid body, will have different velocities. Consequently the voltages induced will be different, and if currents are permitted to flow they will automatically adjust themselves so that the resistance drop in each circuit will completely balance out the voltage induced in that circuit. In the case of conductors of large cross-section, all filaments of which are not linked with the same amount of flux, it becomes necessary to treat the conductor as a bundle of filaments in parallel, and to compute the voltage induced in each separate filament circuit. In such a case it may happen that a certain group of filaments forming a continuous surface are all linked with the same flux, and may therefore be treated as a group. For example, in calculating the skin effect of a tubular conductor where the return is too far removed to affect the current distribution, it is customary to treat the conductor as a nest of concentric hollow cylindrical elements.

In most cases of engineering practise, what constitutes a *turn* is usually so intuitively self evident as to require no explanation. But when tubes of induction are interwound with a circuit there may be some chance for confusion. The following arbitrary definition will therefore apply from the point of view of this paper.

*If it is possible by means of imaginary lines to subdivide a circuit into a network of N cells such that each cell encloses the same flux $\phi$ and in the same direction, then the circuit is said to have N turns with respect to $\phi$.*

On the basis of the above definition, the actual physical loops may be made by either the circuit or by the tubes of induction, and the specification of the number of turns present is entirely arbitrary. In any particular case the induced voltage may be computed on the basis of $N$ circuits in series each linked with a

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flux $\phi$, or of a single turn circuit linked with a flux $N\phi$. To prove the equivalence it is only necessary to note that the voltage round any circuit $C$, which has been so subdivided into $N$ cells 1, 2, 3 . . . $N$, is equal to the sum of the voltages round each cell, all taken in the same direction, thus

$$E_c = E_1 + E_2 + \ldots + E_N$$

If a circuit consists of $n$ concentrated turns linked with a flux $\phi$, then the interlinkages are

$$\Omega = n\,\phi \tag{1}$$

and their rate of change is

$$\frac{d\,\Omega}{d\,t} = n\,\frac{d\,\phi}{d\,t} + \phi\,\frac{d\,n}{d\,t} \tag{2}$$

The term $n\,d\,\phi/d\,t$ of Equation (2) accounts for those changes in interlinkages which are caused by varying the flux through the circuit. It expresses the Law of Electromagnetic Induction deduced experimentally by Faraday and stated as follows:

"*Whenever the total flux through a circuit varies there is an electromotive force induced whose magnitude is proportional to the rate of diminution of the total number of tubes of induction threading the circuit.*"

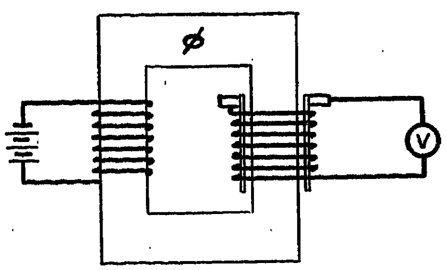


Fig. 1—Changing the Flux Linkages by Unwinding Turns and without Inducing a Voltage

The law is universally true and applies when either or both the magnetic system and circuit are moving. While $n\,d\,\phi/d\,t$ is the general and most concise expression of the Law of Induction, it is nevertheless convenient to expand it into a more useful form in order to facilitate an understanding of its meaning and application. It is perhaps intuitively evident that the total flux threading a circuit may be changed either by varying the density of those tubes of induction already linked with the circuit, or by moving the circuit through the field. However, it is shown mathematically in Appendix I, by means of the Calculus of Variations and Vector Analysis, that $n\,d\,\phi/d\,t$ is composed of these two natural components. Two less general proofs are also included which are not dependent on the processes of mathematical physics, but whose solutions exhibit the same form as that of the general case, and from which the latter may be easily inferred. The expanded form of the Law of Induction developed in Appendix I is

$$e = -N\frac{d\,\phi}{d\,t} = -\sum N\left(\frac{\partial\,\phi}{\partial\,t} + \int_c \mathbf{B}\cdot\mathbf{V}\times d\,\mathbf{s}\right)$$

$$= -\sum N\left\{\frac{\partial\,\phi}{\partial\,t} + \int_c \begin{vmatrix} \alpha & \beta & \gamma \\ u & v & w \\ dx & dy & dz \end{vmatrix}\right\}$$

$$= -\sum N\left(\frac{\partial\,\phi}{\partial\,t} + \int_c B_n\,V\sin\theta\,ds\right) \tag{16}$$

where the summation is to range over all of the circuits of concentrated turns which are connected in series.

The definitions of the symbols are given in the Appendix and in the attached list of symbols.

The first term of $d\,\phi/d\,t$ depends on the position and configuration of the circuit relative to the reference axes, and on the rate of change of the magnetic field. It is independent of the rate of motion or change in configuration of the circuit and is therefore called the "variational component," or referred to as "transformer" action.

The second or "motional" term of these expressions depends on the velocity of the elements of the circuit and on the instantaneous value of the components of flux density at the elements and normal to their planes of motion. It represents the "cutting" action of the moving circuit.

The two terms of the general e. m. f. equation thus have a real physical significance, but they are not invariant to a change of coordinate axes. In some apparatus, as for example in the case of the transformer, there is only one possible choice of axes. But in other instances, either or both terms may be present depending on the choice of reference axes. Thus, in the polyphase induction motor the voltage induced in the rotor is of the type $(\partial\,\phi/\partial\,t + B\,l\,v)$ or $\partial\,\phi/\partial\,t$ or $B\,l\,v$; corresponding respectively to axes taken on the frame, on the rotor, or rotating with the stator m. m. f.

The term $\phi\,d\,n/d\,t$ of Equation (2) accounts for those changes in interlinkages which are caused by varying the number of turns linked with a given rigid distribution of flux. It may appear that there are two possible interpretations to the meaning of $\phi\,d\,n/d\,t$. First, the turns may be changed without cutting the flux, as by winding them around it as indicated in Fig. 1, where the turns are wound onto a drum which revolves about a magnetic core. The electric circuit is completed through a slip-ring and brush. In such an arrangement the turns $n$ are constant until the connection to the slip ring passes under the brush, when the number of turns changes abruptly to $(n \pm 1)$. Thus $d\,n/d\,t$ is infinite at that instant, but otherwise is zero. *No voltage is induced by this process.* A unique d-c. generator based on this process is described in the paper. A variation of the same scheme is in fact employed in every d-c. generator, and will be hereafter referred to as a *substitution of circuit.*

The second interpretation of $\phi\,d\,n/d\,t$ might seem to be the piling up of turns by cutting through the flux, as illustrated in Fig. 2. For such a point of view it is necessary to define a full turn as one linking all

of the specified amount of flux, and a partial or fractional turn as one linking only a part thereof. But such a definition is contrary to the idea of a turn as something necessarily integer, and leads to endless confusion and uncertainty in practise. In fact, if Equation (1) is defined on the basis of concentrated integer turns, then $\phi\, d\, n/d\, t$ cannot possibly admit of this second interpretation; for the several turns which make up the circuit of Fig. 2 are not concentrated, that is, they do not all link the same flux at the same instant. Under the conditions of such a definition each of the several turns must be regarded as separate circuits connected in series, and the changing flux linkages due to the motion of the coil are then fully accounted for by $n\, d\, \phi/d\, t$. It is therefore evident that any attempt to account for the change in interlinkages due to the motion of the circuit through the flux, by any term other than $n\, d\, \phi/d\, t$ would be superfluous and lead to a duplication of result, or else place an unnatural restriction on the definition of $n\, d\, \phi/d\, t$.

Thus it is seen that the interlinkages of a circuit

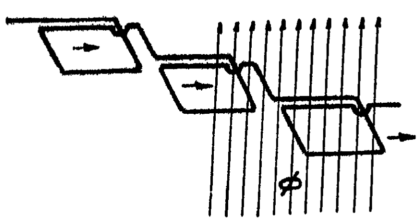

FIG. 2—PILING UP OF TURNS

may be changed in two very different ways:

(1) The flux threading a circuit may be changed either by "transformer" or "cutting" action causing a voltage to be induced according to Faraday's Law of Induction.

(2) The turns linking the flux may be changed in such a way as not to cut through the flux, as by winding on turns or substitution of circuits, thus effecting a change of interlinkages *without* inducing a voltage.

The first of these two methods makes possible the generation of a voltage by electromagnetic induction. The second offers the only possible way for obtaining an average or d-c. component therefrom by means of a dynamo-electric machine.

### THE AVERAGE OR D-C. COMPONENT OF VOLTAGE

The average voltage induced in a non-interrupted circuit over the time interval $(t_2 - t_1)$ is

$$e_{av} = -\frac{n}{t_2 - t_1}\int_{t_1}^{t_2}\frac{d\,\phi}{d\,t}\,d\,t = -\frac{n}{t_2 - t_1}(\phi_2 - \phi_1) \qquad (3)$$

It is evident that $e_{av} \to 0$ if averaged over a sufficiently long period of time, for the flux $\phi_2$ cannot perpetually increase.

Suppose, however, that at time $t_2$ when the flux included by the circuit is $\phi_2$, the interlinkages be reduced to some lower value $\phi_1$ by effecting a substitution of circuit, $\phi\, d\, n/d\, t$; and then that the flux linkages of the new or substituted circuit be increased by increasing the flux. Any number of such cycles may be passed through in succession and the average voltage induced over all of the cycles is

$$e_{av} = -\frac{\Sigma\, n\,(\phi_2 - \phi_1)}{\Sigma\,(t_2 - t_1)} \qquad (4)$$

If every substituted circuit and cycle is alike this reduces to

$$e_{av} = -\frac{\phi_2 - \phi_1}{t_2 - t_1}\, n \qquad (5)$$

Thus a d-c. component of voltage may be obtained over any period of time, merely by providing some arrangement whereby new circuits may be continually substituted as the limit in flux linkages is reached for each. And therefore in any d-c. generator the voltage must be induced by a change of interlinkages $n\, d\, \phi/d\, t$; but the interlinkages must be held within finite bounds by a periodic reduction $\phi\, d\, n/d\, t$.

The most familiar arrangement of this kind is the ordinary d-c. generator, wherein a commutator functions as an automatic switch connecting the armature coils to the external circuit. At regular intervals each armature coil is disconnected from the external circuit on being short circuited by the brushes, and is then substituted back into the circuit, but with reversed connections.

As a second illustration of this method for obtaining a d-c. component of voltage, Fig. 3 shows a generator consisting of a magnetic core $c$, exciting winding $f$, and reversible revolving windings $a$ and $b$. The flux cycle in the magnetic core, furnished by the exciting winding $f$, is a trapezoidal wave as shown in the figure. The generating cycle is divided into four periods, shown on the flux and voltage diagrams as

| | |
|---|---|
| Period No. 1 | $N$ turns on $a$, none on $b$.<br>Flux varied linearly from $-\phi$ to $+\phi$.<br>Constant voltage, since $d\,\phi/d\,t$ = constant. |
| Period No. 2 | Turns transferred from $a$ to $b$. $(d\,n/d\,t.)$<br>Flux constant.<br>Voltage zero, since $d\,\phi/d\,t = 0$. |
| Period No. 3 | $N$ turns on $b$, none on $a$.<br>Flux varied linearly from $+\phi$ to $-\phi$.<br>Constant voltage, since $d\,\phi/d\,t$ = constant. |
| Period No. 4 | Turns transferred back to $a$. $(d\,n/d\,t.)$<br>Flux constant.<br>Voltage zero, since $d\,\phi/d\,t = 0$. |

Of course, whether the voltage generated during period No. 3 is of the same or different sign to that generated during period No. 1 depends on the direction of the windings, but as this is arbitrary it presents no problem.

### THE GENERAL CRITERIA

In the light of the foregoing developments, the following criteria are introduced as a means towards systematically determining the nature of the interlinkages which occur in electrical apparatus, and whether voltages are induced thereby. It is not supposed that these criteria will reduce the analysis to a simple mechanical process, but they are a step in that direction. The proposed rules are:

(a) Choose a set of coordinate axes as convenient and refer all changes in flux, or in position and configuration of the electric circuit, to these axes. (Transformation of coordinates, if properly done, is always permissible.)

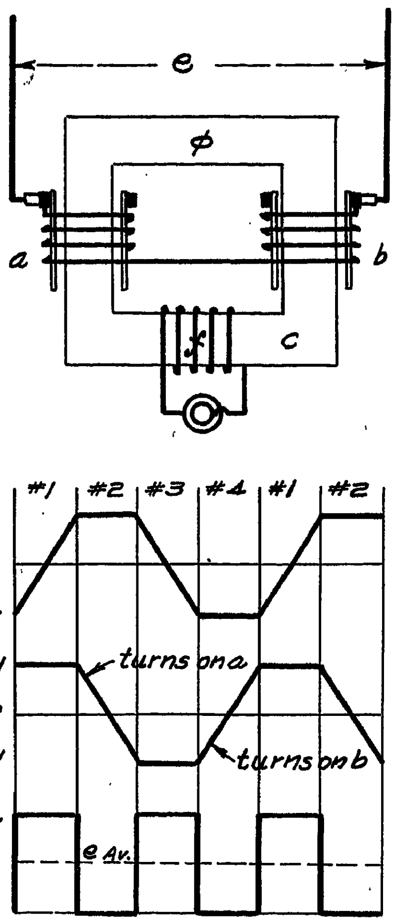


FIG. 3—D-C. GENERATOR MAKING USE OF THE PRINCIPLE IN FIG. 1

(b) If that flux linked with the electric circuit at any instant, *i. e.*, with the circuit fixed, is a function of time with respect to the coordinate axes, it will induce in the circuit a voltage

$$e_1 = - \Sigma\, n\, \partial \phi / \partial t$$

where the summation is to include all those groups of concentrated turns of which the total circuit is composed, and $\phi$ is the flux linked with any group of $n$ concentrated turns. By "concentrated turns" is understood all those turns connected in series which link exactly the same flux, although they may be physically widely distributed and at different places in space.

(c) If any of the elements $ds$ of an electric circuit are moving with respect to the coordinate axes so as to "cut" the flux of the magnetic circuit, they will induce a voltage

$$e = - \sum n \int_c B_n\, V \sin \theta\, ds$$

where

$V$ = velocity of the element $ds$

$\theta$ = angle between the direction of $v$ and $ds$

$B_n$ = component of flux density normal to the plane of $V$ and $ds$

$\int_c$ = the integral for all the elements $ds$ taken completely round the circuit.

(d) Any change of interlinkages which cannot be classified under either (b) or (c) is due to a substitution of circuit, $d\,n/d\,t$, and will necessarily involve some switching operation, sliding contact, or transfer of turns. No voltage will be induced thereby, except in so far as the flux itself is changed; either because the exciting m. m. f. is furnished by the turns themselves, or because they happen to be made of a magnetic material whose shifting changes the reluctance of the magnetic circuit.

(e) It is impossible to induce a d-c. voltage in an uninterrupted circuit. However, a d-c. voltage may be obtained by repeatedly building up the flux linkage by methods (b) and (c) and reducing them by method (d). In this way unidirectional voltages are induced during the increase of flux linkages, but no voltages are induced during their reduction.

### APPLICATIONS

A few examples are discussed under this section of the paper to fix in mind the principles involved, and the method of applying the general criteria. It is pointed out that there is a distinct restriction on the use of coëfficients of inductance quite apart from saturation effects. The sliding contact and homopolar generator are briefly described and analyzed. Finally, a table has been compiled indicating the nature of the interlinkages which occur in some of the more familiar types of electric apparatus.

### COEFFICIENTS OF INDUCTANCE

It is sometimes convenient to express the total flux linkages of a circuit in terms of its coefficients of self and mutual inductance as defined by the equation:

$$N_k\, \phi_k = N_k\, (\phi_k' + \phi_k'') = \frac{N_k^2\, i_k}{R_k} + \sum{}^s \frac{N_k\, N_s\, i_s}{R_{ks}}$$
$$= L_k\, i_k + \Sigma^s\, M_{sk}\, i_s \qquad (6)$$

where

$N_k$ = total turns of circuit $k$

$N_s$ = total turns of circuit $s$

$\phi_k'$ = flux linked with $N_k$ due to $N_k\, i_k$

$\phi_k''$ = flux linked with $N_k$ due to $\Sigma\, N_s\, i_s$

TABLE I
TYPES OF INTERLINKAGE PHENOMENA

| Apparatus | Reference axes | Circuit considered | Change of interlinkages: Increase | Change of interlinkages: Decrease | Voltage generated |
|---|---|---|---|---|---|
| Transformer | On core | Either | $n \frac{\partial \phi}{\partial t}$ | $n \frac{\partial \phi}{\partial t}$ | a-c. |
| Syn generator | On field | Armature | $n\,Blv$ | $n\,Blv$ | a-c. |
| | On armature | Armature | $n \frac{\partial \phi}{\partial t}$ | $n \frac{\partial \phi}{\partial t}$ | a-c. |
| Polyphase ind. motor | On stator | Rotor | $n\left(\frac{\partial \phi}{\partial t} + Blv\right)$ | $n\left(\frac{\partial \phi}{\partial t} + Blv\right)$ | a-c. |
| | On rotor | Rotor | $n \frac{\partial \phi}{\partial t}$ | $n \frac{\partial \phi}{\partial t}$ | a-c. |
| | Rotating synchronously | Rotor | $n\,Blv$ | $n\,Blv$ | a-c. |
| D-c. generator | On field | At brushes | $n\,Blv$ | $\phi \frac{d n}{d t}$ | d-c. |
| | On armature | At brushes | $n \frac{\partial \phi}{\partial t}$ | $\phi \frac{d n}{d t}$ | d-c. |
| Unipolar generator | On core | At brushes | $n\,Blv$ | $\phi \frac{d n}{d t}$ | d-c. |
| Generator in Fig. 3 | On core | At brushes | $n \frac{\partial \phi}{\partial t}$ | $\phi \frac{d n}{d t}$ | d-c. |
| Sliding contact | On core | At contacts | $\phi \frac{d n}{d t}$ | $\phi \frac{d n}{d t}$ | None |

$\phi_k \;= \phi_k' + \phi_k''$
$R_k \;=$ reluctance to $\phi_k'$
$R_{sk} =$ reluctance to $\phi_k''$
$i_k \;=$ current in circuit $k$
$i_s \;=$ current in circuit $s$

Since the identity is to be true for all values of $i_k$ and $i_s$, then

$$L_k = \frac{N_k^2}{R_k} \text{ and, } M_{ks} = \frac{N_k \, N_s \, i_s}{R_{ks}} \tag{7}$$

Now suppose that the currents and reluctances remain constant, but that $N_k$ is varied (as by the winding on of turns in the experiment shown in Fig. 4).

The flux will then vary directly with the turns, $\phi_k' = N_k \, i_k / R_k$, and there will be induced in the circuit a voltage

$$e = - N_k \frac{d \, \phi_k}{d t} = - N_k \frac{d}{d t} \left( \frac{N_k \, i_k}{R_k} + \sum{}^s \frac{N_s i_s}{R_s} \right)$$

$$= - \frac{N_k \, i_k}{R_k} \frac{d N_k}{d t} = - \phi_k' \frac{d N_k}{d t} \tag{8}$$

This result may appear at first sight to violate the conclusion previously reached, *i. e.*, that the Law of Induction is $- N \, d \, \phi / d \, t$ and not $- d \, (N \, \phi) / d \, t$ or $- \phi \, d \, N / d \, t$. However, in Equation (3) the final result follows directly from $- N \, d \, \phi / d \, t$ and has the form $\phi \, d \, N / d \, t$ merely as a matter of coincidence. Had we proceeded from the expression $- d \, (N \, \phi) / d \, t$ our result would have been twice too large.

In Equation (8) $d \, N_k / d \, t$ may be regarded as quite uniform and continuous, as is evident from a study of Fig. 4A. $C$ is the magnetic circuit linked with a coil in which a current $I$ is flowing. At the connection to the slip-ring $R$, the current divides as $i_1$ and $i_2 = I - i_1$ and flows in opposite directions to the brush $b$. This may be considered as a superposition of the full turn currents shown in Fig. 4B. The equivalent turns at full current then are $2 \, I + i_1 = (2 + \theta / 2 \, \pi) \, I$. Thus the effect is exactly equivalent to the hypothetical existence of a uniformly changing fractional turn. Of course if the slip-ring were of non-uniform resistivity the division of current would not be proportional to the angle $\theta$. If it were slit axially the change in $N_k$ would occur abruptly.

The coefficients of inductance as defined by (7) are not sufficient to express the voltage due to a change in flux linkages of the type described above. For under such conditions there results:

$$\frac{d}{d t} \left( L_k \, i_k + \Sigma^s \, M_{sk} \, i_s \right) = i_k \frac{d \, L_k}{d t} + \sum{}^s i_s \frac{d \, M_{sk}}{d t}$$

$$= i_k \frac{d \, L_k}{d \, N_k} \cdot \frac{d \, N_k}{d t} + \sum{}^s i_s \frac{d \, M_{sk}}{d \, N_k} \cdot \frac{d \, N_k}{d t}$$

$$= i_k \frac{d\,N_k}{d\,t} \cdot \frac{d}{d\,N_k}\left(\frac{N_k^2}{R_k}\right)$$

$$+ \sum\nolimits^s i_s \frac{d\,N_k}{d\,t} \cdot \frac{d}{d\,N_k}\left(\frac{N_k\,N_s}{R_{sk}}\right)$$

$$= 2\,\frac{i_k\,N_k}{R_k} \cdot \frac{d\,N_k}{d\,t} + \sum\nolimits^s \frac{i_s\,N_s}{R_{sk}}\,\frac{d\,N_k}{d\,t}$$

$$= 2\,\phi_k'\,\frac{d\,N_k}{d\,t} + \frac{\phi_k''}{N_k}\,\frac{d\,N_k}{d\,t} \qquad (9)$$

In this result the term $2\,\phi_k'\,d\,N_k/d\,t$ is twice too large and the term $\frac{\phi_k}{N_k}\,\frac{d\,N_k}{d\,t}$ should not appear. This discrepancy arises from the fact that the coefficients of

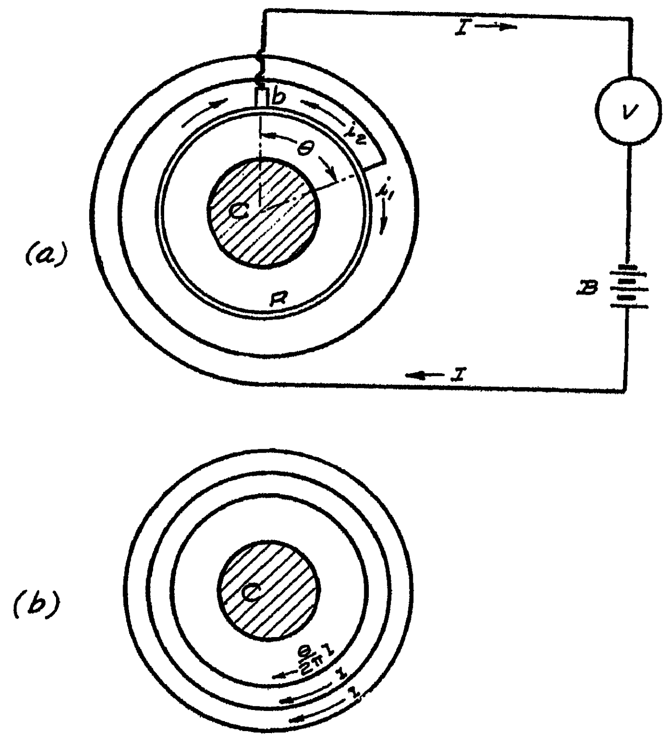


FIG. 4—CHANGING THE M. M F. BY UNWINDING TURNS

inductance $L_k$ and $M_{sk}$ contain by definition the number of turns $N_k$; but this factor is not subject to differentiation when these turns vary in such a way as to change the flux linkages without "cutting" the flux.

On the other hand, if they do vary in such a way as to "cut" the flux it is more reliable and more in accord with the physical facts to attribute the change in interlinkages as due to the changes in reluctances; rather than to introduce the motion of partial turns as previously discussed.

It is therefore evident that the coefficients of inductance are not directly applicable to cases of the type described above.

### THE SLIDING CONTACT

The "Sliding Contact," Fig. 5A consists, in its most simple form, of a circuit completed through, and making a sliding contact with a magnetic core. As the contacts are moved across the core, the total flux linked with the circuit appears to change but no voltage is induced.

This is usually explained as a case of continuous and successive substitution of circuit. The iron core is considered as consisting of an infinite number of conducting filaments in parallel, as illustrated in Fig. 5B, and the process of moving the contacts is equivalent to progressively closing and opening switches at infinitesimal increments.

But in the light of the interlinkage criteria, the fact that no voltage is induced is evident at a glance, for:

a. Choose axes on the core.

b. There is no change of flux, hence $n\,d\,\phi/d\,t = 0$.

c. No moving element of the circuit "cuts" flux, hence $n\,B\,l\,v = 0$.

d. The change of interlinkages is due to a substitution of circuit. Therefore no voltage is induced.

### THE HOMOPOLAR MACHINE

Fig. 6 shows the schematic form of the homopolar or unipolar generator. It consists of a bar magnet $N\,S$ around which revolves a conductor $c\,d$ on rings $R$. The circuit is completed through the upper and lower parts of the slip rings and the brushes $b$, thus forming two circuits in parallel having a common return through the brushes and external circuit. As the conductor revolves the flux in the upper circuit increases and that in the lower circuit correspondingly decreases, but as the flux is in opposite directions relative to the two circuits, the voltage generated is in the same direction. Although the flux through the upper circuit is apparently continually increasing, yet when the conductor reaches the brushes the flux included is the same as it was a revolution previously.

The paradox may be explained by reference to Fig. 6c. Let the conductor $c\,d$ be moved from $A$ to $B$ across the magnet face, generating a voltage. If this conductor could by some means be transferred back to $A$, each time that it reaches $B$, and in such a manner that the flux is not cut in a reversed direction, or the circuit interrupted, then conditions would be ideal for generating a d-c. voltage. The unipolar machine is just such an automatic arrangement for instantaneously effecting a change of interlinkages by a substitution of circuit, and in this respect Fig. 6c may be considered as its "development."

In effect, the unipolar machine is equivalent to making the distance $A\,B$ infinite. But this equivalence is not offered as an explanation of the phenomena of induction in the unipolar generator, for it fails to take into account the actual substitution of circuit which occurs, *an essential feature of any dynamo-electric machine generating direct current.*

Applying the interlinkage criteria:

(a) Choose axes as the frame.

(b) There is no change of flux, hence no voltage $n\, \partial \phi / \partial t$.

(c) The moving element cuts flux and always in the same direction, thus inducing a unidirectional voltage $B\, l\, v$.

(d) There is an automatic substitution of circuit each time that the conductor passes under the brush.

(e) The interlinkages are increased at a regular rate by cutting the flux, and induce a unidirectional

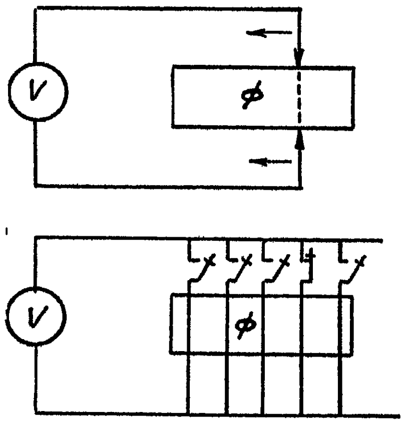


FIG. 5—SLIDING CONTACT AND EQUIVALENT SWITCHING OPERATIONS

voltage; but the interlinkages are reduced to zero by a substitution of circuit each time that the conductor passes under the brush. Thus a d-c. voltage is continuously generated.

## CONCLUSIONS

The phenomena of changing flux linkages and electromagnetic induction in electrical circuits may be classified according to the following scheme:

Thus the flux linkages of a circuit may be changed by changing either the flux or the turns. The flux may be varied either by transformer action or by cutting action. (These are also known respectively as the *variational* and *motional* components of e. m. f.) But whichever action is involved is dependent on the arbitrary choice of the reference axes. Regardless of the way in which the flux through a non-interrupted circuit is changed, it will induce a voltage according to Faraday's Law. And if the period of time is taken

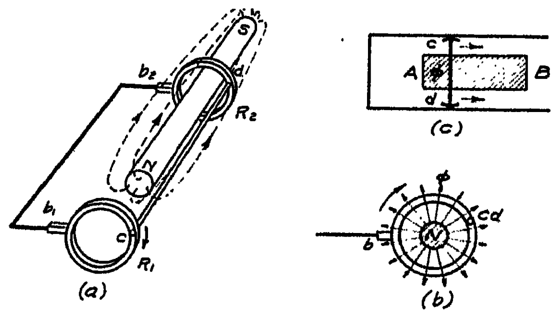


FIG. 6—HOMOPOLAR INDUCTION

sufficiently long, this e. m. f. must be alternating, or zero, for the flux cannot perpetually increase.

But the flux linkages may also be changed by varying the number of turns linked. If this is done in such a way as not to change the flux itself, it is classified as a *substitution of circuit,* and includes the sliding contact, transfer of turns, and commutation or switching operations. *No voltage can be induced by a substitution of circuit.*

The generation of an average or d-c. component of voltage depends on the alternate use of $n\, d\, \phi / d\, t$ and $\phi\, d\, n / d\, t$. That is, the flux linkages must be increased

RATE OF CHANGE OF FLUX LINKAGES

$$\frac{d\,(n\,\phi)}{d\,t}$$

FARADAY'S LAW OF INDUCTION $n \dfrac{d\,\phi}{d\,t}$

(Depending on reference axes)

- Transformer action (or variational e. m. f.) $n \dfrac{\partial\,\phi}{\partial\,t}$ — A-C. VOLTAGE
- Cutting action (or motional e.m.f.) $n \int_c B_n\, V \sin \theta\, ds$ — A-C VOLTAGE

SUBSTITUTION OF CIRCUIT $\phi \dfrac{d\,n}{d\,t}$

1. Sliding contact
2. Commutation or switching operation
3. Transfer of Turns

NO VOLTAGE

D-C VOLTAGE

by increasing the flux and causing a voltage to be induced according to Farraday's Law; but these linkages must be held within finite limits by a periodic reduction with a substitution of circuit.

It is not intended to imply that it is always advantageous in analyzing a given case to follow the above scheme. It may often be more simple to deal directly with $n \frac{d\phi}{dt}$, than with its components. Nor is it suggested that this scheme be adopted to the exclusion of any other consistent classification. Many eminent authorities believe that the seat of the e. m. f. is at the conductor element, and therefore that the most fundamental expression for the Law of Induction is $B L \overline{V}$ where $\overline{V}$ is the *relative* velocity between the conductor elements and the tubes of induction. (In this paper $V$ is the velocity of the conductor element relative to the coordinate system, and therefore is not necessarily the same as the $\overline{V}$ just described.) It is not within the scope of the present paper to discuss in general when tubes of induction have a velocity. Nevertheless, it is often convenient to assign velocities to them—especially when calculating the e. m. f. induced in a finite linear conductor element. (It may be parenthetically remarked that the process of calculating the voltage induced in an isolated element is also equivalent to assuming that the return circuit is at infinity.) Dr. H. B. Dwight recently discussed this $B L \overline{V}$ interpretation of the law, with certain necessary restrictions, in the April 1928 issue of the *Electric Journal*.

Finally, the author wishes to point out that the notions given in this paper have been presented from the engineering point of view, and with no attempt other than that given in Appendix II at correlation with the general mathematical theory of electricity and magnetism.

The author wishes to thank Mr. C. H. Biron for preparing the illustrations.

LIST OF SYMBOLS

$\mathbf{B}$ = $i\,\alpha + j\,\beta + k\,\gamma$ = flux density expressed as a vector.
$B_n$ = Component of $\mathbf{B}$ normal to the plane of $V$ and $d\,s$.
$C$ = Any closed circuit.
$d\,\mathbf{s}$ = $i\,dx + j\,dy + k\,dz$ = element on a circuit $C$.
$e$ = e. m. f.
$e_{av}$ = average value of e. m. f.
$\mathbf{F}$ = $i\,X + j\,Y + k\,Z$ = vector potential defined by $\nabla \times \mathbf{F} = \mathbf{B}$.
$I$ = Current.
$i_k, i_s$ = Currents in circuits $k$ and $s$ respectively.
$i, j, k$ = Unit vectors along the $x, y, z$ axes respectively.
$L_k$ = Self-inductance of a circuit $k$.
$M_{ks}$ = Mutual inductance between circuits $k$ and $s$.
$N_k, N_s$ = Turns of circuits $k$ and $s$ respectively.
$n, N$ = Turns.
$\mathbf{n}$ = Unit vector normal to a surface.
$R_k, R_{ks}$ = Reluctances to fluxes due to $i_k$ and $i_s$ respectively.
$t$ = Time.
$u, v, w$ = Components of the velocity $V$ along the $x, y, z$ axes.
$\mathbf{V}$ = $i\,u + j\,v + k\,w$ = velocity of an element $d\,s$ expressed as a vector.
$V$ = Absolute value of $\mathbf{V}$.
$X, Y, Z$ = Components of the vector potential $\mathbf{F}$.
$x, y, z$ = Any point on a given circuit $C$.
$\alpha, \beta, \gamma$ = Components of flux density $B$ along the $x, y, z$ axes.
$\delta$ = Sign of variation.
$\phi$ = Flux.
$\phi_k'$ = Flux linking a circuit $k$ due to a current $i_k$ therein.
$\phi_k''$ = Flux linking a circuit $k$ due to external currents.
$\theta$ = Angle between $\mathbf{V}$ and $d\,\mathbf{s}$.
$\Omega$ = $n\,\phi$ = flux linkages.
$\nabla$ = $\left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}\right)$ = Grad
$\Sigma$ = Sign of summation, effective as implied.
$\int_c$ = Contour integral taken round any circuit $C$.
$\times$ = Vector or cross product.
$\cdot$ = Scalar or dot product.

## Appendix I

GENERAL EQUATION OF THE LAW OF INDUCTION (Using Vector Analysis and the Calculus of Variations)

Consider an electrical circuit $C$ situated in a field of variable magnetic flux density $\mathbf{B} = i\,\alpha + j\,\beta + k\,\gamma$. Then since $\mathbf{B}$ is a solenoidal vector, $\nabla \cdot \mathbf{B} = 0$, we may define the vector potential $\mathbf{F}$ such that $\nabla \times \mathbf{F} = \mathbf{B}$. It follows by Stokes Theorem that the flux linked with the circuit $C$ at any given instant is

$$\phi = \int\int \mathbf{n} . \mathbf{B} dS = \int\int \mathbf{n} . \nabla \times \mathbf{F}\, d\,S = \int_c \mathbf{F} . d\,\mathbf{s} = L \tag{1}$$

where $\mathbf{n}$ is the unit normal to any surface S of which $C$ is the boundary contour, and $d\,\mathbf{s}$ is an element of the contour $C$.

Now as the circuit moves and changes its shape from $C$ to $C'$, the flux linked therewith also changes; so that in general the flux included by the circuit is a function of the shape and position of the circuit as well as of time $t$. But the shape and position of the circuit is sufficiently characterized by the line integral $L$ as given in Equation (1).

Therefore, functionally

$\phi = f\,(t, L)$ and by the Calculus of Variations (2)

$$\delta\,\phi = \frac{\partial \phi}{\partial t}\,\delta\,t + \frac{\partial \phi}{\partial L}\,\delta\,L = \frac{\partial \phi}{\partial t}\,\delta\,t + \delta\,L \tag{3}$$

since by Equation (1) $\phi = L$ at any given instant, and therefore $\frac{\partial \phi}{\partial L} = 1$.

Now if

$$\mathbf{F} = i\,X + j\,Y + k\,Z \tag{4}$$

and

$$d\,\mathbf{s} = i\,dx + j\,dy + k\,dz \tag{5}$$

we have

$$\delta \int_c \mathbf{F}\,.\,ds = \delta \int_c (Xdx + Ydy + Zdz)$$

$$= \int_c \delta\,(Xdx + Ydy + Zdz)$$

$$= \int_c (\delta\,X\,dx + X\,\delta\,dx + \text{similar terms in } Y \text{ and Z}).$$

$$= \int_c (\delta\,X\,dx + X\,\delta\,dx + \text{similar terms in } Y \text{ and } Z). \tag{6}$$

Integrating by parts

$$\int_c X\,d\,\delta\,x = X\,\delta\,x\Big]_c - \int_c \delta\,x\,d\,X = -\int_c \delta x\,d\,X \tag{7}$$

since at the start and finish of a completed contour $X\,\delta\,x$ has the same value. Also

$$\delta\,X = \frac{\partial X}{\partial x}\,\delta\,x + \frac{\partial X}{\partial y}\,\delta\,y + \frac{\partial X}{\partial z}\,\delta\,z \tag{8}$$

and
$$d\,X = \frac{\partial X}{\partial x}\,dx + \frac{\partial X}{\partial y}\,dy + \frac{\partial X}{\partial z}\,dz$$

$$\therefore \delta \int_c \mathbf{F}\,.\,d\,s = \int_c \Big( \frac{\partial X}{\partial x}\,\delta\,x\,dx + \frac{\partial X}{\partial y}\,\delta\,y\,dx$$

$$+ \frac{\partial X}{\partial z}\,\delta\,z\,dx - \frac{\partial X}{\partial x}\,\delta\,x\,dx - \frac{\partial X}{\partial y}\,\delta\,x\,dy - \frac{\partial X}{\partial z}\delta\,x\,dz$$

$$+ \text{similar terms in } Y \text{ and Z}). \tag{9}$$

Collecting terms and rearranging

$$\delta\,L = \int_c \Big[ \Big( \frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z} \Big) (dz\,\delta y - dy\,\delta z)$$

$$+ \Big( \frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x} \Big) (dx\,\delta z - dz\,\delta x)$$

$$+ \Big( \frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y} \Big) \Big(dy\,\delta x - dx\,\delta y\Big) \Big]$$

$$= \int_c \nabla \times \mathbf{F}\,.\,\delta\,\mathbf{s} \times d\,\mathbf{s} = \int_c \mathbf{B}\,.\,\delta\,\mathbf{s} \times d\,\mathbf{s} \text{ where} \tag{10}$$

$$\delta\,\mathbf{s} = i\,\delta x + j\,\delta y + k\,\delta z \tag{11}$$

is the variation of a point on $C$. Substituting (10) in (3)

$$\delta\,\phi = \frac{\partial \phi}{\partial t}\,\delta\,t + \int_c \mathbf{B}\,.\,\delta\,\mathbf{s} \times d\,\mathbf{s} \tag{12}$$

If this variation in $\phi$ takes place in time $\delta\,t$ the electromotive force induced in the circuit is, by the Law of Induction

$$e = -\frac{\delta\,\phi}{\delta\,t} = -\Big( \frac{\partial \phi}{\partial t} + \int_c \mathbf{B}\,.\,\frac{\delta\,\mathbf{s}}{\delta\,t} \times d\,\mathbf{s} \Big) \tag{13}$$

But

$$\frac{\delta\,\mathbf{s}}{\delta\,t} = i\,\frac{\delta\,x}{\delta\,t} + j\,\frac{\delta\,y}{\delta\,t} + k\,\frac{\delta\,z}{\delta\,z} = iu + jv + kw = \mathbf{V} \tag{14}$$

is the vector velocity of a point on $C$. Therefore,

$$e = -\Big( \frac{\partial \phi}{\partial t} + \int_c \mathbf{B}\,.\,\mathbf{V} \times d\,\mathbf{s} \Big)$$

$$= -\Big( \frac{\partial \phi}{\partial t} + \int_c B_n\,V \sin\,\theta\,ds \Big) \tag{15}$$

where $\theta$ is the angle between $\mathbf{V}$ and $d\,\mathbf{s}$, and $B_n$ is the component of $\mathbf{B}$ normal to the plane of $\mathbf{V}$ and $d\,\mathbf{s}$.

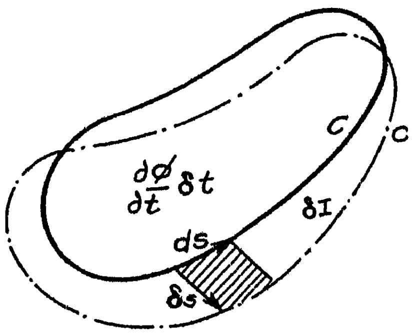


FIG. 7—CLOSED CIRCUIT MOVING AND CHANGING SHAPE IN A VARIABLE FIELD OF FLUX

This general equation of the Law of Induction states that the electromotive force induced in a closed circuit $C$ is equal to

$$\left\{ \begin{array}{l} \text{(The time rate of change in magnitude of} \\ \text{that flux in space which is linked at the} \\ \text{given instant with the circuit } C\text{) + (The} \\ \text{sum for all the elements round the circuit} \\ \text{of the triple product } \textit{length of the moving} \\ \textit{element} \times \textit{component of velocity perpendic-} \\ \textit{ular to the length of the element} \times \textit{compo-} \\ \textit{nent of flux density normal to the plane of} \\ \textit{motion of the element.}) \end{array} \right\} \tag{16}$$

If the circuit consists of $n$ turns the potential induced is $n$ times as great. If several such circuits are connected in series the total potential induced is given by any of the following mutually convertible expressions:

$$e = -\sum n\,\frac{d\,\phi}{d\,t} = -\sum n \Big( \frac{\partial \phi}{\partial t} + \int_c \mathbf{B}\,.\,\mathbf{V} \times d\,\mathbf{s} \Big)$$

$$= -\sum n \Big( \frac{\partial \phi}{\partial t} + \int_c B_n\,V \sin\,\theta\,ds \Big)$$

$$= -\sum n \left\{ \frac{\partial \phi}{\partial t} + \int_c \begin{vmatrix} \alpha & \beta & \gamma \\ u & v & w \\ dx & dy & dz \end{vmatrix} \right\}$$

$$= -\sum n \left\{ \frac{\partial \phi}{\partial t} + \int_c [(\alpha v - \beta u)\, dz \right.$$

$$\left. + (\beta w - \gamma v)\, dx + (\gamma u - \alpha w)\, dy ] \right\} \quad (17)$$

The case of chief practical interest in electrical machinery is that of a coil with parallel coil sides moving through a field which is uniform in a direction along the coil sides perpendicular to the motion, and is zero along the ends. Taking the motion along the $X$ axis, and an effective length of coil $l$ along the $Z$ axis, we have

$$\left\{ \begin{array}{l} \mathbf{B} = i\,0 + j\,\beta + 0 \\ \mathbf{V} = i\,u + 0 + 0 \\ d\,\mathbf{s} = 0 + 0 + j dz \end{array} \right. \quad (18)$$

Then substituting in (17)

$$e = -n\left( \frac{\partial \phi}{\partial t} + \int_c \mathbf{B} \cdot \mathbf{V} \times d\,\mathbf{s} \right)$$

$$= -n\left( \frac{\partial \phi}{\partial t} + \int_0^l \beta_2 u_2\, dz + \int_l^0 \beta_1 u_1\, dz \right)$$

$$= -n\left( \frac{\partial \phi}{\partial t} + l\,\beta_2 u_2 - l\,\beta_1 u_1 \right)$$

$$= -n\frac{\partial \phi}{\partial t} - n\,l\,(\beta_2 - \beta_1)\,u \quad (19)$$

since $u_2 = u_1 = u$ for a rigid coil.

This equation may be derived without the use of the Calculus of Variations as follows, and from it the more general Equation (17) may be inferred.

The flux linked with a coil in the $xz$ plane having parallel coil sides along the $z$ axis is

$$\phi = \int_0^l \int_{x_1}^{x_2} \beta\, dx\, dz = l \int_{x_1}^{x_2} \beta\, dx \quad (20)$$

But if the coil is moving, the limits $x_1$ and $x_2$ are functions of time $t$. Therefore the derivative of (20) must be taken according to the rule for differentiating a definite integral whose limits are functions of the parameter with respect to which differentiation is performed. Thus

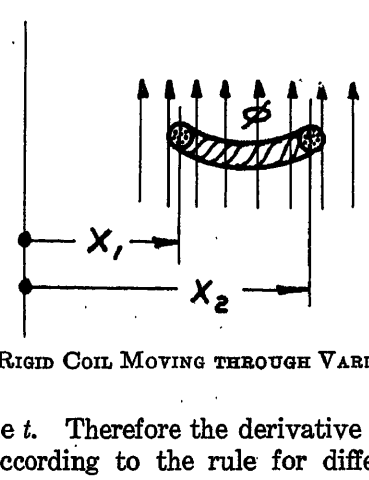


Fig. 8—Rigid Coil Moving through Variable Flux

$$e = -n\frac{d\,\phi}{d\,t} = -n\frac{d}{d\,t}\,l\int_{x_1}^{x_2} \beta\, d\,x$$

$$= -n\,l\left( \int_{x_1}^{x_2} \frac{\partial \beta}{\partial t}\, dx + \beta_2 \frac{d\,x_2}{d\,t} - \beta_1 \frac{d x_1}{d\,t} \right)$$

$$= -n\left( \frac{\partial \phi}{\partial t} + l\,\beta_2 u_2 - l\,\beta_1 u_1 \right)$$

$$= -n\frac{\partial \phi}{\partial t} - n\,l\,u \cdot (\beta_2 - \beta_1) \quad (21)$$

This same procedure may easily be extended to the case of motion in two orthogonal directions.

Still another way of proving this result is to note from Fig. 8 that

$$\phi = f\,(t, x_2, x_1) \quad (22)$$

Then

$$-n\frac{d\,\phi}{d\,t} = -n\left( \frac{\partial \phi}{\partial t}\frac{d\,t}{d\,t} + \frac{\partial \phi}{\partial\, x_2}\frac{d\,x_2}{d\,t} + \frac{\partial \phi}{\partial\, x_1}\frac{d\,x_1}{d\,t} \right)$$

$$= -n\left( \frac{\partial \phi}{\partial t} + l\,\beta_2 u_2 - l\,\beta_1 u_1 \right)$$

$$= -n\frac{\partial \phi}{\partial t} - n\,l\,u\,(\beta_2 - \beta_1) \quad (23)$$

and this method also may be extended to the case of motion in two orthogonal directions.

## Appendix II

### Note on the Induced Voltage in an Element of Circuit

The term $\partial\,\phi / \partial\, t$ of Equation (15) accounts for the voltage induced in the circuit due to the time rate of change of that flux in space which at any given instant is linked with the circuit. It is accordingly computed as though the circuit were fixed in position. From Equation (1)

$e_{variational}$

$$= -\frac{\partial \phi}{\partial t} = -\frac{\partial}{\partial t}\int_c \mathbf{F} \cdot d\,\mathbf{s} = -\int_c \frac{\partial \mathbf{F}}{\partial t} \cdot d\,\mathbf{s} \quad (24)$$

since $C$ is fixed. The total induced voltage for the circuit may then be written in the form

$e = e_{variational} + e_{motional}$

$$= -\int_c \frac{\partial \mathbf{F}}{\partial t} \cdot d\,\mathbf{s} - \int_c \mathbf{B} \cdot \mathbf{V} \times d\,\mathbf{s}$$

$$= -\int_c \left( \frac{\partial \mathbf{F}}{\partial t} + \mathbf{B} \times \mathbf{V} \right) \cdot d\,\mathbf{s} \quad (25)$$

since the dot and cross products of the latter expression are interchangeable.

Equation (25) was derived on the assumption that

the circuit was closed. Since, however, it assigns to each element $ds$ of the circuit a definite portion of the total voltage induced in the complete circuit, it follows that the voltage between any two points $a$ and $b$ of the circuit may be taken as equal to

$$e_{ab} = -\int_a^b \left( \frac{\partial \mathbf{F}}{\partial t} + \mathbf{B} \times \mathbf{V} \right) . d\,\mathbf{s} = \int_a^b \mathbf{E} . d\,\mathbf{s} \qquad (26)$$

where **E** is the electric force at the circuit. To the integrand of (25) may be added any acyclic function $\nabla U$, since the line integral of $\nabla U$ round any closed circuit is zero. Such a function exists when no changes are taking place in the magnetic field or circuit, and is therefore identified physically with the electrostatic potential. For this reason it has been omitted in (26). If the circuit is fixed, then $\mathbf{V} = 0$ and (26) reduces to the familiar rate of change of vector potential expression for the electric force.

$$\mathbf{E} = -\frac{\partial \mathbf{F}}{\partial t} \text{ for } C \text{ fixed} \qquad (27)$$

The advantage of Equation (26) is that the voltage induced in a *part of a circuit* may be calculated quite independently of other parts of the circuit, if the vector potential $F$ is known. The three simultaneous differential equations of $\nabla \times \mathbf{F} = \mathbf{B}$ are not sufficient to uniquely determine **F**. For if **F** is one solution, $(\mathbf{F} + \nabla \psi)$ where $\nabla \psi$ is any acyclic function, is also a solution; since $\nabla \times (\mathbf{F} + \nabla \psi) = \nabla \times \mathbf{F}$. The additional condition which Maxwell arbitrarily imposed was $\nabla . \mathbf{F} = 0$, but this is not entirely satisfactory.

The "second circuital equation" of Heaviside is obtained by taking the curl of Equation (27)

$$\triangle \times \mathbf{E} = -\frac{\partial}{\partial t} \triangle \times \mathbf{F} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (28)$$

However, it is not within the scope of the present paper to discuss the induction of voltage in parts of a circuit. This Appendix has been added merely by way of correlation with the methods found in such texts on Electricity and Magnetism as those by Maxwell, Livens, and Jeans.

## Discussion

**E. E. Johnson:** Mr. Bewley has very well coördinated many of the ideas relating to the subject of electromagnetic induction, and he has also placed the much discussed sliding contact or substitution of circuit on what appears to be a rational basis. Furthermore, the rules which he has set down for determining whether a unidirectional continuously generated voltage may be generated are sometimes very useful, especially when some inventor presents an ingenious device for generating such voltages.

In Appendix I Mr. Bewley discusses the flux linkage of a circuit as well as the voltage generated in that circuit, and he mentions that if this particular circuit contains $n$ turns, the voltage will be $n$ times as great. This brings up the question of what is a turn. Although the concept of a turn is indispensable to electrical engineering, nevertheless in the application of Equation (1) in Appendix I, it is not necessary to think of a turn because if the integration is properly performed, turns do not enter into the expression.

Mr. Bewley, I think, recognizes this, and he brought out the point of the concept of turns, because we use turns so frequently in engineering practise.

It should be pointed out in connection with Equation (16) that the integration is to be performed in the direction that a right hand screw would have to be turned in order to advance in the direction of positive flux.

**L. V. Bewley:** As Mr. Johnson suggests, the idea of a "turn" is only an engineering convenience. Mathematically, the concept is unnecessary and rather inconvenient. Some attempt was made to emphasize this point by the statement in the paper following the definition of a turn. I am indebted to Mr. Johnson for pointing out the direction of the positive normal which applies to the equations and to Fig. 7. The origin of the rule for the direction of contour integration is to be found in the basic derivation of Stokes' Theorem.

For uniform isotropic media at rest, the complete specification of the electromagnet field is given by the pair of symmetrical first and second circuital equations of Heaviside. The latter follows directly from Equation (27) of this paper, by taking its curl as follows:

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{F}) = -\frac{\partial \mathbf{B}}{\partial t}$$

# Progress in High-Tension Underground Cable Research and Development

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Associate, A. I. E. E.

and

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*Synopsis.—This paper deals primarily with the "solid" type paper insulated high-tension cable, the development field still ahead being extensive. A review is given of the progress made during the past four years. The research and development work which made this progress possible is outlined, only the more important features being considered.*

*Particular attention is given to void formation both in the factory and in the field, and methods of arriving at some understanding of this are demonstrated. The conclusion is drawn that void formation under service conditions is inherent in paper insulated "solid" cables. Methods of reducing the size and duration of voids, and increasing their pressure, thereby reducing the possibility of ionization deterioration, are carefully considered. Oil reservoir feed at the joints is of great value in this respect. This is dealt with in detail.*

*Research data on ionization and other characteristics of finished cable and cable materials are presented.*

* * * * *

## INTRODUCTION

PRONOUNCED progress has been made in high-tension underground cable practise during the past three or four years, resulting not only in cable of better quality and reliability but also in greatly improved operating conditions.

The main principles upon which this progress has been based are simple and sound. They are not entirely new and, at least in part, have been recognized by cable engineers for some years past. Knowledge of their relative importance, putting these principles effectively into practise, relegation of less important factors, and confirmation by tests and operating experience are comparatively new acquisitions.

The purpose of this paper is to present as briefly as possible a general outline of the research and development work conducted by one of the manufacturers during the past four years and the observations and conclusions drawn therefrom, as well as from operating sources. High-tension cable is not a self sustaining product. Proper methods of installation and operation are of as vital importance to successful service as are proper design and manufacture. It might be said that we have entered a new era in which there is general recognition of the fact that a high-tension cable is not an inert mass but is physically, electrically, and chemically "alive" and must be so regarded throughout its life. Therefore, no general observations and conclusions bearing upon improved cable service and life can be complete without careful consideration of operating conditions and experience.

It is realized that this is an ambitious program for a single paper of reasonable length and conciseness. In the first place, the authors have at their disposal a mass of research and test data accumulated by a number of workers in their company during the past four or five years. As is usually the case a good part of this is merely mute evidence of a groping for the truth, of experiments tried and discarded. There remains however, a very considerable mass of pertinent data bearing upon the conclusions we wish to set forth.

It would be undesirable, even if possible, to present all of these data herein, for much would be merely repetition of recently published work. Other cable engineers have not been idle during this period and a good part of our work necessarily parallels and in many cases confirms theirs. Where such is the case we will refer to their work and present our confirming conclusions without further proof.

Of the remaining data we will attempt to present only those of particular significance as they relate to the general trend of the paper.

## FUNDAMENTALS OF CABLE PRACTISE

The present trend of high-tension paper-insulated cable practise is towards the principles so clearly brought out by the liquid oil filled cable.[1] These principles, as we see them today, are almost a complete answer to the nearly insurmountable difficulties experienced with the older types of "solid" cable.

Briefly stated:

(a) Removal from the cable of all impurities such as moisture and gas (both free and in solution).

(b) Elimination of voids and prevention of void formation in service.

(c) Maintenance of positive pressure inside the cable at all times.

These three fundamental principles or rules have been more or less recognized for years and are axiomatic. They are given above in the order of their importance.

The whole trouble is that the older types of "solid" cable fall far short of these rules. Few thought so some years ago, but today it is a recognized fact.

The characteristics of "solid" cable, present or past, are inherently such that these three rules cannot be closely followed. Even the latest and most modern types of "solid" cable, such as we will describe in this

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*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

1. For references see Bibliography.

paper, represent a compromise in which are incorporated only the more vital features of these fundamentals. The success of the "solid" cable depends upon our ability to distinguish and meet these vital features.

Bearing this in mind we will attempt to rewrite rules (a), (b), and (c) in the form of compromise rules that will meet the inherently complicated characteristics of "solid" cable. No compromise rule can be axiomatic; therefore, there will always be a reasonable and just difference of opinion regarding the best cable practise so long as "solid" type cable is used. The compromise rules, as we see them, are:

(d) Remove all moisture and similar impurities from the cable.

(e) Remove, as far as possible, all gas (both free and in solution).

(f) Eliminate voids initially and, as far as possible, prevent or reduce void formation thereafter.

(g) Prevent ionization deterioration in the voids that do form.

(h) Maintain, as far as possible, positive pressure inside the cable.

(i) Accepting the fact that low pressure will exist in certain sections of the cable, maintain absolute tightness and integrity of outer covering at these points so that air and moisture cannot enter from outside.

(j) Do not expose the filling oil or compound directly to gas in the reservoirs, joints, or cable, attempting at all times to keep this oil as gas-free as possible. Particularly avoid exposure to air or oxygen.

As the paper progresses the reasons for these rules, many of which are obvious, will be brought out.

### "Solid" vs. Oil Filled Cable

Internal ionization of voids* or gas spaces is now recognized as a vital and dangerous factor. Most of the progress made recently has resulted from a better understanding of void formation during manufacture, installation, and operation, and the causes and effects of ionization when conditions allow it to exist. More will be said of this later.

Our experience with the 132-kv. liquid oil filled cable has been of inestimable value in clearly bringing out these points. The amount of gas in the cable, the internal pressure, and other factors are under control and subject to exact measurement and calculation.

The so-called "solid" type cable is not adaptable to exact engineering study. The high viscocity of the compound and high, variable resistance to flow, longitudinally and radially, offer serious obstacles. We must confess that even with that learned from the liquid oil filled cable a story of just what goes on inside a "solid" cable during void formation and ionization is not yet completely worked out and probably never will be. It is felt, however, that the right line of attack is being followed and that in time the "solid" cable will be under closer control.

This paper will deal primarily with the "solid" cable, not only because it is the type used for all voltages up to 75 kv., but because the development field ahead of us is still extensive. It is a singular fact that this cable, the only type used for thirty years or more, is still in the developmental stage, whereas the liquid oil filled cable, a very recent product, has yielded almost completely to scientific attack, the remaining work being more in the nature of economy, simplification of fixtures, methods of installation, etc. We do not mean to say that the theory and principles are entirely established, but the gaps yet to be filled appear within reach.

No one can say at present what the ultimate development of "solid" cable will be, nor how far its field will be encroached upon by oil filled cable. No doubt there will evolve a legitimate, economic field for each type, but where the dividing line in voltage and current rating will be, time only can tell.

In this respect it might be pointed out that the safe working temperature range of oil filled cable is considerably more than that of "solid" cable. There are so many factors involved that the limits of both types are yet to be scientifically set, but the trend of present cable knowledge indicates that "solid" cable of the higher voltage ratings should, if anything, have its present temperature limits decreased rather than increased.

Since we are making a distinction between the liquid oil filled and "solid" types of cable perhaps a brief description is in order.

### Liquid Oil Filled Cable

The liquid oil filled cable, as its name implies, is filled with a relatively thin oil that remains liquid at all temperatures met with. It is supplied with definite longitudinal channels through which this oil can flow more or less freely. A positive pressure is maintained in these channels at all times after oil filling of the cable is accomplished. During this vacuum and filling treatment all gas is removed and no voids or gas spaces initially exist.

Since pressure is maintained in the channels even during periods of maximum cooling, voids cannot exist for any appreciable length of time in the paper or elsewhere.

The final gas removal and oil filling treatment can be done in the factory on individual reels, or in the field on the line as a whole after splicing work is finished. In either case it is important that positive pressure be maintained at all times thereafter by expansible reservoir feed. In those cases where final filling treatment is given to the factory reels the problem obviously

*The true definition of the word "void" is, empty space free of all matter. In cable engineering the word is used to represent a gas pocket, since a true void cannot exist in cable insulation, due to the nature of the components. Throughout this paper the word void means a gas pocket in which the pressure might be anything from zero to the maximum met with.

remains of installing and splicing these reels without loss of oil pressure, entrance of air, or void formation.

Another very important reason for maintaining positive pressure is that it prevents sucking in of air or moisture at leaky lead pipes or other points of leakage, the cause of many failures in the past in cable of the "solid" type.

### "Solid" Filled Cable

There are all varieties and kinds of "solid" filled, paper insulated cable, the characteristics of the filling compound and the degree of impregnation having much to do with their classification. Some are filled with a compound that is a solid at temperatures below 50 deg. cent., others with a compound that is so thick and viscous at the lower temperatures that for all practical purposes it should be classified as a solid, still others having filling compounds remaining a fair liquid even at the lower temperatures. They all, however, have certain points in common.

The compound must have sufficient viscosity and the cable cross section sufficient compactness to prevent "bleeding" or displacement of compound by air during manufacturing process and when the ends are exposed for sweating connectors, jointing work, etc. Also, migration of compound and piling up of pressure at the bottom of slopes must be avoided.

These requirements are the cause of all our difficulties and troubles. They place serious limitations on the cable and are the main points of distinction between the "solid" and oil filled types.

The compound is so viscous and the resistance to flow radially so great that it is impossible to maintain positive pressure at all points of the cross section. It is also impossible to prevent voids. Void formation in some form or other must be accepted as inherent in paper insulated "solid" cable and it should be designed, handled, and operated with this limitation in mind.

It can be seen from the above definitions that fundamentally the difference between the liquid oil filled cable and the "solid" cable is one of degree. It is simply a physical question of the viscosity of the filling compound and the resistance offered to flow both longitudinally and radially.

The fact that void formation and, frequently, pressures below atmospheric exist in "solid" cable explains many disconcerting failures in the past with cables carefully made of materials having highly desirable electrical characteristics such as low dielectric loss, high dielectric strength, high insulation resistivity, etc. Unfortunately, the physical characteristics and behavior were not so well understood and the problem we have today with the "solid" cable is almost entirely a physical one.

Failures of the type just referred to can be placed in two classes. First, those due to excessive void formation and internal ionization, without external help of any sort. We shall deal with these later in more detail.

The other class of failures is primarily due to entrance of air or moisture through accidental leakage. These leaks might be caused by pin holes and flaws in the lead sheath, formed during manufacture, or by punctures, gouges, or abrasions from foreign matter and sharp projections in the ducts. Electrolysis, chemical corrosion, and porous lead wipes have also furnished a source of leakage.

Whatever the cause of leakage, the results are usually the same. During periods of cooling, partial vacuum is formed in the cable. Air, and sometimes moisture, is forced in driving the compound back on each side of the leak, causing ionization to start, oxidizing the compound, forming additional water and acids, and increasing the dielectric loss. Ionization, in turn, attacks the filling compound and paper, generating gas and additional water and leaving a wax deposit, further enlarging and concentrating gas pockets at this spot.* This cumulative effect continues until eventual failure occurs. In the meantime, as the compound is pushed back, disintegration has spread several feet on each side of the point of leakage, depending upon how long this action has gone on before final failure. When the cable is opened up the part affected has a very dry appearance even though initially it might have been well impregnated. Wax deposit is usually prominent throughout.

In final appearance the two classes of failures can be identical under certain conditions, and since the short circuit arc usually destroys any direct evidence of leakage it is very difficult to distinguish clearly between them. In fact, the mechanism of deterioration and final failure is, doubtless, the same in both cases, the only difference being that the entrance of air (and moisture) initiates and accelerates the action in the second case.

Now, these two classes of failures have been gone into for the purpose of pointing out that it is possible greatly to reduce failures of the first class. The remaining part of this paper will deal with this problem.

Failures of the second class, however, can only be reduced, as far as we can see, in one of two ways,—either by eliminating all possible sources of leakage or by maintaining positive pressure inside the cable, so that air and moisture cannot enter and the leaks can be discovered and repaired before failure occurs. Maintenance of positive pressure at all points of a "solid" cable is apparently a physical impossibility. Perhaps someone will find a practical way of doing this some day. In the meantime absolute tightness of the outer cover is the only remedy.

In this respect, we wish to point out that with the modern, non-solidifying filling compounds tightness of the outer covering is of even more importance. Pressures below atmospheric tend to form when the

*This action will be referred to in detail later.

cable is cool and the liquid filling compound offers no obstacle to the entrance of air and moisture at leaks unless there happens to be positive pressure at these points.

### Petrolatum Filled Cables

Previous to 1919 practically all American manufacturers used a mineral oil or compound containing various percentages of rosin or rosin oil. These mixtures were quite "tacky" and adhesive, staying in place well and more or less preventing migration and "bleeding." It was not appreciated at the time but the most valuable result of this property was that the compound clung to the walls of the cavities in the form of a good thick film, materially reducing the size of the larger voids where ionization trouble always begins and causing a more even distribution of smaller voids throughout the volume.

Most of these mixtures would be considered too stiff and viscous for use today, especially at the lower temperatures, but in other respects had all of the desired physical characteristics. Electrically, they left much to be desired. The dielectric strength was low and the dielectric loss so high and of such variable quality that frequently the cable would not meet the severe American conditions of duct operation and heavy loading, failures due to accumulative heating occurring.

Pressure was brought to bear upon the manufacturers to reduce dielectric loss. This was undertaken without due regard to void formation and ionization, factors then but little understood and considered of no great importance.

The result was that petrolatum was resorted to as a filling medium, either in pure form or with small amounts of rosin or rosin oil. Dielectric loss was thereby effectively reduced but ionization deterioration was accentuated. It was only after several years of field experience and confirmatory laboratory studies that a clear picture of the situation was formed. During this time the evidence was conflicting. The operating record of some systems was quite satisfactory; others showed an increase in failures, depending upon operating voltage, loading, etc. No yardstick was available for measuring results.

About three years ago sufficient evidence had accumulated upon which reliable conclusions could be based. It was then quite clear that high-voltage petrolatum filled cable was inherently more susceptible to ionization trouble than cables filled with rosin mixtures.

This, in itself, is not an alarming statement even though a large amount of petrolatum cable is in service today. Liquid oil reservoir feed at the joints has in part overcome ionization trouble. On some of the higher voltage systems very satisfactory improvements have been obtained in this way. The operating records are comparable with, and in some cases better than older records, and would not indicate in any way a general widespread collapse of petrolatum cable.

At the time the above conclusions were reached it was felt that the situation did not warrant going back to the use of high loss rosin mixture with the limitation in loading and maximum voltage rating that it sets. Experience had shown that both types of compound had undesirable features and progress demanded that better compounds be developed. These new compounds will be described later, but first we wish to discuss the characteristics of petrolatum.

Chemically and electrically petrolatum is all that could be asked for. It is quite stable, has low dielectric loss, etc., but physically it leaves much to be desired. In the first place, viscosity increases rapidly as the compound cools, the solidifying point being reached between 40 deg. and 50 deg. cent. The viscosity is so high below 50 deg. cent. that the resistance to flow both longitudinally and radially in a cable rises enormously and it is almost impossible to fill up voids that form below this temperature during factory impregnation.

The transition from liquid to solid state is, unfortunately, within the operating temperature range and the compound is constantly subjected to this transition as the temperature varies.

Petrolatum is a typical grease with practically no "tackiness" or film adhesiveness. It will not cling to the walls of the larger cavities during contraction. The result is that during the cooling period these larger cavities are drained dry by capillary attraction and remain in this state, surrounded by a rigid wall of stiff compound and fiber, until the temperature again increases.

It is during this period of large, low pressure voids that ionization trouble often begins in service. The mechanism of deterioration where the conditions are such as to allow sustained ionization to exist is more or less the same regardless of the type of compound used. Apparently, petrolatum does not deteriorate more rapidly than other compounds because of any peculiar chemical or electrical property but simply because its physical characteristics allow relatively large low pressure voids to exist for long periods of time, giving wax formation and gas evolution a good chance to get started and spread.

### Improved Type of Compound

An ideal filling compound for "solid" cable would have the following characteristics, as far as our knowledge and experience go:

(a) Very low coefficient of expansion.

(b) Low viscosity at impregnating temperature, increasing to a value just sufficiently high at normal temperatures to prevent "bleeding" during the leading process and during installation in the field, and to prevent migration and building up of excessive pressure at points of low elevation. Solidifying point below the minimum temperature met with in service.

(c) Sticky and adhesive, forming a good strong film.

(d) Low dielectric loss, high insulation resistance, and low corresponding temperature coefficients.

(e) High dielectric strength.

(f) Chemically stable and free of all absorbed gases and other impurities.

In regard to property (a) all liquid compounds have coefficients of expansion within a relatively narrow range and, unfortunately, these are high compared with the coefficients of cellulose fiber, lead, and copper. As far as we know at present little can be gained in this direction.

An all-mineral oil compound, designated as No. 219, was developed about three years ago, having the desirable properties outlined above, with the exception of (a).

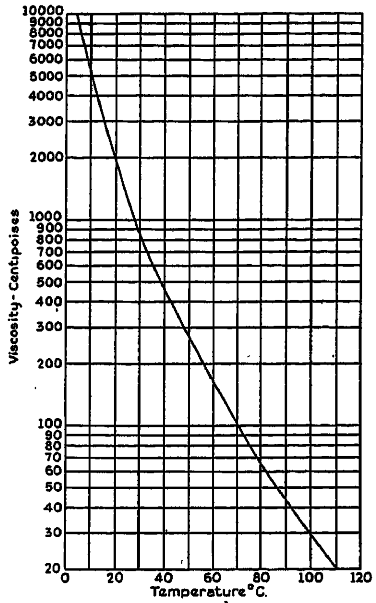


FIG. 1—VISCOSITY OF No. 219 COMPOUNDS

The viscosity-temperature curve is shown in Fig. 1. It will be noted that the compound remains a liquid even at the lower temperatures. The property of adhesiveness or ability to form a thick film cannot be

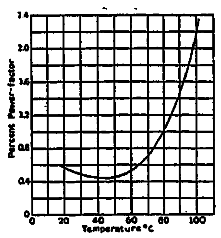


FIG. 2—DIELECTRIC POWER-FACTOR TEMPERATURE OF TYPICAL CABLE AT NORMAL VOLTAGE AT 60 CYCLES

shown clearly with the type of test data we have available. Comparative tests by dipping strips of treated paper in the different compounds and then allowing them to drain at various temperatures show No. 219 and rosin mixtures to have about equal film adhesiveness, while petrolatum in liquid form leaves no film at all.

Fig. 2 shows a dielectric power factor—temperature curve of a typical cable insulated with wood pulp paper and impregnated with No. 219 compound. The dielectric loss and temperature coefficient are both quite low. A typical resistivity-temperature curve of this compound is shown in Fig. 3. In regard to dielectric strength No. 219 compound has the usual high values, 30 to 35 kv. in the standard 0.1 test gap, that all liquid oils free from impurities show. This test is useful

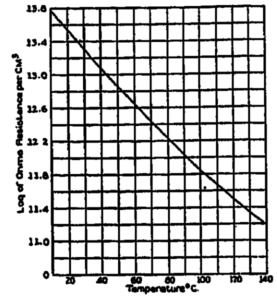


FIG. 3—RESISTIVITY—TEMPERATURE

mainly in checking freedom from suspended impurities but is not otherwise a very useful guide.

In Table I are some typical dielectric strength results on unused cable insulated with wood pulp paper

TABLE I
DIELECTRIC STRENGTH
Typical Cable

Single conductor, 750,000 cir. mils, 0.750 in. paper.

Test (a), 25 deg. cent., 60 cycles:

303 kv. for 5 min., 348 kv. for 5 min., failed at 402 kv. in average time of 2 min.

Test (b), 25 deg. cent., 60 cycles:

214 kv. for 5 min., 235 kv., 259 kv., and 285 kv. each for 1 min., failed at 300 kv. in average time of 1 hr., 20 min.

Standard Short Time Test on Cold Bent Samples

25 deg. cent., 60 cycles

| Type and size | In. thick | Average kv. break |
|---|---|---|
| 3 cond. shielded 350,000 cir. mils round.......... | 0.312 in. | 164. |
| 1 cond., 300,000 cir. mils... | 0.190 in. | 114. |
| 1 cond., 250,000 cir. mils... | 0.220 in. | 134. |
| 1 cond., 200,000 cir. mils... | 0.520 in. | 252. |

and impregnated with No. 219 compound. The increased dielectric strength over older types of cable is not due to the compound alone. Wood pulp paper uniformly and compactly applied is also a contributing factor. The advantages of wood pulp paper are described elsewhere.

## IMPREGNATION

Compound and paper of the right properties and quality are the primary requisites for a high-voltage cable of high initial dielectric strength. There are other factors, however, of equal importance as regards quality of finished cable and durable service which we wish to discuss briefly in their proper order. Of these, impregnation should come first.

A great deal of improvement in impregnation has been accomplished in recent years. In addition to proper compound and paper, the carefully worked out process has added much. Exact control of temperature, more thorough removal of residual gas and moisture from the cable and compound by special process, time and method of soaking, prevention of bleeding during the leading process, etc., have all contributed towards more perfect and uniform impregnation.

To give an idea of this improvement, power factor—voltage curves for a typical unused cable are shown in Fig. 4. There is only slight indication of ionization over a wide range of voltage stress.

It can be safely stated that it is now possible to produce cable which, for all practical purposes, can be regarded as having perfect impregnation, initially, as shipped from the factory. Actually, there is a small amount of gas and void formation present but compared with the amount formed later during installation and service and also compared with the degree of impregnation obtained a few years ago with petrolatum cable, present impregnation can be considered almost perfect, provided, of course, that no incidental mistakes are made in routine manufacture.

Present manufacturing process and methods have greatly decreased such mistakes. This, together with careful inspection and acceptance tests, has reduced the probability of trouble from this source to a very small amount.

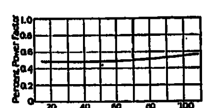


FIG. 4—IONIZATION CURVE OF TYPICAL CABLE AT ROOM TEMPERATURE—60 CYCLES

It is impossible, by any method we know of, to determine directly this small initial total of voids or occluded gas* volume in "solid" cable of the present type, as shipped from the factory. By indirect methods, as described by Emanueli[1, 2] and from theoretical considerations we estimate the total initial void volume at atmospheric pressure, 20 deg. cent. as between 0.10 and 0.20 per cent of the volume occupied by filling compound and fiber.

It is our opinion that this small total percentage of void volume consists of two components, one being the gas normally sealed in the cellular structure of the fiber and the other representing the residual gas from impregnation and leading process. The last mentioned void volume is a variable quantity, depending upon the temperature range and handling to which the cable is subjected before shipment. It can be more definitely controlled and reduced by attaching an expansible oil reservoir to each reel of cable after leading and leaving this attached during shipment, as is done with the oil filled cable. We are not convinced that sufficient benefit is derived thereby to warrant the extra expense, in view of the greater void formation during installation and service.

This question is worthy of further study; also, whether reservoir feed during installation and jointing is warranted, this being a practise learned from oil filled cable. If the cable cannot be held within a comparatively narrow temperature range during shipment and installation, reservoirs would undoubtedly be of real service.

The question of reservoir feed during operation is, to our minds, already answered by wide experience. It is unquestionably of very material benefit, although some doubts regarding its use are not yet entirely cleared up. We wish to deal with reservoir feed in more detail later.

It has been suggested frequently by engineers that perhaps perfect impregnation is not desirable—that it may be better purposely to leave a certain amount of gas in the cable to act as a cushion and assist in maintaining more uniform pressure. This reasoning is right in principle up to a certain point. The gas must, first of all, be kept out of the field of stress and be confined to electrostatically shielded points such as the space between conductor strands and the filler spaces of shielded three-conductor cable.

The real difficulty lies in initially confining the gas to these spaces. We have learned from past experience that entrapped gas is not under control. It cannot be confined to desired points and is just as liable to concentrate at random in sections of the cable length, causing the troublesome so-called "dry spots." We doubt that any manufacturer would court trouble by deliberately leaving gas in the cable. Perfect impregnation must always be the goal aimed at.

Another objection to leaving free gas in the cable is that it will not be permanently maintained in this state unless the exposed compound is gas saturated. Now, gas saturated compound makes it difficult to close up additional voids as they are formed in service. It is much better to have the compound originally gas free so that voids which form will be more quickly reduced by reservoir feed from the joints before ionization damage has a chance to get started.

The principle of a more elastic "solid" cable maintained under pressure is correct but unless practical methods can be devised for definitely segregating the elastic medium (gas) from the compound,* or developing a more elastic sheath,* we should rely upon other ways of accomplishing these results.

*The word "void" as used in this paper represents volumetric dimension and "occluded gas" represents the amount of free gas by weight that fills the void. A high pressure void would contain much more gas than a low pressure one of the same dimension.

*Study is being given to this and several designs have been considered. Economy has so far proved to be one of the chief obstacles.

### Formation of Voids in Service

To assist in obtaining some idea of the formation of voids in service, we will take a specific example. The cable considered (Cable No. 1) will be a 66-kv. single-conductor cable of the following specifications:

Cable No. 1

Cond.—750,000 cir. mils stranded (no core).
Insulation—24/32 in. impregnated paper.
Sheath—9/64 in. lead.
Total area of cond.—0.785 sq. in.
Area of copper—0.588 sq. in.
Area of space between strands—0.197 sq. in.
Total area insulation—4.125 sq. in.
Area of paper fiber—1.650 sq. in.
Area of space between fibers—2.475 sq. in.
Area of lead sheath—1.160 sq. in.

The conditions of operation will be:

(a) Six cables (No. 1) in duct bank, installed at temperature of 20 deg. cent.

(b) Max. load, copper temp. 65 deg. cent., sheath 50 deg. cent. with ambient earth temp. 20 deg. cent.

(c) Load dropped from all cables, which eventually cool down to ambient earth temperature.

As a first step and to simplify the problem we will assume that:

### First Step Assumptions

(a) The cable as installed, before load is applied, is perfectly impregnated with no voids present.

(b) Lead sheath has no elasticity whatever.

With ambient earth temperature still at 20 deg. cent., full load will be applied and the amount of expansion calculated. The volumetric expansion coefficients of the various components are taken as:

| | |
|---|---|
| Lead | —0.000085 |
| Copper | —0.000050 |
| Compound | —0.0007 |
| Fiber | —0.00009 |

The volumetric expansion, considering only the cross section is:

| | |
|---|---|
| Copper | 0.00132 sq. in. |
| Oil in strand space | 0.00620 sq. in. |
| Paper fiber | 0.00535 sq. in. |
| Oil in paper | 0.06230 sq. in. |
| Total | 0.07517 sq. in. |
| Exp. of sheath | 0.01250 sq. in. |
| Stretching of sheath | 0.06267 sq. in. |

If load is dropped and the cable allowed to cool back to 20 deg. cent. a total void formation of 0.0627 sq. in. will exist, since it is assumed that the sheath has no elasticity. This is equivalent to 1.5 per cent of the space normally occupied by compound and fiber.

When load is applied the first time the pressure, of course, increases rapidly until the sheath begins to stretch. It is difficult to measure this pressure accurately for it is a variable function of time and temperature. The maximum pressure reached is in the order of 50 to 60 lb. per sq. in. directly under the lead sheath. It is very sensitive to even slight temperature fluctuations but if the full load temperature could be held steady for a sufficient length of time the pressure would drop gradually and become constant at about 15 to 20 lb. which is, as far as we can determine, the "set point" for the sheath under the conditions as given.

When load is dropped the first time pressure is lost rapidly and a vacuum forms. Assuming no leaks in the system and an absolutely gas-free, non-volatile compound, the voids would be at zero pressure, absolute, and remain so. This is never the case, however, gas being set free and, in consequence, the pressure builds up. The final recovery of pressure would depend upon the degree of gas saturation, stability of compound, and other factors. We regret that no qualitative test data are available with different degrees of gas saturation. It is an example of the large amount of uncompleted work relating to "solid" cable.

Tests show that the recovered pressure would generally fall short of atmospheric at all points and would be very low unless, as we suspect, the following action occurs.

When voids are at very low pressures and the stress above a critical value, ionization occurs, generating or freeing gas from the compound and increasing the pressure until the discharge is automatically extinguished. If the voids are sufficiently small ionization is extinguished in a relatively short while, the action being quite feeble. This will be dealt with in more detail later. We feel that in view of the very low vapor tension of the compound and from tests and other considerations that some such automatic partial recovery of pressure in the voids must take place. If this theory is correct a "solid" cable cannot remain entirely gas free for a very long while in service but probably never becomes entirely gas saturated unless "sustained" ionization of an entirely different and more severe nature takes place. This type of ionization causes the deterioration so frequently found. For this to take place the voids must be sufficiently large and the stress sufficiently high to sustain gas discharge even at relatively high pressure. This also will be dealt with later.

The ability to regain or retain gas pressure involves desirable and undesirable features. To illustrate this let us assume that Cable No. 1, which has now been through one heat cycle and has a total void volume of 0.0627 sq. in. at 20 deg. cent. is filled with a gas saturated, unstable compound. We will assume further that this allows the pressure to return to atmospheric, the cable still being at 20 deg. cent.

If full load is then suddenly applied the cable will heat up more rapidly than the gas can be re-absorbed. Assuming for the moment no re-absorption and a "short-time" elastic limit of 50 lb. per sq. in. for the

sheath the voids would be compressed to 23 per cent of their original volume by the time 50 lb. pressure is reached. The remaining expansion of compound would have to be taken care of by an additional permanent stretching of sheath equivalent to 23 per cent of the original amount, giving a total void volume for two complete heat cycles of 0.077 sq. in.

As additional heat cycles are gone through it is easy to show that progressively less stretching of sheath occurs until a final balance is reached at about the sixth cycle with a total void volume of 0.081 sq. in., equivalent to 1.9 per cent of the space normally occupied by compound and fiber.

This calculation ignores two important factors, the rate of re-absorption of gas by compound and the rate of "viscous flow" of lead sheath between the "short time" elastic limit of 50 lb. and the "set point" of 15 to 20 lb. We have no final data that would allow the inclusion of these two factors but since they must surely cancel each other to some extent the calculation is probably not too greatly in error. Possibly, with a freely "gassing" compound the sheath would be stretched more than this before the final balance is reached.

Assuming no re-absorption at all the sheath would be stretched an additional amount (1.6 per cent) while the pressure is dropping from 50 to 20 lb., giving now a total void volume of 3.5 per cent. This is the absolute maximum that could be reached. We do not believe it would be reached, for some re-absorption of gas must occur during the long period of "viscous flow" of the sheath.

Referring back to the calculation for the first heat cycle the sheath was stretched an amount equal to 1.5 per cent of the total insulation volume. This is equivalent to a radial sheath movement of only 7.0 mils and it was assumed that none of this was recovered after cooling. In the first place the cable is not a perfect cylinder and when the pressure inside drops below atmospheric there is a certain amount of response from the sheath tending to flatten out the distorted areas. This causes some closing of voids. The action is a complicated function of time, temperature, pressure, and distortion of sheath. Our work merely indicates roughly that it exists. We cannot yet assign numerical values.

Considering re-absorption and possible sheath elasticity, we feel justified in taking an average between the limits of 1.9 per cent and 3.5 per cent, giving a maximum possible void volume of 3.0 per cent for a gas saturated cable operating under the conditions given without reservoir feed.

Likewise, we feel justified in reducing the void volume for a gas-free cable from 1.5 per cent to 1.3 per cent to compensate slightly for sheath elasticity.

These observations and calculations are very illuminating when viewed in perspective. They point to several interesting conclusions which will be given in order.

(a) If a cable without reservoir feed, as initially installed, has a degree of impregnation equal to or better than 1.3 per cent total void volume (assuming these voids uniformly distributed along the cable length) its final void volume after "settling down" in service will be between 1.3 per cent and 3 per cent at 20 deg. cent., depending upon the degree of gas saturation. This is based on the assumption that the copper temperature never exceeds 65 deg. cent.

(b) Factory impregnation would have to be of very high order if 1.3 per cent is not to be exceeded in installed cable. Initial impregnation as near perfect as possible is, therefore, very important.

(c) The pressure in the voids at 20 deg. cent. after the cable has settled down might, theoretically, be anything from zero (absolute) to atmospheric, depending upon the degree of gas saturation and the action of temporary ionization.

(d) All other things being equal, any benefit derived from higher gas pressure is counteracted more or less by the voids being larger.

In addition to (d), another and more practical consideration rules out the saturated compound. It is impossible, by any methods we know, to obtain reliably and uniformly the high degree of initial impregnation required.

## Ionization in Service

We have shown in an elementary and approximate way what goes on inside a cable during void formation in service. We have also given a measure of the total amount of void formation to be expected. The question naturally arises as to what relation this might have to the possibility of ionization damage in service.

In approaching this subject we wish to point out that our assumptions are based on a representative, compact cable cross section and a "stringy," adhesive compound that will cause a fairly uniform distribution of voids. If voids concentrate in the cable causing "dry spots," or if due to loose structure relatively large voids form at points of high electrical stress these assumptions do not hold.

The best method of determining the relation between total void volume and ionization can be found in Emanueli's published work.[1] We take the privilege of extrapolating and tabulating this in the form given below:

TABLE II

Impregnated paper insulated cable.
P. F. DIFF. = Difference in power factor from 20 volts per mil to 100 volts per mil, 60 cycles.
% VOID = Total void volume in per cent of insulation volume.

| Atmospheric pressure, 20 deg. cent. | | | |
|---|---|---|---|
| P. F. diff. | % void | P. F. diff. | % void |
| 0.25 | 0.4 | 2.0 | 3.2 |
| 0.50 | 0.8 | 2.5 | 4.0 |
| 0.75 | 1.2 | 3.0 | 4.8 |
| 1.0 | 1.6 | 4.0 | 6.4 |
| 1.5 | 2.4 | 5.0 | 8.0 |

Emanueli's tests were made on a single-conductor oil filled cable with small void volume and of the same general cross section as Cable No. 1, except that it had a hollow core. Presumably, the distribution of voids would be practically the same in Cable No. 1 and his results should hold fairly well in spite of rather broad extrapolation. At any rate, theoretical calculations check Table II surprisingly well. Voids have a tendency to collect (radially) in the largest available spaces. In this case, these spaces are the strand space and that directly under the lead sheath. Doubtless a good proportion of the void volume would be found here and since the strand space is electrostatically shielded and the space directly under the lead sheath is at a point of relatively low stress they do not contribute very materially to the measured ionization loss.

Table No. II would not hold so closely for ordinary three-conductor cable nor for shielded type three-conductor cable because of the different distribution of voids and electrical stress. Similar data are needed for these two types. A fair part of the total void volume of a three-conductor cable seems to concentrate in the filler spaces, forming relatively large voids. In an ordinary three-conductor cable these points are subjected to relatively high electrical stress. Therefore, we feel that test results would show a smaller total void volume than that in Table II for the same ionization loss. In other words, the loose structure is less favorable from an ionization standpoint. The shielded type three-conductor cable, on the other hand, should, if anything, show a larger total void volume than Table II because the filler space is electrostatically shielded. Shielded cable is a great improvement over ordinary three-conductor cable in this respect.

Following this same idea a shield could be placed directly under the sheath of single-conductor cable. It has been tried but its superiority has not yet been demonstrated. We believe it has possibilities in those cases where the sheath is stretched in service.

Standard specifications for single-conductor cable call for a maximum difference in power factor of 0.75 on factory test samples, which from Table II is equivalent to 1.2 per cent void volume. A great deal more experience and study are needed before actual safe limits of power factor difference can be definitely determined. There is no reason to believe, judging from operating experience, that the above rule is not conservative for properly designed cable. At any rate, it is interesting to note that the void volume of 1.6 per cent which this rule allows in the factory falls below the range of the limits, 1.3 per cent to 3.0 per cent, calculated for installed cable without reservoir feed. The data in Table II are based on short samples with voids at atmospheric pressure, whereas the installed 20 deg. cent. void pressure is probably higher or lower, depending upon conditions.

On the foregoing basis we might assume that the cable should operate safely without reservoir feed provided migration does not occur and the temperature cycle never exceeds a range from 20 deg. cent. to 65 deg. cent. Lower temperatures must be met in the winter, unfortunately.

## Low Temperature Operation

Carrying the foregoing calculations a little further by assuming an extreme minimum no load, winter temperature of 0 deg. cent. it is easy to show that the void volume increases to 2.1 per cent for the unsaturated cable and about 4.0 per cent for the saturated cable. Furthermore, the pressure in these voids drops very materially. It is during this period of low temperature operation that accumulative deterioration from ionization gets a good start. Whether ionization damage would occur in either of the two cables considered we cannot say. One thing is certain, however,—if ionization damage occurs at all it will get its start during this period of low temperature, low pressure operation at no load. It is the danger period for high tension cable and every precaution should be taken to guard against it.

Possible safeguards are listed below:

(a) Take the line out of service during periods of no load in winter.

(b) Maintain a higher temperature in the conduit run by always having some cables loaded.

(c) Reduce the maximum allowable copper temperature range both for summer and winter loading.

(d) Use a compound that remains a fair liquid even at the lowest temperature met with.

## Reservoir Feed for "Solid" Cable

Reservoir feed on high-tension cable lines is growing rapidly in this country although certain objections have been raised from time to time. Aside from cost the chief objections have been:

(a) The reservoir oil contaminates the cable filling compound increasing dielectric loss and lowering dielectric strength.

(b) Slow percolation of oil into the cable might cause progressive stretching of sheath and ultimate rupture due to rapid expansion when load is applied.

As far as (a) is concerned we feel sure that, provided the proper grade of oil is used, the contamination that is sometimes found is due to the occlusion or entrance of moisture, oxygen, or both into the reservoir, joint, or cable. A thin, high grade, low freezing point mineral oil should be used. The viscosity of a typical oil of this kind (Oil No. 2) is given in Fig. 6. Wide experience has been had with this oil in intimate contact with many types of insulating materials, including rosin, petrolatum, and the modern cylinder oil cable compounds. In no case has deterioration been found under proper conditions of use.

The harmful results of moisture are apparent. Equivalent results from the presence of oxygen are usually ignored. The cable ends during jointing are exposed to air for a long time and no attempt is made to remove

the absorbed oxygen afterwards. Often, also, the reservoir oil is maintained in direct contact with air, acting as an agent for transmitting an unlimited supply of oxygen into the cable. And yet oxidation of oil under heat with resulting formation of water and acids is widely recognized. It cannot be said how serious this action might eventually be in service but it surely increases dielectric loss to some extent. Riley's paper[3] and the discussion which followed deals with this subject in a most interesting way.

In regard to (b) reservoir feed has now been generally used for several years by some of the operating companies. During this time there has been no noticeable indication of a general swelling of cable sheaths. On the contrary, only a few isolated cases of ruptured sheath have occurred, to our knowledge, and in every one of these cases it was clearly accounted for by excessive pressure at the bottom of vertical risers, or by defective or badly damaged sheath.

Theoretically, it is possible under certain conditions for progressive swelling of sheath to occur within certain limits. Accepting, for the moment, that it does actually occur, no greater proof of the value of reservoir feed is needed for it would prove that the oil surmounted all obstacles and completely penetrated the cable, increasing the degree of impregnation to a high order. It is more than doubtful that such perfect impregnation can be obtained in this way unless the cable is initially well impregnated.

It has already been shown that impregnation must be better than 1.3 per cent to 3.0 per cent (depending upon the occluded gas pressure) before stretching of sheath occurs when load is applied. This is based on the assumption of no reservoir feed. A better degree of impregnation is allowable if reservoir feed is used. This will be demonstrated.

Assume, for purposes of discussion, that oil reservoir feed so thoroughly saturates cable that at some time in the future bursting of cable sheaths becomes pronounced. The remedy in this case is simple and consists of shutting off the reservoirs from the joints while the cable is partly loaded. The reservoirs can be permanently removed and used elsewhere or they can be left in place and used during certain periods for maintaining good impregnation. The line should surely be in better condition than if reservoir feed had not been used at all.

When it is considered that many cable lines in service today have an impregnation in the order of 5.0 to 8.0 per cent and doubtless there are others of even poorer impregnation, it is apparent that reservoir feed has a long way to go on these lines before any question of bursting sheaths will arise.

This statement is based on observation, records of oil absorption where reservoir feed is used, and examination and tests of removed samples. Responsibility for this condition should be shared by both the manufacturers and operators.

Poor factory impregnation and wrong materials have contributed part, rough handling during installation, excessive loading, and short circuit heating have contributed the rest. We know of cases (much rarer today) where short circuit temperatures as high as 150 deg. cent. have occurred. Even a single short circuit of this character would by stretching the sheath radially cause a void volume of 4 per cent or more.

It is not, however, radial stretching of sheath that causes the worst cases of void formation. Longitudinal stretching, forming the characteristic annular, wavy ridges so often found on the sheaths of old cable, is more to blame. This is caused by rough handling, excessive loading cycles, and short circuit heating.

To illustrate this, suppose both the copper and lead of a single conductor cable were at 20 deg. cent., no load. Suppose, further, that the copper heated suddenly to 100 deg. cent. during short circuit while the lead remained practically at 20 deg. cent. Ignoring the effect of stranding the copper would expand 16 in. per 1000 ft. stretching the sheath this amount. Upon cooling back to 20 deg. cent. this 16 in. of excess sheath would form itself into characteristic annular ridges, increasing the total void volume very materially. Migration and re-arrangement of voids would result in the "dry" cable so often found.

Returning to reservoir feed, there is every reason to believe that bursting sheaths in general will never be encountered. It is more probable that a final balance will be reached after the cable line "settles down" to a certain average degree of impregnation after it has absorbed all the oil that it can and that, thereafter, seasonal cycles will be gone through (upon which daily cycles will be more or less imposed) in which the cable will return to the reservoirs the same amount of oil as received. Records kept by the operating companies show a decided trend in this direction and theoretical considerations point the same way.

Since solid cable offers an extremely high resistance to oil flow, reservoirs should be connected along the cable length as frequently as possible; that is, at every joint and the shorter the distance between joints the better from this standpoint.

In the past it has been customary to use very small expansible reservoirs, *re-filling these as the cable absorbed oil.* The modern cable is initially so well impregnated with liquid compound that this practise would be questionable. This type of cable extrudes compound under load in a comparatively short time. If the amount extruded exceeded the available reservoir capacity an excessive pressure would result, bursting the reservoir or expanding the sheath. If this "pumping in" of oil continued long enough the sheath would eventually burst. *Reservoirs of proper capacity and adjustment are most important.*

Reservoir feed is of very material help in preventing so-called migration or formation of "dry spots" in the cable length.

Aside from the benefit to the cable proper liquid oil filling has proved the real solution of the high-tension cable joint problem. Such joints are more uniform in quality and much more reliable than joints filled with solid or viscous compound, particularly when a small positive oil pressure is maintained. The majority of leaks experienced in underground service occurs at or adjacent to the joint, due to porous lead wipes, screw fittings, etc. Positive oil pressure furnishes an opportunity to discover and repair these leaks before damage is done.

### ADDITIONAL CONCLUSIONS REGARDING RESERVOIR FEED

Summed up briefly, some of the additional conclusions we have reached regarding reservoir feed after several years study and observations are:

The pressure at different points in the cable cross section varies from 0 to 125 lb. (absolute) or more during temperature cycles. The duration of these extremes of pressure can be controlled and reduced, but there is no known way of materially reducing the range throughout the paper thickness of solid cables. The pressure range is smallest at those points where the largest voids tend to collect. If the cable has an excess of gas, these larger voids can maintain some pressure, the range depending upon the size of the individual voids and the amount of gas present. The more gas there is in the cable the larger the voids in these positions tend to be, the type of compound used also being a governing factor. Ionization deterioration (when it occurs) is always found in these large voids; therefore reduction in their size and incidentally increase in their pressure, are highly desirable. The larger voids form in those positions that later, after the line settles down, are the main channels or arteries for reservoir feed. If oil penetrates these channels, the voids are reduced in size and ionization is less likely to occur. Excessive stretching of sheath seems improbable simply because a balance is finally reached after some sheath stretching of well impregnated, liquid compound cable and because reservoir feed cannot impart or maintain a sufficiently high degree of impregnation in cables filled with solidifying compound. In those cables having solidifying filling compound, gradual percolation of thin oil from the reservoirs "softens up" the compound, making it possible to reduce voids more easily, although never to the extent reached with liquid compounds.

### RESERVOIR FEED AS A FUNCTION OF TIME

A question that naturally arises regarding reservoir feed is concerned with the penetration of the thin oil into the cable and the transmission of oil pressure as related to distance and time. A final answer to this complicated problem with substantiating data is not available but progress has been made and some light thrown on the subject.

The behavior of "solid" cable in this respect varies widely and depends upon the initial degree of impregnation, the physical characteristics of the filling compound, the compactness of the physical structure, and the character of the temperature fluctuations.

As the cable cools transmission of oil and pressure from the reservoirs are accomplished by a combination of two actions. First, an infiltration or percolation of thin oil through the channels that are formed in the larger spaces. The main channels or arteries are in a general longitudinal direction in the strand space, directly under the sheath and in the filler spaces of three-conductor cable. The second action is a physical pushing back of the heavier compound, mostly in a radial direction, tending to compress the voids in the paper insulation and enlarge the main arteries. A maze of small capillary radial branches to the main arteries are sometimes formed in the paper, particularly in cable filled with petrolatum or other solidifying compound.

This action continues to penetrate the cable length as temperature cycles are gone through, searching out its own path and establishing and enlarging main channels of oil flow as it proceeds. Voids are reduced in this way partly by absorption of gas as the pressure builds up and partly by compression.

Finally a balance is reached after which there is a cyclic exchange of oil between reservoir and cable, depending upon the final degree of impregnation. The length of time required to reach this balance depends among other things upon the type of compound used, degree of initial impregnation, distance between reservoirs, cable contour, rate of sheath stretching, the difference in reservoir pressure and void pressure, rate of gas absorption and, finally, rate of gas generation and other effects of ionization. There are so many factors involved that no one can predict when this balance will be reached. Furthermore, the word "balance" is used in a relative sense. Change in any one of the many factors might upset this. Probably an exact 24-hr. cyclic exchange of oil will never actually exist but only an approximation of this.

Cable lines having reservoir feed are operating on long slopes of as much as 2 per cent grade without any apparent building up of head pressure, or appreciable transfer of oil from one reservoir to another. Where this might occur it can be easily taken care of by balanced pressure reservoir feed. Stop joints could be used to break up the head pressure, although this has so far proved unnecessary except in one or two cases where vertical risers existed. The balanced reservoir feed system will be described in a separate paper soon to be published.

The above theory regarding the penetration of oil into a cable is based on tests and examination of cable lengths removed from service. The additional theory of final balance and cyclic exchange of oil is not fully proved. The calculation of reservoir feed is very complicated. We will attempt to show the headway that has so far been made.

### Calculation of Reservoir Feed

As in the preceding calculations, a start will be made by assuming an initially perfectly impregnated 66-kv. single conductor cable (Cable No. 1). The conditions of operation will be the same as previously, 20 deg. cent. earth, six loaded cables, etc., except that the steady load per circuit of three cables will be 45,000 kv-a., giving a copper temperature of 52.5 deg. cent.

When load is dropped from all six cables they eventually cool down to earth temperature (20 deg. cent.). During this cooling voids are formed. The rate of void formation without reservoir feed in a constant temperature medium can be calculated theoretically with good accuracy. The mathematics involve Bessel functions with the final solution involving Heaviside formulas. Mr. K. M. Miller demonstrated the method of attack in discussing the paper on 132,000-volt cable.[1] Messrs. F. H. Buller and R. H. Park independently developed the same method and carried it to a final solution. This work will soon be published. In view of this and the limitations of space the mathematics will not be dealt with, only the calculated results being given.

The calculations are based on a representative conduit cooling curve, the cables reaching earth temperature practically 72 hr. after load is dropped. The results are given in Fig. 5, and as curve *a* was carried only to a time of 110 min. where cooling is far from complete, the behavior during the first hour of time is shown clearly. Beyond this time the cooling curve approaches the zero line in the form of a simple exponential, the complicated harmonics having disappeared. By the end of two days the rate of void formation is negligible. The total area bounded by this curve represents the total volume of voids (0.577 cu. in. per ft. or 1.1 per cent). This can also be calculated by the simpler method already described.

Curve *a* is practically independent of the type of filling compound used, provided the cable is initially well filled. It represents fairly well the rate of void formation under the given conditions for all types of impregnating compound.

If perfect oil feed is obtained curve *a* also represents the rate of oil feed but due to the flow resistance offered the volume of oil fed into the paper usually lags behind the volume of voids, the amount of lag depending upon the viscosity of the oil, the compactness of wrappings, and the pressure in the feed channels.

We are developing similar methods for theoretically calculating oil feed curves. The problem is very complicated and a solution must be based on approximately assumed laws of oil flow in capillary passages and rate of gas absorption. The degree of accuracy reached in curve *a* can never be expected for calculated oil feed curves, but such calculations will help us to interpret measured results more accurately.

An actual measured oil feed curve furnishes the best method of getting at this problem but here, again, we run into difficulties. It is not feasible to measure an oil feed curve on a cable containing a compound as viscous as No. 219. The rate of feed is too slow for accurate measurement and there are other inherent objections. Such curves can, however, be taken on short cable samples filled with less viscous oil. Curve *b* in Fig. 5 represents a measured curve of oil feed for a cable similar to No. 1 but filled with a thinner oil, designated as Oil No. 1, the viscosity of which is shown in Fig. 6. The curve is an allowable correction of measured results in still air to correspond with the same typical conduit cooling conditions selected for curve *a*. Atmospheric pressure was maintained on the oil in the conductor channels throughout the test by use of a hollow core. The sheath was slightly loosened during the treating process but it is improbable that atmospheric pressure was maintained at this point. Curve *b* represents the radial flow of oil into the insulation thickness at constant pressure. We will first consider radial feed and show later the effects of longitudinal feed. It should be understood that no impossible attempt

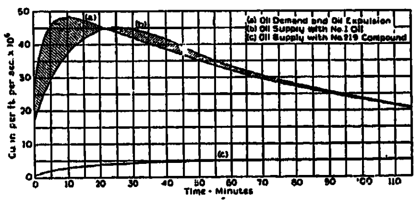



Fig. 5—Cable No. 1, Two Circuits at 45,000 Kv-a. Per Circuit

will be made to calculate oil feed for a cable filled with solidifying compound but only for cable filled with liquid compound of known viscosity.

Returning to Fig. 5, curve *a* will be called the oil demand curve and curve *b* the oil supply curve. Their close agreement even where such small quantities are involved, is not an accident,—it was done purposely. As a matter of fact, the first theoretical calculation of the demand curve might fall above or below the measured supply curve within narrow limits, as determined by the relative values of thermal resistivity, thermal capacity, and coefficients of expansion selected from the known range of such measured values for this type of impregnated paper. There is only one possible combination, however, that will "fit" the calculated demand curve to the measured supply curve and still take care of the known physical conditions, these conditions being that any voids which form during cooling must eventually be closed up, since no voltage stress that might generate gas was applied during the test and the gas saturation point was far lower than atmospheric pressure at 20 deg. cent.

Further work might account for the differences of curves *a* and *b* in some other way but we believe it is due to the supply lagging behind the demand during the first 22 min., thus forming voids. The wall of

compact paper insulation is so thick (24/32 in.) that the oil has a hard time penetrating it with such low pressure in the channels and such short periods of time. Supply curves measured at different pressures in the feed channels agree with this assumption. We are continuing this phase of the study and will publish the details later when more results are obtained with oils of different viscosities. In the meantime the tentative conclusion is offered that this difference is due to void formation.

The area between curves *a* and *b* up to 22 minutes of time represents the maximum volume of voids that are formed. From then on the supply is greater than the demand and the voids are gradually closed up. The area between the curves after they cross must equal the area before they cross.

From the behavior of curves *a* and *b* it appears very probable that at the point and moment each void begins to form there is an extremely low pressure. Gas is then drawn from the compound, filling the void and increasing the pressure, depending upon the amount of gas in solution or the vapor tension of the materials. When the voids begin to reduce in size the pressure increases and the gas is gradually re-absorbed. This would account for the relatively long and slowly decreasing "tail" of the area between the two curves. If this theory is correct the pressure in the voids should be practically atmospheric during the latter part of the "tail" period.

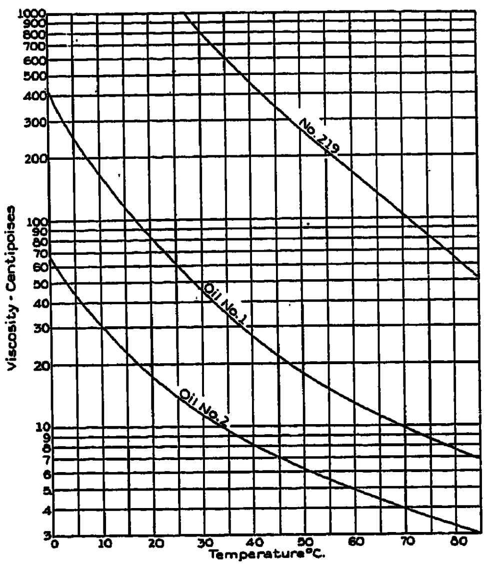



FIG. 6—VISCOSITY OF THREE DIFFERENT OILS

This interpretation of curves *a* and *b* gives us an empirical measure of the rate of gas re-absorption under the conditions that actually exist in the cable.

A study of Fig. 5 is quite interesting. It shows that with a sufficiently thin oil and a compact cross section the voids which form are relatively microscopic in size and are quickly closed up. With a still thinner oil or a sufficiently high pressure practically no voids would form at all, the supply exactly following the demand.

Curves *a* and *b* in Fig. 5 can be used for estimating an oil supply curve for No. 219 oil, provided the relative viscosities and flow resistivities are known. The relation between flow resistivity and viscosity for the type of oil and paper we are considering is shown in Fig. 7. The measurements were made on stacks of carefully impregnated wood pulp paper sheets having parallel slits and arranged to simulate butt tape wrappings. Two thicknesses, 0.05 and 0.3 in., were used. One side of the stack was in oil at atmospheric pressure and vacuum drawn on the other side. From Fig. 7 it will be seen that flow resistivity is approximately proportional to viscosity. Space does not allow presentation of other data bearing on this subject.

From Figs. 5, 6, and 7 oil supply curve *c* in Fig. 8 was

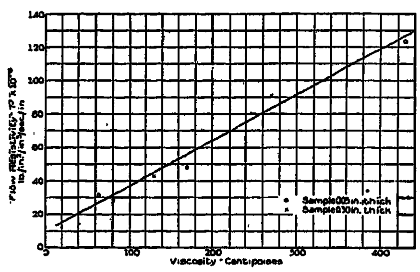



FIG. 7—FLOW RESISTIVITY—VISCOSITY OF 0.005 IN. WOOD PULP PAPER IMPREGNATED WITH AND IN BATH OF No. 219 OIL

estimated for No. 219 oil. The mathematics of oil supply are not yet developed to a point where the accuracy of curve *c* can be vouched for. We believe, however, that it is reasonably close for the purpose of this paper. The oil demand curve *a* is replotted in Fig. 8 for comparison. The radical difference in total void formation and time lag between supply and demand for a heavy oil as compared with a moderately light oil is apparent from Figs. 5 and 8, one plotted in minutes and the other in hours. A short section of curve *c* is re-plotted in Fig. 5 to show this great difference.

With No. 219 oil (ignoring possible mixture in service of thin oil from the reservoirs) maximum void formation (0.47 per cent) is not reached until 6 hr. have elapsed and voids are not reduced to a negligible value until 52 hr. have passed. Quite obviously with a solidifying compound, voids that form during cooling are never reduced to a negligible value except, possibly, after thin oil from the reservoirs has mixed with the compound and softened it. This should require an excessively long time for the two materials do not mix well below 60 deg. cent.

The demand and supply data in Fig. 8 were re-calculated to represent volumetric variation with time, as shown in Fig. 9, curves *a* and *c*. A similar volumetric supply curve *b* is given in Fig. 9 for Oil No. 1. There is such close agreement between curves *a* and *b* that the difference can hardly be noted. Fig. 9 ex-

plains at a glance the great superiority of oil filled cable over solid cable. Curve $b$ might be called the "worst" condition for oil filled cable because it is not difficult to maintain a pressure higher than atmospheric in the feed channels if desired. Curve $c$, on the other hand, represents the "best" condition for solid cable. It is impracticable to maintain atmospheric pressure

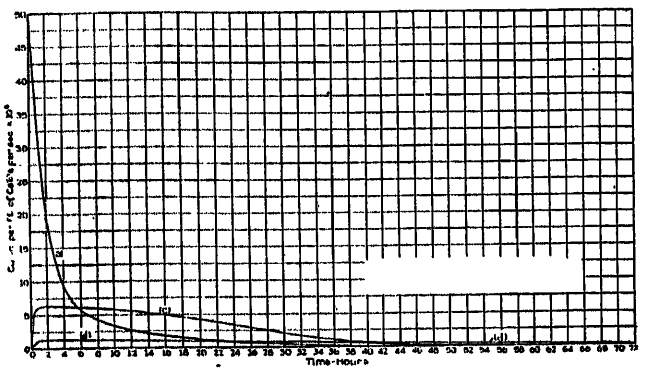



FIG. 8—CABLE NO. 1, TWO CIRCUITS 45,000 KV-A. PER CIRCUIT

(a) Oil demand and oil expulsion
(c) Oil supply with No. 219 oil, 20 deg. cent. earth
(d) Oil supply with No. 219 oil, 0 deg. cent. earth
Atmospheric pressure in channels

in the feed channels throughout the whole length of cable during cooling. At those points where the pressure drops below atmospheric there will be greater void formation and more time lag. Also No. 219 oil is about the thinnest that can be used in solid cable.

### OTHER CONDITIONS OF SERVICE

With atmospheric pressure assumed as maintained in the feed channels by oil reservoirs, we now have a fair picture of what goes on in a solid cable of perfect initial impregnation when steady summer load is dropped and the cable allowed to cool down to earth temperature. During cooling voids gradually increase to a maximum value and then decrease, so that after about 52 hr. time the cable is again brought back to perfect impregnation, assuming no gas generated by ionization in the meantime. This same cable will now be briefly taken through other limiting conditions of service, one step at a time, as listed below. Figs. 6, 7, 8, and 9 will be used as a foundation for this study.

(a) Correction for longitudinal feed.
(b) Dropping full load in summer.
(c) Dropping full load in winter.
(d) Applying full load in summer.
(e) Applying full load in winter.
(f) Effect of daily load cycles.
(g) Effect of seasonal temperature cycles.
(h) Effect of initially poor impregnation.

(*a*) *Correction for Longitudinal Feed.* Longitudinal flow resistance is usually conceived as being, inherently, the determining factor of void formation in service, even in those cables filled with liquid compound. As a matter of fact, radial flow resistance is more of a determining factor. Longitudinal resistance is automatically lowered during operation of initially well impregnated solid cable by stretching of sheath and entrance of thin reservoir oil. Radial resistance, on the other hand, is almost a fixed quantity.

If we assume feeding reservoirs as being 400 ft. apart (it is important to avoid having reservoirs too far apart) each reservoir will feed 200 ft. of cable on each side of its position. From Fig. 8 maximum oil supply is $6.2 \times 10^{-6}$ cu. in. per sec. per ft. of cable. There are approximately 520 drops of oil in a cu. in. Therefore, the maximum amount of oil required from the reservoir to feed 200 ft. of cable amounts to slightly more than one drop every two seconds. Furthermore, this does not flow through the entire 200 ft. but is more or less uniformly absorbed by the paper along the length. Even a very high resistance will take care of this small flow. If longitudinal resistance is higher than that required to maintain the above maximum rate of flow the rate will be lowered to fit this condition and the lag between demand and supply correspondingly greater.

It is difficult to obtain reliable measurements of longitudinal flow representing conditions of reservoir feed in service, where the oil must work down through the joint and cable wrappings to reach the conductor strand space. Resistance along the strand space is small compared with that through the end wrappings. Longitudinal resistance in single conductor cable can be considered as composed of two paths in parallel, one consisting of the joint and cable end wrappings leading to the strand space and the other of the space directly under the lead sheath. Of these two, that directly under the sheath is eventually the most effective and important, although after the thin reservoir oil has had an opportunity to "wash out" channels through the end wrappings, this path is far from being of negligible help.

Unfortunately, from this aspect, more direct access

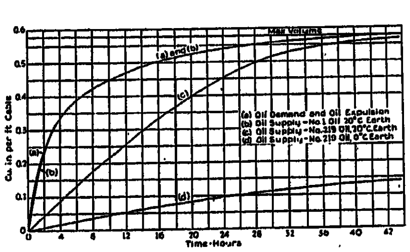



FIG. 9—VOLUMETRIC CHANGE WITH TIME-CABLE NO. 1

to the strand space cannot be obtained with the present standard design of joints. When viewed from another angle, however, this might be fortunate. It is possible that direct access of thin oil to the relatively large strand spaces might cause cumulative piling up of head pressure at the bottom of even slight grades.

From this same standpoint, corrugations or grooves of any appreciable size directly under the lead sheath would, after the thin oil worked in, convert the cable

into an oil filled cable with its attendant problems but without its inherent benefits.

There appears to be at least some promise in compromising by having very small corrugations of the right size under the sheath. This would merely serve to hasten final settling down of the line and reduce but not eliminate the final stretching of sheath. This possibility is being studied with caution. The difficulty of obtaining good initial impregnation is one objection.

The present cable has a tightly applied sheath and before heating the flow resistance directly under the sheath is large as determined by test. Loosely applied sheath is not desirable for there would be too much danger of trapping air during manufacture.

A relatively low longitudinal flow resistance is wanted immediately after installation to prevent excessive time lag between oil supply and oil demand. We believe the most practical way of obtaining this would be initially to "loosen up" cable of this type with circulating current about 25 per cent higher than maximum load current. The line should then be allowed to stand idle for a specified time to give the reservoir oil (under increased pressure) plenty of time to soak in thoroughly. Two or three, or even more, applications of this cycle before operating voltage is applied should be quite helpful.

The formula for calculating longitudinal pressure drop for uniformly distributed oil supply is:

$$P = \frac{s\,r\,l^2}{2\,A} \tag{1}$$

where; $r$ = average longitudinal flow resistance, at 20 deg. cent. in terms of lb. pressure drop per ft. of cable per cu. in. of oil flow per second.

$P$ = total lb. pressure drop along cable length

$l$ = length of cable in ft.

$s$ = average radial oil supply or oil expulsion, cu. in. per ft. per sec.

$A$ = temperature correction factor, assumed as proportional to change in viscosity.

In solid cable the supply, $s$, at any moment varies along the length, depending upon the pressure in the channels at each point. This cannot be included in the above formula until the mathematics of radial feed are more fully developed. Formula (1) is approximately applicable to solid cable only when the pressure drop in the channels is held within a reasonable average range above and below atmospheric. Also, with our present limited knowledge the factor $A$ should be applied with caution.

Summing up the rest that is known of longitudinal flow resistance $r$, it might be stated that, initially, before the cable is heated $r$ is so high that when load is first applied almost the same amount of sheath stretching along the length occurs with and without reservoirs. This much, at least, can be determined by laboratory test. With a tight sheath and before the cable has been heated the value of $r$ is initially about 20,000 for 200 ft. of Cable No. 1 and this represents a flow entirely through the end wrappings and strand space. After the thin oil has washed out a path through the end wrappings $r$ drops to about 1000. The first application of current equivalent to 45,000 kv-a. causes an almost uniform sheath stretching of approximately 0.01 in. increase in diameter. From here on, laboratory tests are not of much value in determining the reduction of $r$ in service. This depends upon the many factors already described and these cannot be fairly included in the laboratory. Any further knowledge of $r$ must be obtained directly in the field. Almost any reduction in $r$ desired can be obtained in the laboratory by sufficient stretching of sheath and "washing out" with thin oil.

The theoretical calculations which will be shown indicate that ultimate stretching will be reached when and if $r$ has been reduced to a value in the order of 100 to 200 under normal conditions of service. Whether this will cause bursting of sheath at the further points from the reservoirs we cannot say. It might be a coincidence, but the few meager field records available seem to show a final value of $r$ lower than 100 after a year or more of operation under head pressure or reservoir pressure much higher than normal. The maximum stretching of sheath in these cases appeared to have been in the order of 0.020 to 0.040 in. increase in diameter. The records, however, were so incomplete that a great deal more study of $r$ is needed.

Under average normal service conditions with 24-hr. daily load cycles the chief trouble would appear to be, not eventual bursting of sheath, but that the sheath will not be stretched sufficiently and $r$ thereby sufficiently lowered to maintain the best possible impregnation.

(b) *Dropping Load in Summer*. Returning to the condition of dropping full load with 20 deg. ambient earth the effects of longitudinal flow will be briefly shown by selecting arbitrary limits of $r$ for comparison.

From Fig. 8 the maximum radial oil supply for Cable No. 1 is $6.2 \times 10^{-6}$ cu. in. per ft. per sec. at an average cable temperature of 37 deg. cent. (The average temperature at any time is directly proportional to the total volume of oil contraction as determined from Fig. 9.) With limiting values of $r$, 2000 and 200, the pressure drop along the channels in 200 ft. of cable necessary to maintain this radial feed can be calculated from Formula (1). These pressure drops are 93 lb. and 9.3 lb. respectively. The first is, of course, entirely out of proportion. It means that with a reservoir pressure of 5 lb. there would be a very appreciable time lag in oil supply over that shown in Fig. 8, and also the maximum rate of supply would be decreased to fit this condition. There are no means at present for estimating this but it would surely require several days at no load to fill up all the voids. This brings out clearly

the desirability of "opening up" the cable initially, before voltage is applied.

The second value, $r = 200$, gives a pressure drop well within the limits of good operation. With a reservoir giving a constant pressure of 5 lb. the pressure in the channels 200 ft. away would be only 4.3 lb. below atmospheric at the time of maximum supply and something better than this at other times. Curve $c$ in Fig. 8 should, consequently, represent the average over-all effect fairly well. The void volume and time lag would be a little greater at the far end than shown in Fig. 8 but less at the near end.

(c) *Dropping Load in Winter.* With the cable gas free and perfectly impregnated (the same theoretical assumption used previously) the oil demand curve $a$ in Fig. 8 still holds if the same load (45,000 kv-a.) is dropped in the winter with an ambient temperature of 0 deg. cent. and copper temperature of 32.5 deg. cent. The estimated rate of supply is shown in Fig. 8 as curve $d$ and the volumetric supply as curve $d$ in Fig. 9. These curves are based on the assumption that flow resistivity (Fig. 7) remains proportional to viscosity at these lower temperatures. The difference from summer conditions, caused by higher viscosity of oil and slower gas absorption, is apparent. The maximum void formation and time lag (0.8 per cent at 24 hr.) are both greater. It would require about 16 to 20 days to fill up entirely all voids with the channels maintained at atmospheric pressure.

Maximum oil supply from curve $d$ is $1.25 \times 10^{-6}$ cu. in. per sec. at an average cable temperature of 18.5 deg. cent. The pressure drop in the channels needed to maintain this supply with a resistance, $r = 200$, would be only 5.5 lb. for 200 ft. of cable. Maximum pressure drop in this case, however, would occur after about 30 hr. when the cable is at an average temperature of 2 deg. cent. and the supply is $0.6 \times 10^{-6}$ cu. in. per sec., giving a drop of 13.4 lb. for 200 ft. of cable. This means that curve $d$ would not hold closely for the far end of the cable, and it would require even a longer time than 16 to 20 days to fill up the voids entirely.

A cable having a resistance, $r = 2000$, would require such a long time to fill up voids that reservoir feed would be of little use at 0 deg. cent.

From the calculations it can be seen that winter operation, from an ionization standpoint, is more severe than summer operation. The practise of increasing winter loads above summer loads makes it even more severe.

(d) *Applying Full Load in Summer.* Curves $a$ in Figs. 8 and 9 also represent oil expulsion curves when load is applied, assuming the cable perfectly impregnated at that time. Fig. 8 shows that the maximum rate of expulsion occurs in 10 min. and is $48.0 \times 10^{-6}$ cu. in. per sec. If the sheath and cable were absolutely rigid the expelled oil would have to return to the reservoirs and the pressure in the channels would reach a maximum value at that time, after which it would fall off rapidly. The pressure drop for 200 ft. of cable, calculated from Formula (1), is shown as a function of time in Fig. 10. The following conditions were assumed:

(a) Perfect impregnation.
(b) Absolute rigidity of sheath.
(c) A value, $r = 100$.

The pressure drop for any other assumed value of $r$ can be determined from the curve by direct proportion.

Fig. 10 shows a brief maximum pressure of 88 lb. (above the reservoir pressure) which drops off to 36 lb. in 50 min., and to 18.5 lb. in 2 hr.

The bursting pressure for pure lead sheath of these dimensions at 20 deg. cent. and with about this speed of pressure application is 125 lb. per sq. in. with considerable swelling first, (¼ in. or more). If the theoretical conditions in Fig. 10 actually occurred the sheath would be stretched very slightly to relieve pressure each time load was applied. The degree of stretching of lead sheath under conditions of transient pressure at

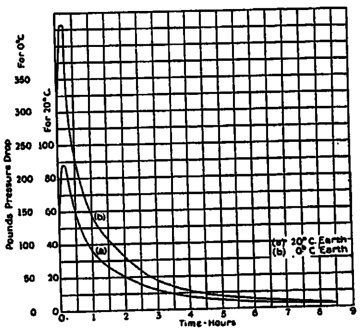



FIG. 10—PRESSURE AT FAR END OF 200 FT. CABLE NO. 1 WHEN LOAD IS APPLIED $r = 100$

various temperatures is not well understood. As a rough guess, the sheath would be stretched about 0.0005 to 0.001 in. in diameter during the first pressure applications shown in Fig. 10. Each succeeding application would cause progressively less stretching as $r$ decreased below its initial value of 100. Whether the sheath would eventually burst is not known.

Standard antimony and tin alloy sheaths have a bursting strength of 200 to 300 lb. per sq. in. under the above conditions but with much less stretching before rupture. This increased strength would appear to come well within the requirements but pure lead sheath is preferred because it will stretch more easily and to a greater degree, automatically lowering longitudinal resistance to the required value and reducing the possibility of rupture.

The theoretical conditions in Fig. 10 do not hold in actual practise. If we assume an initial impregnation of 0.5 per cent at 20 deg. cent., atmospheric pressure (a better impregnation could hardly be expected after installation) it would act as a cushion and smooth off

the pressure peak in Fig. 10. Assuming that compression of the void volume to 1/3 its original size represents a pressure in the channels of 25 lb., which seems fair in view of the high pressure existing in the insulation thickness, we have left a volume of 0.33 per cent to take care of expansion. From Fig. 9, curve *a* this amount of expansion would be reached in 1¼ hr., practically in agreement with the corresponding pressure and time in Fig. 10.

This means that with $r = 100$ and 0.5 per cent initial impregnation the pressure would start from atmospheric and increase gradually to a maximum of 25 lb. in 1¼ hr., after which it would drop off. It cannot be said definitely whether this would stretch the sheath or not. Each application would cause a minute amount of stretching, if any, and a slight decrease in $r$ would stop this altogether.

(*e*) *Applying Load in Winter.* Curve *a*, Figs. 8 and 9 represent the oil expulsion at 0 deg. cent. ambient earth provided the same load (45,000 kv-a.) is applied. The pressure curve corresponding to that at 20 deg. cent. is also plotted in Fig. 10. The maximum theoretical pressure reached in 10 min. is 436 lb. for 200 ft. of cable. Pressure drop, according to Formula (1), varies as the square of the length. The pressure drop would be correspondingly greater for a longer length of cable.

If we assume that initial impregnation of 0.5 per cent at atmospheric pressure still holds at 0 deg. cent. (an optimistically extreme case) a pressure peak of approximately 65 lb. would occur in 2 hr., and then decrease. Lead is much stronger at such low temperatures than at normal temperatures but it is not known whether the above transient pressure would stretch the sheath or not. If so, it would again be a very minute change and many such applications would be required to cause a noticeable effect.

There is very little likelihood of repeated applications of this nature at 0 deg. cent. in service. It has already been shown that a cable line would have to remain idle at 0 deg. cent. for 16 to 20 days before all of the voids caused by dropping load are filled up. In other words, if load is again applied before this time has expired there would be void volume additional to the initial 0.5 per cent.

Cold weather conditions probably determine the ultimate stretching of sheath. We have no way at present of determining this ultimate stretch but bursting of sheath appears improbable. A summary of the work, as far as we have carried it, has been presented, however, hoping that it will assist others in reaching a solution of this difficult problem.

A study is being made of the transient physical characteristics of lead sheath and we hope at a future time to present more definite conclusions.

(*f*) *Effect of Daily Load Cycles.* The lag effect between oil demand and oil supply during 24-hr. load cycles is apparent from previous considerations. From Figs. 8 and 9 this might even be approximately estimated. It means, of course, that in solid cable oil supply can never exactly equal oil demand and during periods of decreasing load a variable void volume will be added to the initial void volume, the amount of lag and nature of the load cycle determining the average total impregnation. This average will be worse for large loads than for small loads.

Maximum void volume will occur near or during periods of no load or low load. Likewise, during periods of high load the voids will be practically closed up. Obviously, high load factor is very desirable from this standpoint. With perfect initial impregnation of Cable No. 1 and assuming a 12-hr. duration of rapidly reached 20 deg. cent. no-load (low load factor) the maximum void volume would be about 0.5 per cent for $r = 100$. With a 4-hr. period of low load, and a gradual decrease in load (high load factor) the maximum void volume would be 0.25 per cent or better.

If accurate hourly records were taken of reservoir feed and pressure, as well as of cable temperature, it would be possible to determine approximately the degree of impregnation and many other interesting things related to the cable.

(*g*) *Effect of Seasonal Temperature Cycles.* There would be no time lag between oil demand and oil supply for slow seasonal temperature cycles (liquid compound filled cables). The important effect of seasonal changes in temperature appears in the time lag of daily load cycles, due to change in viscosity of oil.

(*h*) *Effect of Initially Poor Impregnation.* If the void volume initially in the cable appreciably exceeds the volume of expansion when load is applied there will be little if any extrusion of oil during daily load cycles, depending upon the value of initial impregnation, difference in void pressure and reservoir presence, and the value of longitudinal resistance. Nor will there be any large increase in pressure or stretching of sheath. The cable will continue to absorb oil until a balance is reached. If there is a relatively large quantity of gas still present after balance the extrusion will still be small in comparison with the total expansion.

Obviously, under these conditions the amount of oil absorbed depends upon the relative void and reservoir pressures. With voids and reservoirs both at atmospheric pressure and the cable on level ground no oil would be absorbed at all unless the reservoir pressure is increased.

With a poorly impregnated cable $r$ would remain quite high even after balance is reached, since the sheath would be stretched little if at all.

Poor impregnation materially increases the thermal resistance of the cable; therefore, curve *a* in Figs. 8 and 9, which was calculated for a well impregnated cable, does not hold very closely. For this and other reasons exact determination of the effects of poor impregnation is very difficult.

## Oxidation of Compound

In cable made of proper materials operating within

allowable temperature limits, and assuming that moisture from outside is excluded, the only form of deterioration so far noted, other than that caused by ionization, also results in an increase of dielectric loss.

Riley[3] concludes that this is nothing more than ordinary oxidation of compound, with its accompanying formation of water and acids. He claims that sufficient oxygen to produce this result is normally absorbed by cable during manufacture and installation.

Our work confirms his in effect but perhaps not in degree. We have found cases of as much as a 100 per cent increase in dielectric loss at 60 deg. cent. after two years of operation. Judging from laboratory tests it is hard to see how initially absorbed oxygen could fully account for such an increase, even though the original loss was extremely low, unless the amount initially absorbed was much more than would be expected. Further work might fully confirm Riley's conclusions but it is more reasonable to assume that other actions, such as reaction between paper and compound, or entrance of additional oxygen or water occurred during operation.

We have never found proof of chemical reaction between pure paper and pure compound at operating temperatures. On the other hand, there is abundant proof of oxidation of compound if sufficient oxygen is present.

The only conclusion that can be drawn with our present knowledge is that it is always wise to exclude oxygen as far as it is possible to do so and in this way limit the increase in loss to a safe value. The following precautions are worth while:

(a) Have the cable initially free of oxygen.

(b) Keep the outer covering absolutely tight in service and maintain, as far as possible, a positive pressure in the system.

(c) Do not open cable ends or joints in air when below atmospheric pressure.

(d) Remove, as far as possible, by vacuum treatment the oxygen absorbed during the exposure of jointing work.

(e) Fill the joints and reservoirs with an oxygen-free, unexposed oil.

(f) Avoid exposure of the reservoir oil to air or oxygen during service.

The seriousness of increased dielectric loss in service depends upon the ultimate value of this loss in comparison with copper loss. A loss which, initially, was only 0.5 per cent of the copper loss could increase 500 per cent without being regarded as especially serious. On the other hand an initial dielectric loss of 5 per cent, if increased 500 per cent would seriously affect "hot spot" temperature and bring back the dangers experienced years ago of accumulative heating failures. Obviously, the percentage ultimate increase of dielectric loss is of more importance the higher the voltage rating of the cable.

All oils and compounds deteriorate at high temperature in the presence of oxygen, the rate of deterioration increasing rapidly with temperature. We have exposed No. 219 compound to a temperature of 150 deg. cent. for one week under vacuum without increase in electrical conductivity or dielectric loss. This same exposure in the presence of oxygen increases the dielectric loss to a prohibitive amount for cable work.

A 155-ft. length of 25-kv. three-conductor, shielded type cable, insulated with wood pulp paper and impregnated with No. 219 compound, was put through 230 heating cycles from 20 deg. cent. to 60 deg. cent. at normal voltage by heating the conductor with current. The ends were exposed for some time while the terminals were being applied and no vacuum treatment was used afterwards. No reservoir feed was applied and, of course, the sample dropped below atmospheric pressure while cooling. The dielectric loss, 60 deg. cent. normal voltage, 60 cycles, at the start was 0.094 watts per ft. It increased gradually and appeared to become constant towards the end of the run, the final loss being 0.148 watts per ft., representing an increase of 57.5 per cent. No measurable or noticeable ionization occurred and this increase in loss might be accepted as that due to normal oxygen absorption (it was discovered during test that the terminals leaked slightly). The results are not conclusive, however, because of the relatively short time of test (40 days) and the lack of assurance that moisture might not have entered during test.

It might be claimed that regardless of the fact that no visible ionization damage was present in this test that part or all of the increased loss was caused by the weak, intermittent discharge previously described. We do not think so. Perhaps future work will prove us entirely wrong but it is hard to see how such a faint and relatively brief discharge could cause this effect. According to Hirshfeld, Meyer, and Connell[15] the discharge must also attack the paper fibers before water is formed. This would seem improbable with such a small degree of ionization while, on the other hand, the effects of oxidation are well recognized. We are planning an additional series of tests which will lead to more definite conclusions.

### IONIZATION

This paper, so far, has dealt with methods for the prevention of ionization because there can be no doubt of its injurious action, and every effort should be made to eliminate it by proper design and operation. That it does exist in service there is abundant evidence. For this reason knowledge of its behavior is important.

A great deal of attention has been given to the subject of ionization in cables since it was first identified in 1917, resulting in a large amount of published matter. Some of the outstanding papers on this subject are given in the Bibliography. From these a fairly good knowledge is now available of the voltage stress at which ionization begins as related to size of voids and pressure. Very little is known, however, regarding

the laws of ionization loss in cables as related to size of voids, pressure, and voltage stress. Such knowledge would be of great value in determining the quality of cable, particularly in estimating more accurately the degree of impregnation. It should also throw additional light upon the rate of deterioration.

The effects produced by ionization in a void are, excessive energy loss, production of non-uniform stress distribution in the insulation, chemical changes in compound and paper, and perhaps a migration of the compound from the paper as demonstrated experimentally at very high stresses by Emanueli[2] and Dunsheath.[11]

Hitherto, studies of the mechanism of breakdown of insulation have been confined chiefly to the consideration of phenomena involved in solid or liquid materials. However, in most cases, the failure of high-tension insulation such as that of cable is initiated by electrical discharge through a gas in proximity to the insulation proper. We shall therefore briefly describe some of the characteristics of gaseous conduction which appear to be applicable to the phenomena appearing in cable.

There are always a few ions being produced by penetrating radiation in a gas under ordinary conditions and if a potential difference exists between the boundaries of the space, current will be carried which increases with the applied voltage until it becomes constant, this stage being the condition where the ions are discharged at the opposite electrodes at the same rate as they are being formed in the space. However, the voltage cannot be increased indefinitely without a further change in conditions. This occurs when the ions are speeded up to such an extent that they begin to ionize neutral atoms by collision. At this point the current begins to increase rapidly and ionization and recombination occur so plentifully that radiation may be visible as in corona discharge.

The whole space is now in a comparatively high conducting state and, depending upon the geometry of the enclosure, the nature and pressure of the gas, at a certain current density the whole system becomes unstable and breaks down into an arc-like discharge where the current increases, producing less and less resistance, so that the voltage across the gap falls.

During the first stage before the so-called saturation current is reached, Ohm's law is obeyed, but the current passing is extremely small, being of the order of $10^{-15}$ amperes per sq. cm., so that the energy loss is relatively very small.

A qualitative idea of the change in condition of the gas as the voltage is raised is given by the diagram[17] of Fig. 11 which shows the generalized characteristic of the electrical discharge between metal electrodes. The ordinates and abscissas of this diagram are logarithmic as the current when the arc stage is reached is many hundreds of times the current at the point $O'$, and the voltage at $O'$ is hundreds of times greater than that of the arc.

The source of potential $E$ produces a discharge between electrodes with a voltage drop $V$, a resistance $R$ being in the external circuit, and an ammeter to measure the current $i$. The external characteristic is

$$E = V - R\,i \tag{1}$$

and the internal characteristic of the discharge itself is

$$V = f\,(i) \tag{2}$$

If the series resistance be kept constant and the voltage gradually increased the current is at first small and the volt-ampere characteristic is positive, as shown by the curve $OO'$. Beyond $O'$ the current increases and the voltage decreases, the characteristic of the glow discharge, and at $G'A$ occurs the transition from glow to arc where very large currents may be carried with a very low voltage drop.

Depending upon the value of $E$ and the amount of series resistance in the circuit, the changes from one stage of the discharge to another may be continuous or discontinuous. For instance, in the case of Equation (1), small $R$, the entire glow discharge may be skipped over. In cable containing gas spaces the dielectric acts as a current limiting device so that the characteristic may correspond to that of a discharge with a high series resistance.

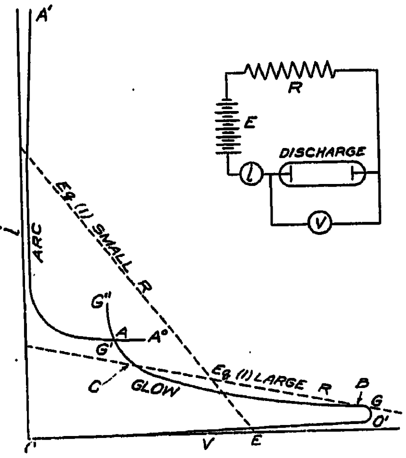



Fig. 11—Volt-Ampere Characteristic of Gas Discharge

It is obvious that in a cable, when a gas pocket is present, as the voltage increases the power factor will also increase so long as the discharge has a positive volt ampere characteristic. When, however, the glow stage is reached, a slight increase of current is accompanied by a very large drop in the voltage. Accordingly the energy dissipated in the gas space decreases because it becomes a relatively good conductor. The damage to the insulation, on the other hand, may be greater, and eventually a spark or destructive arc-like discharge may occur.

The ease with which a discharge may be started in a gas varies very greatly with the pressure, size of enclosure, and the nature of the gas. The variation with pressure of the sparking potential for air is shown in Fig. 12 for two different gaps, 1 mm. and 3 mm.

For any given distance between electrodes there is

some pressure at which discharge occurs at the minimum sparking potential of the gas. Thus with a gap of 3 mm. in air, sparkover occurs at a pressure of 1.5 mm. at about 340 volts. For a gap of 1 mm., the discharge occurs at a pressure of 5 mm. at the same voltage. At pressures above and below these minima, or critical pressures, the voltage necessary to produce breakdown of the gas increases both with increasing and decreasing pressure. Paschen's law summarizes these results by stating that the sparking potential of a gas is a function

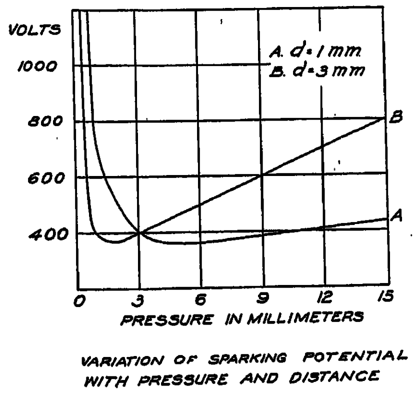



FIG. 12—DIELECTRIC STRENGTH OF AIR AT DIFFERENT PRESSURES AND TWO DIFFERENT GAP SPACINGS

only of the product of the sparking distance by the pressure of the gas, or in other words, of the mass of the gas, between the electrodes. If $p$, the pressure, and $S$, the gap length, are varied in such a way that the product $p\,S$ remains constant, the sparking potential will remain unaltered, and the product $p\,S$ is a constant for any gas. The following table[18] gives data for some of the ordinary gases.

| | Minimum spark potential | $p$ (mm) $\times$ $S$ (mm) |
|---|---|---|
| Air | 341 | 5.7 |
| Nitrogen | 251 | 6.7 |
| Hydrogen | 278 | 14.4 |
| Oxygen | 455 | .... |
| Carbon dioxide | 419 | 5.1 |
| Helium | 261 | 27 |
| Acetylene | 468 | .... |

In order to determine the characteristics of a cable containing gas spaces, measurements were made by E. S. Lee[19] on a length of single conductor cable 2/0 *A W G* (1.06 cm.) with 9/32-in. (0.714 cm.) treated paper insulation which after removal of its sheath was inserted in a brass tube and held centrally by copper spacing rings so as to leave an annular space of 0.120 in. (0.305 cm.) between the paper and the tube. The length under test was 10 ft. and the variation of power factor with voltage was measured with hydrogen, carbon dioxide, and air at different pressures in the annular space. The results are shown in Figs. 13 and 14.

The normal voltage rating of this cable would be about 12 kv. and curve 1 shows the power factor voltage characteristic in its normal state with lead sheath. Curve 2 shows the characteristic with sheath removed and after insertion in the brass pipe to provide an air gap between sheath and insulation. Here the typical break in the characteristic, the so-called ionization point, occurs at about 12 kv. after which the power

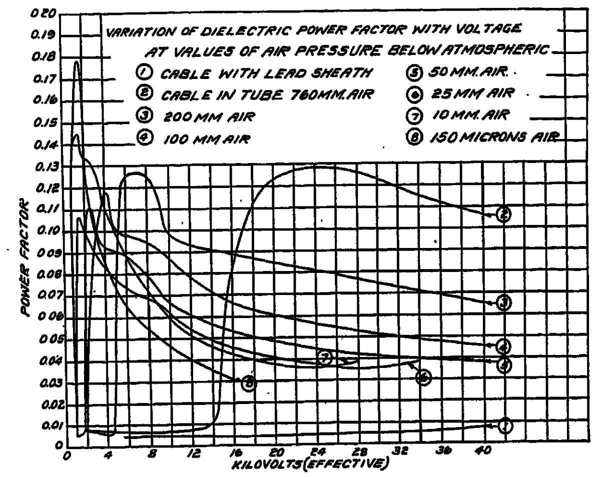



FIG. 13

factor increases rapidly to about 13 per cent and then decreases as the voltage is raised further. At this stage the discharge through the gas is of the glow type and is beginning to result simply in an extension of the conducting sheath inwards towards the insulation.

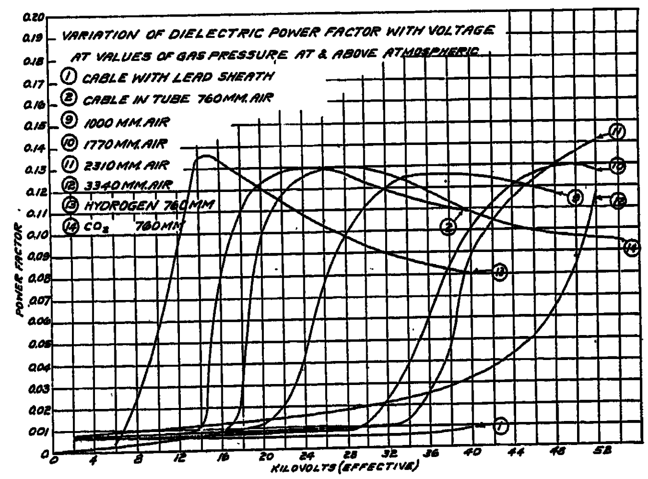



FIG. 14

On reducing the pressure of air the ionization point is lowered until corona eventually occurs at about 500 volts at a pressure of 0.15 mm. as shown in curve 8. In this case after a rapid rise to a power factor of about 18 per cent a decrease to about 3 per cent is found on raising the voltage to 17 kv. Increasing the gas pressure raises the ionization point as shown in curves 9, 10, and 11. Hydrogen (curve 13) shows greater ease of corona formation than air, and carbon dioxide (curve 14) requires a higher voltage than air for initiation of the discharge.

Further consideration of the curves of Fig. 13 shows the possibility of two definite and distinct types of characteristic which may produce failure in the neighborhood of the operating voltage. For instance, consider two cables operated at 14 kv., in one case having a void containing gas at atmospheric pressure and the other with a void having a gas pressure of 25 mm. or less. It is evident that in the first case the corona discharge which is occurring is of the positive volt-ampere type, and results in a relatively high dissipation of energy in the void but relatively less stress across the insulation. In the second case the discharge is of the type with negative volt-ampere characteristic, which results in a lower energy consumption in the void but relatively higher stress across the insulation. When the highly conducting discharge occurs in a void, it is obvious that it is practically equivalent to short circuiting part of the insulation and that the higher stresses set up on the insulation in series with it will develop lateral stresses which probably cause the dendritic markings so often seen near failures.

Which of these conditions will result in the shorter life depends upon several factors, such as the nature of the gas present, the effect of corona on the walls of the void, and the nature of the insulation. With cable materials, such as we are using at present, the low pressure voids are the most dangerous. But it is interesting to see that in the first case the measurement of power factor at 14 kv. and a higher voltage shows an increasing power factor; in the second case measurement above 14 kv. shows a lower power factor. It is, therefore, possible to have two diametrically opposite types of power factor characteristics both caused by the presence of voids in cable and both of which show undesirable quality in the cables.

In order to diagnose more accurately the quality of a cable, therefore, it may be advisable to make power factor measurements at lower stresses than 20 volts per mil, though such a test would probably only be useful after a cable had been subjected to service conditions, factory impregnation now being so good that voids are practically eliminated.

Though experimental evidence is, as yet, incomplete to substantiate completely the effect of these different pressures on the endurance of a dielectric we may, however, tentatively conclude that:

(1) Internal corona in insulation occurs less readily as the gas pressure in voids increases above the critical discharge potential of the contained gas.

(2) The voltage required to produce internal corona varies with the nature of the gas.

(3) The power-factor voltage characteristic shows two distinct parts, one corresponding to the positive volt-ampere characteristic of gaseous discharge, the other to the negative volt-ampere characteristic.

From the general characteristics of insulation, it is very surprising that when ionization weakness develops it does not always accelerate rapidly to ultimate destruction. In the case of cables, for instance, a hotspot may develop and get slowly worse over a period of days before finally breaking down. In some cases the hot spot may actually disappear. We suggest that such a result is caused by the gradual increase of pressure produced in a void by the action of the initial corona in producing an increasing quantity of gas from the compound or paper which tends to stabilize and retard the development of the corona discharge. Such an action may be seen, for instance, in an enclosed mercury arc in quartz, where if the current be increased above normal the arc will fade because of the increased pressure of the vapor.

When a cable fails in service or on life test, there are usually signs of great distress in the vicinity of the break. The compound is decomposed, paper charred, wax is present, and very often irregular dendritic discharge paths are burned in the insulation. Of a different character entirely were three punctures made on samples of liquid oil filled cable of the 132,000-volt type which were tested after removing oil from the hollow core and applying vacuum pumps to exhaust air while under stress. The samples failed at 57 kv., 80 kv. after 3 min., and 80 kv. after 22¼ hr., without development of hot spots and with clean, small punctures. The normal value for breakdown voltage on such cable completely filled with oil is about 450 kv. In the case of the failure under vacuum conditions, pressure could not develop so when weakness developed deterioration proceeded very rapidly.

Similar experience has been obtained with small capacitors during an investigation of characteristics of materials which were assembled in a glass bulb for subjection to refined vacuum treatment and endurance tests. With the bulb sealed off with high vacuum inside, it was found that such capacitors could not be operated above 1200 volts without immediate breakdown. However, upon admitting air at atmospheric pressure they operated at 3000 volts for 1000 hr. and more, a stress of 1500 volts per mil.

Another experiment has been performed, as follows: A glass bulb, 4 in. in diameter, with two electrodes and a mercury manometer attached, containing 50 cu. cm. of standard cable compound, was exhausted, while hot, to good vacuum. A glow discharge was then produced in the bulb by application of 2000 volts from a transformer across the electrodes. Gas was evolved at the rate of approximately 1.2 cu. cm. per hr. and after 28 hr. the discharge stopped and the voltage had to be raised to 2300 in order to start the discharge again. The actual pressure in the bulb rose in 75 hr. to 15 cm. but the voltage necessary to maintain the discharge did not increase correspondingly because of the gradual building up of a conducting deposit which shortened the gap.

We do not wish it to be inferred, however, that we think it advisable to generate gas in a cable in such a way as to displace oil or compound from the insulation

so that the gas space will be enlarged and perhaps ultimately extend from core to sheath. And, of course, we realize that the ideal method is to eliminate voids entirely, and to prevent their formation, but in some cases this may be impracticable. Nevertheless, we do feel that if voids exist it is preferable to have as high a gas pressure in them as possible, and if the whole cable is sealed under definite pressure these spaces need not necessarily grow larger, and may perhaps be constrained to move their boundaries only in a longitudinal direction.

As a result of these observations, we feel that these conclusions may be drawn:

(1) If voids exist in any given insulation it is desirable to have present as high a gas pressure as possible.

(2) Since it may be impracticable to use materials which are capable of maintaining pressures below the critical discharge pressure in insulation, it may be desirable to use those which readily develop as high pressures as possible, though not high enough to cause enlargement of voids which exist because of expansion and contraction phenomena, or to produce gas spaces in otherwise continuous media.

As a matter of fact, if a cable sheath could be constructed and installed with sufficient strength to withstand a pressure of a few hundred pounds per square inch, it should have as high a dielectric strength when filled with gas at that pressure as our present compound treated material. A cable embodying this principle has been described by Messrs. Fisher and Atkinson.[20]

It is also evident, in view of the preceding discussion, that no discharge can occur at any pressure if the voltage drop across a gas space is below the critical discharge potential. For air, this is 341 volts or about 240 volts r. m. s. Therefore, if voids in cable can be restricted to thicknesses across which the drop is less than this no internal corona can occur. Assuming for ordinary cable, an operating stress of 50 volts per mil and a dielectric constant for the insulation of 3.5, this would mean that voids less than 1.4 mils in thickness should cause no trouble. And if such voids could be filled with gas to atmospheric pressure they could be many times this thickness.

## Formation of "X" Wax

The work of Hirshfeld, Meyer, and Connell,[5] on the chemistry of wax formation and changes in insulating materials is outstanding and when completed will greatly assist in clearing up this question. Their work indicates that ionic bombardment forms wax and hydrogen from the compound. It also causes chemical and possibly physical changes in the paper fibers, if sufficiently severe and sustained sufficiently long, yielding water and carbon dioxide among other things. The hydrogen and carbon dioxide react to form more water. It is this water and possibly other factors which cause gradual deterioration of the electrical properties, according to their tentative theory.

The formation of wax appears to be the result of a polymerization or condensation of hydrocarbon molecules with the evolution of more or less hydrogen or hydrocarbon produced by glow discharge, bombardment by electrons, or alpha particles. The theory of this reaction is discussed by S. C. Lind.[21]

This wax is one of the most stable organic compounds yet found and if it could be prepared in large quantity would probably be one of the best of insulating materials.

We have prepared this wax by the action of glow discharge on a wide variety of compounds such as paraffin base oils, asphalt base oils, rosin oil, rosin and petroleum base mixtures, water white oils, such as Nujol and kerosene, and a somewhat similar material from acetylene gas.

Conclusive evidence that wax is not formed by the action of stress alone on the oil is offered by capacitors which have operated for years at a stress of 400 volts per mil without the formation of wax though the oil with which they are filled develops wax by the E. T. L. test or by subjection to corona discharge. Furthermore, wax is never found in cable except at those points where voids are formed.

The formation of wax, therefore, seems to be simply a symptom of "corona disease" and as such is a useful criterion of the quality of a cable and an aid in diagnosing the source of trouble. Existence of wax in a cable after operation shows either that voids have existed as a result of poor initial impregnation or as a result of expulsion of compound or expansion of the lead sheath by heating and cooling cycles of operation.

The real cure for conditions shown by the formation of wax is, of course, elimination of gaseous ionization by methods discussed elsewhere in this paper, and the stability of the compounds to corona is of secondary importance, though we feel that rather than have a compound which will not evolve gas under corona conditions, it may be preferable to have the compound evolve as much gas as possible in order to raise the gas pressure in the void above that at which electrical discharge can occur. However, if oxygen can gain access to the interior of the cable, for instance through the joints, there is danger that water may be produced if corona occurs in the mixture of oxygen and hydrogen.

## Laboratory Cable Treatment

In the latter part of 1923 extensive investigation was started to determine the best methods and materials for cable manufacture. A small treating plant was built in the research laboratory, consisting of a steam jacketed pipe, 5 in. inside diameter and 20 ft. long, attached to which was a compound tank. The outfit was provided with rotary oil vacuum pumps, condensing equipment, liquid air traps, and McLeod gage for measuring the degree of vacuum. With this apparatus, valves and joints being oil sealed, a vacuum of 0.010 mm. could be obtained, and liquid air could be

used to condense any residual water vapor in the apparatus.

Lengths of cable 19 ft. long were treated, two sizes being used: (1) 500,000-cm. conductor with 30/32 in. paper insulation and (2) 2/0 B & S conductor with 9/32-in. paper insulation. Five runs only were made with the larger cable after which the smaller cable was adopted as the standard size for investigation because of its greater convenience for testing and because three lengths could be treated at the same time.

About 50 separate runs were made with this apparatus to determine the effect of variations in (1) temperature of vacuum treatment, (2) time of vacuum treatment, (3) pressures during vacuum treatment, (4) temperature of impregnation, (5) pressure during impregnation, (6) preliminary air dry before treatment, (7) application of high pressure after impregnation, and (8) materials, both paper and compounds.

In order to determine the quality of the cable, with the 9/32-in. insulation, after leading, power factor was measured on one length at 25 deg., 40 deg., 60 deg., 80 deg., and 100 deg. cent. at voltages from 5.6 to 28 kv., and the two other lengths were placed on endurance test at 44 kv., one being operated at room temperature, the other at 85 deg. cent.

Relatively few tests were made with the larger cable but the following illustrates the effect on power factor of using different grades of petrolatum, first the standard factory compound and second, a clear water white material having an initial resistivity about six-fold that of the former, and a third containing a mixture of 78 per cent petrolatum and 22 per cent rosin. The paper used was a standard manila rope paper.

| | Per cent power factor | | | | |
|---|---|---|---|---|---|
| | 25° | 40° | 60° | 80° | 100° C. |
| Standard petrolatum | 0.29 | 0.34 | 0.63 | 1.17 | 1.93 |
| Water white petrolatum | 0.27 | 0.26 | 0.25 | 0.27 | 0.47 |
| Standard petrolatum—22% rosin | 0.48 | .. | .. | .. | 6.60 |

The very low temperature coefficient of the pure petrolatum and the very high coefficient of the rosin mixture seem to be characteristic of such materials.

In the treatment of the 9/32-in. insulated cable the pressure during the vacuum period was varied from 0.01 mm. with liquid air condenser to 25 mm. without liquid air, the time from 77 hr. to 242 hr. Materials from manila paper to expensive linen paper, such as used in capacitors, and compounds from highly refined water white petrolatum to factory petrolatum used 62 times for impregnation, and mixtures of different oils with rosin were used. But no definite relationship to endurance could be established as a result of these variations. The life at 44 kv. of the different samples fell between 5 and 162 hr. The endurance at 85 deg. cent., however, was always much longer than at 25 deg. cent., in some cases reaching one or two thousand hours. Obviously, some essential factor, common to all the experiments, was being overlooked. The longer life at 85 deg. cent. when the expansion of the compound produced a more complete filling of the sheath, the successful operation of capacitors filled with oil at stresses of 400 volts per mil, and the work of Clark and Shanklin[4] and Shanklin and Matson[5] on ionization phenomena suggested that the limiting factor was the presence of voids in the insulation and that these had to be eliminated before any radical improvement could be made. Mr. E. G. Gilson,[22] of the research laboratory, also clearly demonstrated by experiments with petrolatum and cable in both glass tubes and lead sheath the effect of expansion and contraction upon the formation of voids. Temperature drops of 15 deg. cent. and less on tubes 6 ft. long showed separation of the compound with the formation of voids ¼ to 1 in. in size.

Experiments were then made by impregnating cable in the usual manner with transformer oil, but upon removal from the treating tank instead of being leaded they were wound with lead foil and placed in a loose lead sheath which was filled with the same oil. Upon testing samples of this kind submerged in oil, instead of failing in one or two hundred hours at 44 kv. they operated for 1000 hr. and more at 91½ kv., an average stress of 326 volts per mil with no sign of deterioration. Practically no change of power factor with voltage was found up to 100 kv., the power factor of about 0.5 per cent remaining constant. The short time breakdown voltage of this oil submerged cable, however, was only about 15 per cent higher than that of the average petrolatum filled insulation, though the results for a number of samples showed much greater uniformity. Had we taken the so-called dielectric strength as a criterion of quality instead of the endurance test at lower voltages, the advantages of oil filling would not have been so apparent. The difference in the relationship between endurance and dielectric strength shows conclusively that a rapidly increased voltage test to destruction is not a good criterion of the very wide differences which may exist in the quality of cable insulation. It is also very striking that while the petrolatum filled cables operated only a few hours at about 38 per cent of the breakdown voltage the oil submerged cables operated at 66 per cent of their breakdown voltage indefinitely.

There was then no doubt but that a great advance was possible in the design of cable for high voltage operation. However, before we had designed engineering methods for installation, we learned that the Pirelli Company of Milan had also arrived at the same conclusion and had already gone far towards the ultimate solution of the problems involved in engineering application. Accordingly, we combined forces and as a result Chicago and New York now have commercial lines of oil filled cable operating at 132 kv. with only 23/32-in. insulation when the highest voltage in commercial operation with ordinary insulation is 75 kv. with 24/32 in. of insulation.

### Characteristics of Papers and Compounds

In order to ensure the uniformity of the finished cable characteristics and to develop materials of better quality it was necessary to establish definite and significant tests for paper and compound.

The usual mechanical tests and tests for conducting particles for papers were supplemented by:

(1) Determination of power factor of paper alone as a function of temperature.

(2) Determination of dielectric strength of impregnated paper.

(3) Determination of rate of percolation and retention of compounds in paper.

(4) Endurance tests on paper operated as sheets of dielectric in small condensers at a stress of 730 volts per mil, the paper being saturated and submerged in standard compound.

#### Power Factor of Paper

In order to eliminate the uncertainty produced by other factors, such as quality of impregnating material and methods of treatment, the electrical characteristics

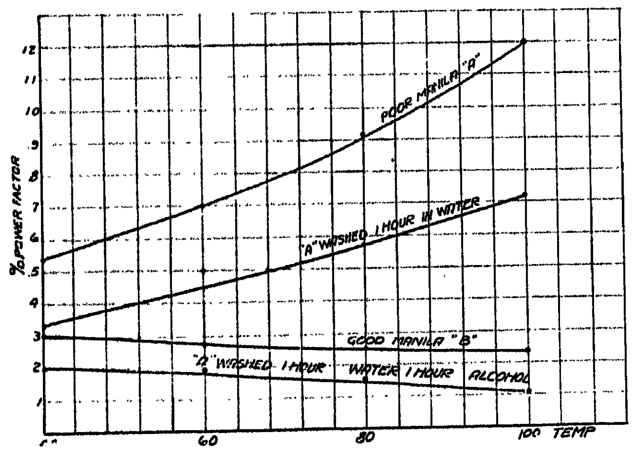



Fig. 15—Effect Upon Power Factor of Washing Paper with Solvents

of the paper itself were determined by making power factor measurements on a small condenser, using three sheets of dried paper as the dielectric between nickel plated iron electrodes. This condenser was arranged to be heated and was inserted in one arm of a Wien capacity bridge manufactured by the General Radio Company, the whole set-up being enclosed in a large box with one or two incandescent lamps burning in it to maintain lower humidity conditions than the open air.

This test as developed by Mr. C. V. Ferguson proved very satisfactory and as a result we were able to eliminate papers which produced a low insulation resistance in the finished cable and to assure uniformity of the product.

The most significant part of the characteristic is not so much the value of the power factor at room temperature as the rate of increase of power factor with temperature. In fact, with the best papers the power factor decreases with temperature up to 100 deg. cent. and this negative slope is more pronounced with wood pulp paper than with manila. The increase of power factor appears to be caused by the presence of conducting or hygroscopic materials and in many cases a poor paper may be improved by washing in water or in organic solvents, the most effective of which appear to be alcohol or chloroform. Fig. 15 shows characteristic curves for a poor manila paper, the same washed with water, with water and alcohol, and a good manila paper.

A positive power factor—temperature characteristic indicates direct leakage of current through the paper, possibly because of superficial films on the outside of the fibers. A negative slope probably means discontinuous conducting media, possibly within the fiber, when because of the current limiting capacity in series with it, the $I^2 R$ loss becomes less as the temperature is increased. The advantage of using an a-c. bridge method for measuring the losses in paper lies in being able to differentiate between these two types of loss, though it is also very much more convenient and easy to apply than a d-c. measurement. Loss because of so called "dielectric hysteresis" in the sense of molecular orientation is considered to be negligible, the more useful point of view being that of Dunsheath,[11] and Joffé,[23] in which the measurable losses correspond to those in a resistance parallel to, or in series with capacity.

#### Dielectric Strength of Impregnated Paper

This test is made by impregnating single sheets of paper for 30 min. in standard compound heated to 150 deg. cent. allowing to stand for one hour and then applying an increasing voltage to the impregnated paper between two disks 2 in. in diameter until puncture occurs. Ten separate punctures are made on each sheet.

The dielectric strength of impregnated paper is proportional to its density and this test is a check on the uniformity of the paper in this respect and upon the retentivity of the paper for compound.

#### Percolation Test

For the study of the characteristics of paper and compounds with respect to impregnation, an apparatus, Fig. 16, was devised by Mr. C. V. Ferguson consisting of a funnel shaped vessel over which could be clamped any desired number of sheets of paper, and to which was attached a graduated glass cylinder with a side tube for vacuum connection.

The percolator and paper are placed in a thermostatic oven at the impregnating temperature and air is drawn through the paper by application of vacuum for two hours. A definite quantity of compound is then placed on top of the paper and the time noted when the first drops appear in the glass tube and after that the rate at which it comes through. The difference between the total amount collected and that placed on top of the paper is a measure of the retentivity of the paper for compound.

Typical results are shown in Fig. 17.

### ENDURANCE TESTS

As a check on the quality of both paper and compound, small condensers are made up, using three sheets of paper about 4½ in. square, impregnated in the compound at 150 deg. cent. for 30 min., and tested at 11 kv. (733 volts per mil for paper 5 mils thick) between brass disks 2 in. in diameter, both paper and

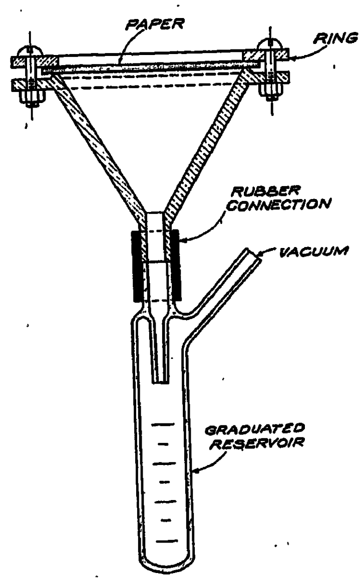



FIG. 16—APPARATUS FOR TESTING RATE OF IMPREGNATION OF PAPER

electrodes being completely submerged in the compound. Usually ten separate cells with similar material are tested at once. The temperature is maintained at 100 deg. cent. so that differences between solid and

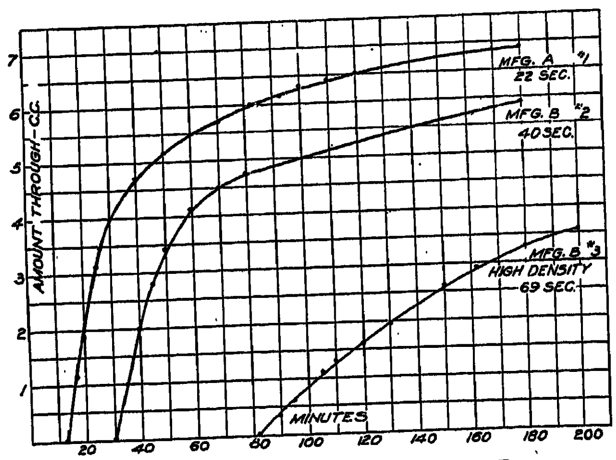


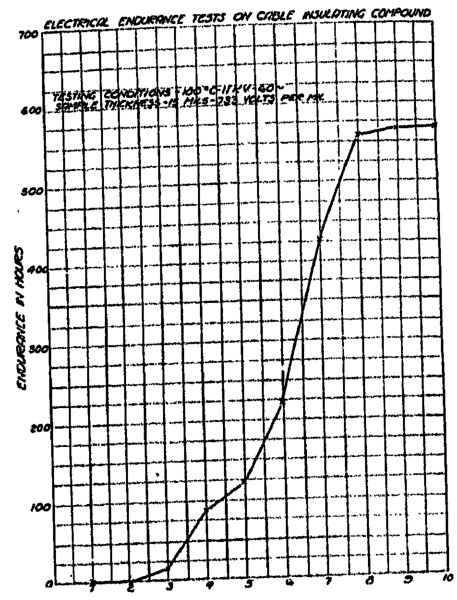

FIG. 17—FLOW OF COMPOUND THROUGH PAPERS OF DIFFERENT POROSITIES

liquid compounds, and the effect of voids are eliminated. The results, expressed as hours of endurance to failure, are plotted in order of their failure, as shown in Fig. 18, which gives an idea as to the uniformity of the materials which are being compared.

This endurance test was used to determine the effect on the dielectric properties of chemical additions to the paper or compound and the effect of mechanical changes in the nature of the paper, such as pinholes, open and closed butt joints, conducting particles, tears in the paper, etc.

In three sheets of paper one sheet might have small holes, open or closed butt joints, or tears, and very little effect would be produced on the endurance.

This, taken with a wide experience of similar nature on actual cable, indicates that there is little to fear with respect to the electrical qualities in having very many more tears present in cable than allowed by the present bending test. Indeed, it is hard to see why tears are




Specimens in order of failure

FIG. 18—ELECTRICAL ENDURANCE TESTS ON CABLE INSULATING COMPOUNDS

any more objectionable than butt joints, unless registration of the tears should also occur, or a relatively large void be formed.

### COMPOUNDS

The most significant of the electrical tests for compound are:

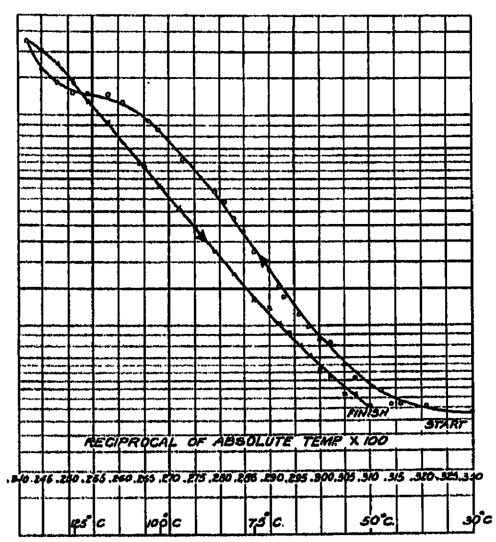
(1) Determination of d-c. resistivity as a function of temperature, and

(2) Determination of rate of decrease of resistivity under exposure to air at 100 deg. cent.

It was found that resistivity measurements made at temperatures below 100 deg. cent. had very little meaning because of the variable water content of an oil. This water is not necessarily driven off at 100 deg. cent. whether the oil is under vacuum or not. In fact, water may remain liquid without ebullition under oil up to many degrees above its boiling point. If water and oil in contact could be prepared free from gas it should be possible to heat water surrounded by oil up to the critical point of water, 365 deg. cent. without conversion to vapor, because there would be no way for the initial development of a vapor phase to

occur. However, in the ordinary oils we found that evolution of water vapor usually occurred at temperatures in the vicinity of 120-130 deg. cent. A plot of the change of resistivity of an oil with temperature is shown in Fig. 19, where a characteristic discontinuity is shown at these temperatures. Similar characteristics have been described by Shrader.[24]

We therefore found it necessary to determine this characteristic up to 150 deg. cent and to make comparisons of resistivity based upon observations at this temperature before we found agreement with the characteristics of the finished cable which had been impregnated with compound heated for many hours in vacuum tanks at temperatures above 100 deg. cent. It will be noticed that the curve of Fig. 19, determined with falling temperatures, shows a lower resistivity than with rising temperature above the critical point and higher below it. This difference is produced by oxidation at 150 deg. cent., and because of this, measurements should be made very quickly in order to eliminate this effect as much as possible. In order to do this a cell was designed by Mr. C. Dantsizen as shown in Fig. 20. When a small volume of oil is heated in the glass cell by a heating coil energized from an insulated X-ray tube filament transformer, about 20,000 volts are impressed on the electrodes from another transformer through a rectifying kenotron and the current read by a microammeter. Such a cell is used for factory control work where speed is a requisite, and complete measurements can be obtained in about 10 min. For more refined work a cell with guard ring, designed by Dr. C. W. Hewlett, to operate in an oil bath at a few hundred volts and for current measurements with galvanometer is used.




CONDUCTIVITY OF PETROLATUM LIQUIDUM AS A FUNCTION OF TEMPERATURE.

FIG. 19

The rate of deterioration is determined by simply exposing a definite quantity of oil in a vessel at 100 deg. cent. to air and removing samples from time to time, measuring their resistivity. This is an important characteristic from the standpoint of permanence during factory operations. When the resistivity of the material in the factory falls below a certain value because of oxidation during the repeated exposures of the treating processes, it is purified by agitation with finely divided Fuller's earth, followed by filtration. This brings the resistivity back to, or better than that of the original material.

Too high a resistivity, obtained by excessive purification, does not appear to yield correspondingly better cable. In fact, endurance tests on some water white oils and petrolatums have indicated inferior dielectric properties.

The unreliability of a dielectric strength test as a measure of oil quality is shown by the results of measurement on a petrolatum after 1160 hr. heating in air at 100 deg. cent. when the dielectric strength fell only to 38.2 kv. from an initial value of 40.2 kv., the sludge formed, however, being allowed to settle. During the same period the resistivity fell from $500 \times 10^8$ to $1.5 \times 10^8$.

## Measurement of Oxidation

An interesting method for the investigation of the changes produced in an oil by oxidation is that described by I. Langmuir[25] in his investigation of "Oil Films on Water."

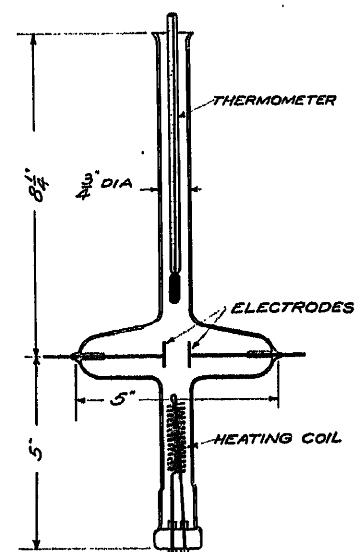



FIG. 20—APPARATUS FOR MEASURING INSULATION RESISTANCE OF COMPOUNDS

A pure saturated hydrocarbon oil has no affinity for water. If a drop of such oil be placed on a clean water surface it remains as a lens without spreading. However, if an acid oil such as oleic acid be used, it will spread out into a film one molecule deep, because the end of the molecule having the OH group dissolves in the water, the other end with only hydrogen atoms exposed being soluble only in the oil. Similarly when an oil is oxidized, the OH group formed will cause the

oxidized portion to spread over the surface of the water. Accordingly by measuring the area of the film formed when a known quantity of oil is dropped on a clean water surface the actual quantity of oxidation may be measured if the size of the molecule is known. This is uncertain in many cases but for approximation purposes these molecules have been assumed to be of the size of a stearic acid molecule.

In order to compare different oils, one gram of oil was dissolved in 10, 50, or more cu. cm. of carefully purified benzol so that two or three-tenths of a cubic centimeter when dropped on water would give a film of about 1000 sq. cm. The benzol rapidly evaporates leaving the oil only on the water. Assuming the cross section of a stearic acid molecule to be $20 \times 10^{-16}$ cm., the amount of oxidized material in the original oil in terms of stearic acid may then be calculated.

The per centage of hydrophils expressed as stearic acid in petrolatum refined to different degrees is given in the following table as well as the resistivities determined at 150 deg. cent.

| | Hydrophils | Resistivity 150° |
|---|---|---|
| Red | 1.82% | $1.82 \times 10^{10}$ |
| Amber | 0.89 | $9.5 \times 10^{10}$ |
| White | 0.35 | $28.5 \times 10^{10}$ |

The effect of light and air upon an insulating oil also produces a corresponding increase in oxidized material and an accompanying increase in power factor as the following figures show:

| | Hydrophils | Power factor |
|---|---|---|
| Original oil | 0.33% | 0.047% |
| 6 hr. in open dish 40 cm. from quartz mercury arc | 2.87% | 0.133% |

A high grade transformer oil heated in air for 120 hr. at 100 deg. cent. developed 4.55 per cent of hydrophil content.

Products of oxidation, therefore, increase the losses in oils very considerably, though other factors, such as sludging or aggregation of oxidized products into discrete particles may greatly modify the relationship between the actual quantity of material oxidized and the resistivity.

The presence of OH groups in an oil is responsible for the formation of emulsions of water and oil. If water is present in insulation containing only pure saturated hydrocarbons, it will occur undistributed in drops which may initiate local breakdown. In the presence of organic acids, however, the water may "chemically" combine so that it will be uniformly distributed throughout the oil and so be in a less dangerous condition. In order to determine whether or not rosin additions acted beneficially in this way, power factor measurements were made upon sheets of undried paper impregnated with oil alone, and with a rosin and oil mixture. The power factor—temperature characteristic is shown in Fig. 21, which shows no essential difference in the characteristics, although it might be expected that if the water were attached to the rosin the power factor would be less than in the case of the oil alone, or that the maximum point on the rosin curve would occur at a higher temperature because of its greater affinity for water. However, it is probable that the oils used for impregnation contain enough oxidized material to take care of any residual water left in the materials after proper treatment.

## Treatment Problems

The use of the methods for determination of power factor and of electrical endurance gives a comparatively simple way of testing modifications of treating methods and the influence of various factors upon life and characteristics of insulating materials.

For instance, the effect of treatment of paper in oil

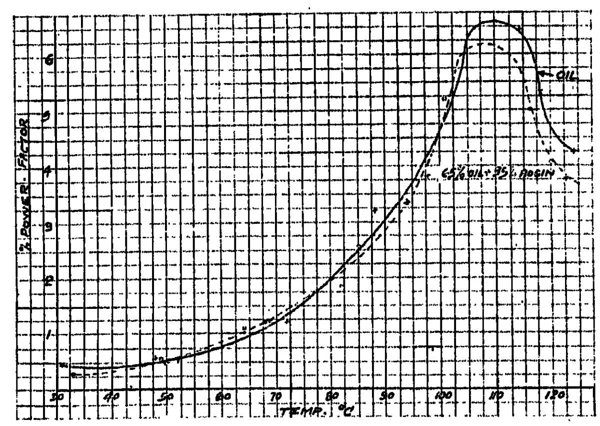


Fig. 21—Dielectric Loss Measurements on Samples of Undried Impregnated Paper

at different temperatures was studied, as shown in Figs. 22 to 24, the chief difference presumably being the amount of moisture left in the paper.

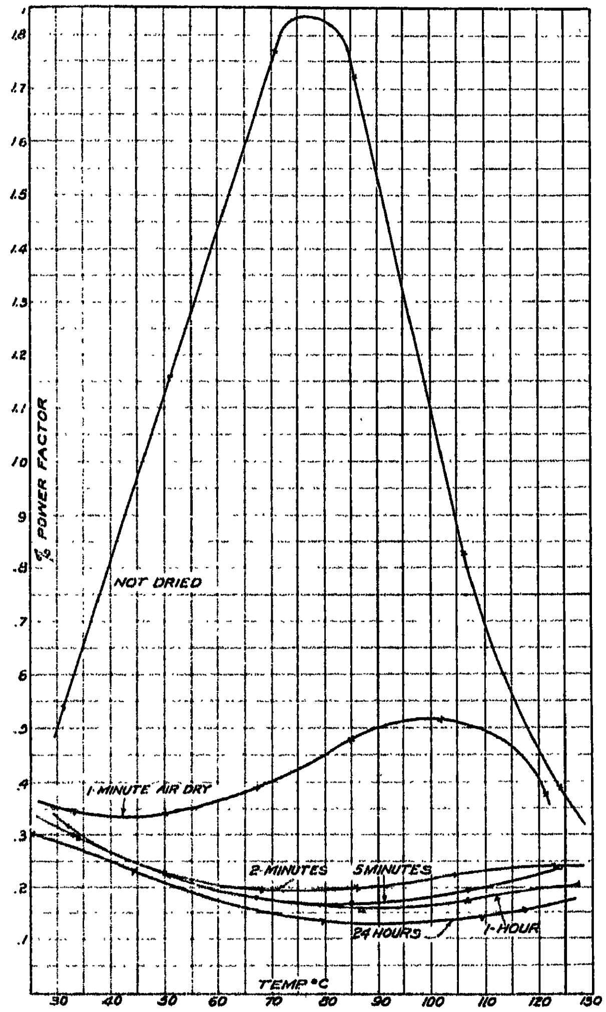
Fig. 22 shows the variation of the power factor of wood pulp paper, as a function of the drying time in air at 110 deg., and Fig. 23, the variation of impregnation time in 118 oil at 80 deg., compared with the characteristics of pre-dried paper impregnated in the same oil for 15 min. Fig. 24 shows the power factor characteristics of similar paper treated in the same oil at 100, 120, and 150 deg. cent. As the temperature is increased the power factor characteristic becomes flatter, and the time necessary for the elimination of water less, so that 15 min. impregnation at 150 deg. gives a better characteristic than 2 hr. at 100 deg. cent.

Fig. 25 shows the relationship between temperature of treatment and endurance of impregnated paper tested according to the condenser method described above. Three sheets of undried manila paper were dipped in petrolatum at the temperatures indicated

for ½ hr. and then placed under stress of 733 volts per mil at 100 deg. cent. until breakdown occurred.

It will be seen that the paper treated at 85 and 115 deg. cent. broke down immediately. In order to equal the life record of the 150 deg. treatment, samples had to be heated for 50 hrs. at 110 deg.

The effect of adding rosin to the oil is shown in Fig. 26 when a steeply rising characteristic is obtained similar to that of moist paper. This indicates that the losses




POWER FACTOR CURVES OF KRAFT PAPER DRIED AT 110° IN AIR FOR DIFFERENT LENGTHS OF TIME

Fig. 22

in a rosin mixture are caused by direct leakage through the dielectric.

The effect produced by eliminating this conduction current is shown in Fig. 27 where a thin quartz plate with negligible losses is inserted in series with a piece of poor quality impregnated paper. The resultant characteristic has lost the high temperature coefficient of loss, as might be expected.

The results of the above experiments show that a high temperature coefficient of power factor, when caused by the presence of moisture, is followed by a short life of the insulation under stress. High power factors produced by other agencies do not show such definite results.

It is also apparent that high temperatures of treatment accomplish the elimination of water much more rapidly than low temperatures and may be more effective than vacuum treatment at lower temperatures.

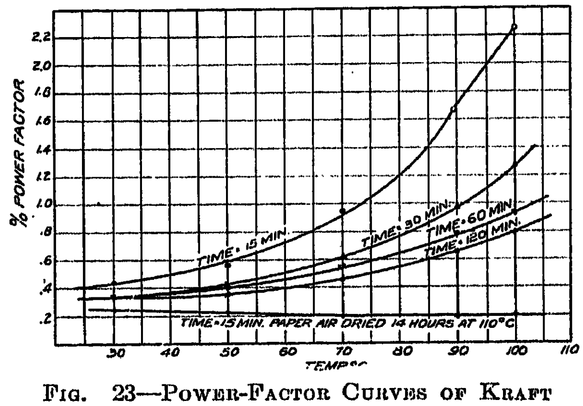



Fig. 23—Power-Factor Curves of Kraft

Paper treated in 118 oil at 80 deg. cent. for different lengths of time and under various conditions

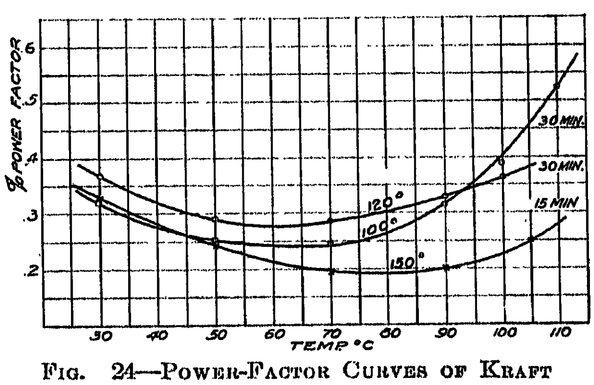



Fig. 24—Power-Factor Curves of Kraft

Paper treated in 118 oil at different temperatures

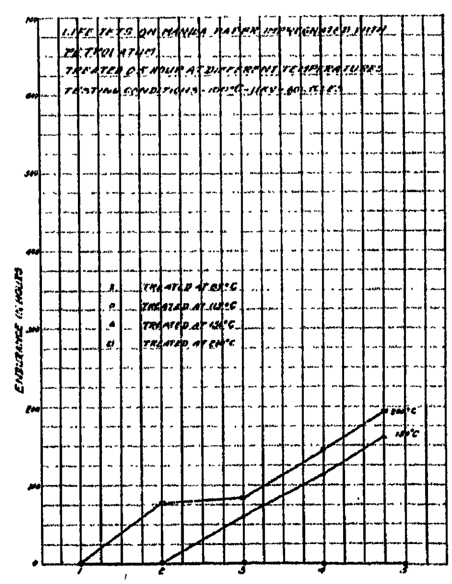
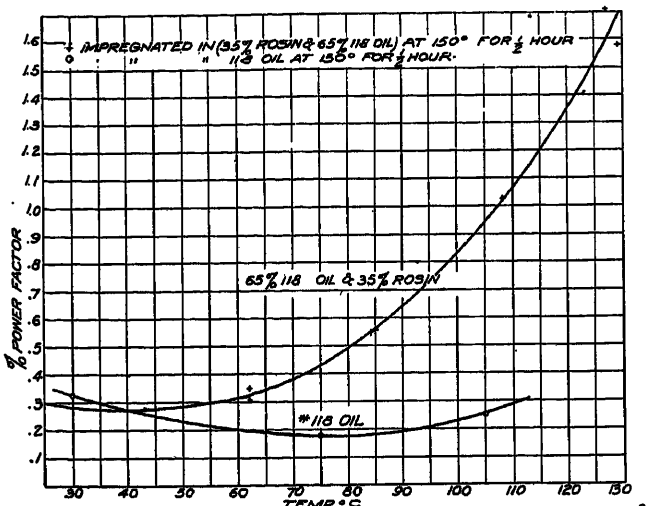


Fig. 25

Specimens in order of failure

However, when using high temperatures, the time of exposure must not be so long as to injure the paper mechanically, nor to oxidize the compound unduly. Compromise between the factors involved is therefore necessary.

In order to determine the characteristics of actual cable exhaust at different temperatures four pieces of lead-sheathed hollow-core cable, 23/32-in. Kraft paper insulation, were vacuum treated and impregnated in the laboratory apparatus under the same conditions except that of temperature, the latter being 80, 100, 115, and 130 deg. cent. A fifth length was treated at 130 deg. with badly oxidized oil, having a resistivity about 1/8




POWER FACTOR CURVES OF KRAFT PAPER AIR DRIED 2 HOURS AT 115°C

FIG. 26

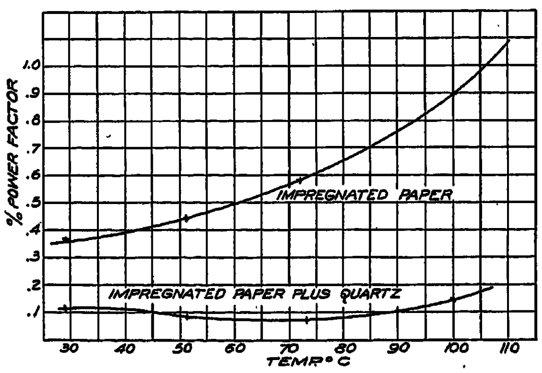



FIG. 27—POWER-FACTOR CURVES OF MANILA PAPER AIR DRIED 2 HOURS AT 110 DEG. CENT. IMPREGNATED IN No. 118 OIL AT 150 DEG. FOR ½ HOUR, SHOWING RESULT OF PUTTING QUARTZ PLATE IN SERIES WITH PAPER

of the regular material. The time of vacuum treatment was 120 hr., and the pressure less than 1/10 mm.

After treatment power factor was measured by the Schering bridge method at temperatures from 25 to 100 deg. cent. at 20 to 76.4 kv. and insulation resistance at 1000 volts d-c.

The following characteristics were obtained:

| Temp. of vacuum treatment | Insulation resistance in megs. per mile | | Per cent Power Factor | | |
|---|---|---|---|---|---|
| | | | 20 volts/mil | | 100 v/m |
| | 25° C | 100° C | 25° C | 100° C | 25° C |
| 80° C | 195 | 8.1 | 0.80 | 6.6 | 0.90 |
| 100° C | 569 | 24.3 | 0.43 | 1.3 | 0.41 |
| 115° C | 823 | 50.2 | 0.43 | 1.1 | 0.40 |
| 130° C a | 1722 | 125. | 0.41 | 0.89 | 0.37 |
| Length treated with oil exceptionally low in resistivity | | | | | |
| 130° C b | 387 | 17.5 | 0.47 | 1.6 | 0.45 |

The following conclusions may be drawn from these results:

(1) Temperatures below 100 deg. cent. are decidedly too low for proper treatment.

(2) The electrical characteristics of the completed cable improve as the temperature of vacuum treatment is increased to 130 deg. cent.

(3) Insulation resistance at room temperature shows greater variation with moisture content than does power factor.

(4) The temperature coefficient of power factor appears to give the most sensitive indication of the presence of moisture.

(5) An excessively high ratio of power factor at 100 deg. to that at 25 deg. indicates incomplete removal of moisture.

(6) A high power factor or low insulation resistance is more likely to be characteristic of the presence of moisture than of poor oil.

Insufficient work has been done in correlating these data with those for other types of cable so it is too early to formulate definite rules for general application.

The principal improvements forwarded by the foregoing tests are as follows:

(1) The development of a uniform high grade of wood pulp paper, having the following advantages over manila:

(a) Greater mechanical uniformity.

(b) Higher and more uniform dielectric strength.

(c) Greater freedom from impurities and conducting particles.

(d) Lower and more uniform power-factor characteristics.

(e) Greater ease of drying.

(f) Better characteristics with respect to impregnating compound.

(2) The securing of insulating oils, uniform in quality, and the maintenance of their uniformity in characteristics during the factory process.

(3) The determination of the magnitude of effects produced by definite additions of impurities or mechanical changes in the insulating materials used.

(4) The determination of optimum conditions for treatment and impregnation.

In conclusion the writers wish to emphasize the fact that they have in this paper correlated the efforts of

many individuals. In addition to those mentioned in the text especial contribution of thought and effort has been made by Messrs. W. S. Clark, W. C. Hayman, C. A. Piercy, E. S. Lee, E. D. Eby, and V. A. Sheals.

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## Discussion

**M. T. Crawford:** Operating experiences discussed by the Underground Systems Committee, N. E. L. A., last week in Pittsburgh, seemed to indicate that the mechanical performance of cables and splices under temperature changes, as referred to in this paper, make it imperative that modern highly impregnated cable be tightly packed and assembled, or else much thicker lead be used on splicing sleeves than has heretofore been the practise. Instances were reported of the swelling and bursting of ordinary ⅛-in. splicing sleeves on first application of load current to 11-kv., and 13-kv. cables. Although these cases represented a very small percentage of the cable recently made, a case in my own experience was so severe that a 1-mi. cable run of five 11-kv., cables could not be operated over half normal loading without excessive bursting of splices, and it was necessary to tap and bleed cable compound from the splices in order to put the cables in full-load operation.

In this case, the only unusual features of the installation were the facts that the cable sheath possessed extra strength due to a content of tin, and observations had been noted at time of factory inspection and tests of loose filler, and of slight looseness of sheath at ends of certain cable lengths.

Assuming, for instance, a cable sheath diameter one-half that of the splicing sleeves, and the same thickness and quality of lead in each case, (this approximating current practises) the wall of the sleeve will be subjected to twice as great a unit stress as the wall of the sheath, providing the cable is constructed loose enough to permit ready longitudinal flow of compound and transfer of pressure into the splices. The fact that the great majority of recent solid-type cable installations has not burst splices must be due to the fact that the cables are built so tightly that this ready longitudinal flow is not permitted, and the pressures developed are therefore largely taken up by slight stretching of the sheath in increments of the cable length. However, from this paper I conclude that ready flow of compound along a cable and some looseness of assembly, together with maximum impregnation, may be desirable features in solid type cables, for electrical reasons.

I should like to ask the authors:

1. If the above conclusions are in their opinion correct, and if fully impregnated cables *are* better with slightly loose construction, should not present splicing standard practise be changed, to include sleeves of proportionate strength to that of the sheath, or else be provided with reservoirs or expansion chambers filled with oil, even in the moderate voltage class solid-type cables.

2. If looseness of construction in solid-type cables is not a desirable feature, should not specifications cover this point more specifically in cables which are to be fully impregnated, in order that standard splices will not be damaged by an occasional variation in tightness which heretofore has not been considered of importance.

**F. A. Brownell:** A comparison of tests made on cable 5 years ago with the present might be of interest, to show the tremendous advancement that has been made in the art in the past few years.

The maximum time obtained on a test of 5 samples, 5 years ago, was 23 hr. at 192 volts per mil and 5 hr. at 224 volts per mil; present tests give 226 hr. at 202 volts per mil (without failure) and 95 hr. at 224 volts per mil.

Analysis of cable after tests made 5 years ago showed heavy wax formation, the entire test section was filled with dendritic designs and heavily carbonized spots, evidence of extreme heating and a rank odor which indicates heavy overstressing were found. The tests that we are making at present show the cable to be practically free from wax and relatively free from dendritic designs, except in the region of the failure. We do not get heating due to ionization that we did in the past. We have had cables on test for 25 hr. at 260 volts per mil with practically no rise in temperature.

One great advancement has been in the general use of shielded-conductor cable for voltages over 25 kv. In this type of cable the stresses are at right angles to the conductor at all times, thus eliminating the tangential stresses and the ionization and heating that occurred in the center filler space of the belted cable.

The soaking in of the compound with time, is no doubt a most important detail in manufacture. If the paper is not thoroughly saturated the free oil is drawn from between the layers into the fibers of the paper, after the cable has been in service for some time. This leaves voids in the cable which in time are ionized. We find plenty of evidence of this in cable that has been taken from service.

We find in many cases that in the region of failure on laboratory tests the insulation is water stained, which checks the findings of Hirshfeld, Meyer, and Connell.

Extreme care should be used in handling oil for joint filling. We have tested oil that has been shipped in sealed containers and it has shown a dielectric strength of 15 kv. Consequently all of our oil is tested before being used.

I should like to ask the authors what they consider the maximum safe voltage for a "solid" single-conductor cable.

I believe most of us are in agreement with the authors that the short time breakdown test is of little value and consider it a waste of time to make them.

I want to bring up the subject of wrinkled cable. The best tests that we have ever obtained were on cable that was rejected due to wrinkles. Analysis of failures on 12 samples tested showed that none of the failures occurred in or were due to wrinkles, although some of the cable was extremely wrinkled. I trust that manufacturers will not use this as an argument for selling us wrinkled cable. My object for bringing up this subject is to ask the authors if they can visualize what will happen in these wrinkles after the cable has been in service for a few years.

**E. W. Davis and W. N. Eddy:** The data tabulated on page 366 are of considerable interest in indicating the relative difficulty of removing moisture from the oil filled hollow-core type of cable with the sheath in place. In the course of a laboratory investigation the following impregnating procedure was applied to a piece of single conductor solid core cable insulated with 24/32 in. of wood pulp paper, without the lead sheath. The cable was subjected to 50 hr. atmospheric drying at 105 deg. cent., 12 hr. vacuum drying at 80 deg. cent., 3-mm. pressure, 10 hr. saturation at 80 deg. cent. with a semi-solid compound, and then cooled to room temperature. While this procedure appears far less rigorous than the authors' 120 hr. at 0.1-mm. pressure, the cable gave a power factor of 1.8 per cent at 100 volts per mil, 100 deg. cent. For a solid cable such a power factor is not considered sufficiently high to indicate the presence of an excessive amount of moisture. It is far lower than the hollow-core cable after 120 hr. at 80 deg. cent. and 0.1-mm. pressure.

Fig. 26 suggests that the addition of rosin to the compound might be expected to increase greatly the influence of temperature on the power factor. That this may not always be the case is indicated by the following experiments with single conductor No. 2/0 cable insulated with 9/32-in. wood pulp paper. Pieces of this cable impregnated in the laboratory with a cylinder oil (very similar in viscosity to the authors' No. 219 compound) gave a power factor of 1.1 per cent at 100 volts per mil, 80 deg. cent. while the power factor of the compound itself was 1.6 per cent at 80 deg. cent. Using the same impregnating procedure in each case, similar cable was impregnated with other compounds. A mixture of the same cylinder oil after some oxidation (compound power factor of 2.2 per cent with 15 per cent of a wood rosin) resulted in a cable power factor of 5.5 per cent and a compound power factor of 6.8 per cent. A mixture of the same cylinder oil unoxidized (1.57 per cent power factor) with 15 per cent of a gum rosin resulted in a cable power factor of 1.7 per cent and a compound power factor of 1.01 per cent. (All power factors given are at 80 deg. cent.) Tests on the two rosins separately showed no wide difference in electrical characteristics. The only difference in the mixing procedure was that the wood rosin was added to the cylinder oil with both materials at 120 deg. cent. and stirred at that temperature for 30 min., while the gum rosin in a wire basket was submerged in the cylinder oil with the latter at 120 deg. cent. until complete solution and for 30 min. thereafter.

The authors suggest that the electrical quality of cable insulation may not be so sensitive to the presence of torn tapes as is sometimes assumed. In the laboratory, high-voltage cables are continually being tested to destruction. During the last three years not one of these failures has been found due to torn paper. Only one was due to wrinkled paper. As a laboratory failure is not burned so badly as a service failure any torn paper at the fault would usually be easily seen.

**Herman Halperin:** Two leading European cable engineers recently made statements to me which are in agreement with the authors, that is, the development field ahead for the "solid" type of high-tension cable extensive is still quite excessive. These engineers cited considerable work being done in this direction in Europe.

On the basis of theoretical and field investigations in Chicago in the past four years in connection with oil-filled joints, I cannot be as hopeful as the authors are for the maintenance of impregnation of entire lengths of cable having solid insulation by means of reservoir feed at the joints.

In connection with a recent examination of over 100 samples of three different makes of 66-kv. single-conductor cable, which had been in service for 20 months, it was found that the oil from joints fed from gravity reservoirs traveled only 5 or 10 ft. into two of the makes, while it was present 20 or 30 ft. away from the the joints in the third make. The presence of the oil was determined by those present, including the manufacturers' representatives, by the appearance of the insulation, and perhaps there was some oil still farther in the cable, but this amount would be so small that it could do very little good in connection with the filling of void spaces during decrease in load.

On the other hand, there was one length of cable installed in a tunnel which is usually almost completely filled with water, but in connection with some special work the water was removed. Oil from the joint at the top migrated down the vertical length of cable (a distance of 85 ft.) and thereby caused a hydrostatic pressure of 35 lb. per sq. in. inside the cable. The sheath cracked at a defect. The distance the oil traveled was a few hundred feet, but the pressure of 35 lb. per sq. in. is considered very excessive for single-leaded sheaths.

If the pressures on the oil in the joint are to be limited to some value of a few pounds up to 10 to 15 lb. per sq. in., it seems to me that the oil will travel entirely too slowly to affect void formation materially during temperature drop in the middle of insulation 200 or 300 ft. away from a joint. There is the great resistance to longitudinal flow and then, even were there fairly free channels along the conductor and sheath, there is large radial resistance to flow of oil.

Our experience confirms some of the authors' theories in that we have found that with lengths of the same vintage of cables installed on two different lines and operating through different temperature ranges, the migration of oil from the joints into the cable was proportional to the temperature ranges. Furthermore, we have found with both single and three-conductor cables that while the total amount of oil taken is increasing, the rate of absorption is decreasing slightly each year. It is hoped that the authors' theories that the sheath will not rupture eventually are true.

Our experience with some three-conductor, 33-kv. cable and the 66-kv. cable, which was furnished us in the past three to

eight years giving considerable trouble in service, was that neither the operating temperature nor temperature range had any primary influence on the rate of failures. When such cable, however, is greatly improved and the product is uniform, it appears from theory that the temperature range will have an effect; but at present I do not agree with the authors that the present temperature limits should be decreased.

Another point is on the third page, where the authors say that when holes occur in the sheaths the water is allowed to come into the insulation and usually wax and ionization is found. Our experience does not agree with this generalization.

**T. F. Peterson:** It is quite gratifying to me to see that some of the conclusions reached by the authors are in line with contentions that have been held by myself and others in the Brooklyn Edison Company and other utilities using reservoirs on solid types of insulation.

It would seem to me, however, that the time has come for a really rational consideration of impregnation and reservoir use, covering cable throughout the entire range of operating voltage. We in the electrical industry are not unlike those in other spheres of life; we have a tendency to move in cycles; the pendulum motion seems to be quite characteristic of our motion. We get into a certain line of throught, run away with it, and finally slow down and retrace our steps. An illustration of this was had a few years ago when power factor of cable was predominant in our minds. All efforts were exerted to reduce the power factor of insulation. Finally compounds were introduced which, though they corrected power factor, ultimately gave rise to wax formation, and so had to be modified or abandoned.

It seems to me now that the question of ionization, after having been so fully studied, is going to lead us into a similar condition. Though we should bear all pressure on the higher voltage cables and endeavor to eliminate voids and air spaces as much as possible in this range, it does not necessarily follow that the same principles should be applied to the lower voltage cables. I have in mind a range from about 600 volts to 13,000 to 15,000. We have had illustrations of cable sheath and joint failure due to expansion of compound. Others have experienced difficulties with compound emerging from oil impregnated low-voltage cable into air-filled potheads, and the like. What is the solution to this? Obviously, reservoirs on such cable systems are impractical, since, for the most part, hard compound is used for filling joints.

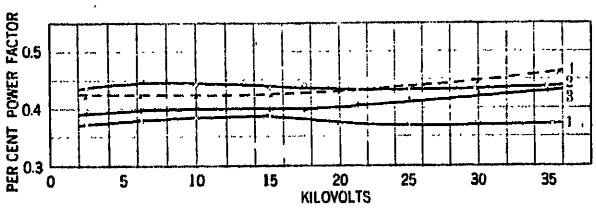
My contention is that in such cable it is perfectly permissible to allow a certain small percentage of air within the cable to provide for the expansion and contraction of compound without the development of extreme pressure or very low vacua. The cable can be designed so that these void spaces will not have detrimental effects at operating voltage, and will also serve to eliminate most of the difficulties that have been experienced. Therefore, in the case of low voltage cable, say to 13,000 volts, it would seem that 2 per cent of air running up probably to 6 and 7 might be allowable.

Carrying this thought a step further, it might be argued that we ought to have no impregnation but air, throughout the cable length. However, the factor entering and making that impossible is, of course, thermal conductivity of the insulation. With all air, the carrying capacity of the cable would be materially reduced. Nevertheless, it is perfectly possible to strike some happy balance of air content with which we will not run into the difficulties that have been cited this morning, but which will also yield desirable thermal characteristics of the insulation.

The range from perhaps 20,000 to 60,000 volts will probably be taken care of with a semi-solid insulation with reservoirs at joints and above that, of course, the regular oil-filled cable would serve.

I am stressing this point because I think it is necessary in this connecton to view also the tests that are applied to cables in determining the quality rating. Obviously if we think it permissible to have a certain percentage of air in lower voltage cables we shouldn't judge their serviceability by ionization characteristics, as in the case of high voltage cable, nor should we judge their serviceability by accelerated life tests that are obtained at gradients probably three or four times those used in operation.

**C. L. Dawes and P. H. Humphries:** At the Harvard Engineering School we are studying the progressive changes in cable characteristics as they undergo accelerated life tests. Five 10-ft. samples of 300,000-cir. mil, 3/16-in. wall, single-conductor cables were put on test at an average gradient of 225 volts per mil. These samples were submitted by four different manufacturers, one manufacturer submitting two samples. Four have already failed. An examination shows that in nearly every case the tapes adjacent to the copper were perforated by ionic bombardment. Moreover, the portions of the paper which lay over the interstrand spaces showed either deposits of wax or charred compound. This would seem to bear out the statement that voids have a tendency to collect in the strand spaces. This would also indicate that the interstrand spaces were not entirely shielded electrostatically. Otherwise, the ionic perforations and disintegrated compounds would not have occurred as we found them.




POWER FACTOR *vs.* VOLTAGE CURVES OF CABLE

Frequency — 60 cycles. Temperature — 22 deg. cent.

| Curve | Life in hours |
|---|---|
| 1 | 0 |
| 2 | 83¾ |
| 3 | 170½ |
| 4 | 470½ |

We are able to confirm the statement that for all practical purposes a cable may be perfectly impregnated as shipped from the factory. Among the cables under test, the initial power-factor curve of one is shown as 1 in Fig. 1 herewith. Curve 2 gives the power-factor curve after 83 ¾ hr. test; Curve 3 after 170½ hr. test, and Curve 4 after 470½ hr. test. The cable has been on test to date 532½ hr. and although its characteristics have been obtained daily, no appreciable change in the power-factor from that given by Curve 4 has been found. The relation of power and capacitance to voltage has also assumed constant values. The cable to date after 532½ hr. test has shown no indications of failure, whereas the last cable of the other four failed at the end of 124½ hr. Hence, it appears that the perfectly impregnated cable has long life.

On page 346 it is stated that a shield under the sheath of single-conductor cable may be tried. We have made tests on three such cables, submitted by three different manufacturers. The same manufacturers submitted simultaneously, cables nearly similar but without shielding tapes. The complete report of our tests was made to the N. E. L. A. Underground Systems Committee Meeting at Louisville, March 22, 1928. Even after severe bending tests, the cables with the shielding tapes showed no marked superiority over those having no shielding tapes.

Referring to Fig. 11, the authors show a decrease in voltage across a gas discharge after the current has reached a critical value. This is undoubtedly true where the discharge occurs in a comparatively long tube as indicated. However, we have investigated the discharge characteristics of a number of air films

as shown in Figs. 8 and 9 of our paper. The voltage increases to a maximum and constant value. Under the highest ionic current densities which we have been able to produce (4 micro-amperes per sq. cm.) there has never been any indication of a drop in voltage with increasing current. Hence it does not appear probable that the reversed type of curve occurs in cable insulation. Perhaps this is due to the more rapid recombination which occurs in restricted gas films. We are in complete accord with the statement that very little is known regarding the laws of ionization loss in cables. We have been attempting for some time to determine the laws of ionization of gas films and some of our results are given in our paper. When a few more data are obtained, we hope to be able to state definitely the effect of pressure, gap length, frequency, etc., on the loss.

We are particularly interested in Figs. 13 and 14 which give power-factor characteristics of a cable with artificial gas films at different pressures. Reference is made to the relation of negative slope in the power-factor curve to negative volt-ampere characteristics of the gas. This is contrary to our experience. We have investigated the properties of such gas films down to 10-in. pressure, and have never obtained a negative volt-ampere characteristic. The variation of the power factor curves is readily explained by Equation (12) in our paper, which may be rewritten as follows:

$$\text{P. F.} = \frac{1}{C\,\omega}\left[K + K_1\left(\frac{1}{E} - \frac{E_0}{E}\right)\right]$$

The rate of increase with voltage of the capacitance $C$ increases rapidly with decrease in pressure. Hence, decrease in pressure lowers the power-factor curve as the voltage increases. The first term in the brackets is a function of the solid dielectric, and does not change with pressure or voltage. $K_1$ decreases with decrease in pressure. For example, at atmospheric pressure, $K_1 = 39.3$; at 10-in. pressure, $K_1 = 10$. $E_0$ obviously decreases with pressure. This can be readily seen in Figs. 13 and 14. Hence $E_0/E$ is smaller for lower pressures than for higher pressures. Thus at the lower values of $E$, the parenthetical term is larger for the lower pressures than for the higher pressures. As $E$ increases $E_0/E$ becomes smaller in comparison with unity and the parenthetical term becomes more nearly constant. The lower value of $K_1$ for the lower pressures now causes the power-factor curve to decrease at a more rapid rate. Hence, the curves at different pressures should cross one another which they actually do in Figs. 13 and 14, the curves for lower pressures ultimately having the lower values of power factor.

**R. W. Atkinson:** I find myself in very good agreement with most of what is given in the paper by Messrs. Shanklin and Mackay.

As to specific points in the paper, I will comment on one only. Mention is made of the relation between the volume of void space, as defined, and the amount of ionization. We have considered that a given amount of ionization represents a considerably greater void space than is indicated by the authors. I think that the difference is largely in the method of measurement of the void space. As the authors point out, this is not a simple matter to determine, and I think that probably both of us should study that question more and find out whether the void space is actually that given by the authors or whether it may be an amount perhaps several times as large as that.

I should like to say a word in summary of the trend of development in the future, an analysis which is right along the line that the authors have given. In fact, I have taken part of it from there. I think the future development is going to be along two or three or four distinct lines, each one being carried forward and ultimately perhaps resulting in things rather different, or perhaps the trend may bring the different lines together. The oil-filled cable development is one of those. That development, as the authors point out, is going to be largely in the perfection of manufacturing and installation, and in the reduction in cost. In addition, there will be the development of stop joints that are much less expensive and can be placed at closer intervals.

Then another trend of development will be in the direction of an approximation of the oil-filled cable, an attempt to retain the conveniences of the solid type and the electrical advantages of the oil-filled type. There is no doubt there will be important developments in that direction.

An important direction of development of the solid type of cable will be in the mechanical characteristics of the compound so as to minimize void formation. As pointed out by different authors, an attempt to find a compound which is immune to the effect of the ionic bombardment has not proved at all promising.

The development in both the oil-filled and the solid type of insulation seems to be toward minimizing that bombardment by minimizing the voids. Of course we will have improvement in the characteristics of the paper and in the general method of application, and of course we will have improvements in the general matter of increasing uniformity and in the obtaining on all cables of results substantially equal to those obtained on the best of them.

There is one other phase of development that has been remarkable in its rate of activity recently and which undoubtedly will continue in the same direction. I refer to the shielded type of cable. Some may wonder why the shielded type of cable, since it has been known as many years as it has, did not take large commercial importance for some time, and then having taken a hold, has gone with such extraordinary rapidity. I think three reasons have contributed to that: first, of course, the very successful results of the earlier experience with the shielded type of cable; then, the recognition of its value; and finally, the fact upon which I commented at the N. E. L. A. Convention in 1923, that the better we make our insulation, the more advantage we obtain by the shielded construction. As we develop the quality of our insulation, the field for the shielded-type cable becomes greater and greater, and this is going to continue and bring its use down into the lower voltages as well as increase the predominance it already has in the higher voltage range.

**E. S. Lee:** There is an achievement recorded in this paper which is most inspiring.

In a most intensive effort to obtain better cable insulation many samples of different materials made up with different treatments were tried by subjecting them to 44 kv. at room temperature until they failed. The life of the many different samples fell between 5 hr. and 162 hr., which was certainly not encouraging.

However, similar samples maintained at a temperature of 85 deg. cent. continued to live under the applied voltage for many hours, reaching in some cases 1000 or 2000 hr. All that one had to do to get the sample to fail was to open the heater switch and let the ambient temperature come down to that of the room, after which the sample was sure to fail within a few hours.

What was the reason for this remarkable difference and long life at the higher temperature? Investigation revealed that it was simple due to the treating compound being liquid at the higher temperature and solid at room temperature. The suggestion of using a liquid oil filler was immediate.

Subsequent tests proved that by using liquid oil as a treating material the voltage rating could be doubled. This was a most remarkable achievement in insulation design and one not very frequently met. However, the difficulties of using liquid oil prevented the immediate application of so signal an advance.

Opportunity was later afforded by the New York Edison Co. and the Commonwealth Edison Co. for the installation of oil-filled cable for 132-kv. circuit. The cable installed was for double the voltage of the 66-kv. cable then in operation but had only 77 per cent of the insulation thickness. The electrical performance of that cable has been perfect.

The reason for the achievement is in the uniform and perfect filling of the insulation throughout the operating temperature range. The result came from intensive application in the

laboratory by observing carefully all the related phenomena. As it turned out, the only equipment necessary was a high-voltage transformer, a voltmeter, a clock, means for connecting the transformer to the cable sample, and means for holding the cable sample at 85 deg. cent. I point this out as encouragement to those who may not have extensive equipment available but who still may be in position to contribute worthily.

**N. E. Buck:** (communicated after adjournment) Oil reservoirs connected to the joints are probably of benefit to the cable adjacent to the joints in the following ways:

1. Gas bubbles can be expelled from the cable ends, and find their way into the reservoir. On the contrary, there can be no transfer of free gas from the reservoir to the cable.

2. The principal oil-flow in the cable will take place in the filler spaces, between sheath and insulation, or between conductor strands. Gas bubbles which may be present in these channels will probably be carried along with the oil flow. Thus when the heat cycle causes oil to be returned to the joint, the free gas which flows with it will become trapped in the reservoir. The converse effect cannot take place, as pointed out in (1).

3. The pressure, and therefore the amount of dissolved gas, will be lower when oil is being drawn into the cable than when it is being forced out. The net result will be a removal of dissolved gas.

From the above it is clear that there will be a marked tendency for the amount of free and dissolved gas to diminish, in the cable adjacent to joints equipped with reservoirs.

The flow of oil through the body of the insulation is probably small, and the gas entrapped in the capillaries is highly resistant to motion. The only way in which oil reservoirs at the joints could have a beneficial effect on the cable insulation remote from the ends would be to maintain a high enough pressure to keep the gas dissolved.

**W. A. Del Mar:** (communicated after adjournment) The authors reiterate the statement that a cable should be gas free, but neither they nor anyone else has offered any evidence to support this theory.

The statement that reservoir oil contaminates the cable compound and lowers dielectric strength is contrary to theory and experience. Indeed, operating companies report that the cable near the reservoirs is greatly improved in quality.

There is a notable absence of reference to the properties of cable compound with respect to gas absorption, although statements are made about the effects of gas dissolved in oil. This is a matter of great importance as it is known that oil dissolves gases in proportion to the pressure applied and that some oils, unlike water, hold less gas at low than at high temperatures. The published data on the subject are conflicting and experimental work is beset with difficulties.

**D. W. Kitchin:** (communicated after adjournment) Some work on the subject of gas and "x" wax formation done in our laboratory about three years ago is in agreement with the conclusion of the authors and may be of interest. It was possible to produce gas and "x" in all the materials mentioned by the authors except acetylene, which was not tested, when the arrangement was such that corona was *certain* to occur. A rigorous demonstration that stress alone could in no case form "x" or gas was more difficult. Tested in a glass apparatus in which corona could not occur in the evacuated space over the materials, none of them showed any effect in the liquid or molten state, but some greases, notably petrolatum, produced both gas and "x" fairly long periods of stress although initially no voids could be detected visually.

Petrolatum is plastic and requires a certain minimum pressure to make it flow. When allowed to cool in a highly evacuated vessel, the weight of the grease was not sufficient to make it contract absolutely in one piece and voids, in some cases microscopic ones, were produced.

After altering the apparatus so that the previously evacuated grease cooled under a mercury head of an inch or so, sufficient to overcome the stiffness of the compound, *all* the materials could be stressed indefinitely without change. These results serve to confirm the conclusions of Shanklin & Mackay on "x" formation but they also suggest some definite ideas on the value of the Electrical Testing Laboratories' plate test as a criterion of compound stability, which it seems appropriate to express at this time. Many cable engineers are familiar with the conflicting results obtained from this result. Successive tests on the same sample of compound often show contradictory results and often compounds form "x" on the test but not in service, and vice versa. Why does the plate test fail to furnish an infallible criterion of compound stability? Because the action of corona or ionized gas is necessary to cause the "x" and gas production, and corona can take place only when voids or gaseous spaces are present. The methods involved in performing the plate tests do not insure either the *presence* or *absence* of voids, so that whether a material fails or not in that test is largely a matter of chance. It is true that plastic materials like petrolatum are more likely to contain voids when placed between the plates than a viscous oil, and if a large number of plate tests are made on different compounds and the results treated by statistical methods a certain trend may be shown which might have been predicted by merely noting the physical properties of the materials.

The things we want to know and which the plate tests indicate only vaguely are, (a) what compounds form voids most readily, and (b) given the presence of corona, what compounds are most stable to it? To answer (a), measure directly the coefficient of expansion and the plasticity (not the viscosity), or most simply let the compound cool in a long glass tube and note the formation of voids in the body of the material as described in the papers of Shanklin and Mackay, and of Davis and Eddy.

To answer (b) subject the several compounds deliberately to corona under *comparable* conditions and measure the production of gas during a definite time interval.

Since deterioration due to corona is a *surface* action, the apparatus should insure the exposure of a *definite constant* area and the same stress should be applied in each case. A device of the type used in the Detroit-Edison investigation, but arranged to subject the material to corona instead of X-rays is proposed. Data secured in this way should give us a real measure of merit of the various compounds instead of the accidental results of a hit-or-miss method. We need more rational and scientific criteria of compound stability if we are to make progress in the development of better cables of the non-oil filled type. Also, since the plate test gives variable results under its uncontrollable conditions, the proposal to include it in specifications is fundamentally unsound. More scientific tests are suggested above to substitute for the plate test.

**G. M. J. Mackay:** In answer to Mr. Crawford's questions we wish to say that we feel that low-voltage cable should be free from looseness of assembly in order to assure a good mechanical structure and should be as completely impregnated as possible. Installation of reservoirs or expansion chambers at the joints would, of course, eliminate the troubles caused by expansion but the expense may not be justified. Reinforced sleeves or alloy metal at the joints should be sufficient to overcome the difficulty. Enlargement of the cable sheath will then take care of the expansion of the compound.

Specifications covering the type and strength of joint in the case of these cables should be useful to the operator in safeguarding his installation.

Mr. Brownell's testimony on the improvement of cable during the last few years is very gratifying. We feel, however, that it is too soon to make a prediction about the maximum safe voltage for a "solid" single-conductor cable. We rather expect, however, to see the principle of the oil-filled cable extended to lower

voltages than the raising of the present limit for the "solid" type.

Wrinkles, we are afraid, if not the cause of breakdown on test, will eventually be the spots where voids will occur during operation with the eventual formation of destructive ionization.

The table on page 366, mentioned by Messrs. Davis and Eddy, was inserted to emphasize the effect of temperature in drying paper insulation rather than the difficulty of eliminating water. The time involved was arbitrarily chosen and is longer than that necessary to attain constant conditions at 100 deg. cent. The high vacuum may have been disadvantageous, as far as rate of water evolution is concerned, because a current of air caused by leakage, indicated by a higher pressure, may help in sweeping the water vapor out through the condenser and pump. Their tests on rosin and oil combinations are very interesting, apparently showing the betterment of characteristics when oxidation by exposure is lessened by elimination of stirring while the materials are at high temperatures.

While we agree with Mr. Halperin that oil feed is not a perfect solution of the difficulties attendant upon the use of solid cable, we feel that it certainly very materially improves its endurance. His evidence that oil was found 20 or 30 ft. away from the joint indicates that voids would have existed in this region if oil were not supplied, and in this case were eliminated by the oil. The longer the cable operates the freer the oil flow becomes, whereas without oil, conditions would not improve.

Mr. Peterson is quite right, we think, in saying that there is no necessity for carrying all of the high-voltage technique to the construction and operation of low-voltage cables. Gas, if at sufficiently high pressure, should cause no danger from ionization in low-voltage cables but free admission of air would be dangerous on account of oxidation of the compound with resultant increase of power factor and formation of water.

Messrs. Dawes and Humphries have made some important and instructive criticism of the ionization characteristics in gas spaces. We agree with them that at the current densities involved the discharge itself does not have a negative volt-ampere characteristic and that it is unnecessary to assume one in order to obtain the characteristics shown in Fig. 13. Their equation will give the same form of curve.

We do feel, however, that at any given pressure of a gas once ionization has appeared it is not possible to increase the voltage above a certain limit. The lower the pressure the lower this voltage becomes and in the curves of 6, 7, and 8, Fig. 13, it is only a few hundred volts. The total voltage cross the cable and tube is only 500 when ionization exists. Because, however, the cable insulation is intended for 13,000 volts, this means that if the full voltage were suddenly put across such insulation the gas space would act practically as a good conductor, extending the sheath inwards, when almost the entire voltage would be impressed upon the impregnated paper insulation. While then the discharge itself does not have a negative volt-ampere characteristic, if the full voltage of 13 kv. be impressed across the impregnated paper and series air gap, in a sense the resulting characteristic may be said to have a virtual negative volt-ampere characteristic.

Mr. Del Mar has apparently misunderstood us when he quotes us as stating that reservoir oil contaminates the cable compound and lowers the dielectric strength. Only when the reservoir oil is in contact with air so that it deteriorates by oxidation is such an effect possible. Otherwise, of course, it is of the greatest advantage to use oil feed.

We are in complete agreement with Mr. Kitchin's criticism of the inclusion of so-called stability tests for compounds in present specifications.

# Some Problems in High-Voltage Cable Development

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Member, A. I. E. E.

and

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## 1. INTRODUCTION

A LARGE amount of research work on high-voltage cable is being carried on throughout the country in various laboratories by both the manufacturer and operator. It is generally recognized that a considerable amount of duplicate effort can be avoided and general progress increased by comparison and common discussion of the independent investigations.

In order to encourage such co-operation it was thought desirable to present briefly a few of the problems along this line that are receiving attention in the laboratory of a manufacturer.

## 2. MEASUREMENT OF DIELECTRIC LOSS AND POWER FACTOR

In order to obtain greater sensitivity, the high-voltage bridge introduced by Dawes and Hoover[1] was installed for use in conjunction with the dynamometer method already available. This involved the design and construction of a high-voltage air condenser. As some features of this condenser may be of interest it is described in Appendix I.

Inherently the bridge methods are single-phase and therefore not so well adapted to three-phase measurements. However, the connection shown in Fig. 2 which has been developed for this purpose, has been found satisfactory, about its only disadvantage being that a setting without the cable connected must be taken for each test voltage.

## 3. DETERMINATION OF INSULATION QUALITY

At present the determination of the insulation quality of high-voltage cable is an important problem for both the cable manufacturer and operator that is not yet satisfactorily solved. Everyone concerned with such cable is familiar with cases where cable withstood satisfactorily every test imposed by the manufacturer and operator but failed to give satisfactory service after installation.

Considerable experimental work is in progress with the object of improving the quality of cable insulation. It is obvious that the reliability of the experimental indications is entirely limited by the reliability of the criterion of quality that is used.

At one time a "short-time breakdown test" (increasing voltage rapidly enough to cause insulation failure within 30 min.) was considered a reliable indication of the insulation quality. More recent experience combined with extensive laboratory tests has shown the limited value of this test as compared with the "over-voltage life test." The test results in Table I are given as examples of the greater sensitiveness of the life test to insulation quality.

However, a considerable experience with the over-

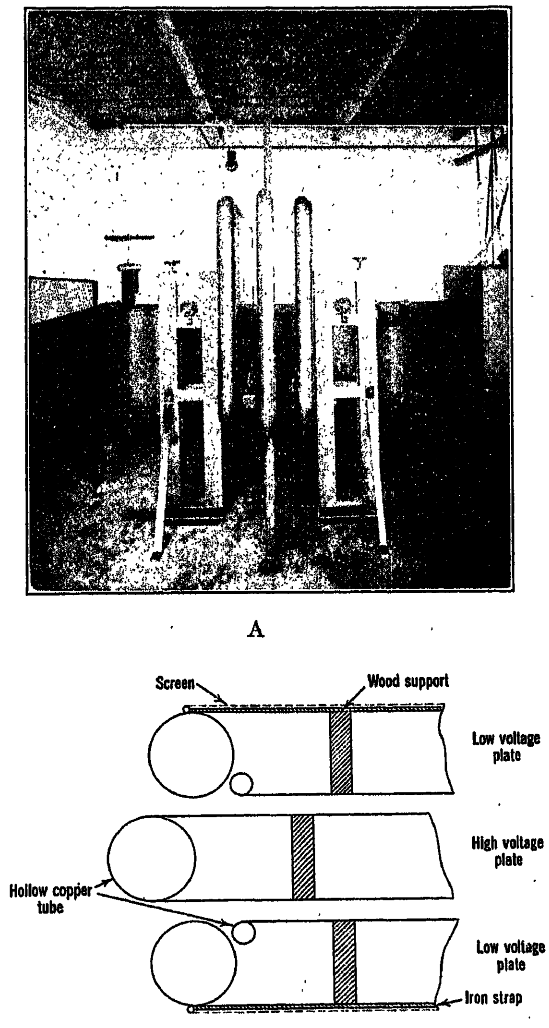


FIG. 1—NO LOSS HIGH-VOLTAGE AIR CONDENSER

voltage life test suggests that even this test may not always give a true indication of relative quality. It is becoming accepted that the cause of most premature failures in service is fundamentally the presence in the insulation of voids that are formed by the shrinkage of the compound when it cools. While the service conditions (extreme temperature changes with the in-

* Simplex Wire & Cable Co., Boston, Mass.

1. A. I. E. E. TRANS. 1926, Vol. XLV, *Ionization Studies in Paper Insulated Cables—I.*

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

sulation approaching a constant enclosing volume condition) encourage this void formation, the life test conditions (constant temperature with the insulation at the ends of the sample exposed to constant pressure) tend to discourage the void formation. Due to the different contraction and viscosity characteristics of different compounds, this tendency may vary with different compounds.

Table II shows the results of life tests on cable saturated with three different compounds at each of several testing conditions. It is seen that the life is in general increased by conditions unfavorable to void formation and decreased by conditions favorable to void formation, the tendency varying with the compounds. In connection with the influence of condition No. 2 on the cylinder oil, it has been generally found that when the life test is interrupted for several hours the life will be increased, the cause being apparently

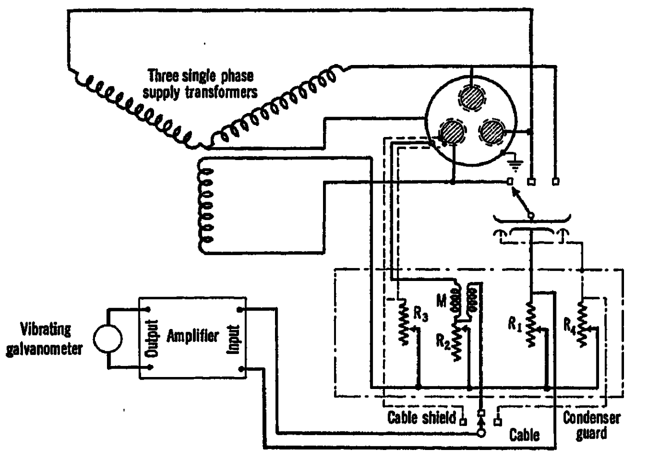


FIG. 2—A THREE-PHASE CONNECTION OF THE DAWES BRIDGE

the same in both cases—the application of sufficient heat to liquefy the compound and improve the void distribution. The relative independence of the rosin compound to testing conditions also merits attention.

Investigation of the life test is in progress with the object of improving its reliability, possibly by some such modification as No. 4.

The principal expense of the life test is that of preparing the ends of the sample. While high voltage cable terminals are available they are expensive and almost impossible to handle without overhead crane facilities. Fig. 3 shows a laboratory terminal that was developed to overcome these disadvantages in connection with a life test at 180 kv. on single conductor cable samples—750,000 cir. mils, 24/32-in. wall.

The ionization test or variation of power factor with voltage at room temperature has received considerable attention as an indication of insulation quality. Fig. 4 shows a comparison between the life test on several cables and an ionization test made just before the life test. While differing in type, dimensions, impregnating procedure, and treatment previous to the life test, the cables were all impregnated with the same compound. The ionization tests were made at 24 deg. cent. and the life test at 20-26 deg. cent.

It is seen that 17 of the 19 samples in Fig. 4 indicate a relation between the ionization test and the life test. A relatively flat power factor—voltage curve seems to correspond in general with a relatively long life. However, the two re-saturated samples give an unusually flat power-factor curve but poor life. They contradict

TABLE I

COMPARISON BETWEEN SHORT TIME BREAKDOWN AND LIFE TESTS

All short time breakdown tests made in accordance with 1926 Specifications for Impregnated Paper Insulated, Lead Covered Cable by Association of Edison Illuminating Companies, Section 20.

All 3-conductor cable tested with 3-phase voltage.

a—3-Conductor 350,000 cir. mils, 19/64 in. Wall, 9/64 in. Jacket.

| | Construction No. 1 | Construction No. 2 |
|---|---|---|
| Short time breakdown at room temp., 10-ft. sample | 201 kv. for 20 sec. | 201 kv. for 15 sec. |
| Life test at 96 kv. room temp., 10-ft. sample | 3.5 hr. | 12.5 hr. |
| Life test at 78 kv. room temp., 250-ft. sample | 22 hr. | 99.5 hr. |

b—Single cond. 750,000 cir. mils, 24/32 in. Wall—all samples 10 ft.

| | Impregnation No. 1 | Impregnation No. 2 |
|---|---|---|
| Short time breakdown at room temp. | 290 kv. | 310 kv. |
| Life test at 180 kv | 2.5 hr. | 30 + hr. |

*Test discontinued—no failure.

TABLE II

OVERVOLTAGE LIFE TESTS ON 10-FT. SAMPLES OF 1 × 2/0, 9/32-in. WALL IN HR. LIFE AT 65,000 VOLTS

| Compound | Condition No. 1 | Condition No. 2 | Condition No. 3 | Condition No. 4 |
|---|---|---|---|---|
| 70% Petrolatum B / 30% Transformer Oil C | 30 | 24 | 215 | 1.0 |
| Cylinder Oil | 50 | 700 | .. | 1.6 |
| 85% Cylinder Oil / 15% Rosin | 9 | .. | 0.5 | 2.2 |

Each figure the average of 3 tests.

Condition No. 1—Tested at room temp. 25-30° C. with thin oil in ends.

Condition No. 2—Tested after 300 hr. at 25-30° C., otherwise same as No. 1.

Condition No. 3—Tested at 40° C. ambient, otherwise same as No. 1.

Condition No. 4—Sample ends sealed at room temp., with pothead compound, then sample plunged in ice and voltage applied. Final sheath temp. 3-6° C.

Although not typical of factory practise, the saturation and handling of each sample was identical. All samples were cut from the same length of cable.

the relation suggested by the 17 samples. That is, while the ionization test may be a fairly reliable indication of insulation quality on certain cables, it is equally unreliable on other cables. Re-saturation may result in an excellent ionization curve but a deficient life.

Fig. 5 shows the power factor of the same 19 samples at 50 volts per mil. Apparently such a value is even less reliable than the ionization test as an indication of insulation quality.

## 4. Air Resistance of Paper

One of the most important properties of the paper is its compactness or denseness. This is usually measured by timing the passage of a given volume of air through a definite area of the paper under a known pressure

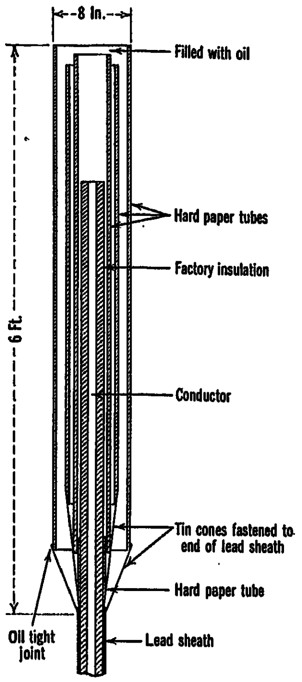


Fig. 3—180-Kv. Life Test Terminal

and is known as the air resistance of the paper. The most familiar units in this country are the seconds necessary for 100 cu. cm. of air to pass through one

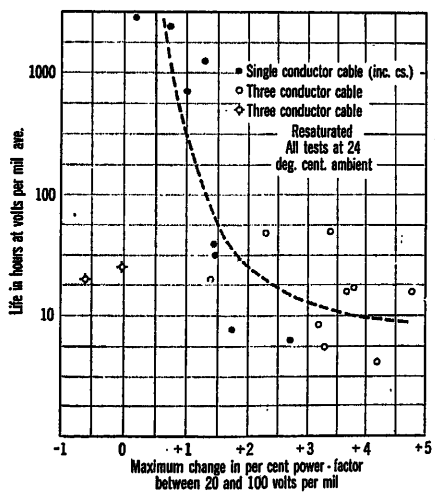


Fig. 4—Comparison between Life and Ionization Test

square inch of paper under 1.25 lb. per square inch pressure.

Fig. 6 shows that the air resistance has a pronounced effect on the dielectric strength of the saturated paper in sheet form and that the effect is independent of the kind of fiber, wood pulp or manila. The papers shown in Fig. 6 represent 12 different manufacturers and therefore should be representative.

In Table III are given the results of life tests on several different cables, both cables of each pair being alike in all respects except the air resistance of the paper. The results show that the paper air resistance

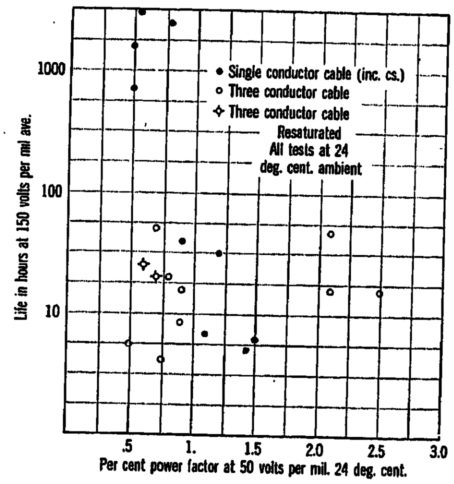


Fig. 5—Comparison between Life and Power-Factor Tests

has a pronounced influence on the dielectric strength of the cable.

Fig. 7 shows that the air resistance also has a marked influence on the penetration of the compound into the paper, the higher the former the slower the latter. The penetration rate of the compound is important to the manufacturer in its influence on the time necessary for saturation of the insulation.

## 5. Selection of Compound

As the power factor of the compound has a controlling

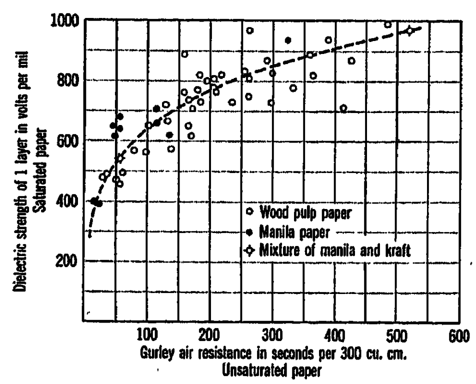


Fig. 6—Influence of Air Resistance on Dielectric Strength of Paper

influence on the power factor of the cable, the deterioration of the compound power factor with heating is of importance.

Without the presence of air, it is insignificant in comparison with the deterioration in the presence of air.

Fig. 8 shows the power factor deterioration of some representative compounds in the presence of air.

Each sample of compound was in the same shape container so that the area exposed to the air was always the same. All containers were immersed in an oil bath. Free natural circulation of air through the oven was provided. The curves show that there is material difference between compounds and that sometimes a blend of compound will show more deterioration than its constituents separately. While the compounds containing rosin show more deterioration at first, they eventually show marked improvement.

TABLE III
INFLUENCE OF PAPER AIR RESISTANCE ON DIELECTRIC STRENGTH OF CABLE

All life tests at room temperature, ambient.
A. R. = air resistance of unsaturated paper in Gurley secs. per 300 cu. cm.

| Size of cable | Life test voltage | Cable No. 1 A. R. | Cable No. 1 Hr. life | Cable No. 2 A. R. | Cable No. 2 Hr. life |
|---|---|---|---|---|---|
| 1/750,000 $\frac{24 \text{ in.}}{32}$ | 180 kv. | 300 | 30 + | 35–300† | 1.0 |
| 1/ 2/0 $\frac{9 \text{ in.}}{32}$ | 52 kv. | 138 | 116 | 30 | 49 |
| 3/350,000 $\frac{19 \times 9 \text{ in.}}{64}$ | 96 kv. 3 phase | 140 | 12.5 | 40 | 3.5 |
| *3/350,000 $\frac{19 \times 9 \text{ in.}}{64}$ | 78 kv. 3 phase | 140 | 99.5 | 40 | 22 |

*Sample 250 ft., all others 10 ft.

†Inside $\frac{12 \text{ in.}}{32}$ of wall A. R. = 300

Outside $\frac{12 \text{ in.}}{32}$ of wall A. R. = 35

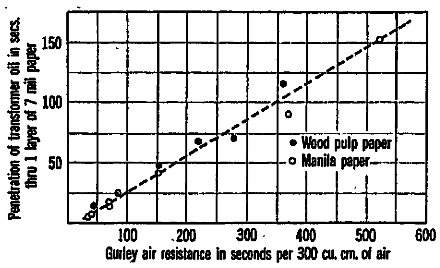


FIG. 7—RELATION BETWEEN OIL PENETRATION AND AIR RESISTANCE

As the formation of the voids that are so harmful to the quality of impregnated paper insulation is caused primarily by the contraction of the compound on cooling, the volume change of the compound with temperature is important. Fig. 9 shows this characteristic for a few typical compounds. It is seen that all the compounds are alike when in a liquid state, but that when they solidify the volume change approximately doubles. Therefore, on cooling from 50 to 25 deg. cent., a petrolatum should form more voids than a cylinder oil.

The viscosity of the compound below 50 deg. cent. is of importance because of its great influence on the formation and distribution of the shrinkage voids (2) and on the drainage of the compound. Fig. 10 shows this property on some typical compounds. It is seen that the movement of a petrolatum through the insulation should decrease sooner and more abruptly with decreasing temperature than that of a cylinder oil.

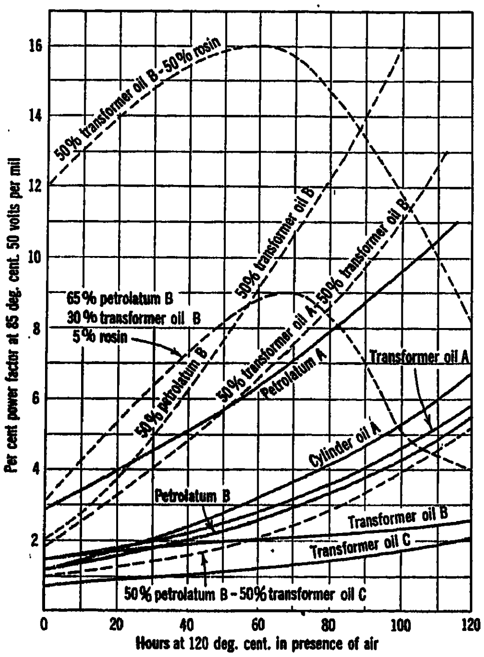


FIG. 8—ELECTRICAL DETERIORATION OF COMPOUNDS

Therefore, the petrolatum should begin to form voids at a higher temperature than the cylinder oil.

As no available viscosimeter was found that would give this range of viscosity, one was developed. It is substantially a weighted plunger moving down into a cylinder slightly larger than the plunger. The cylinder is full of compound which is displaced by the plunger and forced up through the annular opening between the plunger and cylinder.

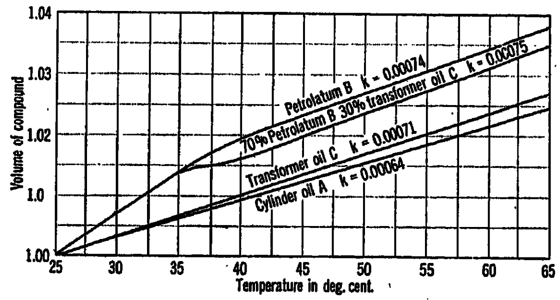


FIG. 9—CONTRACTION OF COMPOUNDS
$K$ = Volume change per deg. cent.

Because of greater simplicity in preparation and testing, miniature samples of insulation (thickness of 50 mils and less) are in considerable use for laboratory comparison of compounds. When tested immersed, these samples represent almost ideal conditions for void elimination and when tested unimmersed, their

failure is usually caused by expulsion of compound. Because of the extreme contrast between each of these conditions and service conditions, it is believed that results of this test are of doubtful value in comparing compounds.

It is now recognized that the formation of gas and $X$ by compounds in the presence of corona (ionized voids) can only be eliminated by the development of a compound of sufficient chemical stability to withstand such

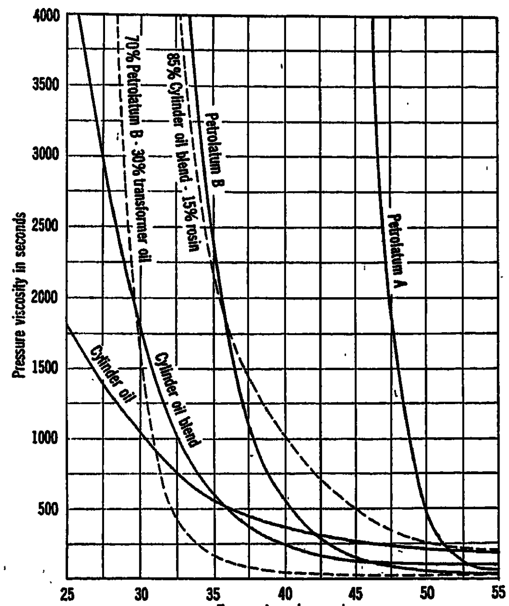


Fig. 10—Viscosity of Compounds

conditions without forming gas. As the immediate development of such a compound is not promising, improved control of the void formation and distribution merits careful consideration as an available means of minimizing the dangerous gas formation and harmless $X$ formation.

## 6. Mechanism of Void Formation and Ionization

Under this heading are given some preliminary conceptions and test results that, while admittedly not conclusive, were thought of sufficient interest for subjection to general discussion.

*A—Influence of Void Pressure and Size on Ionization.*

The upper solid curve in Fig. 11 shows the dielectric strength of air at atmospheric pressure. While the greater part of this curve was taken from an Institute paper (3), the relations have been well established by several investigators independently. The application of Paschen's law (breakdown of air constant as long as the product of the gap length and absolute pressure is constant) to this curve gives the dielectric strength at other pressures as shown.

The dotted lines represent the voltage across each void in insulation 300 mils thick at 100 volts per mil total stress. When this voltage exceeds the ionization voltage (solid curves), the void will be ionized. The curves show that increase of void pressure and more complete distribution of voids (many small instead of few large voids) both tend to decrease the possibility of ionization.

*B—Void Pressure versus Air Content.*

Air can exist in the insulation in two forms, in solution in the compound or separated. It is harmful only when it is separated as it is then subject to ionization.

The solubility of air in insulating oils is such that at atmospheric pressure, the oil will take into solution from 10 to 15 per cent or approximately 13 per cent of its volume of air, the latter also at atmospheric pressure. The influence of temperature is so slight that it can be disregarded for approximate calculations. In terms of the volume of air at atmospheric pressure, the solubility is directly proportional to the absolute pressure; in terms of the volume of air at the pressure under consideration, it is independent of the pressure. For convenience in the following discussion, the solubility will be expressed in the latter terms. In these terms, if the solubility is 13 per cent at atmospheric pressure, it is 13 per cent at any pressure. All air in the compound in excess of 13 per cent will be in the separated form (void).

An air content of 3 per cent at atmospheric pressure would mean that with decreasing pressure, no air would separate out above 17.5 cm. absolute $\left(\frac{3 \times 76}{13}\right)$. Hereafter, the pressure at which the air begins to separate

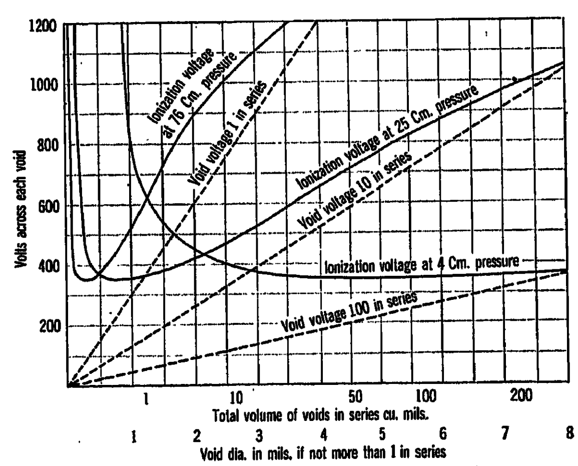


Fig. 11—Influence of Void Size Pressure and Arrangement on Ionization

Thickness of insulation—300 mils
Total voltage—30,000 volts
All voids the same size

out will be called the critical pressure of the compound. It is entirely dependent on the air content of the compound. There should be no separated air at pressures greater than the critical pressure.

If the air content exceeds 13 per cent at atmospheric pressure, the critical pressure will be greater than atmospheric and there will be separated air at atmospheric pressure and, therefore, voids even before the

cable is cooled. Thus, an air content in excess of 13 per cent at atmospheric pressure is definitely undesirable. Hereafter, only air contents less than this will be considered.

Let $V_c$ = the volume of compound immediately surrounding a void, the compound that is subjected to a decreased pressure resulting from the formation of the void.

$V_v$ = the volume of the void.

$P_v$ = pressure of the void in cm. absolute.

$a$ = air content of the compound in fraction of compound volume, the air at atmospheric pressure.

maximum solubility of the air in the compound = 13 per cent.

The total air content of $V_c$ at any pressure $P_v = \frac{76\,a\,V_c}{P_v}$ from Boyles law.

The maximum dissolved air content of $V_c$ at any pressure $P_v = 0.13\ V_c$. The separated air content of $V_c$ at any pressure $P_v = \frac{76\,a\,V_c}{P_v} - 0.13\ V_c$. As the latter must fill the void,

$$V_v = \frac{76\,a\,V_c}{P_v} - 0.13\ V = \left(\frac{76\,a}{P_v} - .13\right) V_c$$

Rearranging

$$P_v = \frac{76\,a\,V_c}{V_v + 0.13\ V_c}$$

$$\text{Let}\ \frac{V_c}{V_v} = R \qquad \text{Then}\ P_v = \frac{76\,a\,R}{1 + 0.13\ R} \tag{1}$$

Let $P_p$ = critical pressure of compound

$$\text{By definition} \qquad P_p = \frac{76\,a}{0.13} \tag{2}$$

Equations (1) and (2) indicate that $P_v$ is dependent on $a$ and $R$; as the latter increases, the value of $P_v$ increases, approaching the critical pressure $P_p$ as a limit. That is, the void pressure cannot be greater than the critical pressure of the compound. It will be less because of the incomplete distribution of pressure through a stiffening compound—the same characteristic that was instrumental in forming the void in the first place. The stiffer the compound, the more incomplete will be the pressure distribution outwards from the void, the less the quantity of air available in the compound, and the lower the void pressure.

If $V$ = viscosity of the compound and $K$ the proportionality factor

$$R = \frac{K}{V} \tag{3}$$

When $R = 1000$ pressure distribution can be taken as reasonably complete as from (1) $P_v$ = 99 per cent of $P_p$.

If a pressure of 20 sec.* (thin transformer oil) is taken as representing complete pressure distribution

$K = 1000 \times 20$ and (3) becomes $R = \frac{20000}{V}$

allowing the approximate determination of $R$ for any viscosity.

The curve in Fig. 12 was plotted for a pressure viscosity of 145 by solving (3) for $R$, then solving (1) for $P_v$ and obtaining the corresponding total void volume possible without ionization from Fig. 11. The relation shown in Fig. 12 should be independent of the viscosity-temperature characteristic of the compound because, other conditions being equal, the voids should begin to form at the same viscosity regardless of the temperature.

This curve is of considerable interest because it

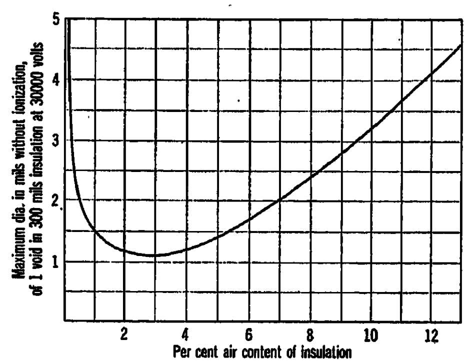


FIG. 12—INFLUENCE OF AIR CONTENT ON IONIZATION

indicates that in general, an air content in the neighborhood of 10 per cent is more desirable than one around 2 per cent.

*C—Void Size versus Moisture.*

For the same total void volume, the size of each void is dependent on the distribution of the voids, many small or few large. It is evident that the more complete distribution (smaller voids) is desirable because it decreases the tendency to ionization.

Table IV shows the results of some tests made with a glass tube approximately 9 in. long, ¼ in. diameter, a stop cock at each end. Each test was made by completely filling the tube between the stop cocks at 60 deg. cent., then closing the stop cocks and allowing the compound to cool to 25 deg. cent., when the resulting voids were counted. It will be noted that the dry compounds gave definitely less complete distribution (larger voids) than the compounds containing moisture.

This indication suggests an interesting advantage of compounds containing rosin. Because of the well-known tendency of rosin to evolve moisture when heated, such a compound is more liable to contain

*—pressure viscosimeter described above.

moisture in the cable and, therefore, to have smaller voids than a no-rosin compound.

When an enclosed volume of compound shrinks, the molecules of the compound are torn apart at some point (their cohesion is overcome), and a void is formed there. As the shrinkage continues, this void increases in size. As the surface of the void enlarges, some expenditure of energy is necessary to rearrange the molecules at the boundary of the void and compound. As the void becomes larger, this expenditure of energy increases. When the energy necessary to enlarge the void becomes greater than the cohesion of

TABLE IV
VOID DISTRIBUTION IN COMPOUNDS
Container 9 in. long by ¼ in. diameter. Cooling range 60 to 25 deg. cent.

| Treatment previous to test | Total number of voids in container | Relative average dia. of each void |
|---|---|---|
| *Cylinder oil No. 1 + 15% rosin* | | |
| a—After 1 hr. at 120 deg. cent. | 45 | 0.68 $d_1$ |
| b—After the original mixing | 116 | 0.50 $d_1$ |
| c—"b" after drying | 14 | $d_1$ |
| d—"c" after addition of moisture | 3600 | 0.16 $d_1$ |
| *Cylinder oil No. 2* | | |
| a—Fresh from original container | 320 | 0.4 $d_2$ |
| b—"a" after drying | 20 | $d_2$ |
| *Petrolatum No. 1* | | |
| a—After drying | 12 | $d_3$ |
| b—"a" after addition of moisture | 3000 | 0.16 $d_3$ |
| c—"b" after drying | 14 | 0.96 $d_3$ |
| *Petrolatum No. 2 + 15% rosin* | | |
| a—After drying | 6 | $d_4$ |
| b—After 15 hr. at 200 deg. cent.* | 75 | 0.43 $d_4$ |

*Rosin content reduced to 8 per cent by heating.

the compound, molecules somewhere else in the compound will be torn apart and a new void formed. This would indicate that a decrease in cohesion should increase the number of voids and, therefore, decrease the size of each void. As the surface tension of water is materially less than that of the compounds, the presence of moisture in the compound may appreciably decrease its cohesion. Further investigation of these relations is in progress.

## Conclusions

1. Reliability of insulation quality determination is essential to satisfactory progress in high-voltage cable development. The most reliable test available at present for this purpose is the overvoltage life test, but there is some evidence that even this test may not always give a true indication of insulation quality. Efforts are being made to increase its reliability by causing failure on test by exaggeration of the causes of failure in service, rather than by increase of the operating voltage alone.

2. While the ionization test is usually in fair agreement with the overvoltage life test as an indication of insulation quality, there are some instances, as that of a resaturated cable, where an indication of quality by the ionization test is not confirmed by the life test.

3. The air resistance of the paper used has a material influence on the insulation quality of the completed cable, regardless of whether the paper is wood pulp or manila.

4. While voltage tests on miniature samples of insulation are of considerable value along some lines, they are not believed to be a reliable basis for comparison of compounds.

5. The interdependence of the contraction-temperature and viscosity temperature characteristics of the compound in their controlling influence on the formation of voids should receive more general consideration.

6. Tentative indications are found that a decrease in air content of the insulation might increase the tendency to ionization because of the resulting decrease in void pressure.

7. Data are shown suggesting that ionization might be decreased by the presence of moisture in the insulation because of the resulting more complete void distribution and consequent smaller size of each void.

## Appendix I—High-Voltage Air Condenser

A photograph and some details of the condenser are shown in Fig. 1. The high voltage plate is 58 in. by 80 in., the two low voltage plates each 50 in. by 72 in. At a spacing of 3 inches the total capacitance is 572 $\times 10^{-6}$ farads, the calculated and tested values checking very closely. No appreciable loss has been found in the condenser.

In order to minimize the expense and time of construction, (1) each plate surface was made in two pieces, the seam being smoothed with solder so that it could be distinguished only by the difference in color, and (2) copper tubing was used for the edges of the plates as shown in Fig. 1. This greatly simplified the construction and has not been found objectionable in any way.

# Discussion

**C. L. Dawes and P. H. Humphries:** We are particularly interested in the experiences in removal of moisture and air. The equations preceding (1) in this paper show that in order to reduce the volume of absorbed air, the pressure must be very low. As stated in our paper,[1] we submitted comparatively small volumes of compounds to 2 mm. vacuum and 105 deg. cent., for 8 to 9 hr. before forming the sample. For considerable time after beginning, the foaming is so violent that we cannot apply full vacuum at once as the oil will be drawn over into the pump. The foaming and bubbling may continue 4 or 5 hr. Recently, on advice from a manufacturer, we attempted to prepare a sample of oil in two hours. The electrical tests showed it to contain both moisture and air in such comparatively large quantities that it was worthless for our purpose. Messrs. Davis and Eddy state that when miniature samples of compound are tested immersed the conditions for void elimination are almost ideal. We have tried several times to prepare samples in the open air, by immersing the electrodes very carefully. We have never been able under such conditions to prepare a sample which did not have such rapid increases in power factor that we considered the sample worthless with regard to its being representa-

1. *Ionization Studies in Paper-Insulated Cables-II*, by C. L. Dawes, H. H. Reichard, and P. H. Humphries, see p. 382.

tive of the compound itself. We were driven to devising the impregnating apparatus which we describe. We now go still further in the matter of vacuum and are able to obtain vacuum so close to absolute that we cannot read the differences of the meniscuses of the mercury. I do not believe that all manufacturers appreciate the large capacity of impregnating compounds for absorbing air and moisture.

**J. B. Whitehead:** Messrs. Davis and Eddy have called attention to the influence of air resistance on the dielectric strength. Most of us are agreed that the principal function of impregnated paper insulation is not only to hold the oil but also to serve as a barrier. It is, therefore, easily understood that if a paper has a lower air resistance, it has bigger openings in it, and therefore there is a greater cross-section of free liquid passages, and greater opportunity for secondary ionization to start in the oil itself. This secondary ionization in the oil results in an increasing conductivity, and so can result itself in an increasing power factor. It is, therefore, a factor in the deterioration of cable. This may explain the fact that beyond the question of the degree of impregnation, the character of the compound may have a bearing on the life of cables merely in its relation to voltage. If we have the process of secondary ionization going on and it is facilitated in some way, it means not only the presence of temporary loss, but it must also mean a progressive change in the character of the compound.

**Wm. A. Del Mar:** (communicated after adjournment) This paper puts on record a number of interesting facts of the kind that laboratory workers too often keep to themselves, largely because each one is too unimportant to be made the subject of a paper.

The relation between air resistance and dielectric strength of paper is checked very closely by our own tests, but that between air resistance and time of oil penetration shown in Fig. 7, does not always hold. The air resistance of paper is a function of the length of fibers, the hydration of the cellulose, and the amount of surface calendering. If the different air resistances are obtained by altering these three variables and not one, as was apparently done with the papers used by the authors, a curve is sometimes obtained showing a consistent decrease of time of oil penetration with increasing air resistance. In other words, it is quite possible to obtain a highly air-resistant paper which impregnates rapidly and conversely a very porous paper which impregnates slowly.

The authors make an excellent case for cylinder-oil compounds by showing both the lower thermal contraction and the lower and more uniform viscosity over the working range of temperatures (Figs. 9 and 10). These characteristics with the superior stability when used with rosin, are strong arguments for cable made with such compounds. Their argument in favor of moisture has the merit of originality and further developments will be awaited with interest.

**F. M. Clark:** (communicated after adjournment) Field service tests for determining insulation quality are not only slow in producing results, but in many cases are expensive and may lead to erroneous conclusions due to lack of definite knowledge of service conditions. The question is ultimately one for the chemist, a determination of the behavior of material under varying conditions, especially under electric stress. This phase of chemical behavior has been very largely neglected. The complexity of the oil and cellulose has served to concentrate chemical attention to the identification of a molecule, or molecules, rather than the behavior under conditions of interest to the "dielectrician."

The authors lean toward the overvoltage life test as a means of determining service quality, although frankly recognizing its defects. It does not appear possible, with our present state of knowledge, to eliminate any of the numerous tests in use without sacrificing something in the knowledge desired. Air-resistance tests on the paper alone merely give information concerning the "barrier action" of the solid insulation. Such tests throw no light on the stability of the treated product under low-voltage application. Initial power factor and power factor—temperature tests give an indication of the effectiveness of the drying process. Power factor—voltage tests throw some light on ionization tendency. The oxidation of the treating compound in air gives some information on its chemical and electrical stability, but such tests should be carried out at temperatures lower than 100 deg. rather than at 120 deg. cent. as done by the authors. The rate of oxidation is important, but just as important is the knowledge of the amount of water produced by the oxidation reaction. Oils and petroleum compounds vary according to their individual chemical characteristics in the products formed by oxidation. High voltage—high temperature tests on small treated samples give an idea of the chemical and electrical stability of the compound—paper combination, but a set-up to throw light on the behavior under ionization is difficult to obtain. Thermal expansion of the compound, air content of the compound, and other properties are tests which cannot be eliminated in the proper selection of cable material. Each property is of value and must be given proper consideration.

With regard to Fig. 8 of the article, it appears that the injurious effect of rosin is apparent only in the early stages of an oxidation test. The mixture of rosin and compound is eventually tending to approach the characteristics of the other constituents present. I should like to ask the authors if such behavior is shown in tests run at 100 deg. cent. or lower. The tendency of rosin to eliminate water when heated in air may give considerably different results with tests run at lower temperatures. I would also request the authors to describe the physical and chemical characteristics of the oils and petrolatum used in the experiments. This is necessary for a proper understanding of the results illustrated in Fig. 8.

The solubility of air in insulating oils is given as 10-15 per cent. Subsequent discussion is based on these figures. I should appreciate information from the authors concerning the test on which this amount of air solubility is based. Work which we have done in Pittsfield, and which I hope soon to present to the Institute, indicates that the air content of oils and compounds varies considerably from one type of compound to another. It is only in the lighter oils such as those approaching the transformer-oil type that the air solubility reaches values such as those given by the authors.

The authors state that the size of the void formed is a function of the water content of the oil or petrolatum and suggest an explanation based on the relatively high surface tension of oil as compared to water. Our experience has indicated that the surface tension of oil against air, at 25 deg. cent. is considerably less than the same value for water. Surface tension of oils under such conditions appears to be approximately 30 dynes per sq. cm., varying slightly with the state of oxidation, but never approaching the much higher value possessed by water. I would suggest that the smaller voids produced in the presence of water are caused by the precipitation of dissolved water from the oil as the temperature falls, such precipitation assisting in the formation of gas bubbles and securing a wider distribution of voids than is obtained with the dry oils. Such moisture separation, of course, would be equally, if not more dangerous to cable insulation than would be the void formation itself.

**E. W. Davis:** We are interested to learn that Mr. Humphries found two hours vacuum treatment of the compound to be insufficient for his miniature samples of sheet insulation. In the past, using paper of low air resistance, we have made many miniature samples with no more than two hours previous vacuum treatment of the compound. Our electrical tests on these samples indicated almost complete elimination of voids. For instance, some of them withstood 750 volts per mil for 18 months

without failure.[2] In fact similar results were obtained without any previous vacuum treatment of the compound.

Mr. Del Mar suggests that our different air resistances were obtained by variation of only one of the several factors influencing air resistance. This was not the case. All air resistance shown in the paper was obtained by variation of all the factors. While we have done some work on the separation of the factors it was considered too incomplete for publication.

We are glad to see Mr. Clark's recognition of the fundamental importance of the development of a reliable criterion of insulation quality. We believe this to be the greatest need of the industry today.

Mr. Clark suggests that the oxidation tests on the compounds (Fig. 8) should be made at 100 deg. cent. instead of 120 deg. cent. If the only purpose of the tests were to study the oxidation of the compound in service we would agree with him. However, in order to study the deterioration of the compound in the factory tanks we believe such tests should be made at temperatures greater than 100 deg. cent. It may be found necessary to make tests at several temperatures. What tests we have made at 100 deg. cent. show the recovery of the rosin compounds to be much less than at 120 deg. cent. and in some cases entirely absent.

Mr. Clark asks for a more detailed description of the compounds shown in Fig. 8. It probably should have been noted in the paper that the compound designations are the same in Figs. 8, 9, and 10. Thus Figs. 9 and 10 give considerable information as to the physical characteristics of the compounds in Fig. 8. Only wood rosin was used. The Saybolt Universal viscosity at 85 deg. cent. is 126 sec. for petrolatum *B*, 245 sec. for cylinder oil *A*, and 38 sec. for transformer oil *B*. While we cannot give a complete chemical description of all the compounds we can say that the petrolatums, cylinder oils, and transformer oils of Fig. 8 are all representative of the better grade paraffin-base insulating compounds that are being supplied the paper cable industry. The principal purpose of Fig. 8 was to show the wide difference in oxidation represented by such a group of typical compounds.

The air solubility figure of 10-15 per cent was based on tests with a known volume of air above and in contact with a known volume of carefully prepared oil which in turn was supported by mercury. By means of the mercury the air and oil were maintained at constant pressure until the air volume showed no further decrease. While our tests gave some indication of the influence of compound type they were not continued far enough to be conclusive.

The assumed value was purposely taken for "insulating oils" and not all types of compound. In this discussion disregard of the unknown but suspected decrease of solubility with increasing viscosity is not believed to influence the ultimate conclusion in any way (No. 6 on page 379). In fact it seems that the inclusion of this decrease in the discussion would tend to accentuate the relations considered. We are indeed glad to hear that Mr. Clark is soon to publish data on the influence of compound type on the solubility of air as available information on the subject is far from complete.

We agree with Mr. Clark that our suggested explanation of the moisture effect may not appear entirely sound. However, we do not feel able to accept his explanation without further investigation.

2. Further test results on these samples given in A. I. E. E. Trans., 1927, p. 215.

# Ionization Studies in Paper-Insulated Cables—II

BY C. L. DAWES,*
Member, A. I. E. E.

H. H. REICHARD,†
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and

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*Synopsis.—This is the second report to be presented before the Institute of the research investigation of ionization phenomena in paper-insulated, high-voltage cables, which is being conducted at the Harvard Engineering School under the auspices of the Impregnated Paper-Insulated Cable Research Committee. This paper is a report of progress and will be followed by further reports as the work progresses.*

*The power dissipated as ionization loss in cables is much more harmful than the power dissipated in the solid dielectric. The paper presents methods of separating this ionization loss from the total dielectric loss occurring in high-voltage impregnated paper cables, the methods being verified experimentally. It consists essentially of seven parts as follows:*

*(1) A description of the apparatus and the method of impregnating cable paper whereby test specimens having practically no occluded gases can be produced. It is necessary to prepare and test samples in this manner in order to determine the law of their dielectric properties over a wide range of voltage gradient. It is only by knowing this law that it becomes possible to separate the ionization loss in the cable from the total dielectric loss. These experiments show that with three typical but widely different types of impregnating compounds, the power factor and capacitance are constant up to gradients as high as from 250 to 300 volts per mil; the power loss is proportional to the voltage squared under these conditions. These results are illustrated with curves.*

*(2) In order to verify the law of ionization loss in cables, as determined from the analysis of their dielectric-loss curves, it is necessary to know the character of the ionization-loss curve for thin air films under conditions of "restricted ionization." Such ionization-loss curves have been determined experimentally and the characteristics of two air films, of different thickness, are given in the paper. Above the ionization voltage, the ionization loss is a linear function of the voltage.*

*(3) By employing the fact that in a cable the power loss in the solid dielectric varies as the square of the voltage, even above the ionization voltage, it is a simple matter to separate this loss from the total dielectric loss in a cable, the remainder being the ionization power loss. The ionization power-loss curve concaves upwards directly above the ionization voltage. After all the gas films have become ionized, the curve becomes linear. This is confirmed by a synthetic curve determined by the data obtained in (b). This analysis is given for four different cables having widely varying degrees of ionization. As a result of this analysis, the power-loss curves of cables may now be expressed by a very simple equation. It is suggested that the constants of these equations be used as a basis for the quality of cables.*

*(4) Cable models, of glass and intervening air spaces, were made and their dielectric-loss curves determined as with the actual cables. The analysis of the curves gives results identical with those obtained with actual cables. This seems to verify the results obtained with the cables themselves.*

*(5) It is possible to express the power-factor curve by a simple equation, from which its maximum, etc., may be determined. In reality the power-factor curve consists of three simple curves.*

*(6) The energy current of a cable may be analyzed into three simple components, one of which is determined by the loss in the solid dielectric and the other two by the ionization characteristics of the cable.*

*(7) The character of the capacitance curve, which varies in different cables, is determined by the positions and thicknesses of the gas films. Further experimental data and study are necessary before an accurate interpretation of these curves can be made.*

## INTRODUCTION

THIS paper presents further results of research work being conducted at the Harvard Engineering School in the study of Ionization in High-Voltage Impregnated Paper Cables. The first paper was presented at the Midwinter Convention in 1926. The work is conducted under the auspices of the Cable Research Committee which is a subcommittee of the National Electric Light Association, the American Institute of Electrical Engineers, and the Association of Edison Illuminating Companies. The National Electric Light Association provides the Research Fellow for the work. The work is under the immediate supervision of a Cable Research Committee made up of members of the faculty of the Harvard Engineering School and one of the authors is chairman of this committee.

In this paper are presented further laws of ionization phenomena which occur in high-voltage paper-insulated cables. These laws are substantiated by experimental data taken under widely different conditions.

## PROCEDURE

The dielectric-loss measurements which are given in this paper were made on a bridge, the principle of which was given in the first part of the paper.[1] Since that time, however, many improvements have been made in the bridge which have increased its accuracy and ease of manipulation. Detail of this bridge and its accessories will be described in a paper§ which will shortly come before the Institute, hence will not be repeated here.

As is well known, the dielectric medium of paper-insulated cables consists of impregnated paper, the impregnating material, and thin gas films which may be air or gases from the impregnating material, all in series. In this paper, the insulating paper and the impregnating material will be referred to as the solid dielectric and the air and vapors as gases.

The variation of the dielectric characteristics of the solid material with voltage is quite different from the

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1. For numbered references see Bibliography.

§*Some Problems in Dielectric Loss Measurements* by C. L. Dawes, P. L. Hoover, and H. H. Reichard.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

variation of the dielectric characteristics of the gaseous films with voltage. This fact provides a means of separating each of these two losses from the total loss in a cable. This is important, for, as is well known, the energy which is dissipated in the gas films is very insidious and destructive in its effects on the dielectric and hence is much more harmful to the life of the cable than the energy dissipated in the solid material. Thus the energy which is responsible for tree designs and ultimate failure of the cable may be determined quantitatively and used as a criterion of cable quality. However, before it is possible to make this separation of losses, it is necessary to determine experimentally the individual dielectric characteristics of both the solid material and the gas films as functions of voltage gradient.

## Dielectric Characteristics of the Solid Material

The separation of the losses within the cable dielectric can be accomplished only when the dielectric characteristics of cable paper, impregnated under conditions that remove essentially all occluded gases, are known.

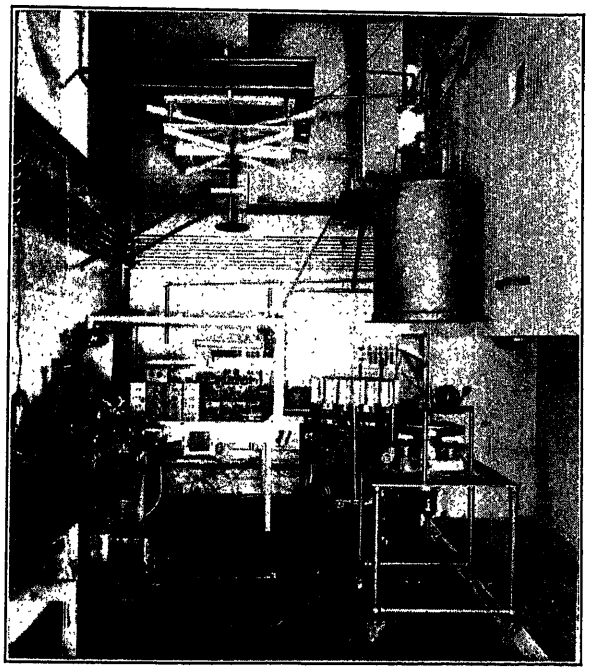

Fig. 1—Photograph of Paper-Compound Impregnating Apparatus

Thus a solid dielectric having no perceptible ionization is obtained.

Although the dielectric characteristics of paper and impregnating compounds have been determined experimentally a number of times and such results have been published occasionally, the authors know of only two instances[2,3] where the experimental work was conducted under such carefully controlled conditions that the results might be used to confirm the theories which are developed in this paper. In order to substantiate these theories beyond question, it seemed necessary to conduct a series of experiments on cable papers impregnated with different types of compounds under carefully controlled conditions. To conduct these experiments properly, it became necessary to design and construct special apparatus for impregnating and testing. Fig. 1 shows a photograph of the bridge and the impregnating apparatus with the dome unbolted from the base of the apparatus and raised. A diagram is shown in Fig. 2A.

The sample is made and tested in a circular brass vessel, Figs. 1 and 2A, the vessel itself forming the lower electrode. As this lower electrode is connected to the

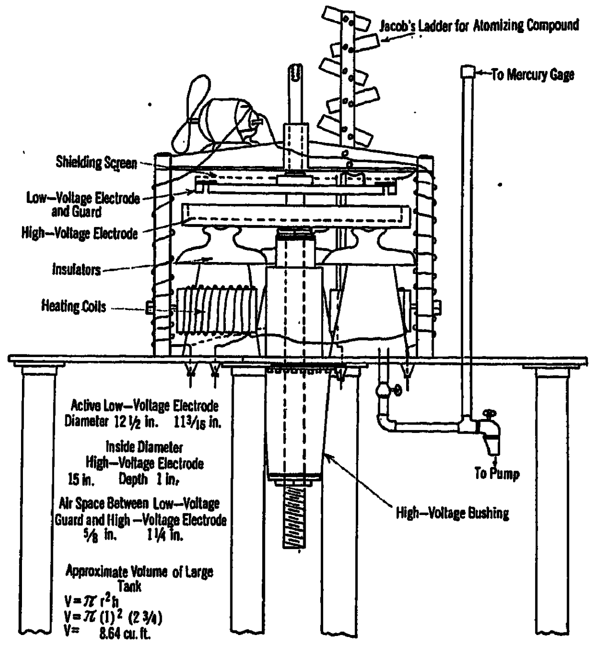


Fig. 2A—Diagram of Paper-Compound Impregnating Apparatus

high-voltage side of the transformer secondary, it is set on three high-voltage, pin-type insulators. The inside diameter of the electrode is 15 in. (38.1 cm.) and the depth is 1 in. (2.54 cm.).

The low-voltage electrode is a circular brass disk ½ in. (1.27 cm.) thick and originally 12.50 in. (31.8 cm.) diameter, but later reduced to 11.187 in. (28.4 cm.) to permit the obtaining of voltage gradients up to 300 volts per mil. This electrode is surrounded by a guard ring 1-in. (2.54 cm.) radial thickness separated from it by 0.1 in. (2.54 mm.) and made a unit with the electrode by means of strips of bakelite which hold the two together. The low-voltage electrode is also shielded above with copper screening attached to the guard rings (Fig. 2A). This low-voltage electrode unit is fastened rigidly through a block of horn fibre insulation to a 1-in. (2.54 cm.) brass rod which runs in a rigid bearing above. This keeps the two plates in exact alinement at all times. The upper electrode can be raised and lowered by means of a ⅜-in. (0.95 cm.) brass rod attached to this larger brass rod and extending outside the top of the dome through an air-tight gland, Fig. 2B. The details of the

method of making this joint tight are also shown in Fig. 2B.

To permit high-voltage tests to be made while the prepared sample is within the impregnating apparatus, a 35,000-volt bushing is set in the base plate of the apparatus to carry the connections from the high-voltage electrode to the high-voltage side of the power supply, Figs. 1 and 2A.

The low-voltage lead is carried through the base plate by means of an automobile spark plug. This lead is shielded throughout its entire length. The iron-work of the apparatus itself is a portion of the shielding and is balanced to the potential of the low-voltage electrode with the remainder of the guard circuit. Connections to the heating units and to a fan motor are also made through automobile spark plugs. Wires to the ther-

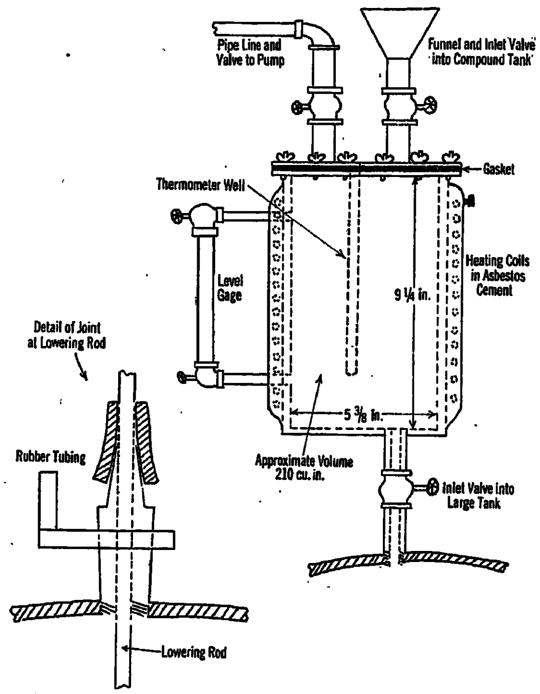


FIG. 2B—DETAIL OF SMALL COMPOUND TANK AND METHODS OF PRESSURE SEALING

mocouples are carried into the apparatus through bushings sealed with special compounds.

The entire apparatus is covered with a boiler-iron dome, which bolts to the base plate. This dome, when in use, is heat lagged. A special rubber gasket is used to seal the joint between the dome and base plate. Great difficulty was experienced in making the apparatus sufficiently tight to obtain the necessary high vacuum. It was particularly difficult to make the high-voltage bushing joint air tight. This difficulty was finally overcome by designing a type of seal similar in principle to that used for the lowering rod (Fig. 2B).

A small, totally enclosed tank, having a capacity of approximately 0.91 gal. (3.44 liters), is fastened to the top of the dome, (Fig. 2B). This tank is provided with a thermometer well and a glass gage for determining the amount of compound at any time. The tank is surrounded with resistor wire to permit heating of the compound, and is heat lagged. A connection to the vacuum pump is made so that the compound in the tank may be subjected to vacuum as well as temperature, before it runs down to the sample. The compound is allowed to run down into the dome to the testing vessel by means of a valve. A "Jacob's ladder" (Figs. 1 and 2A) is provided so that the compound trickles slowly down into the test vessel, thus exposing a very thin layer to the vacuum for a considerable time before it reaches the sample.

The procedure is to place 12 circular pieces of wood-pulp cable paper in the brass test vessel (Figs. 1 and 2A) and subject them to a temperature of 105 deg. cent. and a high vacuum for several hours to remove the last traces of moisture.[3] The upper test electrode must be raised above the paper during this process to permit the air and moisture to escape readily. While the paper is being subjected to this drying and evacuating process, the compound in the small totally enclosed chamber on top of the dome is maintained at 2 to 3 mm. vacuum and at 105 deg. cent. by means of the current in the resistor wire which surrounds it. The paper alone was subjected to the temperature of 105 deg. cent. and 2 to 3 mm. vacuum for a period of approximately 9 hr. The valve in the pipe at the bottom of the small tank was then opened slightly. This permits the compound to trickle slowly down to the test vessel over the "Jacob's ladder," the vacuum of 2 to 3 mm. still being maintained. A thin layer of the oil is thus exposed to the vacuum for a considerable time, so that as far as possible all occluded gas is removed. This impregnating process requires something like one-half hour before the test plate can be lowered down to the specimen by means of the rod which extends through the top of the dome. After the test plate had been thus lowered, the specimen was allowed to remain at a temperature of 105 deg. cent. and a vacuum of 2 to 3 mm. for a period of not less than 9 hr. before it was ready for test. The samples tested consisted of 12 circular sheets of wood-pulp paper, each having a diameter of 15 in. (38.1 cm.), so that they just fitted within the test vessel. Their approximate thickness was 6.5 to 6.7 mils (0.165 to 0.170 mm.) per sheet.

Three different compounds were chosen for impregnating the paper and producing test samples. These compounds were supplied by three different cable manufacturers. Their viscosity curves are shown in Fig. 3. The first compound is a light oil, specially prepared, so that amorphous constituents are removed. It has a flow point of approximately − 20 deg. cent. This is designated as Compound A; the second is a paraffin-base cylinder-oil compound of medium viscosity and is designated as Compound B; the third is of a petrolatum base, containing a small percentage of rosin and is designated as Compound C. Although Fig. 3 shows that for temperatures down to 60 deg. cent.

and even less, the viscosity of Compound C is considerably less than that of Compound B, yet at room temperature the viscosity of Compound C is very much greater than that of B, being very plastic.

These three compounds, therefore, not only include the ranges of viscosity of compounds generally used for paper-cable impregnation, but are fairly representative of different types of compounds used in practise. Hence, any general deductions from results obtained with these three compounds will be more likely to apply to compounds in general.

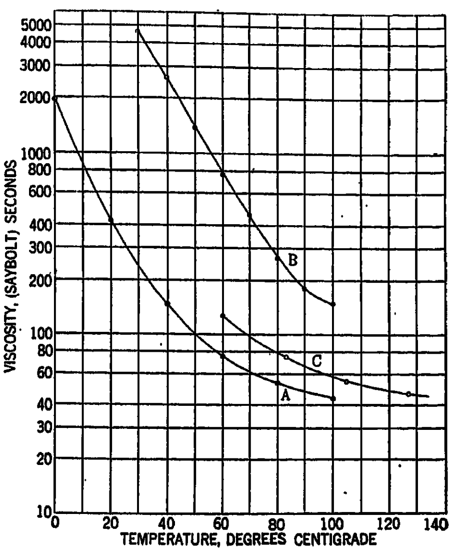


FIG. 3—VISCOSITY CURVES FOR COMPOUNDS A, B, AND C

The method of impregnation has already been described. Tests were made immediately after impregnation and at atmospheric pressure, since it is practically impossible to conduct high-voltage tests in a vacuum

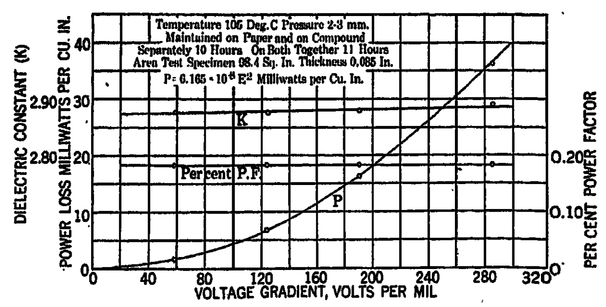


FIG. 4—CHARACTERISTICS OF WOODPULP PAPER IMPREGNATED WITH COMPOUND A, FREQUENCY = 60 CYCLES, TEMPERATURE 19.5 DEG. CENT.

because of flashovers within the apparatus. After a sample has been properly prepared in a vacuum, test results are not affected by conducting the tests at atmospheric pressure. This has been confirmed by Emanueli.[2] Tests must, however, be conducted within three or four days after removing the vacuum, for if the sample stands too long, moisture commences to permeate it. Tests were made over a considerable range of frequencies and temperatures. For the purpose of this paper, however, data taken at room temperature and at a frequency of 60 cycles per second only are necessary. A paper giving the complete results of all these tests will be published later.

The power factor, the dielectric loss in milliwatts per

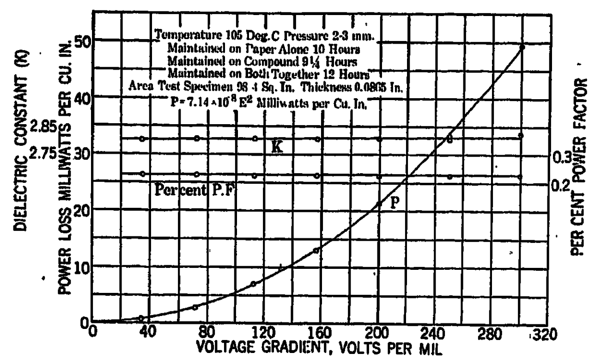


FIG. 5—CHARACTERISTICS OF WOODPULP PAPER IMPREGNATED WITH COMPOUND B, FREQUENCY = 60 CYCLE, TEMPERATURE 22 DEG. CENT.

cu. in., and the dielectric constant of compound A as functions of voltage gradient are given in Fig. 4. The frequency is 60 cycles per second. Similar characteristics for compounds B and C are given in Figs. 5 and 6.

The time of drying the paper in vacuum, of heating

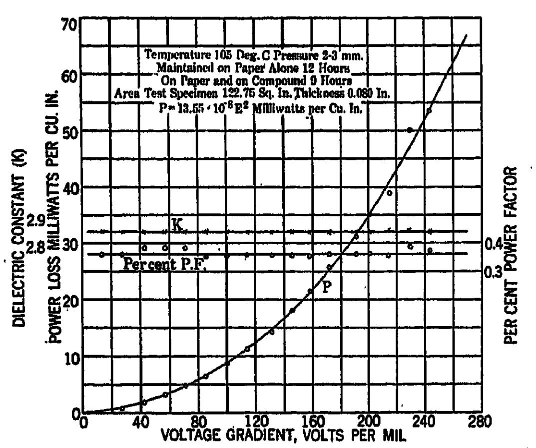


FIG. 6—CHARACTERISTICS OF WOODPULP PAPER IMPREGNATED WITH COMPOUND C, FREQUENCY = 60 CYCLE, TEMPERATURE 24 DEG. CENT.

the compound in the small tank, the temperature, etc., are given with each figure.

In none of the figures is there any indication of an ionization voltage. The power factor and dielectric constant and hence capacitance remain essentially constant at all voltages. At the higher voltage gradients, a very slight increase in power factor and capacitance was observed during the time required to balance the bridge. We have shown that this increase is due to the slight increase in temperature caused by dielectric loss in the sample. The following three conclusions may be drawn from these curves:

With the most careful impregnation, which removes nearly all traces of occluded air, at gradients up to 250 to 300 volts per mil, and at room* temperature, the power factor is essentially constant, the capacitance is essentially constant, the power loss varies as the voltage squared. This last conclusion follows from the other

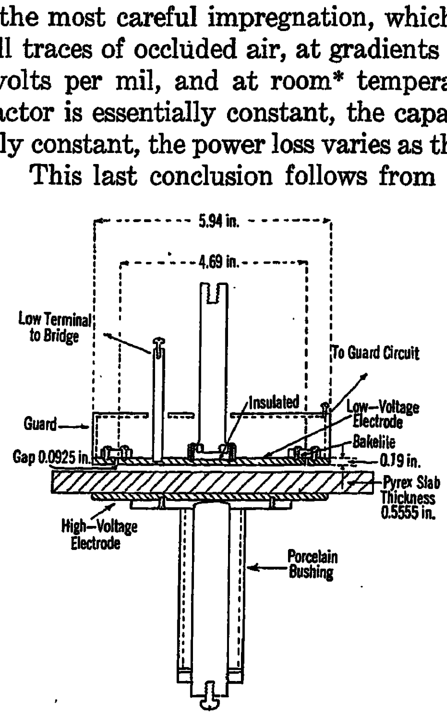


FIG. 7—DIAGRAM OF ELECTRODES USED IN TESTING THIN LAYERS OF AIR

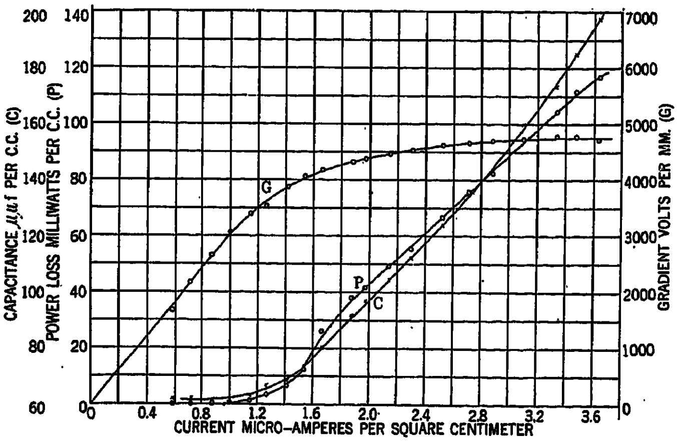


FIG. 8—CHARACTERISTICS OF 14.75 MIL (0.3745 MM.) AIR FILM, FREQUENCY = 60 CYCLES, TEMPERATURE 19 DEG. CENT.

two. These three properties of impregnating compounds are very important, for they form the basis of the methods of analysis which are given later in this paper.

## DIELECTRIC CHARACTERISTICS OF GAS FILMS

Gas films form a portion of the cable dielectric. It is important to know the character of their dielectric-loss curves as a function of voltage gradient, for this gives additional means of verifying the ionization-loss curves that result from analyzing the dielectric-loss curves of cables.

In the first paper (I)[1], Figs. 29 and 30 showed some of the dielectric properties of a 2-mm. air film under conditions of "restricted ionization." Since that time we have continued further the investigation of the electrical properties of air films independently of the Cable Research. The apparatus has been improved considerably and the measurements are now made with much greater precision than formerly. This work is a research in itself and will require a separate paper for its presentation.

Since gas films, as well as solid materials, constitute the body of the impregnated cable dielectric and their electrical characteristics form one basis for the method of analysis that follows, two sets of typical curves for such air films are given. A diagram of the apparatus used for the experimental determination of these curves is given in Fig. 7. The air film is tested under con-

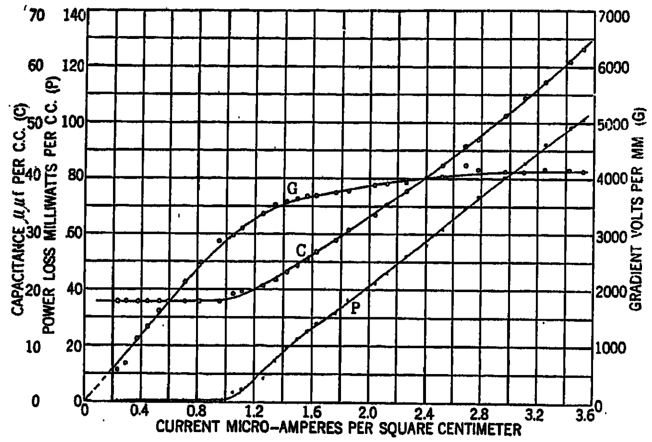


FIG. 9—CHARACTERISTICS OF 27.65 MIL (0.703 MM.) AIR FILM, FREQUENCY = 60 CYCLE, TEMPERATURE 21.5 DEG. CENT.

ditions of "restricted ionization," the current flow being limited by a slab of Pyrex glass 0.555 in. (1.411 cm.) thick. The lower surface, which is silvered, rests on the high-voltage electrode.

The low-voltage electrode has a diameter of 4.69 in. (11.91 cm.). The air films were tested in practically the same manner as that used for the compounds. The testing electrodes and associated apparatus shown in Fig. 7 were enclosed in a vacuum-tight iron chamber almost identical with that used for the compounds, except that the height is less. Hence, it is possible to control temperature, pressure, humidity, etc. The dielectric characteristics of the Pyrex alone were first measured. The dielectric characteristics of the air film and the Pyrex in series were next measured. The electrical characteristics of the air film are then determined by deducting at any given current the characteristics of the Pyrex alone from the characteristics of the air film and Pyrex in series.

In Fig. 8 are given the following characteristics of a 0.3745-mm. (14.75-mil) air film as a function of current: the voltage gradient, the power loss per cu. cm., and the capacitance per cu. cm. Similar characteristics for a 0.703-mm. (27.65-mil) air film are given in Fig. 9.

*At temperatures above from 35 deg. to 50 deg. cent. this is not true. Further discussion will appear in a paper devoted to the characteristics of cable compounds.

These curves are given as typical of the characteristics of air films under the conditions of "restricted ionization."

The voltage gradient below ionization is proportional to the current. After ionization has begun, the rate of increase of voltage gradient becomes less and less until the voltage gradient ultimately becomes substantially constant. This characteristic of gas films has already been described in Part I.[1] It is also in accordance with the investigations of Townsend,[5] although his investigations were made with direct current and at much lower pressure. The fact that the voltage gradient across the gas films becomes constant is important in that it causes the power factor to reach a maximum and then to decrease with further increase of voltage on the cable.

After ionization begins, the increase of power loss in the air films is practically proportional to the further increase in current, as shown in Figs. 8 and 9. That is, the relation between power loss and current is linear. In a large number of measurements under various conditions, this has invariably been found to be the case. A very slight curvature occurs in the lower portion of the curve, but as a whole the characteristic is practically a straight line.

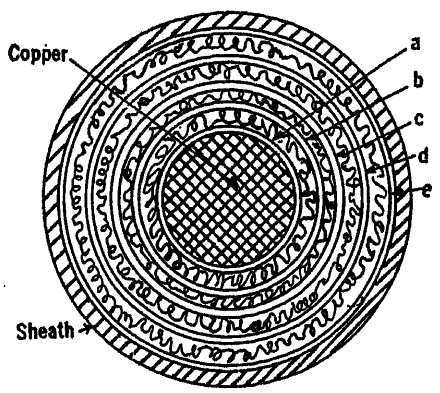


Fig. 10—Successive Gas Films within Cable Dielectric

Hence, the power loss in the solid dielectric varies as the square of the voltage; above the ionization voltage the relation of the power loss in the gas films to the current is linear. These two relationships give a method of separating the two losses in the cable dielectric.

The relation between capacitance and current in ionized air films is almost identical with the relation of power to current. That is, after ionization has begun, the increase of capacitance with current is also linear. The rate of increase of capacitance depends, however, on the thickness of the air film, the rate of increase of capacitance becoming less with decrease in the thickness of air film. These capacitance relationships are also important in that they may give a basis for determining the character of the gas films within the dielectric of impregnated paper cables.

## Power Characteristics of Gas Films Distributed in Cable Dielectric

Within the cable dielectric, the gas films are distributed indiscriminately throughout the insulation. There may be a greater number in some places than in others, due to voids occurring during impregnation; gas films may be produced near the sheath, due to wrinkling and bending. This is all well known.

Each of these gas films will become ionized as soon as its voltage gradient exceeds a certain critical value. This critical gradient depends on the thickness of the film and also on pressure. The power loss characteristic of each film then becomes linear, similar to the characteristics shown in Figs. 8 and 9. With films of equal thickness, those nearer the copper first become ionized, and as the voltage is raised, the films further and further from the copper ionize successively. With films of varying thickness, it is possible that some films further from the center will ionize sooner than some nearer the center. Probably this effect occurs only occasionally, but in any event it does not in any way affect the general relationships which we are attempting to prove.

The character of the total power-loss curve is, however, determined by the radial positions of the gas films. As a whole, the gas films near the copper ionize first and the others throughout the dielectric ionize successively from the copper outwards as the voltage is raised.

In Fig. 10 a cross-section of an impregnated paper cable is shown. For simplicity, five concentric gas films $a$, $b$, $c$, $d$, and $e$ are shown distributed radially outwards through the insulation. Gas film $a$ is adjacent to the copper and $e$ is adjacent to the sheath. Now consider that the voltage on the cable is being raised and that the change in capacitance of the cable is so small that the current is practically proportional to voltage. Below the ionization voltage, the loss in the gas films is zero. When the voltage gradient across $a$ reaches its critical value at $E_1$, Fig. 11, the power loss in this film will be linear with further increase in voltage, as is shown by the straight line $a$. As the voltage is further raised, film $b$ becomes ionized at voltage $E_2$ and the relation of its power loss to voltage is given by the straight line $b$. In a similar manner, films, $c$, $d$, and $e$ ionize at voltages $E_3$, $E_4$, and $E_5$ and their power characteristics are given by lines $c$, $d$, and $e$, Fig. 11.

At any voltage, the total power loss in the gas films is obviously the sum of the ordinates of these power curves at that voltage. Thus, in Fig. 11, between voltages $E_1$ and $E_2$ the power loss is given by that portion of curve $a$ alone. Between $E_2$ and $E_3$ a portion of curve $b$ is added to $a$, etc. Up to $E_5$ the power curve $P$ is a broken line. In the actual cable, however, the gas films relatively are much smaller in magnitude and very much more numerous than shown in Fig. 10.

Hence, that portion of the curve up to voltage $E_5$ will be a smooth curve.

After the last gas film has become ionized, the power curve $P$ becomes a straight line. This is true, irrespective of the slopes of the curves $a$ to $e$. That is, the point $f$ on the curve is the criterion of complete ionization of all the gas films within the cable. Not long ago, it was thought that this condition was reached when the

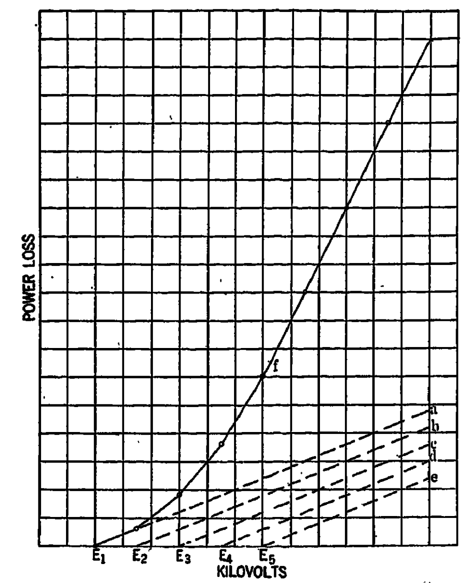


FIG. 11—POWER LOSS IN CABLE DIELECTRIC DUE TO SUCCESSIVE GAS FILMS

power-factor curve tended to become horizontal. That this is erroneous has already been shown in (I).[1]

## ANALYSIS OF CABLE POWER CURVE

Fig. 12 shows the three well-known characteristic curves of a 10-ft. length of 500,000-cir. mil, 3/16-in.

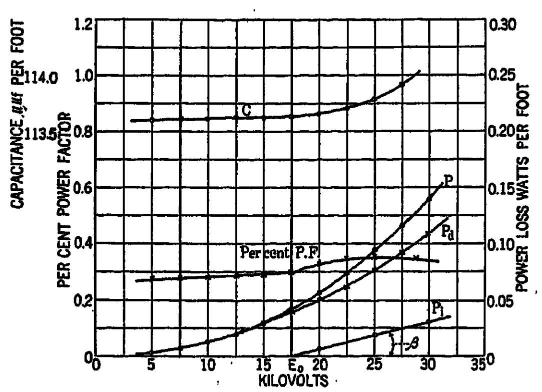


FIG. 12—CHARACTERISTIC CURVES OF CABLE NO. 14

500,000-cm., single-conductor 3/16-in. paper cable with metallized shielding tape 10-ft. length, frequency 62.1 cycle, temperature = 20 deg. cent.

(4.76 mm.) paper, 8/64-in. (3.18 mm.) lead, single-conductor cable with a metallized shielding tape, which is designated as Cable 14. The power curve $P$, the power-factor curve, and the capacitance are plotted against kilovolts. An examination of these curves shows that this cable has relatively little ionization.

In Fig. 14 this same power curve $P$ ($a\ b\ d$) is plotted on logarithmic paper. Up to 17.5 kv., the ionization voltage (see Fig. 12), this logarithmic power characteristic is a straight line having a slope of 2.0. This shows

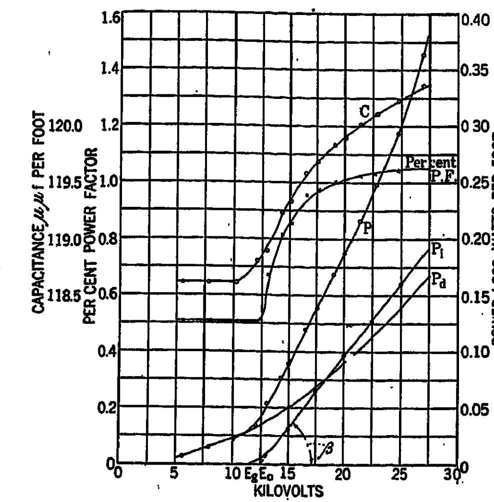


FIG. 13—CHARACTERISTIC CURVES OF CABLE NO. 12

500,000-cm., single-conductor 16/64-in. paper cable with copper shielding tape 10-ft. length, frequency 60.6 = cycle, temperature = 20 deg. cent.

that below the ionization voltage the power loss varies as the square of the voltage. This loss must all occur in the solid dielectric, so that this portion of the curve confirms the results obtained with the compounds and given in Figs. 4, 5, and 6. A discontinuity occurs at point $b$, the ionization voltage, and the characteristic from $b$ to $d$, although still a straight line, now has a slope of 2.3. Above the ionization voltage, the total loss in this cable varies as the voltage to the 2.3 power.

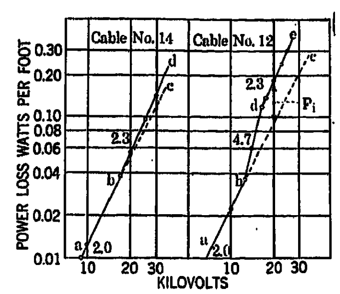


FIG. 14—LOGARITHMIC PLOTS FOR FIGS. 12 AND 13

If it be assumed that above the ionization voltage the power loss in the solid dielectric still continues to vary as the square of the voltage, the loss in the solid dielectric will be given by the line $b\ c$ where $b\ c$ is a continuation of line $a\ b$. This assumption is justified by the results obtained with the three compounds which are given in Figs. 4, 5, and 6.

A small error is introduced in considering $a\ b\ c$ a straight line. After ionization begins, the voltage across the gas films increases slowly and soon becomes constant and the rate of increase of the voltage across

the solid dielectric must become greater. (See Fig. 20 in Bibliography 1.) If power loss were plotted as a function of current, the line *a b c* would be straight. It follows that if the total voltage were proportional to the current, the line *a b c* would be straight. The departure of the relation of current to voltage from the linear relation is accounted for by the change in capacitance of the cable. If the capacitance remained con-

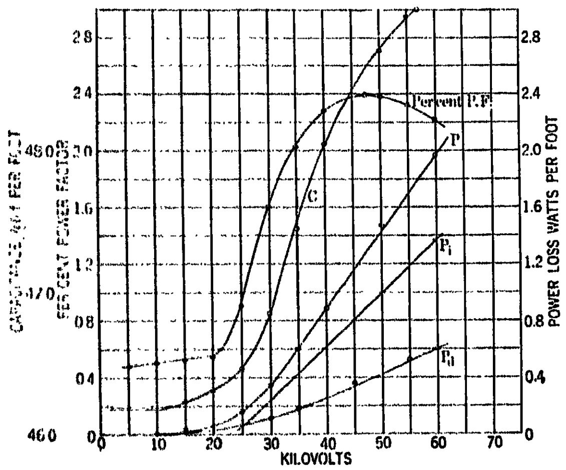


FIG. 15—CHARACTERISTIC CURVES OF CABLE B

No. 00 single-conductor, 9/32-in. paper cable with 10-ft. length, frequency = 60 cycle, temperature 20.4 deg. cent.

stant, the voltage and current would be proportional to each other and the line *a b c* would be straight. A study of Fig. 12 shows that the maximum change of capacitance per foot with cable 14 is from 113.6 to 113.9 $\mu\,\mu$ f., a change of 0.3 per cent. In cable *A*, Fig. 16, the change is 5.2 per cent. The 0.3 per cent is obviously negligible. It so happens that the largest percentage changes in capacitance occur when the ionization losses are greatest, so that under most conditions the error introduced into the ionization losses by the foregoing assumption is small. If conditions warrant it, obviously it is not difficult to make this correction for values of power above the ionization voltage. Let it be desired to find the true power dissipated in the solid dielectric at any voltage $E_1$. Multiply $E_1$ by $K_c$, where $K_c$ is the ratio of the capacitance at voltage $E_1$ to the initial value of capacitance. The true value of power for voltage $E_1$ is the value corresponding to the voltage $K_c\,E_1$. In this paper the authors do not consider it necessary to make this correction.

It follows that in Fig. 14 the line *a b c* gives the power loss in the solid dielectric and the line *a b d* gives the total loss. The difference between the two must be the power loss in the ionized gas films.

This difference is transferred and plotted in Fig. 12 as the curve $P_i$. It is a linear curve and its equation is of the form

$$P_i = K_1\,(E - E_0) \qquad (1)$$

where $K_1$ is the slope of the line ($= \tan\beta$, Fig. 12), $E$ the voltage, and $E_0$ the intercept of the line with the voltage or $X$-axis. $K_1\,E_0$ is the intercept of the line, when extended, with the power or $Y$-axis. These relationships are shown in Fig. 18. The equation of the curve, $P_i$, Fig. 12, is

$$P_i = 2.38 \times 10^{-6}\,(E - 17{,}500) \text{ watts per ft.}$$

The loss in the solid dielectric for this cable is given by the curve $P_d$, Fig. 12, and the equation of the curve is

$$P_d = 1.21 \times 10^{-10}\,E^2 \text{ watts per ft.} \qquad (2)$$

The equation for the total power-loss curve $P$ is the sum of (1) and (2) and is given by

$$P = K\,E^2 + K_1\,(E - E_0) \text{ watts per ft.} \qquad (3)$$

Fig. 13 gives the power, power-factor, and capacitance curves of a 500,000-cir. mil, 16/64-in. (6.35-mm.) paper, 8/64-in. (3.18-mm.) lead, single-conductor cable with a copper shielding tape. This cable is designated as 12. A comparison of these curves with those of Fig. 12 shows that the cable has more ionization than cable 14. Fig. 14 also shows the power curve of the cable, Fig. 13, plotted logarithmically. As before, the line *a b* gives the loss in the solid dielectric before ionization commences, and the broken line *a b d e* gives the total loss. Under the assumptions already made, the straight line *a b c* gives the loss in the solid dielectric, and the loss in the ionized gas films is the difference of the curves *a b d e* and *a b c*. This difference is plotted in Fig. 13, giving the curve of ionization power $P_i$. The dielectric power curve $P_d$ is also plotted in Fig. 13. The equation of these two curves is

$$P_i = 1.28 \times 10^{-5}\,(E - 12{,}500) \text{ watts per ft.} \qquad (4)$$

$$P_d = 2.24 \times 10^{-10}\,E^2 \text{ watts per ft.} \qquad (5)$$

Fig. 15 gives the power, power-factor, and capaci-

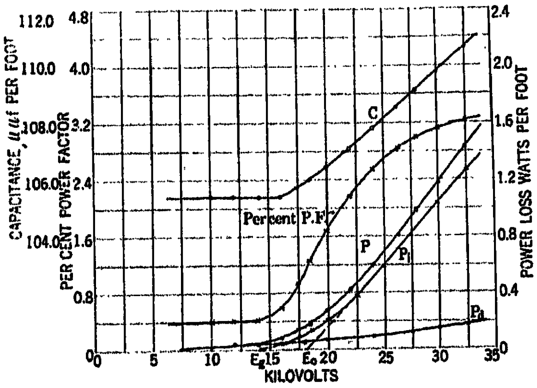


FIG. 16—CHARACTERISTIC CURVES OF CABLE A

500,000-cm. single-conductor ¼-in. paper cable, 10 ft. length, frequency = 58.3 cycle, temperature = 20 deg. cent.

tance curves of a 00, single-conductor, 9/32-in. (7.14 mm.) paper cable, which is designated as Cable *B*. A study of these curves shows that the cable has a much higher degree of ionization than the other two.

The power curve for this cable is plotted logarithmically in Fig. 17. As before, the broken line *a b d e* gives the total power loss and the straight line *a b c* gives the dielectric power loss. The difference of the

two, which gives the ionization power loss, is plotted in Fig. 15 as curve $P_i$. Likewise, the dielectric power curve $P_d$ is plotted in Fig. 15. The equations of these two curves are

$$P_i = 3.81 \times 10^{-5} (E - 24{,}000) \text{ watts per ft.} \quad (6)$$

$$P_d = 1.67 \times 10^{-10} E^2 \text{ watts per ft.} \quad (7)$$

Fig. 16 gives the power, power factor, and capacitance curves of Cable $A$, a 500,000-cir. mil, 8/32-in. (6.35-mm.) paper, 8/64-in. (3.18-mm.) lead, single-conductor cable.

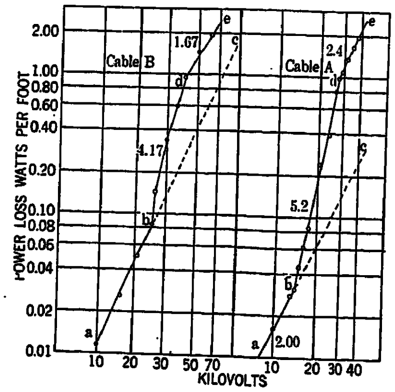


FIG. 17—LOGARITHMIC PLOTS FOR FIGS. 15 AND 16

This cable has a very much higher degree of ionization than the other three. The power curve for this cable is plotted logarithmically in Fig. 17. Again, the total power is given by the broken line $a\ b\ d\ e$ and the dielectric power by $a\ b\ c$. The difference of the two, which gives the ionization power, is plotted in Fig. 16 as curve $P_i$. Likewise, the dielectric power curve $P_d$ is plotted in Fig. 16. The equations of these two curves are

$$P_i = 8.8 \times 10^{-5} (E - 18{,}000) \text{ watts per ft.} \quad (8)$$

$$P_d = 1.62 \times 10^{-10} E^2 \text{ watts per ft.} \quad (9)$$

The foregoing characteristics of four cables were selected from among a large number of similar characteristics, as representing four cables having varying degrees of ionization from small values to large values. It is to be noted that in every instance the ionization curve $P_i$, obtained analytically, is identical in character with the curve of Fig. 11, obtained synthetically. The curve actually intercepts the voltage axis at the ionization voltage of the cable $E_g$. Due to the successive ionization of the series gas films, the first part of the curve is concaved upward. Eventually, after ionization is complete, it becomes a straight line which when extended intersects the voltage axis at $E_0$.

It is to be noted in cables 14 and 12 (Figs. 12 and 13) that the difference between $E_g$ and $E_0$ is small. In fact, so far as it is possible to tell, $E_g$ and $E_0$ coincide. This would seem to indicate that the gas films are not distributed throughout the dielectric, but rather concentrated near one radius, so that complete ionization occurs almost instantaneously.

For comparison, the characteristic constants of these four cables are tabulated below.

| Cables | 14 | 12 | B | A |
|---|---|---|---|---|
| $K$ | $1.21 \times 10^{-10}$ | $2.24 \times 10^{-10}$ | $1.67 \times 10^{-10}$ | $1.62 \times 10^{-10}$ |
| $K_1$ | $2.38 \times 10^{-6}$ | $1.28 \times 10^{-5}$ | $3.81 \times 10^{-5}$ | $8.8 \times 10^{-5}$ |
| $E_0$ | 17,500 | 12,500 | 24,000 | 18,000 |
| $E_g$ | 17,500 | 11,500 | 24,000 | 14,000 |
| $K_1\ E_0$ | $4.05 \times 10^{-2}$ | $16.0 \times 10^{-2}$ | 0.953 | $15.8 \times 10^{-2}$ |

This tabulation shows that the cables with highest ionization loss have the largest values of the ionization constant $K_1$, which is the slope of the ionization curve. (See Fig. 18.) Cable $A$, whose value of $K_1$ is 6.9 times that for cable 12, has a much smaller dielectric loss constant $K$. The value of the ionization voltage $E_g$ is important. The value of $E_0$ is significant, for it determines in part the ionization loss. The product $K_1\ E_0$ is the intercept on the power axis of the ionization-loss curve, Fig. 18. This product is not significant, since a low value of $K_1$ and a high value of $E_0$ are both desirable. For example, the value of $K_1\ E_0$ for cable 14 is $4.05 \times 10^{-2}$, that for cable 12 is $16.0 \times 10^{-2}$, and that for cable $A$ is $15.8 \times 10^{-2}$. Yet if ionization power is a criterion of the worth of a cable, 14 is superior to 12 and both obviously are far superior to $A$.

Incidentally, however, under some conditions $K_1\ E_0$ gives the value of the ionization power which corresponds to maximum power factor as will be shown later.

The authors feel, however, that the following constants are criteria for cable quality:

$K$ is important, for it determines the loss per foot in the solid dielectric.

$K_1$ is very important, for it defines in a large measure the ionization loss which is progressively harmful to the cable.

$E_g$ is important, for it defines the voltage at which

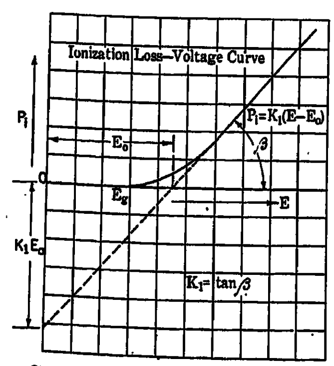


FIG. 18—GRAPHIC ANALYSIS OF POWER LOSS

ionization begins, and its value is closely related to $E_0$.

Another advantage of the foregoing analysis is that it gives a simple method for the determination of the power dissipated in cables for voltages far in excess of values determined experimentally. It is merely necessary to extrapolate two straight lines, the line $a\ b\ c$, Figs. 14, 17, to obtain the power loss in the solid dielectric, and the straight line $P_i$, Figs. 12, 13, 15, 16, to obtain the power dissipated in ionization.

The power loss $P_d$ in the solid dielectric is only harmful

because of its heating effects. It limits the kv-a. rating of the cable and, if excessive, may cause thermal disintegration of the dielectric. It is a function of the dielectric properties of the impregnating compound and paper in combination. The constant $K$ is a criterion of this loss and is readily determined for any cable by the foregoing method of analysis.

The power loss $P_i$ in the gas films is not only harmful because of its heating effects, but also because it is accompanied by ionic bombardment of the solid dielectric. This was shown in Part I[1]. This loss is accompanied by perforations of the paper, carbonization, chemical change in the compound, and tangential stresses. These effects are all cumulative and frequently result in tree designs and localized burn-outs. The constant $K_1$ is a criterion of this loss and is also readily determined by the foregoing method of analysis.

Hence, it may become desirable to rate cables, not only on the power loss per foot, but also on the ratio of ionization to dielectric loss, or similar factors.

When comparing the ionization power loss per foot

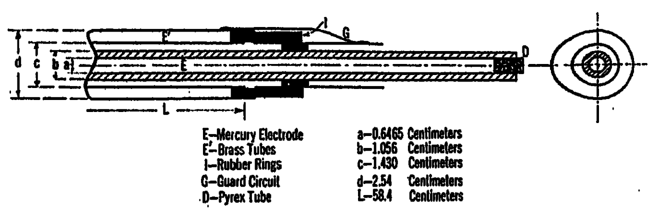


FIG. 19—GLASS-AIR CABLE MODEL, SINGLE AIR SPACE

in different cables, their cross-sectional size must also be taken into consideration. For example, Cable $A$ (Fig. 16) shows much higher ionization power loss than Cable $B$ (Fig. 15). Yet the fact that Cable $A$ has 500,000-cir. mil copper and Cable $B$ has only 133,000-cir. mil copper, and the volume of insulation in Cable $B$ is considerably less than in Cable $A$ must be taken into consideration. A more complete discussion of such factors will be presented when more data are available.

## CABLE MODELS

The authors have analyzed the characteristics of a large number of cables and in every instance the analysis has given results which are in accordance with those just described. It was deemed advisable, however, to apply these same methods to a cable model composed of Pyrex glass tubing and intervening air spaces. This was done in Part I of the paper to substantiate the theory of restricted ionization. The advantage of such models is that they are composed of a definite solid dielectric whose characteristics are very stable. The gas films are definite and the geometry of both the solid dielectric and the air films is known. Moreover, it is possible to determine the dielectric properties of the glass alone, which leaves only one unknown quantity, the dielectric properties of the gas film. Thus, data taken with such a cable model and analyzed in the same manner as that used with the foregoing cable data, should give results which are similar. This is found to be the case.

In Fig. 19 is shown a simple cable model. In all cable models Pyrex tubing was used, since the variation of power and power factor with temperature is known to be small.[4] The cable model, Fig. 19, is composed of a Pyrex tube having inside and outside diameters of 0.6465 cm. (0.254 in.) and 1.056 cm. (0.416 in.). This is surrounded by brass tubing whose inner diameter is 1.430 cm. (0.563 in.). This brass tubing is in three sections, a center section whose length is 58.4 cm. (23.0 in.), which corresponds to the active sheath of a cable, and the end sections $G$ which are guards. The two guard sections are separated from the active section by saw cuts and are held in alinement by outside bakelite cylinders, so that there is no solid dielectric in the dielectric field under measurement. This brass sheath is completely surrounded by a second brass tube whose inside diameter $d$ is 2.54 cm. (1.0 in.). This outer tube is connected to the guard sections, so that the entire active section of brass tubing is completely shielded. The glass tubing is filled with mercury, which constitutes the center electrode of the model. Provision is made for filling the space between the glass tube and the brass sheath with water. This short-circuits the air space and permits the dielectric characteristics of the Pyrex tube alone to be determined.

Fig. 20A gives the electrical characteristics per meter of the Pyrex tube alone, the space between the inner brass tube and the Pyrex tube being short-circuited with water. The capacitance and power factor are substantially constant. A few air bubbles in the water did cause a very slight rise in power factor. However, this effect in the determination of the ionization loss in the air space is almost insignificant. Hence, since these few bubbles would have no appreciable effect on the principles which the authors are attempting to prove, the time and difficulty involved in their removal did not seem justified. The current curve $i$ is linear and the power increases as the square of the voltage. These relations appear to be true generally of dielectrics at room temperature, provided all occluded gas is removed.[4]

Fig. 20B gives the power factor, the power curve $P$, and the capacitance curves per meter of the model without water in the air space. These curves are typical of dielectrics containing occluded gases. The curve $P_G$ gives the power loss in the glass taken from Fig. 20A. For any given voltage, Fig. 20B, the value of $P_G$ is obviously not equal to the value of power given by this same voltage, Fig. 20A, for in Fig. 20B the total voltage is equal to the vector sum of the voltages across the glass and air film in series. Correction was made by finding the current $E\,C\,\omega$ at any voltage $E$, Fig. 20B, and then taking from the power curve $P_G$, Fig. 20A, the loss corresponding to this same value of current. This method disregards the loss in the glass due to the harmonics which are produced in the current wave by ionization. Since, however, the loss in the glass is

small compared with the ionization loss, little error is introduced into the curve $P_A$, Fig. 20B.

The curve $P_A$, Fig. 20B, gives the power loss in the air film, $P_A$ being obtained by deducting the power loss in the glass, given by curve $P_G$, from the total power-loss curve $P$. The resulting ionization power-loss curve $P_A$ is identical with those obtained for cables. From the ionization voltage $E_0$, the curve begins to be concave upwards in accordance with the synthetic curve, Fig. 11, based on the fundamental characteristics of the solid dielectric and gas films in series. That

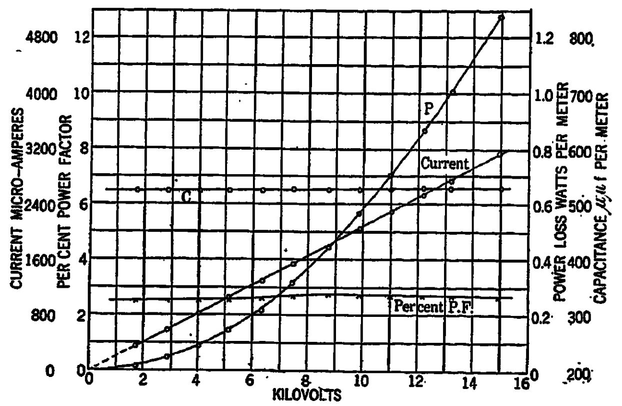


FIG. 20A—CABLE MODEL, GLASS DIELECTRIC SINGLE AIR SPACE—CURVES

Frequency = 60 cycle, temperature = 20 deg. cent.
$P = P_G$ = Power loss in glass = $56 \times 10^{-10} E^2$ watts per meter

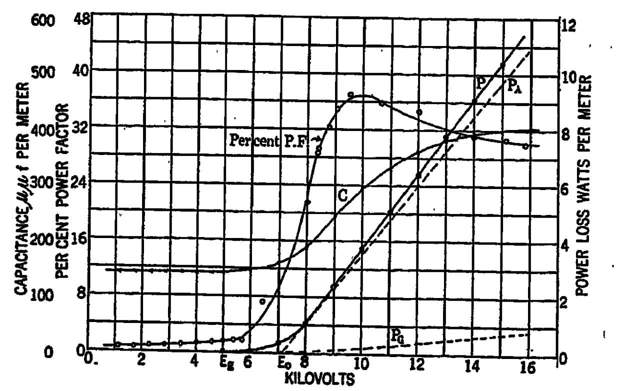


FIG. 20B—CABLE MODEL, GLASS-AIR DIELECTRIC SINGLE AIR SPACE—CURVES

Frequency = 60 cycle, temperature = 20 deg. cent.
$P$ = Total power loss, $P_A$ = Power loss in air (ionization loss) = $12.24 \times 10^{-4} (E - 7100)$ watts per meter, $P_G$ = Power loss in glass

is, it appears to be almost certain that the relation of ionization power loss in a cable to voltage is a linear relationship after all the gas films have become ionized.

In order to simulate more nearly the conditions existing in a cable, in which the gas films are distributed throughout the insulation, a second cable model of Pyrex tubing was constructed having three concentric gas films in series. This produces very nearly the type of cable illustrated in Fig. 10. A diagram of this model is shown in Fig. 21. $B$ is a polished brass rod having a diameter of 4.78 mm. (0.188 in.). It is surrounded by a Pyrex tube $G_1$ having a 1.0 mm. (0.0394 in.) wall, giving an air space $A_1$ of 1.6 mm. (0.063 in.). $G_2$ is a second Pyrex tube surrounding $G_1$, giving an intervening air space of 1.75 mm. (0.0689 in.). $G_2$ is surrounded by a brass tube $B_1$ having an inside diameter of 1.9 cm.

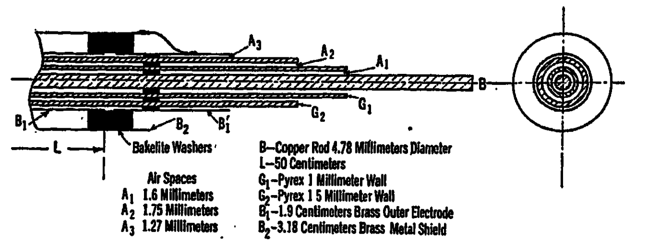


FIG. 21—TRIPLE AIR SPACE CABLE MODEL

(0.748 in.), which gives an air space $A_3$ of 1.27 mm. (0.05 in.) between $G_2$ and $B_1$. $B_2$ is another brass tube surrounding $B_1$ and acting as a shield. $B_2$ is connected to $B_1'$, which is identical to $B_1$ but separated from it by a saw cut. Hence $B_1'$ acts as a guard ring. The effective length $L$ of $B_1$ is 50 cm. (19.68 in.). It will be noted from Fig. 21 that none of the dielectric used for spacing the tubes is within the dielectric field under test.

The power, power-factor, and capacitance curves per meter for this model, as functions of voltage, are given in Fig. 22. With this type of cable model, it was hoped that the discontinuities in the power curve, which occur as the successive air spaces become ionized, would be

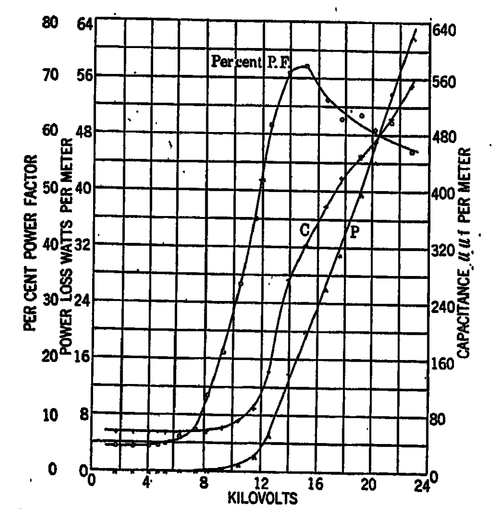


FIG. 22—CABLE MODEL, GLASS-AIR DIELECTRIC TRIPLE AIR SPACE CHARACTERISTICS

Frequency = 60 cycle, temperature = 19 deg. cent., $P$ = Total power loss per meter

apparent in the power curve. Such a theoretical curve has already been shown in Fig. 11. A study of Fig. 22, however, shows that the loss in the glass alone, as compared with the ionization loss, is not even noticeable in the power curve. Below the ionization voltage, for example, the power curve as far as one can tell coincides with the voltage axis. Hence, it would not be expected

that the discontinuities which occur as successive air spaces are ionized would be shown on this power curve. It is to be noted, however, that except for the usual curved portion at its beginning, the power curve is linear. This is still further confirmation of the authors' theory of the relation of ionization loss to voltage in cables.

The capacitance curve, Fig. 22, does show, however, discontinuities, which undoubtedly are due to the successive ionization of the air spaces.

*Power-factor Curve.* It follows from Equation (3) that the power factor

$$\text{P. F.} = \frac{K E^2 + K_1 (E - E_0)}{E^2 C \omega} \tag{10}$$

where $\omega$ is $2\pi$ times the frequency.

If the variation of capacitance $C$ with voltage is known, Equation (**10**) gives a means of extrapolating the power-factor curve. With most cables, the capacitance $C$ varies only a few per cent with voltage, so that the law may be very closely approximated. It thus becomes possible to determine graphically the maximum point of the power-factor curve without actually carrying the cable to the high stresses that would frequently be necessary, if this maximum were to be determined experimentally.

If Equation (**10**) be differentiated with respect to voltage, the equation becomes

It is interesting to note that if the term $E \dfrac{dC}{dE}$ is small compared with $2C$, the point of maximum power factor occurs when $E = 2E_0$. This condition is closely approximated with Cables 12 and *B*, Figs. 13 and 15. (See also Fig. 23.)

If this value, $E = 2E_0$, be substituted in Equation (**1**), the ionization power under these conditions of maximum power factor becomes $K_1 E_0$, which is the intercept of the ionization power curve with the power axis. (See Fig. 18.)

Equation (**10**) may be divided into three terms.

$$\text{P. F.} = \frac{K}{C\omega} + \frac{K_1}{E C \omega} - \frac{K_1 E_0}{E^2 C \omega} \tag{12}$$

The first term is nearly constant and is equal approximately to the power factor of the cable below the ionization voltage $E_0$, since the second and third terms vanish below the ionization voltage. The second term, whose denominator is equal to the charging current of the cable, is approximately a rectangular hyperbola lying in the first quadrant. The third term, whose denominator is equal to the volt-amperes of the cable, is a hyperbolic type of power function of the form $y = x^n$, where $n = -2$. This therefore represents an "inverse square" curve which lies in the fourth quadrant and approaches zero with increasing values of voltage.

$$\frac{d\,(\text{P. F.})}{dE} = \frac{E^2 C\,[2KE + K_1] - [KE^2 + K_1(E - E_0)]\left[2CE + E^2 \dfrac{dC}{dE}\right]}{E^4 C^2 \omega} \tag{11}$$

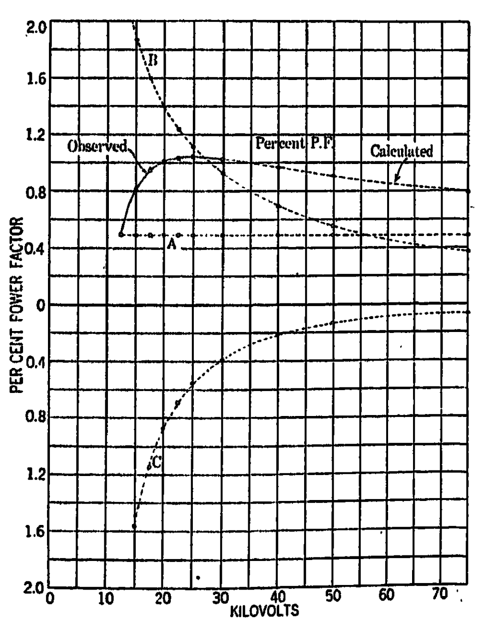


**Fig. 23—Analysis of Per Cent Power Factor Cable No. 12**

Per cent power factor $= A + B - C = 100\left(\dfrac{K}{C\omega} + \dfrac{K_1}{E C \omega} - \dfrac{K_1 E_0}{E^2 C \omega}\right)$

Each of these three terms, Equation (**12**), is plotted in Fig. 23, using the data of Cable 12, Fig. 13. The magnitude of each term is given. It is obvious that the second term is responsible for the descending portion of the power-factor curve, which occurs when the voltage is carried to sufficiently high values. The computed power-factor curve is carried well beyond the experimental curve.

It thus becomes possible to extrapolate the power-factor curve of a cable to values of voltage which it may be inconvenient or even impossible to obtain experimentally.

*Current Curve.* The variation of the total current in a cable with voltage requires but little comment. The current is

$$I = E C \omega \tag{13}$$

since in any ordinary cable the effect of the energy current on the absolute value of the total current is negligible.

It is, however, interesting to study the component currents in a cable.

The energy current,

$$I_e = \frac{P}{E} \tag{14}$$

substituting the value for $P$, Eq. (3) in Eq. (14)

$$I_e = \frac{K\ E}{C} + K_1 - K_1 \frac{E_0}{E} \tag{15}$$

Neglecting change in capacitance, the first term is proportional to the voltage and is the solid-dielectric energy current. The second and third terms are the two components of the ionization energy current. That is,

$$I_i = K_1 - K_1 \frac{E_0}{E} \tag{16}$$

The first term is constant, as shown in Fig. 24. The second term is a rectangular hyperbola which lies in the fourth quadrant, as shown by the curve $I_i'$, Fig. 24. As the voltage $E$ increases $I_i'$ decreases. Hence the ionization energy current itself is a rectangular hyperbola having the current axis as one asymptote and a horizontal line $K_1$ units from the voltage axis and parallel to it as the other axis. The ionization energy current thus approaches $K_1$ as a limit, as shown in Fig. 24. Therefore, the energy current of a cable may be analyzed into three simple components, one practically proportional to the voltage, a second which is constant, and hence independent of the voltage, and a third which is inversely proportional to the voltage. Fig. 25 shows the ionization energy currents and the total energy currents for cables 14, 12, and $A$. The relative degree of ionization of these three cables can be readily determined by comparing these curves.

*Capacitance Curves.* The curve showing the increase

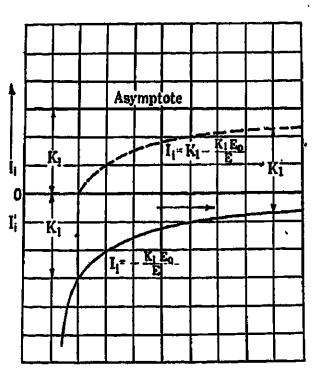


FIG. 24—GRAPHICAL ANALYSIS OF IONIZATION ENERGY CURRENT

$$I_i' = K_1 + I_i = K_1 - \frac{K_1 E_0}{E} = \frac{P_i}{E} = K_1 \left( 1 - \frac{E_0}{E} \right)$$

$$I_e = \frac{P}{E} = K\ E + K_1 - \frac{K_1 E_0}{E} = \frac{K\ E^2 + K_1\ E - K_1\ E_0}{E}$$

in capacitance in cables after ionization begins may take three different forms. It may be practically a straight line, Fig. 16; it may be concave, upwards, Fig. 12; or it may be concave downwards, Fig. 13.

In Figs. 8 and 9, which give the electrical characteristics of flat air films, the increase of capacitance with voltage, after ionization begins, is substantially linear. The capacitance curve is, therefore, similar in character to the power curve. Also the rate of increase of capacitance with current increases as the thickness of the air film decreases. This may be seen from a comparison of Figs. 8 and 9. The rate of increase of capacitance for the 0.3745-mm. (14.75-mil) air film, Fig. 8, is greater than for the 0.703-mm. (27.65-mil) air film, Fig. 9.

In the cable, the gas films are in series with the solid dielectric and in series with one another. The resulting capacitance of the cable is determined, therefore, from a

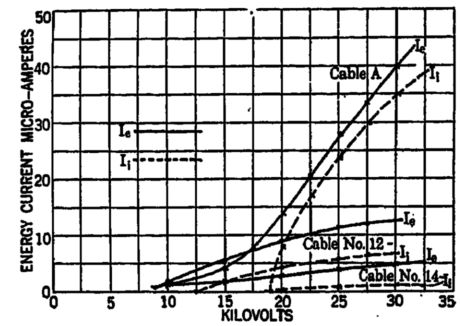


FIG. 25—$I_e$ AND $I_i$ FOR CABLES 14, 12, AND A

reciprocal relationship of individual capacitances in series. Hence, after ionization begins, the resultant capacitance of the cable is determined by the reciprocal relationships of the fixed capacitance of the solid dielectric, and the variable capacitances of the gas films which in themselves ionize successively, and after ionization each has a linear increase in its capacitance.

So far, the authors have not been able to express the resultant capacitance in a mathematical relationship which would show clearly the effects of various factors on the resultant capacitance of the cable. The relationship is not simple by any means. However, our study so far has shown us that the different types of capacitance curves, Figs. 12, 13, and 16, depend on the thicknesses of the gas films and on their positions within the dielectric. Before a more exact interpretation of the capacitance curves can be given, we must have further data on the capacitance relationships in very thin gas films, supplemented by further tests on cable models.

The matter is still being studied and it is hoped that these investigations will result in our being able to determine from a capacitance curve of a cable the relative magnitudes and the locations of the gas films within the cable dielectric.

## CONCLUSIONS

1. The power dissipated as ionization loss in cable insulation is more harmful than the power dissipated in the solid dielectric.

2. At room temperature and up to from 250 to 300 volts per mil, the power factor and capacitance of cable insulating paper impregnated with typical compounds are essentially constant with variation in voltage and the power loss varies as the voltage squared, provided the impregnation is conducted in such a manner as to remove practically all occluded gases.

3. The relation of power loss to voltage in thin gas films is substantially linear for voltages above the ionization voltage.

4. From the relationship in (3), it may be shown that in single-conductor cable insulation above the ionization voltage the relationship of ionization power to voltage is a curve which is concaved upwards until all the gas films are ionized. With further increase in voltage the relationship becomes linear.

5. By applying (2) to the power curve of a cable, it becomes possible to analyze it into two components, one giving the loss in the solid dielectric and the other giving the loss due to ionization. The ionization power curve so determined becomes linear and similar in character to the curves determined by (4) and (3).

6. It thus becomes possible to express the power curve by a simple equation and to extrapolate it, if necessary.

7. The constants of the power equations may be criteria of cable quality.

8. The analysis of power loss into two components, one giving the power dissipated in the solid dielectric and the other giving the power dissipated in ionization, is confirmed by tests made with glass cable models.

9. It becomes possible to express the power-factor curve by a simple equation and to analyze it into three simple components. Also the power-factor curve may be extrapolated and the point of maximum power factor can be determined.

10. The energy current of a cable can be analyzed into three simple components, a term nearly proportional to the voltage, a constant term, and a term inversely proportional to the voltage.

11. The character of the capacitance curves depends on the thickness and distribution of the gas films within the cable insulation.

The authors are indebted to D. W. Roper, F. M. Farmer, and W. F. Davidson, the members of the Paper-Cable Research Committee, for their suggestions and counsel during the progress of this investigation; to the several cable companies for their cooperation in supplying cable samples and compounds; and to Prof. H. E. Clifford of the Harvard Engineering School for his advice and suggestions during the progress of these investigations.

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## Discussion

**E. W. Davis and W. N. Eddy:** Because of the well known need for a criterion of insulation quality that is more reliable than the power factor—voltage relation and as convenient to determine, it is encouraging to see that the problem is receiving the attention of capable investigators like Professor Dawes and his assistants.

Preliminary inspection of the paper gives the impression that the authors' ionization loss $P_i$ might be superior to the power factor—voltage relation as a quality criterion. However, the calculation of $P_i$ from a considerable amount of available dielectric loss data indicated that $P_i$ is no more reliable or sensitive than the power factor.

The authors base their method of separating $P_i$ from the total loss on the assumption that without ionization the losses increase with the square of the voltage (the power factor showing no change with increasing voltage). This assumption appears reasonable for the authors' purposes because their discussion is limited to room-temperature data. At higher temperatures several different investigators have found in some cases the power factor *decreasing* with increasing voltage, indicating that the losses are not increasing as rapidly as the square of the voltage. This is probably due to the effect of the temperature and the liquidation of the compound on the anomalous properties of the insulation. While the rate of increase of loss with increasing voltage has been found to increase with temperature in the case of insulation impregnated with some compounds, it has been found to decrease in the case of similar insulation saturated with other compounds. Such data would indicate that the common interpretation of a flat power factor—voltage curve as meaning no ionization, may be rather limited in application.

It is suggested that the influence of temperature and character of the insulation on the rate of loss increase with voltage might well be considered for future investigation by Professor Dawes.

**W. B. Kouwenhoven:** At the Johns Hopkins University we are conducting an investigation to determine the correlation, if any, between the absorption under continuous potentials and the losses under alternating voltages of the same magnitude. In our experiments we are planning to study the continuous- and alternating-current characteristics of the impregnating compound alone, and of the paper alone. Then we will impregnate the paper with the compound and obtain the resulting properties of the impregnated paper.

We considered very carefully the use of flat samples for this work, and in fact designed apparatus for testing flat samples. After further study, however, we decided to use wound samples with the paper wound under tension, as in a cable, and we have redesigned our equipment to use this type of specimen.

Although Mr. Dawes' curves of power factor against voltage, Figs. 4, 5, and 6, are flat and show apparently that there are no voids present in his specimens, I still think that in a flat sample there is more danger of voids than in a tightly wound specimen.

In this type of apparatus it is always difficult to obtain a vacuum-tight seal. In the apparatus used by Drs. Whitehead and Marvin in making their study of *Anomalous Conduction as a Cause of Dielectric Absorption*, we were confronted by the problem of sealing. We experimented with rubber and a number of other materials with more or less unsatisfactory results. The best thing that we found for a vacuum-tight seal was a wax consisting of three parts of rosin mixed with one part of beeswax. This wax is made by melting the rosin, adding the beeswax, and mixing thoroughly. Before hardening, it should be poured into molds. It may be removed by slightly warming the bottom of the mold. In applying this wax to a joint two methods may be used. It may either be cut into strips and laid on the joint and melted in place by the use of a bunsen flame, or it may be melted in a small dish and applied by means of a brush. The surface should be warm enough to melt the wax, but care should be taken

not to burn it. Both surfaces of the joint are usually coated with the wax and the surfaces placed in contact. Then the joint is warmed by means of a bunsen flame sufficiently to soften the wax after which it is allowed to harden. The joint is then ready for use. When it is desired to break a joint the wax is easily softened by heating.

**T. F. Peterson:** The authors have done a very valuable work in making these tests on various samples of cable, compiling the data, comparing them with experimental models, and finally plotting the data according to logarithmic coordinates and then determining equations. These equations, however, are empirical and must be used as such. All too often experimenters, and especially those using the results of experimental work, are prone to take these empirical equations and use them as if they actually represented the functional relations of the phenomena involved.

I want to point out that in the case at hand these phenomena are very complex and are not necessarily represented as simply as Professor Dawes has done. About a year ago we had two papers on subjects similar to these being presented this morning, and at that time I derived some theoretical equations for the power factor—voltage curves of insulation containing air. These, of course, were based on uniform distribution of voids and uniform size. Although the expressions derived were rather complicated, they served very well in plotting curves similar to those which have been attained by Professor Dawes in this case.

I am citing this because it leads to a criticism of one of the figures in the present paper. To my way of thinking, the dotted curves of Fig. 11 should not be straight lines throughout their entire length, nor should they be parallel. What is actually represented in each of these curves is the loss in certain air films. As the voltage increases, the power factor or the equivalent phase angle between the current through air film and the voltage across it varies and, therefore, we do not have a straight line or linear relation between the power loss in the air space and voltage until a voltage is reached somewhat in excess of the initial ionization voltage for that void. That would mean that at the outset these curves would be sloping upward until finally a point is reached where the voltage is considerably in excess of the initial ionization and then we can say that the power factor in the void space is unity and, of course, the curve would be a straight line from that point on.

Inasmuch as this represents the loss in the void space at a certain radius from the center of the conductor, it is obvious that, as the ionization progresses outward, there will be more air (based on uniform distribution), *i. e.*, more void space in ionization, and hence the constant coefficient which would represent the slope of each of these curves will be variable and will be greater the greater the distance out from the center of the conductor. These curves ought, therefore, to have increasing slopes as we progress from $E_1$, $E_2$ to $E_5$.

The other point that I am not quite clear on is immediately below that particular curve. The statement is made that it is not generally conceded now that the power factor curve does not have a tendency to become flat when ionization is complete.

I might say that the work I have done seems to indicate that the curve does flatten out when ionization is complete. In fact, I should say that ionization is complete just a little in advance of the flattening out of the power factor curve. This can be shown pretty definitely by means of the mathematical equations.

**J. B. Whitehead:** It is more or less tacitly assumed in all three of the papers presented in this session that the invariable cause of the rise in the power factor—voltage curve is the presence of air. Certainly in these days no one will controvert the fact that the presence of air in the cable will give a rise in the power factor curve and probably in the vast majority of cases it is the principal cause.

However, we have shown that in laboratory samples both air and moisture can cause a rise in power factor curves, in insulation, which on dismantling appears to be extremely well impregnated.

Further we have shown that a decreasing power factor curve, a curve in which the power factor falls with voltage, usually to be found at the higher temperatures, is in very well impregnated insulation.

The flat curves shown by Professor Dawes, and which we have obtained many times, are shown by him as at relatively low temperature. If these samples were carried up in temperature, say to 60 or even to 80 deg., I venture to predict that he would find a decreasing power factor curve. In one of our papers, we have offered a speculative suggestion as to an explanation of this.

In contrast with the foregoing, I call attention to the variation which may occur from an entirely different cause, namely, the quality of the paper. In experimenting with some different qualities of paper, we have found that under the same conditions of evacuation and drying it is not possible to obtain the same final value of conductivity of the paper. In the various samples we have taken this final conductivity as a convenient indication of the amount of moisture remaining in the paper.

In those papers in which the conductivity cannot be reduced to the standard which we have adopted as pertaining to excellent paper, we have found that this decrease in the power factor curve at the upper temperatures is not found. We, therefore, emphasize the fact that the shape of the power factor curve may be varied considerably, due to other causes than that of large air voids. It is worth speculation as to whether the ionization test, as we have it today, might not be extended and modified to embrace tests at higher temperatures as an indication not only as to the original state of the paper as regards air in large voids, but also as regards the presence of moisture which would cause an increasing power factor—voltage curve. I take it that everyone will agree that a decreasing power factor—voltage curve is eminently desirable.

Professor Dawes has made an excellent case for a derivation of his very simple law of loss due to gaseous ionization. In view of all the factors that must enter in the gaseous ionization, I think it is extremely remarkable that this result should come out.

It has been shown long since that in the ionization of gas films at the moment when ionization begins there is a considerable increase in the pressure of the confined gas states. This pressure is not due to the temperature rise but due to the fact that a greater number of ions arises immediately.

Temperature will have a bearing on the loss due to ionization. Also above a certain voltage, ionization in an air film and accompanying oxidation undoubtedly reach saturation or constant values.

We have reported experiments in which the character of the gaseous discharge in a gas film changes very definitely after a short time, and we have attributed this to the fact that whatever oxygen is present in the gas film is consumed or rather goes into combination with the adjacent insulation and as a consequence the energy loss following such a process, *i. e.*, during the subsequent constant condition, must be less than it was during the time that it was arising.

It would appear that these various matters must play some part in the process of ionization and in the behavior of cable insulation.

The decrease in power factor shown by Messrs. Shanklin and Mackay for artificial gas films is quite different in character from that I referred to earlier, *i. e.*, the decreasing power factor curve of good impregnated paper. In the latter case, the values of power factor involved are of very much lower order of magnitude than those pertaining to the gas films, and I should like to ask a comment from one of the authors as to their explanation of the decreasing power factor of these gas films. It seems to me that it is a result of the stable condition of ionization which I have referred to as a saturated condition of ionization, in which practically all molecules are being ionized and where there is no progressive change thereafter.

**W. V. King:** (communicated after adjournment) As a com-

ment on the paper of C. L. Dawes, H. H. Reichard, and P. H. Humphries on *Ionization Studies in Paper Insulated Cables* may I submit the results of analyzing the dielectric loss curves on two separate samples of type H-3 conductor cable as tested by the Research Bureau of the Brooklyn Edison Company?

The solid dielectric losses vary approximately as the square of the voltage. The equations expressing these losses as a function of the voltage are

$$P_{d_1} = 12.5 \times 10^{-4}\, E^{2.08}$$

and $$P_{d_2} = 10.5 \times 10^{-4}\, E^{1.99}$$

where $P_d$ = Solid dielectric loss in watts per foot

and $E$ = Potential in kv.

The ionization voltages for the two samples are

$$E\,o_1 = 21.3 \text{ kv.}$$

$$E\,o_2 = 17.0 \text{ kv.}$$

The voltages at which ionization will commence will depend upon the pressure in the void, the size of the enclosure, and the different distribution of the voids from the copper conductor to the lead sheath. Many investigators have shown that an increase in the void pressure and a distribution of many small voids instead of several large ones both tend to raise the ionization point.

The total dielectric losses above ionization voltage vary as the 2.31 power of the voltage which compares very favorably with the results of Dawes, Reichard, and Humphries on a single-conductor cable with metallized shielding tape and designated as cable 14 in their paper. The equations expressing the total dielectric losses above ionization voltage are

$$P_1 = 6.18 \times 10^{-4}\, E^{2.31}$$

and $$P_2 = 4.27 \times 10^{-4}\, E^{2.31}$$

The ionization losses, after ionization is complete, do not vary exactly as a linear function of the voltage. However, a very small error will result if the points are averaged and a straight line is drawn to represent the variation of ionization loss with voltage after ionization is complete.

**C. L. Dawes, P. H. Humphries, and H. H. Reichard:** Referring first to the discussion of Messrs. Davis and Eddy, the authors did not intend to convey the impression that $P_i$ should be the sole criterion of cable quality. They are of the opinion, however, that it may be *one* of the factors which determine cable quality. Experimental work to determine the relation of cable constants to cable life is now being conducted and when sufficient data are obtained, it may be possible to state more definitely the relation of $P_i$ to cable life.

In our paper we found that the loss in the solid dielectric does vary as the square of the voltage, but the method of analysis is not at all dependent on this fact. A careful study of the method shows that it is necessary merely to assume that solid-dielectric power loss follows the same law above the ionization voltage as it does below the ionization voltage. We realized that at the higher temperatures the power factor did not necessarily remain constant with increase in voltage and a footnote to this effect is given at the bottom of the fifth page. As stated in the first paragraph of the Synopsis, this is a report of progress and the authors hope to present a paper somewhat later which will deal more specifically with phenomena at higher temperatures and some of the other matters mentioned by Messrs. Davis and Eddy.

We hope that Dr. Kouwenhoven and his associates will use cylindrical samples rather than flat samples, since this will give an excellent opportunity to compare the results obtained with the two types of samples. If their results differ from ours, there will be opportunity to reconcile the differences. On the other hand, if the results are in accord, it will show that either flat or cylindrical samples may be used for such tests. Dr. Kouwenhoven feels that there is greater opportunity for voids with flat samples than with cylindrical samples. As a matter of fact, we chose flat samples because we believed they gave less opportunity for voids. With a flat sample it is possible to expose the paper and compound to a high vacuum and to apply the compound to the paper without the upper electrode being in contact with the paper. This electrode is then lowered while still in the high vacuum. This process prevents the occluding of any considerable amount of air within the sample. Since these experiments were performed, improvements in sealing the apparatus together with a pump of larger capacity have enabled us to obtain vacuums as close to zero as we can read the mercury meniscus, so that the amount of occluded air is now almost zero. It is difficult to seal this apparatus with wax and resin as Dr. Kouwenhoven suggests, on account of its construction. We now find that porthole rubber makes very satisfactory gaskets and is very convenient to use.

I agree with Mr. Peterson that our results are to some extent empirical, but to date even the fundamental work on dielectrics has, for the most part, been empirical. We are submitting the results as we find them and as is frequently the case with pioneer research, later investigators may be able, with improved methods, to add refinements to the relationships which we now offer. We do believe, however, that the characteristics of both solid and gaseous dielectrics are more fundamental than Mr. Peterson realizes. For example, the fact that the voltage across an ionized gas ultimately becomes constant with increase in current is one of the fundamental discoveries of Townsend, and for the most part is very likely due to the effect of space charge. Mr. Peterson's criticism that in Fig. 11 the power lines are not necessarily parallel is justified although the figure is not incorrect; the slope of the different power curves should have been shown as being different in order to generalize the discussion. As a matter of fact, the curves were not parallel in the original sketch, but somehow the figure finally appeared in the paper with the lines drawn parallel. The general theory concerning the curved and straight portions of the resultant curve, however, still holds, as is stated in the paper, "This is true irrespective of the slopes of curves *a* to *e*." The individual power curves should be substantially straight, however, in spite of Mr. Peterson's reasoning to the contrary. He appears to rest his agreement on the origin being at $E_o$. As a matter of fact, in the region under discussion, the rate of increase of voltage $E$ decreases until it becomes zero (that is, $E$ becomes constant) and at the same time the power factor increases until it becomes nearly constant. These changes are such that in the equation for power, $P = E\,I \cos\theta$, the term $E \cos\theta$ is nearly constant in this region for any given gas. The coefficients which represent the slope of each of the curves in Fig. 11 depend on the thickness of the gas film and on the pressure. The coefficient does not depend on the geometrical position of the void in the cable insulation as stated by Mr. Peterson. The slope of the resultant curve obviously increases as the voltage is raised until all the voids become ionized as stated in the paper.

We do not recall having stated in a previous paper that the power-factor curve does not have a tendency to become flat when ionization is complete. We have shown, however, that the power-factor does reach a maximum and then actually decreases and that the reasoning as to the character of the power-factor curve previous to our work had been fallacious. Mr. Peterson, however, seems to be in accord with our conclusion that ionization is complete at point *f*, Fig. 11, which is in advance of the "flattening out," or better, the maximum value of the power-factor curve.

Like Dr. Whitehead, we find with the different cable compounds both increasing and decreasing power-factor curves at the higher temperatures and we intend to analyze our results under these conditions as the work progresses. The committee, as well as ourselves, believes it wiser to confine our efforts for the time being to work at room temperature, until some of the many problems that have already arisen have been solved. We shall then consider the matter of higher temperatures. We agree with Dr. Whitehead that it does appear to be remarkable that the

power curve for ionized gases, at least its lower portion, is so simple. Some of the reasons for this have been brought out earlier in this discussion. At the higher current densities, however, the voltage and power factor tend to become essentially constant, which would make the power proportional to the current after this condition is reached.

Referring to Dr. Whitehead's query as to the cause of the decreasing power factor of gas films, we intend to analyze the electrical characteristics of gas films in a later paper. We do not feel that the current density reached in occluded gas films is sufficiently high to produce the condition of high saturation as Dr. Whitehead suggests. At this time the increase in the power factor of the gas films themselves appears to be due to the increased power loss and the enormous increase in capacitance of the films themselves. The increase in capacitance is due to increased space charge. If the air film be considered as a series circuit having resistance $R$ and capacitance $C$ in series, $\tan \theta = 1/R\,C\,\omega$, where $\theta$ is the power-factor angle and $\omega = 2\,\pi$ times the frequency. With increase in current, $R$ will first increase rapidly to a maximum and then decrease slightly to a nearly constant value. $C$, however, will always increase rapidly with voltage. Hence $\theta$ ultimately decreases and the power factor increases with voltage. With the current densities which we have been able to attain, the power factor reaches a maximum value of 0.75 or 0.80 and then becomes nearly constant. It is the combination of the increasing power factor of the gas films and the constant power factor of the solid dielectric which gives the increasing and then decreasing power factor of the two in series, such as is shown by the curves in Figs. 13 and 14 of the Shanklin and Mackay paper.

Mr. King's results are in part a confirmation of our own work. We have found in some instances recently that the ionization power loss $P_i$ increases more rapidly with voltage than is indicated by the linear relationship. This was found to be due in part to heating of the cable while taking measurements. However, after this factor had been eliminated, the $P_i$ curve of some cables that had been undergoing accelerated life tests still showed a slight upward curvature. We attribute this to the losses caused by the creepage currents which result from tangential electrical stresses. This loss probably does increase more rapidly with voltage than the linear relationship shows. Mr. King's results seem to bear this out, since it is well known that tangential stresses are large in unshielded 3-conductor cables. It would be interesting to learn of the experiences of others in this connection.

# Reduction of Sheath Losses in Single-Conductor Cables

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*Synopsis.—The use of single-conductor, lead-covered cable for high-voltage, three-phase transmission lines results in sheath losses ranging from 25 to 300 per cent of the conductor losses for cables installed in separate ducts, unless special methods for the reduction of the losses are used. Some of these methods, while practically eliminating sheath losses, cause a-c. sheath potentials which may be injurious. In this connection the authors have developed a new scheme of bonding and a new bonding device, which appear to have marked advantages.*

*This article consists of a general discussion relative to the reduction of sheath losses with special reference to laboratory tests and field work done on 132 miles of single-conductor cable of the Commonwealth Edison Company.*

*The economics of sheath losses and of the methods for their practical elimination are discussed. The theories of sheath losses and induced voltages are outlined and correlated, and new formulas and curves are developed. An analytical and graphical comparison of sheath bonding connections is presented. Investigations are reported on tests regarding the nature and extent of possible corrosion of sheaths caused by a-c. sheath voltages.*

## I. INTRODUCTION

WITHIN recent years the use of single-conductor lead-covered underground cable for three-phase voltages ranging from 11 to 132 kv. has been rapidly increasing. The principal reasons are:

(a) To transmit large quantities of power, for which three-conductor cable would be unwieldy;
(b) To obtain phase isolation;
(c) To gain advantage of the inherently higher unit dielectric strength of the insulation in single-conductor cable.

The growing importance of single-conductor transmission cable is shown by the fact that the percentage used as compared to the total cable used in the United States increased from 4 per cent in 1926 to 8 per cent in 1927.[1]

If the cables are installed in separate ducts and operated with solidly bonded sheaths, the sheath losses will range from 25 to 300 per cent of the conductor losses, thereby considerably decreasing the carrying capacity and increasing operating costs. If the sheaths are made discontinuous and bonded in special ways to prevent sheath currents, problems arise to provide satisfactory insulating joints, bonding apparatus and bonding connections, and to limit the sheath corrosion that may be caused by unneutralized sheath voltages.

In 1926 the Commonwealth Edison Company began the installation of single-conductor underground cable as three-phase, 60-cycle, 66-kv. lines and as 12-kv. leads to transformers feeding these lines. Since calculations showed that if the sheath losses were eliminated, the carrying capacity of these two sizes of cable would be increased approximately 20 and 70 per cent respectively, with a corresponding increase in the cost of the installed cable and conduit of 1 or 2 per cent, it was decided to make some trial installations with various kinds of insulating sleeves, bonding apparatus, and methods of bonding. Because the results of the preliminary installations were favorable, this practise was extended to all 66-kv. lines and 12-kv. leads in 1927.

The authors developed a new device, namely, a three-phase sheath bonding transformer, and also a new sheath bonding connection, which appear to have marked advantages over previous bonding devices and connections.

At the end of 1928 the various devices and the most promising schemes of connections were installed on 114 miles of 66-kv. cable and 18 miles of 12-kv. cable.

A general consideration of the entire problem of eliminating sheath losses and reducing sheath voltages on single-conductor a-c. cables will be attempted in this article with special reference to the work done by the Commonwealth Edison Company in the past two years.

## II. ECONOMICS

Preliminary findings, which have been confirmed, show that the elimination of the sheath losses in three-phase, 60-cycle, 66-kv. lines consisting of single-conductor 750,000-cm. cables installed in conduits with other cables, increases the average yearly carrying capacity from approximately 50,000 to 60,000 kv-a., or about 20 per cent. Since the total investment cost of the installed cable is approximately $60,000 per mile of line or about $1.20 per kv-a. mile, the increased investment value resulting from the elimination of sheath losses is, therefore, approximately $12,000 per mile of line. An additional saving is made in the cost of reduced sheath losses which is about $1000 per mile of line per year, assuming 55,000-kv-a. average yearly rating and a 60 per cent load factor. Capitalized at 10

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1. See Bibliography.

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per cent per year, this represents an additional investment saving of $10,000 per mile of line.

Extra investment is required for (1) insulating joints, (2) insulated cable saddles and bond wire, (3) special fire-proofing to minimize corrosion due to a-c. voltages, and, perhaps, (4) bonding devices such as reactors or transformers. The additional investment cost required per manhole is about $65 for the first three items and $125 to $250 for all four items. Using even all four

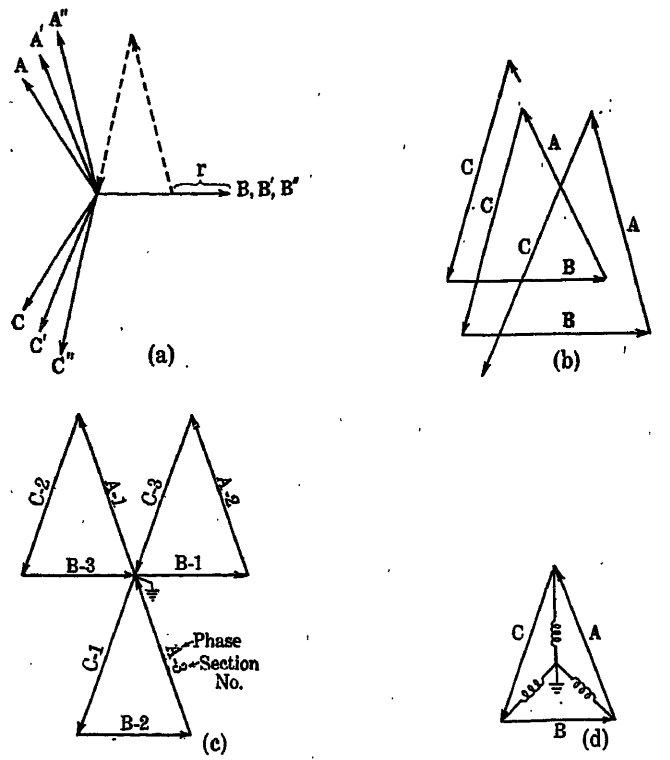


FIG. 1—SHEATH VOLTAGE VECTOR DIAGRAMS

(a) Induced voltages for equilateral rectangular and flat cable spacings, Note, $\Sigma E'' = r \neq 0$
(b) Continuous cross bonding, one circuit of Fig. 2c
(c) Cross-bonding, Fig. 2D
(d) Continuous cross-bonding, star-connected transformer, Fig. 2G

items and assuming twelve manholes per mile, the average additional cost of elimination of sheath losses is approximately $1500 per mile. This is only one-eighth of the increased value of a 66-kv. line due to the increased rating. The cost of the losses saved would pay for the additional investment cost in one or two years.

If 66 kv. cables only were installed in a given conduit, the elimination of sheath losses would result in an increase of carrying capacity of 30 per cent, while the extra cost would be the same.

On lower voltage, higher current cables, the gain is much larger. For the 12-kv. leads feeding the transformers of the 66-kv. lines, the standard practise in Chicago is to install three 1,750,000 cm. cables per phase. All nine cables are usually in one conduit. In this case the increase in carrying capacity obtained by the elimination of sheath losses is about 70 per cent (see cable 2 in Table I).

TABLE I

CALCULATED RATINGS, INDUCED SHEATH CURRENTS, LOSSES, AND VOLTAGES FOR TYPICAL CABLE INSTALLATIONS

| Item No. | Location | Oper. volt kv. | Cable: Conductor Size cir. mils | Conductor Diam.-Inches Int. | Conductor Diam.-Inches Ext. | Thickness 64th inch Insul. | Thickness 64th inch Lead | Diam. over lead Inches | Fre-quency | Cable Arrangement: Spacing | Inches between centers | Ratio for sheath to copper, % Currents | Ratio for sheath to copper, % Losses | Kv-a. rating, sheaths solidly bonded | Results of eliminating sheath losses: Increase in carrying capacity Kv-a. | Increase in carrying capacity % | or Decrease in copper temp. °C | Induced voltage per 100 ft. of sheath§ Outer cables | Induced voltage per 100 ft. of sheath§ Middle cable |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | N. Y. City | 11 | 600,000 | 0 | 0.89 | 12 | 7 | 1.5 | 25 | O* / O O | 1.7 | 3 | 1.2 | 9,400 | 58 | 0.6 | 0.4 | 0.44 | 0.44 |
| 2 | Chicago | 12 | 1,750,000 | 1.0 | 1.88 | 20 | 9 | 2.8 | 60 | O O O | 6.0 | 37¶ | 220¶ | 12,300 | 8,700 | 70 | 40 | 4.50 | 3.52 |
| 3 | N. Y. City | 44 | 500,000 | 0 | 0.82 | 30 | 8 | 2.0 | 60 | O* / O O | 2.3 | 14 | 12.5 | 34,000 | 1,400 | 4 | 2 | 0.95 | 0.95 |
| 4 | N. Y. City | 44 | 500,000 | 0 | 0.82 | 30 | 8 | 2.0 | 60 | O O O | 6.0 | 30¶ | 56¶ | 32,500 | 4,500 | 14 | 6 | 2.62 | 1.94 |
| 5 | Paris | 60 | 297,000 | 0 | 0.63 | 36 | 6.3 | 1.95 | 50 | O† / O O | 2.1 | 8.1 | 4.0 | 31,000 | 420 | 1.4 | 1.4 | 0.47 | 0.47 |
| 6 | Chicago | 66 | 750,000 | 0 | 1.00 | 48 | 9 | 2.85 | 60 | O O O | 6.0 | 40¶ | 85¶ | 50,000 | 10,000 | 20 | 6 | 2.35 | 1.83 |
| | Chicago | 132 | 600,000 | 0.75 | 1.20 | 46 | ‡ | 3.15 | 60 | O O O | 6.5 | 57¶ | 100¶ | 95,000 | 25,000 | 26 | 10 | 2.36 | 1.83 |

*In one duct
†Jute covered and buried in contact
‡Double sheath, 100 mils plus 90 mils thick
¶Average for all three cables
§Based on kv-a. rating with *no* sheath losses

The maximum allowable length of conduit between manholes is another important economic consideration. The cost of a manhole suitable for 66-kv. cable and joints is about $500. The additional cost of three 66-kv. joints and oil reservoirs plus the joints on lower voltage cables, fireproofing, racking, etc., will average $600 per manhole. Each manhole eliminated along a typical 66-kv. line will represent, therefore, a saving of $1100, besides additional savings in the unit cost of pulling and handling longer cable lengths.

Induced sheath voltages are directly proportional to cable length. Because of a-c. electrolysis and other reasons, it may be impracticable or destructive to operate the cables with sheath voltages exceeding 12 volts to ground or 22 volts between cables. Without bonding devices on 66-kv. cables, for example, this limit is reached with about 500-ft. lengths, or under practical conditions with about 13 manholes per mile. If a series type of bonding device as shown in Fig. 2E, or the cross-bonded, star-ground type of connection as shown in Fig. 2G is used, the lengths may be increased 100 or 60 per cent, respectively, without increasing the voltages between sheaths and ground, and with only negligible sheath losses.

The Commonwealth Edison Company has found it desirable to increase the maximum conduit lengths up to the length of a city block, about 660 ft. This reduces the average number of manholes per mile from 13 to 9, thereby saving over $4000 per mile of line. The increased investment cost for the necessary bonding devices for connection Fig. 2G, that is, one in every other manhole, is approximately $600 per mile. In view of this large saving, the company is trying over 90 H & M sheath bonding transformers for installation in new conduit built in 1928, where the sections of conduit are 550 to 700 ft. long.

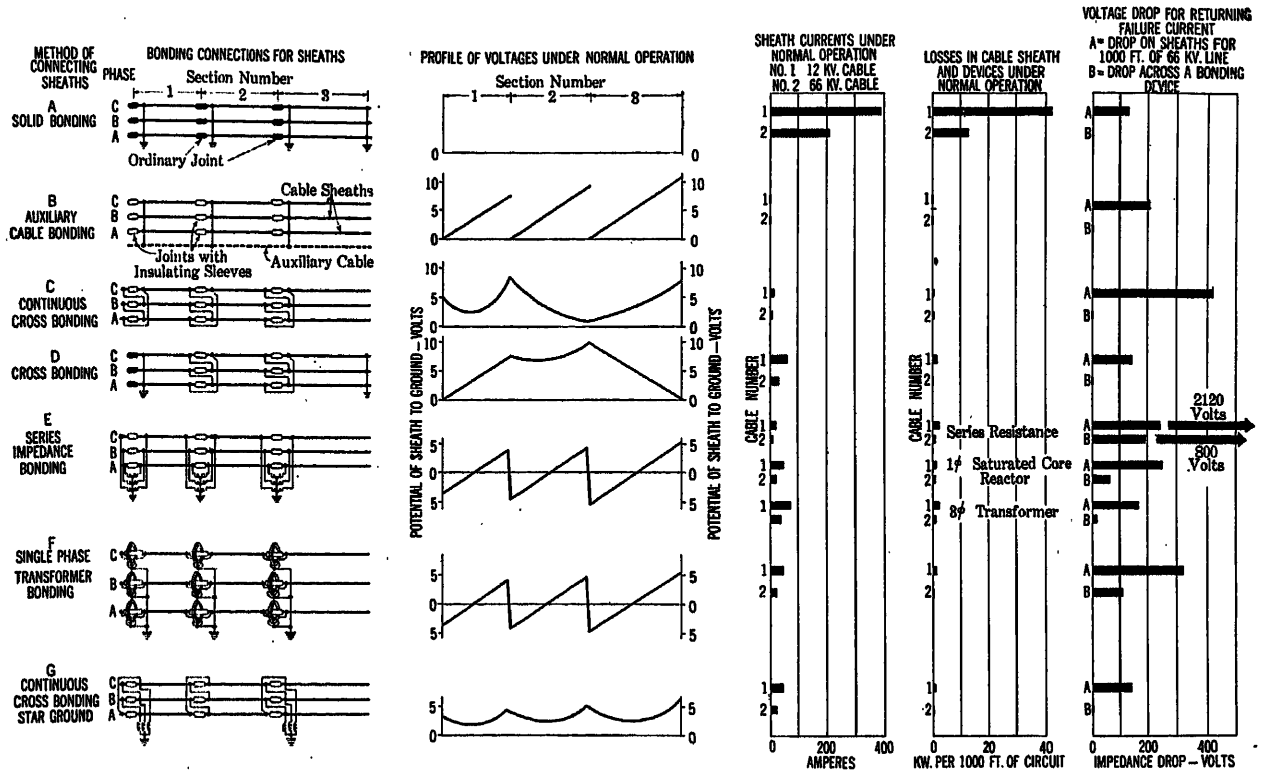


FIG. 2—COMPARISON OF CONNECTIONS AND RESULTING SHEATH VOLTAGES, CURRENT, AND LOSSES FOR 12-KV. AND 66-KV. THREE-PHASE 60-CYCLE SINGLE-CONDUCTOR CABLES

1. Lengths of sections 1, 2, and 3 are 330, 400, and 470 ft. for 66-kv. cables and 55 per cent as long for 12-kv. cables
2. Cable (1) 1,750,000 cir. mils, 12 kv.; flat arrangement with 6 in. between centers; 20,000-kv-a. load per circuit
3. Cable (2) 750,000 cir. mils, 66 kv.; flat arrangement with 6 in. between centers; 60,000-kv-a. load per circuit
4. Voltage profiles are for outer cables on connections *B*, *E*, and *F*, and profiles are for section 1-*C*, 2-*B*, and 3-*A* for connections *C*, *D*, and *G*.
5. Impedance drops are for fault current of 3000 amperes returning over sheaths with manholes every 400 ft.

## III. SHEATH VOLTAGES AND LOSSES

### A. *Sheath Voltages*

The induced sheath voltages in single-conductor lead-covered cables on a-c. circuits vary logarithmically with the ratio of distance between cable centers to the sheath radius, directly with the frequency and magnitude of the conductor currents, and directly with the length of section between insulating joints. When cables are closely spaced, the adjacent and remote portions of the sheaths are unequally affected, a condition commonly called the "proximity effect." For cable sheaths in contact the proximity effect becomes very pronounced, but for cables installed in separate ducts with six or seven inches separation between

centers, it is small. The proximity effect has little effect on sheath voltages and is usually neglected. Various formulas for sheath voltages and losses have been developed; these formulas, neglecting proximity effect, have been collected and extended in the Appendix.

For purposes of illustration, the calculated induced voltages for typical kinds of cables are shown in Table I, and for the 66- and 12-kv. cables of the Commonwealth Edison Company in Fig. 2, the calculations being made on the basis that no other influence, such as other single-conductor circuits, iron pipes, etc., is present. For cables in contact the voltages per 100 ft. at rated loads are less than one volt, while for cables in separate ducts the voltages are 2 to 4½ volts.

For equilateral cable spacing the induced sheath voltage vectors are equal. For flat and rectangular spacings the voltages induced in the outer cable sheaths are usually larger than the voltages induced in the middle cable sheath, and, in addition, the phase angles of the induced voltages are no longer 120 deg. apart (see Fig. 1A and Appendix).

At line terminals the configurations of lengths to potheads are usually complicated and the induced voltages are greatly affected by other equipment, such as the busses which close the current loops. Because calculations are difficult and usually give results that are too low, estimates based on past experience are the most reliable guides in such cases.

*B. Sheath Losses.*

If the single-conductor cables are solidly bonded, the induced voltages cause currents in the sheaths. When the spacing of sections of cables in a circuit is uniform between adjacent locations where the cables are solidly bonded, the induced voltage is neutralized as generated and there will be practically no voltage between the sheaths.

In addition to the effect of the conductor currents, the sheath currents are mutually inductive between themselves, so that in general the sheath current cannot be obtained by dividing the induced sheath voltage (open circuit) by the vector sum of the sheath resistance and sheath reactance. The sheath currents must be solved for simultaneously; and, as shown in the Appendix, the currents and losses in outer cables in flat or rectangular spacing are generally unequal.

In order to give an idea of the magnitude of sheath currents and losses, the ratios of currents and losses in the sheath as compared to the conductor are shown in Table I. In general if the sheath losses are eliminated and the cables are in separate ducts, the carrying capacity is increased 15 to 70 per cent for usual cable sizes, or for a given load the copper temperature is decreased from 5 to 45 deg. cent. For cables in contact, the sheath losses, including the proximity effect, are 1 to 15 per cent of the conductor losses. (Precise calculations for cables in contact are difficult because of proximity effect.)

The sheath currents and losses are graphically shown in Fig. 2 for the 66- and 12-kv. cables of the Commonwealth Edison Company.

## IV. Methods of Reducing Sheath Losses

When each phase of a given three-phase line installation is to be carried in a separate metal-covered cable, one of the following methods might be used for materially reducing the sheath losses:

1. Large increase in sheath resistance and solid bonding;
2. Large increase in conductivity of protective covering and solid bonding;
3. Two-conductor cable for each phase and solid bonding;
4. Special sheath bonding connections and devices.

Since the first three methods are impracticable or have only a limited application, they will be discussed briefly.

1. By increasing sheath resistance, the sheath current is proportionately decreased and the sheath loss is decreased approximately inversely as the resistance. However, no suitable plastic metal having a resistivity several times that of lead is known.

2. If the conductivity of the protective covering is greatly increased by adding copper wire armoring, for example, the current in the covering will approach the conductor current in magnitude. The conductivity of the armoring can be made so large that the losses will become less than the sheath losses with solid bonding and no armoring. For cables in adjacent ducts, the cost of the armoring is several times the cost of special bonding methods. The armoring might be used to advantage on submarine cables or on widely separated cables where the sheath voltages would be excessive if special bonding connections were used.

3. If a two-conductor "D" or concentric cable is used for each phase, with the two conductors connected to opposite ends of transformer phase winding, the two currents will have equal and opposite effects on the sheath and cause no induced voltage. This method requires special transformers (6-phase), but appears to have some merits for use at 10 or 15 kv. This scheme, however, also has the advantage of allowing continuation of operation with open delta connection when one cable fails. For 22 kv. and higher voltages, the tendency is for the cable size to become impracticable and for the cost per kv-a. of capacity to become considerably more than for single-conductor cable with special bonding.

## V. Sheath Bonding Methods

*A. General Considerations.* All special methods of sheath bonding have as their principal purpose the elimination of sheath losses. Some methods are designed also to result in smaller voltages between sheaths and to ground. Many bonding methods are possible, although only a limited number of connections is

practicable. Various connections (see Fig. 2) will be discussed later in detail.

For a given cable arrangement, conductor current, and system frequency, the sheath voltage *induced* per cable length is a fixed quantity. The voltage from end to end of a cable sheath cannot be reduced except by allowing sheath currents to flow and to neutralize partially or totally the voltages in the sheath where they are induced. It is possible, however, by the bonding devices and connections later described to *shift* the relative position of the induced voltage vectors of the several cable sheaths in such a manner that the voltages between sheaths and to ground are reduced.

The major considerations in the selection of connections, methods, and devices for special bonding are as follows:

1. Elimination of sheath losses and increase of cable current-carrying capacity;
2. Reduction of normal induced voltages between sheaths and to ground to keep corrosion due to a-c. voltages at a minimum;
3. Limitation of abnormal sheath voltages during failure to the lowest possible values.

The above objects must be accomplished *without* causing the following objectionable features:

1. Excessive losses in the sheath bonding devices;
2. Introduction of triple or other harmonic currents into the sheath circuit causing inductive interference with telephone circuits;
3. Interference with proper current drainage to prevent d-c. electrolysis; also adverse effect on operation of the a-c. sheath bonding method by flow of stray d-c. currents;
4. Excessive size, weight, space, or cost of bonding devices.

*B. Solid Bonding.* If the sheaths are joined with ordinary joints and solidly bonded (Fig. 2A), the installation is simple and avoids the introduction of new methods and apparatus. The sheath losses, however, will be of such size, especially if the cables are in separate ducts or farther apart, that the reduction in carrying capacity and the increase in sheath losses and operating costs are excessive, as previously indicated and as shown by the graphs in Fig. 2.

*C. Bonding One End Only to Auxiliary Cable.* Sheath losses may be eliminated by connecting only one end of every length of sheath to some auxiliary cable (Fig. 2B) or to the sheaths of other cables in the same conduits. This connection requires either the use of an extra duct and an auxiliary cable at considerable expense, or placing dependence on other cables which may not be permanent. One bond is more apt to become open by accident or mistake than bonds at each end, and the cable sheath may be left "floating" with the possibility of acquiring potentials dangerous to cable maintenance or to personal safety.

This method, however, has been used by the company for special lengths at line terminals consisting of sections of cable going to potheads, transformer neutrals, delta ties, etc. In these cases the induced voltages are irregular in magnitude and phase, and the cable sections do not lend themselves readily to other schemes of bonding. The usual practise of the Commonwealth Edison Company is to place one or more insulating sleeves in the sheaths of these special lengths. One end of each insulated section is connected to the station ground bus, and in case the section has a pothead, the pothead end is the end that is usually grounded. The end of the line in regular conduit is solidly bonded in the last manhole and also connected to the same ground bus.

*D. Cross Bonding.* Two methods of cross bonding have been suggested. In 1914 L. Emanueli[2] devised a method of sheath bonding in which the cable sheaths were cross-bonded continuously along the complete line, (Fig. 2C). With such a method and irregular cable lengths (which are of necessity always present), it is evident that the voltage vector diagram will not repeatedly retrace a triangle; instead, it will form three erratic triangular helices, one for each sheath circuit (Fig. 1B). The voltages to ground are uncontrolled and may become excessive, depending on the chance succession of unequal cable lengths. (See voltage profile in Fig. 2.) Also, during failures it is not desirable to confine the returning failure current to a single cable sheath for the full length of the line between the failure and the terminal, on account of the excessive voltage that would exist on the cable sheath near the failure due to the high impedance of one cable sheath. (Note last column of Fig. 2.)

W. E. Kirke and H. R. Searing[3] devised a method of cross-bonding, (Fig. 2D), in which the cables are solidly bonded in every third manhole and transposed in the two intermediate manholes. With this method the sheath voltages of the three sheath circuits between solid bonded points each trace the three sides of the triangle, starting at ground potential and returning to the same potential. If the cable spacing is not equilateral or if the cable lengths are not equal, the induced voltage triangles will not close and a residual voltage remains which is neutralized by circulating current over the three cable lengths (see "r" Fig. 1A and Equation 4 of Appendix). Since the impedance of three cable sheaths in series is presented to a relatively small differential voltage, the resulting sheath currents caused by unequal cable lengths are in general quite small; and the sheath losses (see Fig. 2) are usually not more than 1 or 2 per cent as great as for solidly bonded sheaths.

The ground potential is definitely fixed in every third manhole at the corner of the voltage vector triangle. As shown in Fig. 1C, the complete vector diagram for all three sheath circuits consists of three triangles with one common point (ground). The maximum voltage between sheath and ground is equal,

therefore, to the induced voltage of a length of cable as shown in the voltage profile (Fig. 2D).

This method has the advantages of simplicity and low cost, since it requires insulating joints in only two-thirds of the manholes and no additional bonding devices. Where conduit lengths are very unequal and where lines are frequently cut and interchanged, as on station properties, this method is often inconvenient.

Cross-bonding has been used in over 90 per cent of the work of the Commonwealth Edison Company. However, the company has installed insulating joints in all manholes in order that other bonding schemes could be used later if the sheath voltages with cross-bonding proved too high.

*E. Reactance Bonding.* In 1914 L. Emanueli[3] and in 1920 P. Capdeville[4] set forth the general principle of making a 50 per cent reduction in potential between sheaths and to ground by using impedances connected in series with the cable sheaths, the coils being interconnected and grounded at the mid-points (see Fig. 2E). The impedance of the devices is made considerably higher than the impedance of the sheaths, with the result that very little current flows and the voltage drop is almost entirely in the device. Emanueli used single-phase transformers for the purpose, while Capdeville describes both resistance and reactance (Fig. 2E), preferring a single-phase iron cored reactance designed for enclosure inside the cable joint sleeve.

A method of bonding cable sheaths through saturated iron core reactance coils has been devised by R. W. Atkinson.[5] The coils are connected in series with the cable sheaths, and their midpoints may also be interconnected and grounded (Fig. 2E). A second purpose of these coils is to limit the value of abnormal voltage which can exist on the cable sheaths due to excessive current in one or more of the conductors during a failure or other cause. For this purpose the coils are designed with closed iron cores proportioned to operate normally at or near the saturation point. The coils normally draw only a small exciting current. If, however, the voltage which is induced in the sheath and applied to the coil tends to rise to an excessive value due to abnormal conductor current, the iron core will saturate and the effective impedance of the reactor will greatly decrease. This will allow a very large current to flow which will neutralize the excessive induced voltage in the sheath. Abnormal voltages from this cause are limited to a small amount over the saturation voltage of the coil, perhaps 200 to 300 per cent of normal.

Although reactance coils accomplish their intended functions they introduce several objectionable features, among which the following may be named:

1. Since they are single-phase and operate near the saturation point, triple harmonic exciting currents are introduced into the sheath circuits, and may cause inductive interference on parallel telephone or signal circuits. (Triple harmonic currents equal to 15 per cent of the exciting current were found in test on such devices.)

2. It frequently happens that cable sheaths carry stray direct currents which may easily be of the same order of magnitude as the normal exciting current of the reactors (10 to 20 amperes). The iron core may become saturated with d-c. flux, increasing losses in the sheath and reactor and causing even harmonics in the exciting current, which may result in telephone interference.

3. During return flow of failure current along the sheath circuit the voltages or impedance drops across the coils are limited only by flux saturation in the iron cores. These voltages are added in series with the sheath voltage drop along the entire length of an isolated section of a line, and the total voltage drop, including the single-phase coils, may become excessive (see Table II and Figs. 2 and 6).

4. The three separate devices increase the investment cost and in addition require considerable manhole space.

TABLE II
COMPARATIVE VOLTAGES AND LOSSES FOR VARIOUS SHEATH BONDING CONNECTIONS

| Connection | Fig. 2 | Calculated* losses in per cent of losses with solid bonding | | | Calculated sheath voltage drops along 66-kv. line during failure†-volts | |
|---|---|---|---|---|---|---|
| | | Cable sheaths | Device | Total | Across device | Per 1000 ft. of line |
| Solid bond | A | 100 | .. | 100 | .. | 125 |
| Aux ground | B | 0 | 0 | 0 | .. | 200‡ |
| Continuous cross bond | C | 0.2 | .. | 0.2 | .. | 415 |
| Cross bond | D | 3 | .. | 3 | .. | 135 |
| Series resistance | E | 0.5 | 5 | 5.5 | 800 | 2120 |
| Sat. core reactor | E | 1 | 2 | 3 | 65 | 250 |
| 1-$\phi$ transformer | F | 0 | 2 | 2 | 100 | 320 |
| 3-$\phi$ transformer | E | 1.5 | 0.9 | 2.4 | 14 | 160 |
| Cross bond—star | G | 1 | 0.5 | 1.5 | .. | 135 |

*Average maximum limit.
†Isolated circuit with 3000 amperes returning on its sheaths with man holes assumed 400 ft. apart.
‡Assumed other cables in three adjacent ducts.

*F. Resistance Bonding.* Resistances may be connected as shown in Fig. 2E. They have several times the losses of equivalent reactance devices and must have large thermal storage to withstand line failure currents of several thousand amperes. Metal grids would probably be required and manhole water conditions would require enclosing them in a case. All these factors necessitate excessive size, and hence relatively large cost and inconvenience in installation.

Resistance bonding does not have the desirable voltage limiting characteristic for minimum voltage drop during the return flow of line failure currents. The summation of the $IR$ drops for a section of isolated line during the flow of failure currents might be several thousand volts per mile of line (see Table II and Figs. 2 and 6). Finally, resistance bonding would greatly

complicate protection against d-c. electrolysis, making effective "sheath drainage" practically impossible.

*G. Partial Current Flow Bonding.* Either series resistances or reactances may be chosen of such values that any desired percentage of the sheath current with sheaths solidly bonded will be allowed to flow. With such a scheme there is partial neutralization of the induced sheath voltages and thus a compromise is made between sheath losses and sheath voltages. This reduction may be made in addition to the 50 per cent obtained by grounding the coil mid-taps as in Fig. 2E. In no case can a reduction of more than 50 per cent in sheath voltage be obtained without introducing sheath losses.

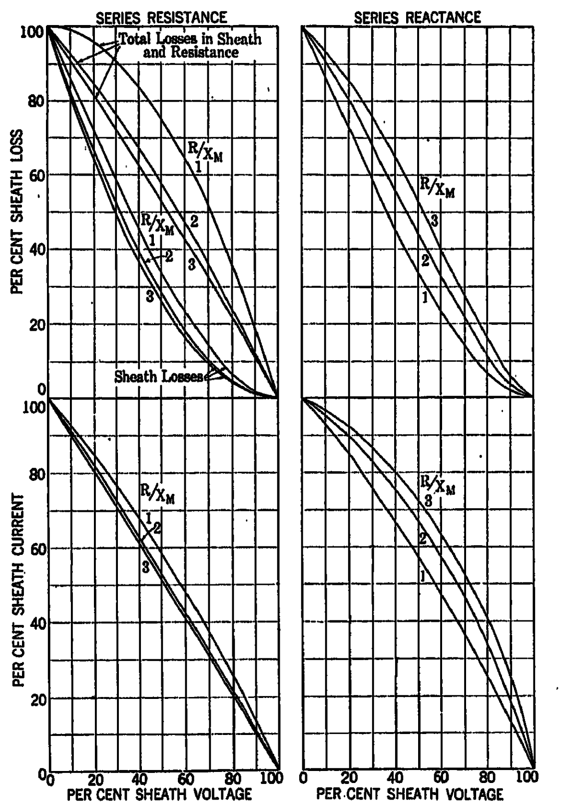


FIG. 3—SHEATH CURRENT, SHEATH LOSS, AND TOTAL LOSS *vs.* SHEATH VOLTAGE

Using series resistors or reactors in single conductor cable sheath
The ordinates give values as compared to the values which obtain for solid bonding, while the abscissas give voltages as compared to those which obtain when no sheath current flows

Fig. 3 shows in percentages the possibilities of this method for ordinary conditions where the ratio of sheath resistance to reactance varies from 1 to 3—the values usually encountered with 60-cycle cables.

If resistance bonding is used and numerical values are considered it is quickly discovered that the heat losses in the resistor itself are very large. For example, if sheath potentials are limited 50 per cent by this method, then, as shown in Fig. 3, the sheath currents are reduced to 50 or 60 per cent of the values which they would attain with solid-bonding, and the total losses in both the sheath and the resistor are from 50 to 70 per cent of the value for solid-bonding. Of these losses about half occur in the resistors and for typical cables are easily of the order of 200 to 300 watts per resistor.

If reactance bonding is used and sheath potentials are limited to 50 per cent of the induced voltages by this method, the currents are 60 to 70 per cent of the values with solid bonding, and sheath losses are 45 to 60 per cent of values for solid-bonding. The losses in the reactors will be small. The reactors usually must have an air gap in the core to prevent saturation of the iron.

With either resistance or reactance a serious practical disadvantage of this method of sheath voltage reduction is the necessity for separate individual adjustment of every device to values appropriate for the various sheath lengths if uniform conditions are desired along the line. In all other respects the connections, advantages, and disadvantages are the same as already discussed for reactance coils and resistors.

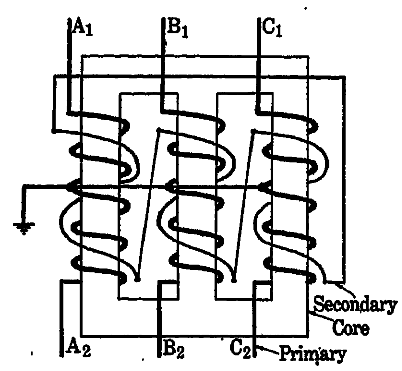


FIG. 4—WIRING DIAGRAM OF THREE-PHASE H & M SHEATH-BONDING TRANSFORMER

Series primary-wound secondary type for use in connection *E* of Fig. 2

*H. Single-Phase Transformers.* Both foreign[2,4] and American engineers have suggested single-phase transformers for sheath bonding (Fig. 2F). The iron core is placed either inside or outside the joint sleeve and around the copper conductor, which acts as a one-turn primary. The secondary winding is connected across the insulating sleeve so that its induced voltage exactly opposes the induced sheath voltage and prevents the flow of sheath currents. Individual adjustment of every coil is required. The method has serious mechanical disadvantages and introduces also practically all the disadvantages described for single-phase iron cored reactors.

*I. H & M Sheath Bonding Transformers.* In order to retain the advantages of 50 per cent reduction in the voltages between cable sheaths and to ground by using connection 2-*E*, or a 40 per cent reduction by using the newly devised connection 2-*G*, and at the same time not to incur the disadvantages listed for reactance or resistance bonding, the H & M three-phase sheath bonding

transformer has been devised by the authors. This transformer consists essentially of three primary coils wound on the legs of a three-phase iron core. The secondary winding may consist of an individual coil on each of the three core legs, the coils being delta-connected as shown in Fig. 4. This type of transformer is called the wound secondary type. A cheaper method of obtaining the secondary is to place a bar of copper around each end of the entire core, as shown in Fig. 5. This design is called the bar-secondary type. It is less effective electrically during cable failures.

The normal three-phase reactance of the primary coils is high, only a small exciting current flows, and losses are negligible. (The losses in the transformers themselves are only a fraction of one per cent of the losses which would obtain with solid bonding. See Table II.) The secondary coils function normally only as a tertiary winding preventing the flow of triple harmonic currents in the sheaths. When there is a line failure the single-phase failure current divides in

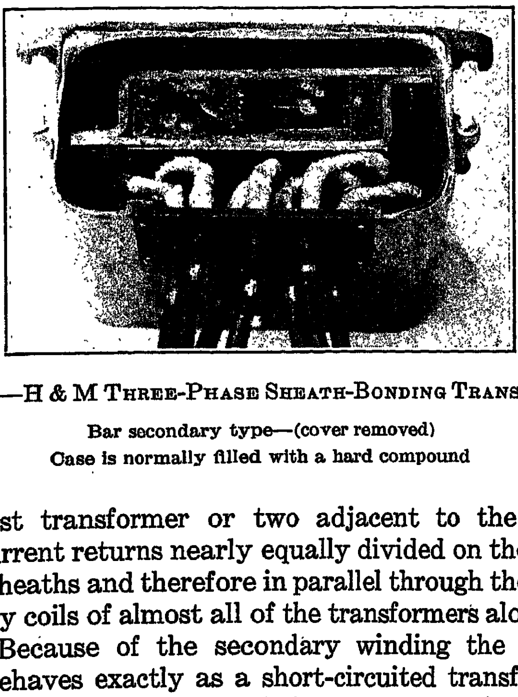

FIG. 5—H & M THREE-PHASE SHEATH-BONDING TRANSFORMER

Bar secondary type—(cover removed)
Case is normally filled with a hard compound

the first transformer or two adjacent to the fault. The current returns nearly equally divided on the three cable sheaths and therefore in parallel through the three primary coils of almost all of the transformers along the line. Because of the secondary winding the device then behaves exactly as a short-circuited transformer, and the series reactance of the transformer is due to leakage flux only (not saturation flux) with the result that the reactance drop is minimum in value. Consequently, in the case of cable failure current returning along the sheaths, the transformers cause only a very small increase in the total impedance drop, as indicated in Figs. 2 and 6.

Stray direct currents flow according to the resistance of the sheath circuit, and divide about equally in the three coils. The d-c. flux is in parallel in the three legs of the iron core and must return through an air path, and thus these stray direct currents flowing through the transformers will have no undesirable effect on the a-c. characteristics. Also, the transformer coils present very small resistance to the flow of direct current, and introduce no complications into d-c. electrolysis mitigation. The three-phase transformer allows large economies in size, weight, and convenience of installation.

To summarize, the three-phase transformers have the following characteristics: high three-phase impedance, minimum single-phase impedance, very low losses, practically no current harmonics, no interference with stray d-c. flow or electrolysis mitigation, and economy of installation.

The primary coils of the three-phase transformer may be connected in series with the sheaths and the mid-taps grounded as shown in the series reactance or resistance methods (Fig. 2E) or in the single-phase transformer method (Fig. 2F). With the three-phase transformer, however, the 50 per cent reduction in sheath potential is obtained without incurring the disadvantages inherent in the other devices.

In the scheme of connections devised by the authors and shown in Fig. 2G, the cable sheaths are cross-bonded continuously throughout the entire line, and the center of the vector triangle of sheath voltages is fixed at ground potential by star connected transformers spaced at approximately equal intervals along the line. The transformers may be installed in every manhole, in every second manhole, or in every fourth manhole. They should not, however, be connected at intervals which are a multiple of three cable lengths, because with such a connection the vector triangles have the ground point shifted over to the corner, instead of being fixed at the center. If the transformers are connected at too great an interval, the voltages during normal operation will not be sufficiently controlled as indicated in Fig. 1B; furthermore, impedance drops would be excessive during failures because the returning current must travel on one sheath circuit until it can divide at a transformer. Under practical conditions every second manhole appears to be the most desirable interval.

This connection permits a considerable saving in investment cost for the following three reasons: only half as many coils are required as for other connections; the coils are wound for about 60 per cent as much voltage as for the series connections; and only three coil leads are required for the sheath bonds.

Under practical conditions the maximum sheath voltages to ground are reduced to about 60 or 65 per cent of the induced voltages instead of to 50 per cent as with the series connections (see Table II and Fig. 2E and G).

*J. General Comparison of Voltages and Losses.* In order to make a general comparison of the various sheath bonding connections, Fig. 2 and Table II have been prepared. The profiles apply only to the normal voltage on the cable sheath. The maximum voltages occur in all cases in the manholes where the insulating sleeves are installed.

Table II and Fig. 2 give also a comparison of the abnormal voltage rises along the sheaths of a three-

phase 66-kv. line. It is assumed that a typical fault current (3000 amperes) is confined entirely to the three sheaths (in parallel) of the given line for all cases except for continuous cross-bonding where the fault current is confined to the one sheath circuit, and for auxiliary bonding where current returns on sheaths of three adjacent cables. In the practise of the Commonwealth Edison Company, the maximum distance to which the fault current will be confined entirely to a given line is one-half mile and occurs in conduit built in 1928. The company's practise for single-conductor cables as well as other cables is to bond them to other cable sheaths wherever possible, thereby reducing the impedance of the return path for the fault current. For the same reason the sheath bonding transformer neutral points or mid-taps are bonded solidly to other cables in the manhole wherever possible. On the 12-kv. leads to transformers the maximum length has been 2000 ft., the usual length, however, being only 500 or 600 ft.

In most of the sheath bonding methods under practical conditions a small amount of sheath current and losses are present. Some losses are present also in sheath bonding devices. The sum of these losses for the various connections under average field conditions is given in Table II and Fig. 2.

*K. Field Tests and Experiments on Connections.* Voltage tests have been made on all 66-kv. and 12-kv. single conductor cables bonded for elimination of sheath losses and on several short sections of line specially bonded for tests. In all cases the measured sheath voltages check those calculated by the ordinary logarithmic formulas (given in the Appendix) within about 10 per cent. The degree of accuracy of the voltage formulas is apparently greater than is required due to the uncertainties and complications of actual installations.

Tests have been made on connections *A, B, D, E,* and *G* of Fig. 2 and on some other less promising connections not shown. The results of all these tests verified the accuracy of the theories and formulas.

No measurable difference in sheath voltages was obtained with any of the connections with the cable dry or submerged in water. This does not indicate that leakage currents did not flow between the sheaths, but merely that the total amount is small compared to solid bonding currents and that because of symmetry of connections very little leakage current flows through the bonding devices.

No resistance devices were tested, but there is apparently no good reason to expect any departures from theory. Tests were made using single-phase Atkinson saturated core reactors and H & M three-phase sheath bonding transformers, both devices being rated at 22 volts per phase. In one case the latter device has withstood return failure current from a line service failure about one-half mile distant with no apparent effects.

Fig. 6 shows the much lower voltage drop for single-phase current flowing equally in all three coils of a three-phase transformer as compared to three single-phase saturated core reactors or resistors in parallel. The higher current values are similar to those which would occur in the field for return failure currents.

The vectorial relations shown in Fig. 1A or the summation voltage Equation 4 of the appendix for flat spacing was checked by disconnecting cable sheaths at one end of a group of cross-bonded 66-kv. cables installed in three nearly equal lengths of conduit. With one set of bonds open at the end of a group of three lengths, the voltages between cables were nearly zero, while the voltage between each sheath end and ground was about 35 per cent of the average voltage induced per sheath length. When the bonds were closed, sheath currents measured about 10 per cent of the solid-bonded values in spite of practical equality of the cross-bonded lengths.

A confirmation was made of the interesting fact that

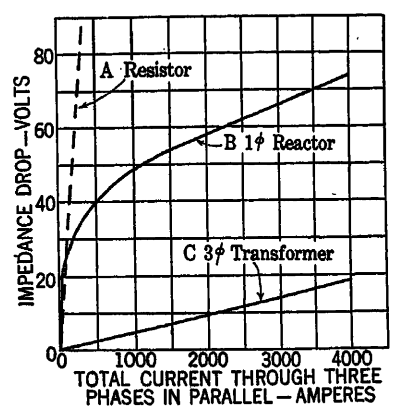


FIG. 6—IMPEDANCE DROP DURING SINGLE PHASE LINE FAILURE FOR SHEATH BONDING DEVICES

*A* = Resistor or open core reactor
*B* = Three 1-phase saturated core reactors in parallel
*C* = 3-phase H & M sheath bonding transformer (wound secondary type)
(Normal rated phase voltage, 22 volts for all devices)

for solidly bonded cables in either flat or rectangular spacing the voltage summation is not zero. The residual voltage which is cumulative along the entire line causes current to circulate between the three sheaths in parallel and the earth or surroundings, entering the cable sheaths at one end of the line and leaving at the other. This circulating current was measured for a 66-kv. line and totaled about 1/30th of the conductor current. The actual numerical value is of little significance since the impedance of the earth return is not known.

## VI. INSULATING JOINTS

In any method of eliminating sheath losses by special bonding, insulating joints are required for interrupting the electrical continuity of the sheath circuit. An insulating joint should be mechanically rugged, impervious to moisture, fluid tight under all operating conditions of internal pressure and vacuum, and should have an insulating ring or surface of sufficient length to

prevent excessive current leakage across the insulator. Also, it must operate satisfactorily when totally submerged in manhole water for years at a time.

A number of insulating sleeves has been devised. The leading types are as follows:

(1) An assembly of parts mechanically held together;

(2) A lead sleeve soldered directly onto a metallic coating on the ends of a porcelain insulator tube;

(3) A material such as Bakelite molded on brass.

The latter method, as incorporated in the H & T insulating sleeve, has been used for almost all the work of the Commonwealth Edison Company. In their practise the brass end rings of the sleeve are soldered to lead tubing which forms a part of the containing sleeve for the joint or slip-over sleeve for installation on the cable sheath. A picture of a longitudinal cross-section of the device as part of one-half of a 66-kv. joint sleeve is shown in Fig. 7. No leaks have occurred in these insulating sleeves in service, and several which had

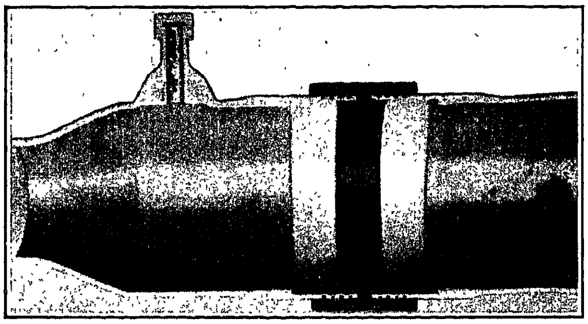

FIG. 7—CROSS-SECTION OF ONE-HALF OF 66-KV. JOINT SLEEVE WITH H & T INSULATING SLEEVE

operated for over 2 years under water with voltages of 15 to 35 volts across the Bakelite were inspected and all found to be in good condition.

When a common oil reservoir is used for the three joints, it is necessary to put insulators in each of the three oil pipe lines. Insulators suitable for this service are available. In one design of the H & T sleeve, the nipple for the oil pipe is molded directly into the Bakelite ring. This avoids the necessity for an additional insulator in the oil pipe line and at the same time it may be used as one of the filling nipples for the joint sleeve.

## VII. A-C. ELECTROLYSIS

Any method of eliminating sheath losses results in the presence of a-c. potentials between sheaths and to ground. The principal problem introduced by these voltages is the production or increase of sheath corrosion.

For lines in regular conduits, the Commonwealth Edison Company has operated single-conductor cables from a few months to two years with a-c. sheath potentials at full load of 5 to 12 volts to ground and 8 to 20 volts between sheaths. For lengths to potheads the voltages have been about the same, excepting for one temporary installation, where they were 100 per cent larger. In addition, a large number of laboratory tests has been made and is still being conducted for the purpose of determining the chief factors in corrosion produced by a-c. voltages and the practical maximum value that these sheath voltages should be allowed to attain. These tests and field observations have demonstrated that under field conditions corrosion due to a-c. voltages may occur. The principal factors appear to be as follows:

(a) Current density, which is dependent on the voltages between cable sheaths and to ground and on the resistivity of the surrounding manhole water, conduit, and fireproofing.

(b) The chemical nature of the water, ducts, and fireproofing.

(c) Superimposed d-c. potentials.

(d) Temperature.

The field studies have been made on cables installed in the company's ordinary conduits which are made by pouring concrete around precast concrete ducts to form a monolithic mass. The usual distance between duct centers is 6 or 6½ in.

As the company's standard fire-proofing has been rope and cement, the first tests were made with that covering. In the laboratory and field it was found that severe pitting was produced under the rope leaving distinct chainlike rope marks. Pitting occurred also under bubbles of air entrapped next to the sheath with cement directly applied with cheese-cloth binding. Since it was apparent that such pockets would be very difficult to eliminate entirely,—one air bubble or crack might cause concentration and a pit,—the opposite course was pursued by providing a uniform cushion of unimpregnated asbestos tape wrapped next to the sheath, over which was applied standard rope and cement fireproofing. This distributed the area of attack, laboratory tests showing that the severity of the pitting for given a-c. potentials was reduced at least 50 per cent, as compared to what was obtained with only rope and cement. It was discovered in the laboratory that certain asbestos preparations, cements and dips, had a very pronounced action in promoting corrosion, chiefly, it appeared, as a result of forming a good electrolyte and increasing the current density for a given voltage.

Tests were made in the field to determine the relative resistivities of the several elements for the purpose of obtaining an approximate idea of the important factors affecting the current densities. The range of resistances found was as follows:

RESISTANCE PER FT. OF CABLE LENGTH*—OHMS

| | Max. | Min. | Ave. |
|---|---|---|---|
| Bare cable sheath in manhole water........ | 35 | 1.5 | 10 |
| Wet fireproofing on cable.................. | 100 | 10 | 50 |
| Cable sheath in submerged ducts........... | 1500 | 250 | 400 |

*690 sq. cm. of surface per foot of cable (cable overall diameter—2.8-in.)

Laboratory tests have been made on ⅛-in. lead wires in test tubes with the wires submerged in waters taken from nine manholes and with the voltages such that the average current densities were maintained at 8.6, 12, and 20 milliamperes per sq. cm. The test tubes were kept at temperatures of 18 to 25 deg. cent., which are typical temperatures of manhole waters in the summer. The tests, which were run with 150 cells for periods of 300 hours, showed that corrosion occurs with a large variation in severity. It seems that the only general conclusion that can be safely drawn in these tests is that waters which are practically neutral chemically and of high specific resistivity will cause the smallest amount of corrosion with a-c. voltages.

Increased corrosion at higher temperature was determined by running duplicate test cells at room temperature and at 45 deg. cent. with current densities of 10 milliamperes per sq. cm. It was found that corrosion progressed two to four times as rapidly at the higher temperature. In all tests in small cells it was found that the active chemicals in the water were used up more or less rapidly and that with a larger volume of water, or with renewed water, the corrosion was greatly increased. In the field, of course, available water supply is often very large.

In some laboratory tests, conditions at the duct mouth were simulated. The cables were protruding from the dummy conduit with 6 in. between centers, the protruding portions being fireproofed with rope and cement. It was found that excessive corrosion could take place in the protruding portions with a potential of 10 volts between the sheath and ground, or 20 volts between cables. It was found also in these tests, and confirmed by a special field installation, that the rate of corrosion increased with the temperature. Other tests showed that the resistivity of manhole waters and cement ducts or fire-proofings decreased about 15 per cent, if the temperature is increased from 15 to 25 deg. cent. and 50 per cent for a change from 10 to 40 deg. cent. The increase of corrosion with temperature appears to be due to both the stimulation of chemical activity and to an increase in current density.

Cables that operated a year with voltages of 5 to 12 volts between sheath and ground and in conduits which were normally dry have been found upon removal to have only a very thin layer of corrosive products, probably only one or two mils thick, which could be easily rubbed off with a rag. Other cables which had operated a year and a half under water with maximum potentials of about 12 and 24 volts to ground showed pitting. The maximum depths of the pits were found in the manholes under rope and cement fireproofing and were approximately 4 and 15 mils deep, respectively. The cable that was in the ducts showed only surface discoloration of 1 or 2 mils thickness except in one case where the cable had operated with a sheath potential of 18 volts to ground and corroded to depth of over 20 mils at two isolated spots. It appears that the cable at these spots was intermittently wet.

Hayden[6] found that a-c. electrolysis could be greatly reduced, even entirely removed or reversed, by superimposing a d-c. voltage about 1.5 per cent of the a-c. voltage with the sheath negative. From tests made in the company's laboratory on cable samples and on lead wire in the test tubes it was found that the converse may be equally true and that a positive d-c. potential of 0.5 volt combined with a-c. potentials of 10 or 20 volts may be destructive out of all proportion to the sum of the effects to be expected from either potential alone. Apparently, when single-conductor cables are operating with induced a-c. potentials on the sheaths, special precautions will be necessary to keep the sheaths at zero or negative d-c. potentials relative to the surroundings.

As a result of their studies, Hayden and the Bureau of Standards[7] and other investigators have concluded that only under very unusual circumstances will a-c. corrosion exceed 1 per cent of the theoretical effect of an equal direct current. Most of the results are based on weighing the samples to detect the total amount of removed metal. But with a-c. electrolysis just as with d-c. electrolysis, the chief damage has been found to be pitting of the sheath surface. These pits may penetrate entirely through the lead sheath long before the loss of weight uniformly distributed would become serious. Pitting may be caused by impurities in, or adhering to, the sheath, by differential aeration at the bottom of the pit due to corrosion products left behind, or by breaks in the coating of corrosion products which protect and reduce corrosion over most of the surface.

It appears that so-called self-corrosion, which may be caused by chemicals from the surrounding soil or conduit structure, by differential galvanic action, or by non-uniformity or impurities in the lead, may be increased by a-c. potentials. This applies also to the increased action at the air-water line.

From two years of laboratory and field experience it appears that for cables submerged in water, an a-c. potential of 12 volts between the cable sheath and ground is a practical safe limit for Chicago. It is realized, however, that conditions vary in different cities or parts of one city and that conclusions based on experience gained over such a short interval of time may have to be altered as a result of future data.

Similar corrosion problems may be encountered when lead-covered cables are buried in the earth as has been done occasionally in Europe and the United States. The effects of soil corrosion itself have been studied and a good summary appears in Technologic Paper No. 368, Bureau of Standards, 1928. If the sheath is covered by a fibrous covering, then there is still the possibility of sheath corrosion on account of the covering not remaining impervious to moisture. The various methods for reducing sheath losses and possible corrosion described in this article for cables in ducts are equally applicable to buried cables.

## VIII. Conclusions

1. For cables separated as in ducts the elimination of sheath losses results in increased load ratings of 15 to 80 per cent, and decreased total cable losses of 20 to 75 per cent.

2. Experiments have verified the calculated sheath voltages for the various bonding connections.

3. If sheath voltages on regular sections of cable must be reduced because of corrosion due to a-c. electrolysis, it appears very desirable to use the bonding connections shown in Fig. 2E or 2G for which the three-phase sheath bonding transformer seems most suitable.

4. Satisfactory insulating joints are essential for the practical elimination of sheath losses and successful designs are now available.

5. A-c. electrolysis is a complicated phenomenon and few definite conclusions can be drawn. Field tests and experience in Chicago indicate that 12 volts to ground is a practical safe limit.

### Acknowledgment

The authors wish to express their appreciation of the very helpful assistance given by Messrs. D. W. Roper, Karl Horine, and their assistants in obtaining the field and laboratory data.

# Appendix

## Sheath Voltages, Currents, and Losses

BY K. W. MILLER

1. *Symbols and Assumptions.* The formulas for sheath voltages, currents, and losses given in Table III and the numerical values plotted in Figures 8, 9, 10, and 11 are based on the following assumptions:

1. No proximity effect.
2. Mathematically perfect cables and spacing, and equal sheath resistances.
3. Conductor currents accurately 3-phase and equal.
4. Open-circuited sheaths not connected at more than one point and solidly bonded sheaths not grounded at more than one point.
5. No disturbing magnetic or conducting bodies within the field of influence.

*B*-phase conductor current is taken as the reference vector in all cases. Phase rotation is *A*, *B*, *C*. The following abbreviations are used.

$E_1$ and $I_1$ = sheath voltage and sheath current in cable A

$E_2$ and $I_2$ = sheath voltage and sheath current in cable B

TABLE III

FORMULAS FOR SHEATH VOLTAGES, CURRENTS, AND LOSSES FOR SINGLE-CONDUCTOR CABLES OPERATED 3-PHASE*

| Equation Number | Cable Arrangement Number and Diagram | I | II Equilateral | III Rectangular | IV Flat | V 2-Circuit | VI 2-Circuit |
|---|---|---|---|---|---|---|---|
| | | * (2) (1) ←S→ | (A) (B) (C) ←S→ | (A) (B) (C) ←S→ ↕S | (A) (B) (C) ←S→←S→ | (A) (B) (C) (A) (B) (C) ←S→←S→ ↕S | (A) (B) (C) (C) (B) (A) ←S→←S→ ↕S |
| | SHEATHS OPEN CIRCUITED: $I_1 = I_2 = I_3 = 0$ | | | | | | |
| 1 | $E_1/jI_B =$ | $-x$ | $\frac{x}{2}(-1+j\sqrt{3})$ | $\frac{1}{2}[(-x+a/2)+j\sqrt{3}y]$ | $\frac{1}{2}[(-x+a)+j\sqrt{3}y]$ | $\frac{1}{2}[(-x+b/2)+j\sqrt{3}y]$ | $\frac{1}{2}[(-x+b/2)+j\sqrt{3}y]$ |
| 2 | $E_2/jI_B =$ | $+x$ | $x$ | $x$ | $x$ | $(x+a/2)$ | $(x+a/2)$ |
| 3 | $E_3/jI_B =$ | | $\frac{x}{2}(-1-j\sqrt{3})$ | $\frac{1}{2}[(-x+a/2)-j\sqrt{3}y]$ | $\frac{1}{2}[(-x+a)-j\sqrt{3}y]$ | $\frac{1}{2}[(-x+b/2)-j\sqrt{3}y]$ | $\frac{1}{2}[(-x+b/2)-j\sqrt{3}y]$ |
| 4 | $\frac{(E_1+E_2+E_3)}{3jI_B} =$ | $0$ | $0$ | $a/6$ | $a/3$ | $\frac{1}{6}(a+b)$ | $\frac{1}{6}(a+b)$ |
| | SHEATHS SOLIDLY BONDED: $I_1+I_2+I_3=0$ | | | | | | |
| 5 | $E_r/I_B =$ | $0$ | $0$ | $+\frac{jRa}{6(R+jZ)}$ | $+\frac{jRa}{3(R+jZ)}$ | $+\frac{jR(a+b)}{6(R+jZ)}$ | $+\frac{jR(a+b)}{6(R+jZ)}$ |
| 6 | $\frac{I_1}{I_B} =$ | $+\frac{jx}{R+jx}$ | $-\frac{jx(-1+j\sqrt{3})}{2(R+jx)}$ | $\frac{-(1-\sqrt{3}N)+j(M+\sqrt{3}}{2(M+j)(N+j)}$ | | | |
| 7 | $\frac{I_2}{I_B} =$ | $-\frac{jx}{R+jx}$ | $-\frac{jx}{(R+jx)}$ | $\frac{-j}{(N+j)}$ | | | |
| 8 | $\frac{I_3}{I_B} =$ | | $-\frac{jx(-1-j\sqrt{3})}{2(R+jx)}$ | $-\frac{(1+\sqrt{3}N)+j(M-\sqrt{3})}{2(M+j)(N+j)}$ | | | |
| 9 | $\frac{W_s}{I_B^2R} =$ | $\frac{2x^2}{R^2+x^2}$ | $\frac{3x^2}{R^2+x^2}$ | $\frac{3(M^2+N^2+2)}{2(M^2+1)(N^2+1)}$ | | | |
| 10 | WHERE: | | $y = x$ <br> $Z = x$ | $y = (x+a/2)$ <br> $Z = (x-a/6)$ | $y = (x+a)$ <br> $Z = (x-a/3)$ | $y = (x+a+b/2)$ <br> $Z = (x+a/3-b/6)$ | $y = (x+a-b/2)$ <br> $Z = (x+a/3-b/6)$ |

*Single-phase operation, Case No. 1, is given for comparison.

$E_3$ and $I_3$ = sheath voltage and sheath current in cable C

$\Sigma E/3$ and $E_r$ = residual voltages per unit length of circuit for connections (*e. g.*, see Fig. 1A)

$W_s$ = Total sheath losses in cables of one circuit

$R$ = Resistance of sheath per unit length of cable

$s$ = Spacing between adjacent cable centers

$r$ = Mean radius of cable sheaths

$j\,x$ = $2\,j\,\omega \log_e s/r$

$j\,a$ = $2\,j\,\omega \log_e 2$

$j\,b$ = $2\,j\,\omega \log_e 5$

$M$ = $R/y$ and $N = R/z$

$y$ and $z$ as specified in Equations 10.

If the foot is chosen as the unit of length then at 60 cycles $a = 1.594$, $b = 3.705$, and $x = 5.30 \log_{10} s/r$, all times $10^{-5}$ ohms per foot to neutral. The values of $a$, $b$, $x$, $y$, and $z$, and the voltages given in Fig. 8 for other frequencies and lengths may be found by direct proportion. The value of $x$ may be found from Fig. 8 by multiplying the values of curve (II $A$, $B$, $C$) by $10^{-4}$.

*2. Discussion.* The expression for the voltage drop per unit length of a conductor of resistance $R$ in a magnetic field, due to alternating currents flowing in itself and in parallel conductors is of the general form

$$E = I\,R + \sum^{n} 2\,j\,\omega\,I_n \log c/d \qquad \textbf{(A)}$$

where the symbols and limiting assumptions need not here be defined. If "proximity effect" (or the unequal induction for points around the circumference of closely spaced cables) is neglected, it is possible to set up $n$ simultaneous equations similar to Equation (**A**) for the $n$ cable sheaths. These equations can be expressed simply in terms of the spacings between cable centers and the mean sheath radii. They may be solved simultaneously to obtain the unknown sheath potentials and currents in terms of the conductor currents providing the assumed conditions are such that the current summation for the group is zero. (This is always true for the assumed three-phase conductor currents. In order to insure zero current summation in the sheaths the ideal case is usually considered with sheaths isolated or not grounded at more than one point.)

With similar or identical assumptions this method of solution has often been used by various authors. Sheath voltages with open circuited sheaths, even with a large number of mutually inductive cables, can easily be computed by this method. The sheath voltage formulas given in the upper portion of the table can be found in various works on this general subject.

For convenience in computing and for easy comparison, the magnitudes of these induced voltages are plotted in Fig. 8. The curves are labeled with Roman numerals corresponding to the cable arrangements shown in Table III and with letters to identify the phases to which the curve applies. Thus, for example, III-$A$, $C$ denotes voltages for $A$ and $C$ phases of rectangular spacing. Some curves are identical for different arrangements of cables.

The error in induced voltages due to neglect of the proximity effect is estimated to be less than 10 per cent for cables in contact and the error rapidly decreases with wider spacings.

The algebra involved in solving the $n$ simultaneous equations for sheath currents with solidly bonded sheaths and any cable arrangement, other than equilateral, is quite laborious. It is gratifying to find that for most ordinary cable arrangements it is possible to derive general type formulas (given in the lower part of Table III) for sheath currents and losses covering all the three-phase cable arrangements shown in the table and, in fact, several other arrangements of one or two circuits having geometrical phase symmetry about a line or a point.

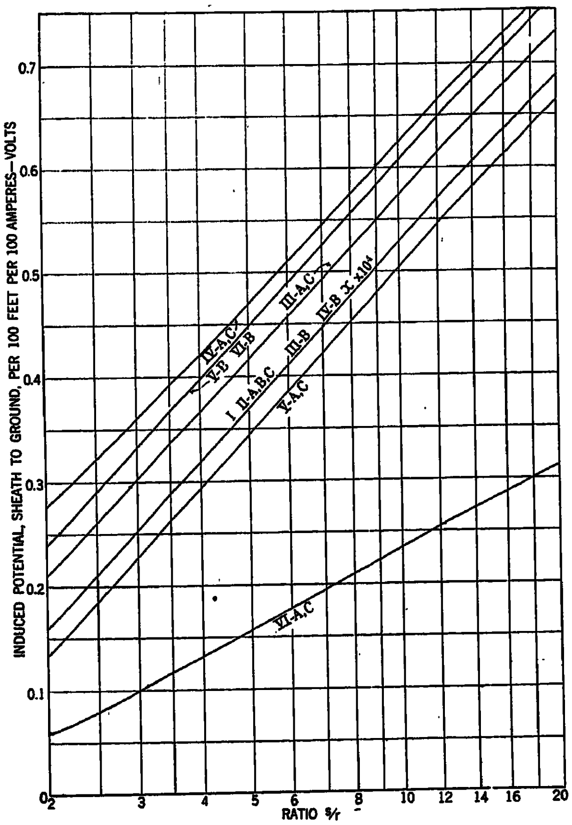


FIG. 8—SHEATH VOLTAGE TO GROUND FOR VARIOUS SPACINGS

The current formulas tabulated were derived separately and also from more general arrangements which, for the particular cases, degenerated into the expressions given. In particular, the general expressions for two parallel circuits became identical to that for one circuit when the two individual circuits were greatly separated. Also those for one circuit with equilateral spacing reduced to expressions identical to

the well known relations derived in other ways by several investigators and given for arrangement II in the third column of the table. The limiting conditions for zero and infinite sheath resistance are properly satisfied.

Therefore, subject to the approximations discussed, it becomes possible to express sheath voltages, currents, or losses for a large variety of typical cable arrangements in the form of master equations or curves and to solve for the numerical values after finding a few easily obtained ratios. Obviously the equations are applicable to similar arrangements with phases rearranged in cyclical order or to reversed phase rotation by interchanging $A$ and $C$.

It is interesting to note that in general, even with solid bonding, residual sheath voltages may still be present, a conclusion reached by others and in harmony with experiment. Also, in general, the losses in the outer cables of flat or rectangular spacings are unequal. Since resistance $R$ cannot be factored out of the expressions for sheath losses there is no value of "effective spacing," as far as losses are concerned, which is independent of sheath resistance.

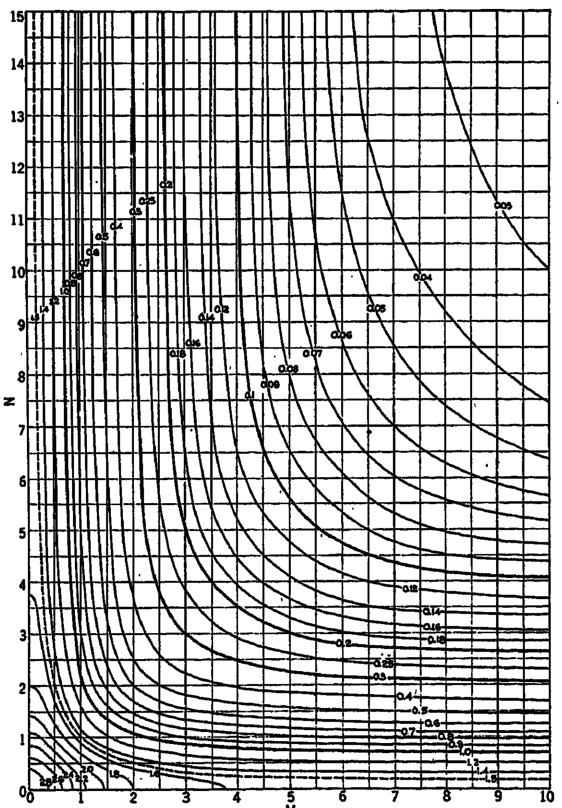

FIG. 9—RATIO $\frac{W_S}{I_B^2 R}$

For convenience in computation Equation (9) has been plotted in Fig. 9. It is easily shown that $W_s/I_B^2 R$ is equal to the magnitude of $(I_1^2 + I_2^2 + I_3^2)/I_B^2$. This enables us to find the total sheath losses for one circuit by computing the losses $I_B^2 R$ in one sheath if full conductor current flows in it and then merely applying the ratio to this $I_B^2 R$ value. Similarly the losses in sheaths $A$ and $C$ can be found by applying (to $I_B^2 R$) the ratios of $I_1^2/I_B^2$ and $I_3^2/I_B^2$ plotted in Figs. 10 and 11. To obtain the current squared ratios it is only necessary to form the simple ratios $M$ and $N$ for the particular cable arrangement as expressed in Equations (10) of the table. Entering Figs. 9, 10, and 11 with $M$ and $N$ as abscissas and ordinates, the current squared ratios may be read off at once by interpolating between the curves. To find the ratio of currents $I_2^2/I_B^2$ for the middle cable, phase B, either Fig. 10 or 11 may be entered with the value of $N$ *only* and the ratio read at the point of intersection of the ordinate $N$ with the dotted 45 deg. line. A check may be obtained by comparing the sum of ratios for the three phases with the total from Fig. 9.

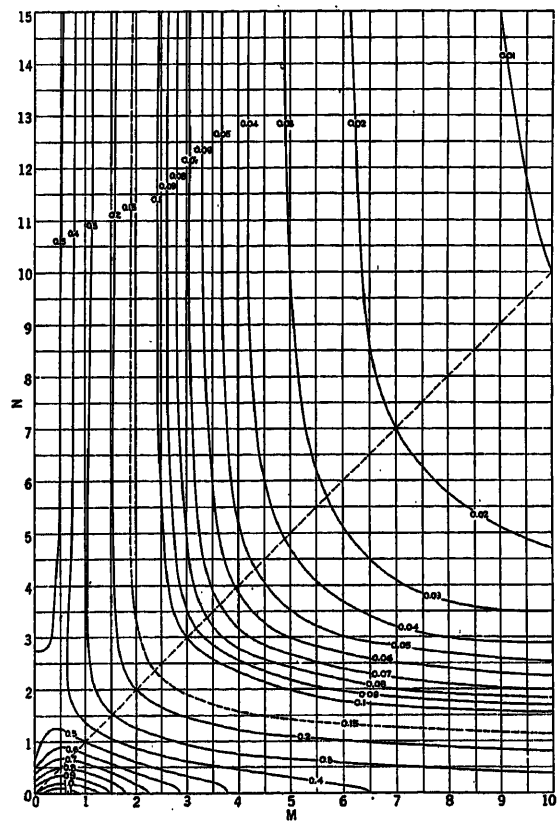

FIG. 10—RATIO $\left|\frac{I_1}{I_B}\right|^2$ FOR A-PHASE

It will be found that for cables in contact the losses inclusive of proximity effect will ordinarily be less than 25 per cent in excess of the values computed by the approximate formulas or curves while for cables in separate ducts the error is much less. Also in practical field installations the sheaths are grounded at more than one point so that the residual sheath voltage causes a current to flow over the line and return through earth or surrounding cables. In general it is impossible to predict the effective impedance of this earth return

but the sheath losses will be increased a few per cent by the additional current required to consume this "residual" voltage.

The values of the "effective" conductor resistance and reactance with solidly bonded cables are equal to the real and $j$ components respectively of the following equations for the effective conductor impedances $Z$:

$$Z_A = (R_c + j\,X_c) - (I_1 R - E_r)/I_A \qquad (11)$$

$$Z_B = (R_c + j\,X_c) - (I_2 R - E_r)/I_B \qquad (12)$$

$$Z_C = (R_c + j\,X_c) - (I_3 R - E_r)/I_C \qquad (13)$$

where $R_c$ and $X_c$ are respectively the conductor resistance and the conductor reactance from the cable center to the mean sheath radius. The other vectorial sheath voltages and currents and sheath resistance $R$ are already defined. A study of these equations indicates the existence of unequal transformer action between phases (except for the equilateral case where the equations reduce to the familiar simple form). Complete discussion of these equations lies outside the subject matter of the present article.

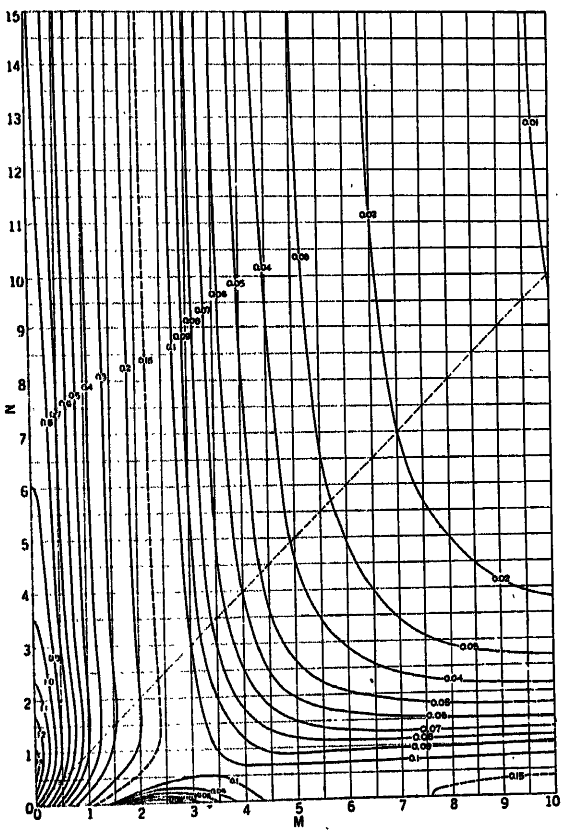

FIG. 11—RATIO $\left|\frac{I_3}{I_B}\right|^2$ FOR $C$-PHASE

*3. Examples.* As an illustration of the use of the equations and curves an example will be given for 66-kv. 3-phase, 60-cycle, 750,000 cir. mils single conductor cable, one circuit, flat spacing, operating at 60,000 kv-a. or 525 amperes per phase. The necessary dimensions are as follows: Spacing—6 in., overall diameter—2.8 in., sheath thickness 9/64 in., from which the following constants are easily obtained:

| | | |
|---|---|---|
| Mean sheath radius | $r$ | $= 1.33$ in. |
| $x = 5.30 \times 10^{-5} \log_{10} 6/1.33$ | | |
| or from Fig. 8 curve (II $A, B, C$) | | $= 3.46 \times 10^{-5}\ \Omega$/ft. |
| Constant | $a$ | $= 1.59 \times 10^{-5}\ \Omega$/ft. |
| Sheath resistance | $R$ | $= 9.3 \times 10^{-5}\ \Omega$/ft. |

Therefore from Equation IV-10, Table III:

$$y = (x + a) = 5.05 \times 10^{-5}$$

$$z = (x - a/3) = 2.93 \times 10^{-5}$$

$$M = R/y = 1.84$$

$$N = R/z = 3.17$$

$I_B^2 R = (525)^2 \times (9.3 \times 10^{-5}) = 25.65$ watts/ft. cable

Making these numerical substitutions into the formulas (1, (2), (3), and (4):

$E_1 = (-\,0.023 - j\,0.00491)$ volts/ft. of cable

$E_2 = (\quad 0 + j\,0.01816)$ volts/ft. of cable

$E_3 = (+\,0.023 - j\,0.00491)$ volts/ft. of cable

$\Sigma\,E/3 = (\quad + j\,0.00278)$ volts/ft. of cross bonded line.

from which the numerical magnitudes are

$E_1 = 2.35$, $E_2 = 1.82$, $E_3 = 2.35$ and $\Sigma\,E/3 = 0.28$ volts per 100 ft. of line. These values of $E_1$, $E_2$, and $E_3$ could have been found directly from Fig. 8 curves IV-$A$, $C$ and IV-$B$ by multiplying the values of $E$ (at $s/r = 6/1.33 = 4.5$) by the ratio of currents 525/100.

Numerical substitution into Equation IV-5 gives:

$E_r = (0.00076 + j\,0.00253)$ volts/ft.

$= 0.264$ volts/100 ft. of solidly bonded line

Equations (6), (7), and (8) yield:

$$I_1 = 215 - j\,28 = 217 \text{ amperes}$$

$$I_2 = -\,48 - j\,151 = 158 \text{ amperes}$$

$$I_3 = -\,167 + j\,179 = 245 \text{ amperes}$$

from which sheath losses are:

$$W_1 = 4.37 \text{ watts/ft.}$$

$$W_2 = 2.32 \text{ watts/ft.}$$

$$W_3 = 5.58 \text{ watts/ft.}$$

which check with

$W_s = 12.26 =$ total sheath losses by Equation (9).

If, as is usually the case, only the sheath losses are required they may be found directly by entering Figs. 10, 11, and 9 with $M = 1.84$ and $N = 3.17$. The sheath to conductor current ratios squared for cables A, B, C, and total are found to be 0.17, 0.091, 0.22, and 0.48 respectively. Multiplying $I_B^2 R = 25.65$ watts by these ratios we find practically the same sheath losses $A = 4.35$, $B = 2.30$, $C = 5.65$, and total $= 12.30$ watts per foot as before.

It is of interest to compare sheath voltages and losses for this same cable with various cable arrangements, conditions being otherwise identical. The results are

easily obtained from the curves and are as follows:

| Cable spacing | Induced sheath voltage per 100 ft. line | | Sheath losses watts per ft. | | | |
|---|---|---|---|---|---|---|
| | A & C phases | B phase | A phase | B phase | C phase | Total per circuit |
| II Equil. | 1.82 | 1.82 | 3.08 | 3.08 | 3.08 | 9.24 |
| III Rect. | 2.06 | 1.82 | 3.59 | 2.69 | 4.36 | 10.64 |
| IV Flat. | 2.35 | 1.82 | 4.35 | 2.30 | 5.65 | 12.30 |
| V | 1.69 | 2.30 | 6.28 | 2.82 | 8.86 | 17.95 |
| VI | 0.76 | 2.30 | 2.80 | 2.82 | 2.80 | 8.47 |

The average error in using the graphical solution is estimated to be within ± 5 per cent. Uncertainties of cable spacing, sheath thicknesses, and temperatures, etc., make further accuracy unnecessary.

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## Discussion

**H. R. Searing:** The United Electric Light & Power Company has used the cross-bonding method (*D* in the paper) for several years with entire success, and can subscribe to the limit of 20 volts between sheaths set forth by the authors. We adopted this value as a limit in order to be certain that sheaths would not attain dangerous potentials during short-circuit conditions, but careful inspection has not disclosed any deterioration of sheaths due to a-c. electrolysis.

We feel that care should be taken in drawing conclusions regarding a-c. electrolysis in that the sheath deterioration observed, especially under fireproofing, in manholes, may be caused at least in part by other agencies than induced a-c. currents.

We wish to correct a statement in Table I, item 4, regarding cable arrangement. These cables are arranged rectangularly as shown by the authors' cable arrangement III.

For several years we have felt that more satisfactory operation could be obtained by the use of single-conductor cables. Recent cable development has indicated that triplex cable of the oil-filled type may be supplied for operation at 66 kv. and below. In view of this development, it is our feeling that the use of single-conductor cable would, in general, be limited to circuits operating above this voltage.

Triplex cables are to be preferred from a regulation standpoint. The regulation of circuits using single-conductor cables, especially where they are installed in separate ducts, is much poorer than that with triplex cables of equal rating. This difference in circuit regulation tends to complicate operation of a system having both types of cable.

**E. R. Thomas:** Mr. Riley, (in a paper on sheath losses in single-core cable which was printed in the November 1927 issue in the *Journal* of the Institution of Electrical Engineers, London) gave results of measurements on flat spacing of cables for various ratios of spacing to sheath radius. He found, contrary to the equations of Messrs. Halperin and Miller, that the losses in each of the two outer sheaths were the same. Several years ago I developed a procedure for calculating the sheath current and losses for the general case of unequal spacing and by applying these formulas to the cable constants used by Mr. Riley I obtained very good agreement.

I am in agreement with the paper regarding the magnitude of the voltages induced per loop unit of length between pairs of sheaths, but I am not in agreement with the computed voltage to ground for each sheath, which for the middle cable sheath for the three cases of equilateral, rectangular, and flat spacings are shown to be the same value. Accurate experimental data would be very desirable to determine whether sheath losses are unequal in the two outer sheaths in the case of flat spacing and what the values of induced voltages to ground are for various arrangements.

**P. S. Creager:** I have made some preliminary design calculations on a proposed 26-kv. single-conductor cable. The scheme of bonding studied was the same as the one shown in Fig. 2c of the paper, namely, cross bonding with every third joint grounded. The primary purpose of the study was to determine the voltage across an insulated joint under fault conditions. This factor was considered to be of paramount importance as the proposed cable would probably be installed in a run with several other cables.

As in this type of cable, faults would be single-phase to ground, the method of symmetrical components appeared to be necessary for a solution. The scheme as developed by Mackerras, Lewis, and Bekku in the *General Elec. Rev.*, with certain modifications and approximations to meet the different conditions, was applied to the problem. As a first approximation, all of the fault current was assumed to return on the sheath. It was found necessary to take account of both the resistance and the reactance of the sheath. It was also found that the resultant voltage across a joint had to be found by combining the induced voltages with the impedance drops due to the returning fault current. The final result, while admittedly approximate, gave a value of about 300 volts across a joint under the conditions specified. While no calculations were made for normal operating conditions, some rough mental calculations on the equations used gave results of the same order of magnitude as those given by the authors in their paper. The writer takes this as a confirmation of the general method used and of the admitted approximations which he made.

**H. C. Lewis:** I should like to add something on the question of regulation previously referred to by Mr. Searing. The regulation is of considerable importance in the case of parallel cables. Inasmuch as it is not always practical to select ducts, and to get equal duct spacing, unequal impedances may result which will cause great difficulties in paralleling cables. In some cases this will make it impractical to parallel cables due to improper division of load. Information is desired from the authors as to how they actually took care of this difficulty.

**P. D. Morgan and S. Whitehead:** (communicated after adjournment) In discussing the economic aspects the authors have made several important assumptions the validity of which is not proved in the paper. These are as follows:

a. That the life of the cable is the same and the liability of failure no greater due to corrosion and to the insertion of insulating sleeves with their complicated oil attachments, etc., than when solid bonding with continuous lead is employed.

b. That the labor cost of installation and upkeep is no greater than with solid bonding.

c. That the danger of electrostatic sparking and consequent ignition of gases accumulated in manholes is negligible.

If these assumptions are only partly true the net economic result may be quite different from that stated in the paper.

We have recently carried out an investigation into methods of laying single-conductor cable for 60 kv. and above in connection with the high-voltage "grid" system now under construction in Great Britain, and an account is shortly to be published in the *I. E. E. Jl.* (England).

The British Electrical and Allied Industries Research Association, as a result, decided to recommend wherever possible the laying of cables in triangular spacing with regular transposition, as close together as possible and with solid bonding and earthing at every joint. As an alternative where special conditions render it desirable, flat spacing with regular transposition at each joint and solid bonding and earthing of the sheaths at every third transposition (*i. e.*, at every complete turn of the lay) may be employed. Where two or more three-phase circuits have to be laid in close proximity, mutual inductive effects between them will be eliminated by coordination of the frequencies of transposition. For example, where two circuits are employed the frequencies of transposition will be in the ratio of 3 to 1. These considerations apply both to cables laid in ducts and direct in the ground, the latter method being the usual practise in this country.

A comparison of the above with the authors' method reveals the following features:

*Advantages of authors' method:*

a. Increased permissible loading under certain conditions.

b. Reduced sheath losses.

*Disadvantages of authors' method:*

c. A-c. electrolysis of sheath of unknown amount.

d. Increased initial and maintenance costs due to insulating sleeves, transformers, etc.

e. Increased interference with nearby communication circuits due to the conductors not being transposed. (This is chiefly of importance where the power and communication cables are in the same trench, such as when supplying private telephone lines for the power-supply authority.)

f. Possible danger due to electrostatic sparking and to personnel, caused by the voltage between sheaths and to earth.

For cables laid direct in the ground, the authors' method would be difficult to apply in practise as in order to obtain the increased permissible loading it would be necessary to separate the cables. For cables in ducts, as an alternative to the authors' method we are investigating the possibilities of obtaining increased loading by a reduction in the thermal resistivity between the cable and duct wall. Advantage (b) above is not considered of much importance economically as on a high voltage system the copper losses (of which the sheath losses are only a fraction) are generally much less than the losses in the transformers, etc.

Regarding the disadvantages mentioned, it is considered that (f) above may be a real menace as extensive explosions have sometimes occurred due to gases accumulated in cable tunnels.

On the theoretical side of the paper the authors' remarks as to the inequality of sheath losses in the outer cables in flat and rectangular spacing, and in the existence of a "residual" voltage to earth with solid bonding, are in agreement with recent theoretical and experimental work in this country. Our theoretical formulas differ slightly from those of the authors, but it is believed that the differences are not material.

The authors mention "proximity effect" losses. We incline to Cramp's nomenclature "sheath eddy losses," *i. e.*, losses occurring due to eddy currents whose vector sum is zero over the cross section of the sheath, and find it more convenient to estimate these separately although their magnitude is affected by the presence of circulating sheath currents. We agree that they may nearly always be neglected with high-voltage cables.

With regard to the impedance of the earth return for residual currents, the present writers' experience with overhead lines inclines them to the view that this is not greatly affected by the presence of other currents, the "balanced" components, and may therefore be calculated by the methods developed for overhead lines (*e. g.*, Rudenberg, Pollaczek, or Carson). Further attention is being given to this matter.

With regard to the effect of copper-wire armoring, it is more usual in this country, in the few cases where non-ferrous armoring is used, to use copper tape. The present writers have incorporated the effect of such armoring into their theory of sheath losses, but this section will not be published in the account referred to. Our conclusion is that with tape armoring, reduction of losses below those for plain lead sheaths by reducing the resistance of the tape is not practicable. As the resistance of the armoring decreases, the losses rise to a maximum and then fall. At small spacings of the cables it is impracticable to attain even the maximum loss by decreasing the resistance of the tape. At very large spacings, the maximum is shifted towards higher resistances so that the fall of loss with resistance can be realized but it is only at impracticably large spacings that a reduction below the unarmored case can be obtained with tape of practicable dimensions. With wire armoring, it is, of course, easier to obtain a low resistance armoring. Nevertheless, our opinion is that, in order to obtain a reasonable reduction of losses even with wire armoring, it would be necessary to introduce rather heavy armoring, particularly if bronze wires are used as is more usual in this country.

It is to be observed, with pleasure, that at the present time, agreement is being reached between different countries as to the method of evaluating sheath losses.

**Herman Halperin and K. W. Miller:** Mr. Searing has pointed out that his company has selected 20 volts between cable sheaths as a safe operating limit because this results in safe values of sheath potentials during failures. It appears that in general emphasis should be given to the a-c. electrolysis factor since the possibility of corrosion from this cause is present continuously while the cables are loaded.

Regarding the spacing of the cables shown in Table I, nine-tenths of our own single-conductor cables are installed with rectangular spacing but conditions were shown for flat spacing since that is the worst condition from the standpoint of both sheath losses and voltages.

We have not yet encountered the problem of poor regulation or poor load division between single-conductor and three-conductor lines. Our 66-kv. lines are comparatively short (1½ to 10 mi.) and, with transformers included, the line impedance is usually a small fraction of the total. Load division between lines or reversal of power flow is accomplished by tap changers on the transformers working under load. We have not yet attempted to parallel single-conductor and three-conductor cables at the same voltage. Where two or more single-conductor lines are paralleled it is important to keep the lengths nearly equal for cables installed in separate routes or to properly transpose the cables to minimize mutual effects between circuits when they are in the same conduits.

Replying to Mr. Thomas, equations and experimental data have been presented by several investigators, recently by Dr. Arnold in the January issue of the *Journal* of *I. E. E.*, and we have also our own field data all showing unquestionably the inequality of sheath currents and losses in the outer cables with flat or rectangular spacings. The existence of residual voltage induced along a solidly bonded line with flat or rectangular spacings has been theoretically and experimentally confirmed. A residual voltage is also induced along a line with sheaths cross bonded (copper conductors not transposed). We have measured this voltage in the field and found it to check with theory.

It is interesting to know that Mr. Creager has checked our work for the cross bonding connection in Fig. 2D and found that the voltage drops along the sheath during a single-phase failure is of the same magnitude as computed by us. The insulating sleeves are never called upon to withstand more and usually

less than the voltages given in the last column of Table II during a typical fault. Excepting for resistance bonding they can easily and safely be subjected to abnormal voltages of this order of magnitude even when submerged in water.

Mr. Lewis' questions on line impedance and parallel operation have already been answered.

Messrs. Morgan and Whitehead have contributed some very interesting information on English practise and raised some very practical questions. If sheath potentials are limited to reasonable values (we have found 12 volts to ground is practical for our duct conditions) then sheath corrosion due to a-c. electrolysis will be little or nothing. All cable sheaths are exposed to the hazards of chemical corrosion, d-c. electrolysis, and mechanical damage which, like possible a-c. corrosion, if present, are usually random and localized in occurrence. All factors considered, placing, of course, a reasonable limit on a-c. sheath potentials, it does not appear that cable life is appreciably reduced below the age of obsolescence or a natural insulation life by operating with small a-c. sheath potentials. Our wide field experience is confirming this.

The additional installation cost for eliminating sheath losses was included in the economic study and is only 1 or 2 per cent of the total line cost. The economic savings both in increased line carrying capacity and decreased losses are so large and real as to demand serious attention.

In this country on typical installations of single-conductor cables the sheath losses would be from 50 to 250 per cent of copper losses if the sheaths were solidly bonded. Decreasing the thermal resistance exterior to the sheath does not avoid the extra cost of the sheath losses. Also, for a given permissible maximum operating copper temperature and within practical limits of reduction of thermal resistance, the percentage increase in rating made possible by eliminating sheath losses is still very attractive.

It appears that there are three factors in English practise which tend to minimize sheath losses with solid bonding:

a. The cables are generally comparatively small and have high sheath resistance.

b. The usual frequency of 50 cycles is lower than the standard 60 cycles in the United States.

c. It is feasible to lay buried cables at close spacings or in contact.

All these factors decrease the necessity or advantages of special sheath bonding.

Oil-filled joints with their attachments are an item entirely independent of methods of sheath bonding. Maintenance costs are not affected by special sheath bonding. Occasional inspections are required by all cables for miscellaneous reasons and special bonding of single-conductor cables has not increased inspection costs. We have at present over 3000 insulating sleeves in service and there has not been one leak or defect developed in the 2½ years of service with the type of sleeve generally used by us.

Telephone cables are seldom run in the same conduits with power cables. Even if they were there is nothing inherent in any of the special sheath-bonding schemes which prevents transposing the copper conductors at any desired intervals—in fact such transpositions are standard on all of our lines.

The danger from sparks igniting gas is rather remote. The sheath voltage is far too low to cause "static" discharges. The cables are insulated from each other by fireproofing in the manholes and cannot come in contact in separate ducts.

When the copper conductors are transposed, "residual" sheath voltages for solid bonding or any sheath-bonding connection will also be transposed out. Then, unless the sheaths are very substantially grounded at intermediate points, no "residual" current will flow. Field measurements for cables in ducts have shown that the leakage resistance to ground, even with cables submerged in water, is so high compared to sheath resistances as to have little effect on sheath potentials with any of the sheath connections. When this is borne in mind, it is seen that transposing copper conductors along a long line will allow the residual sheath potentials to trace a vector triangle instead of being consumed by current flow as would occur with an untransposed line. Unless the length of line between transpositions is short, appreciable "residual" a-c. sheath voltages to ground can result even with solid bonding.

The authors are in entire agreement with the remarks on copper armoring. The method was included for completeness in listing all possible methods available for reduction of sheath losses.

# Losses in Armored Single-Conductor Lead-Covered A-C. Cables

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*Synopsis.—The losses occurring in single-conductor armored cables whose sheath and armor are bonded and grounded at more than one point are—besides the copper loss and the dielectric loss—the circulating-current losses in the sheath and in the armor, and the additional iron losses in an armor of magnetic material. The circulating-current losses are due to the currents induced in the sheath and armor circuits by the fluxes linking these circuits when these circuits are closed by the bonds or grounding connections. Since the losses obtained in such cables are widely variable, depending on the cable design, both designers and operating engineers desire to know the loss magnitudes obtained in different cases and to have an understanding of the factors tending to give low losses.*

*In this paper the sheath and armor losses have been analyzed for a steel-wire armored cable, a copper-wire armored cable, a steel-tape armored cable, and a cable enclosed in an iron pipe, and have been compared with the losses occurring in a plain lead-covered cable without armor. The data used include test data and calculated values.*

*It is shown that a steel-wire armored 350,000-circular mil single-conductor copper cable with sheath and armor short circuited by low-impedance bonds had a total loss (exclusive of dielectric losses), of 2.8 times the conductor loss, by test in a single-phase 60-cycle circuit at 4 ft. cable spacing, at 260 amperes. The corresponding loss in a similar cable without armor, with lead sheath short circuited, was 2.4 times the conductor loss. Thus the steel-wire armored cable had an additional loss due to the armor of about 20%, this relatively small loss increase being due to (a) an armor of individual wires wound with a large lay, so that the path of circumferential magnetic flux at the armor is broken up by many non-magnetic spaces, and the component of magnetizing force along the armor wires is a small part of the resultant magnetizing force; (b) low-resistance circulating-current circuits. A steel-tape armor or an iron pipe surrounding a single-conductor cable usually introduces higher losses.*

*A cable similar to the steel-wire armored one but having an armor of copper wires has a calculated total loss (exclusive of dielectric loss) of only 1.3 times the conductor loss, in a single-phase circuit at 4 ft. spacing, at 60 cycles, the armor current itself being 92 per cent of the conductor current.*

*The reduction of losses due to diminished cable spacing in a single-phase circuit is shown. When the cable spacing is reduced from 10 ft. to 1 ft. the overall losses (exclusive of dielectric losses) for a steel-wire armored 350,000-circular mil cable are reduced about 8%, while those for a similar cable with copper-wire armor drop only about 3%.*

*A loss-calculating procedure for determining the losses in single-conductor armored cables in a single-phase circuit, or in a balanced three-phase circuit with equilateral spacing, is presented. The procedure involves (a) the calculation of the circulating currents in sheath and in armor by formulas derived, which takes into account the effect of armor iron on reactance values, (b) the computation of the circulating-current losses as $R\,I^2$ values, (c) the determination of the extra iron losses in the armor with the aid of suitable test data, or by calculations when sufficiently reliable. Test data on effective armor resistance and on extra iron losses are given for a steel-wire armored cable. These data are applicable to loss calculations for certain steel-wire armored cables. Corresponding test data for cables with steel-tape armor or with other types of magnetic armor have not been obtained. In the absence of these test data the calculations are less reliable.*

*The relative total annual costs of a steel-wire armored cable and of a similar copper-armored cable, both cables having a conductor cross section of 350,000-circular mils, were compared. The lower operating cost of the copper-armored cable was found to have a lower total annual cost than that of the steel-wire armored cable at load factors above 60%, the steel-wire armor having the advantage economically at lower load factors.*

* * * * *

## INTRODUCTION

IT is the object of this paper to present the results of test data and of calculations on losses in certain armored single-conductor lead-covered cables, and with the aid of the data to point out the effects of the chief design variables on the total losses, and on the principal loss components. The relative merits of steel-wire armor and copper-wire armor are discussed both in respect to losses and total annual costs at different load factors. A calculating procedure is given for the determination of the circulating currents in sheath and armor circuits for a pair of cables in a single-phase circuit or for three cables in a symmetrical single-phase circuit (triangular spacing) with balanced three-phase circuit currents, the formulas derived taking into account the increase in inductance values due to a magnetic armor. Test data on effective armor resistance and on extra iron losses for a steel-wire armored cable are given such that with their aid, and with the aid of the circulating-current formulas, the losses in certain steel-wire armored cables may be computed. It is not intended in this paper to give data or formulas broadly applicable to the calculation of losses in cables with steel tape armor or with other types of magnetic armor differing widely from the steel-wire armor tested, although the circulating-current formulas are applicable to a variety of cables with magnetic armor.

It is usually considered desirable in practise to bond together and ground by means of low-impedance connections the sheaths and armors of single-conductor armored cables at more than one point so as to keep the armor and sheath at substantially ground potential. Therefore the cases considered throughout most of the paper will be those for cables whose sheath and armor are bonded with low-impedance bonds from cable to cable (or grounded) at several points. The bonding

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and grounding, of course, introduces circulating currents due to the fluxes passing between cables or between cables and ground.

Under present practise the case of *armored* cables not bonded or grounded (*i. e.*, having no circulating currents) is of more academic than practical interest. Nevertheless data and discussions are included for cables having the sheath or the armor open circuited, or both sheath and armor open circuited.

Throughout the paper, unless otherwise mentioned, cables in a single-phase circuit, equivalent (for the purpose of this paper) to cables in a balanced three-phase circuit in a symmetrical triangular arrangement at the same spacing are dealt with. Unsymmetrical multi-conductor layouts of steel-armored cables have not been covered. In such layouts the circulating currents and losses will differ for the different cables of the group. However, the average loss per cable will often not differ much from the loss calculated for each cable of a pair of going and return cables at the proper spacing. It seems that more thorough-going researches might profitably be made into the subject of losses in steel-armored cables in unsymmetrical multi-conductor layouts.

In analyzing the circulating currents in single-phase circuits in this paper the sheaths and armors of the going and return cables are considered as bonded together at two or more points by short bonds of negligibly small impedance. Any circulating currents flowing through the earth by reason of the particular system of grounding employed are not taken into account here. Since the effect of such earth currents upon the total circulating-current losses will usually be sufficiently small to be negligible (provided highly conducting short bonds with low-resistance joints are used between the cables) and since the need for low-impedance bonding connections is generally recognized, the loss data based on tests and calculations made without earth currents are considered as representative of actual cases in good practise where ground connections are used.

*The component losses* occurring in single-conductor a-c. cables with lead sheath and iron armor, when both sheath and armor are bonded and grounded, so as to give circulating currents in sheath and armor, are

(a) conductor loss (copper loss)
(b) lead sheath circulating-current loss
(c) armor circulating-current loss
(d) eddy-current loss in lead
(e) extra eddy-current and hysteresis losses in armor
(f) dielectric losses

This paper deals primarily with the lead sheath and armor losses, *i. e.*, with items b, c, and e, which have not been so well understood and have formed the subject of recent investigations. The other component losses, which in the main have been better understood for design purposes, will not be analyzed here and will be only briefly referred to at the end of the paper.

## Circulating-Current Losses in Lead Sheath and in Armor

The circulating-current loss in the lead sheath (item b) is due to the circulating current induced in the sheaths of adjacent cables by the flux passing between the cables when the sheaths are bonded together or grounded at two or more points. In the case of a pair of going-and-return cables (without armor) whose lead sheaths are bonded at both ends, the sheath circulating current follows the path indicated in Fig. 1 by the arrows *a b c d e f g*. The flux inducing this current is indicated by $\phi_1$ and $\phi_2$. The sheath circulating-current loss is then equal to the sheath resistance times the square of the sheath circulating current.

The armor circulating-current loss is produced in a similar manner by the circulating current induced in the armor circuit by the flux passing between adjacent cables when the armors of the cables are bonded together (or grounded) at more than one point. In addition there will often be eddy-current and hysteresis

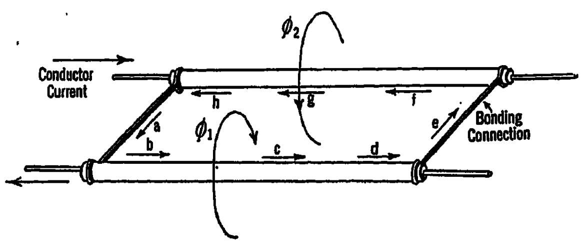


Fig. 1—Circulating Current in Sheaths of a Pair of Lead-Covered Single-Conductor Cables (not Armored) in a Going-and-Return Circuit

Circulating current is indicated by arrows *a b c d e f g h*. The current is induced by fluxes $\phi_1$ and $\phi_2$ linking the loop indicated

losses in the armor to be considered separately.

For a circulating-current path of very low resistance the circulating current will approach the conductor current and will then be practically opposite to it in phase; *i. e.*, the so-called phase angle will be nearly zero. Increasing the resistance of the circulating-current path will diminish the circulating current and materially increase the phase angle. When there are two circulating-current paths a change in the resistance of one of the circulating-current paths will change the currents and phase angles for both circulating currents. The above matters are, of course, not new, having been covered in earlier literature (see bibliography).

The behavior of armored single-conductor cables in regard to losses due to circulating currents in lead sheath and in armor may be analyzed by referring to the known characteristics of unarmored lead-covered single-conductor cables carrying circulating current in the lead.

*Unarmored Lead-Covered Single-Conductor Cables.* When the sheaths of cables are bonded at the ends, sheath circulating-current losses will be obtained

increasing with spacing between going and return conductors and varying with sheath resistance.

Let us examine more closely the effect of sheath resistance on circulating current and losses for a particular cable as indicated by curves of current and losses in the sheath in Fig. 2. The values are calculated for cable spacings of 1 ft. and of 4 ft. for a single-phase circuit, and cover a range of sheath resistances of from 0 to 0.4 ohm per 1000 ft. per cable. At the maximum sheath current, equal to the full conductor current, (*i. e.*, for the hypothetical case of zero sheath resistance) the circulating-current loss is obviously zero. While

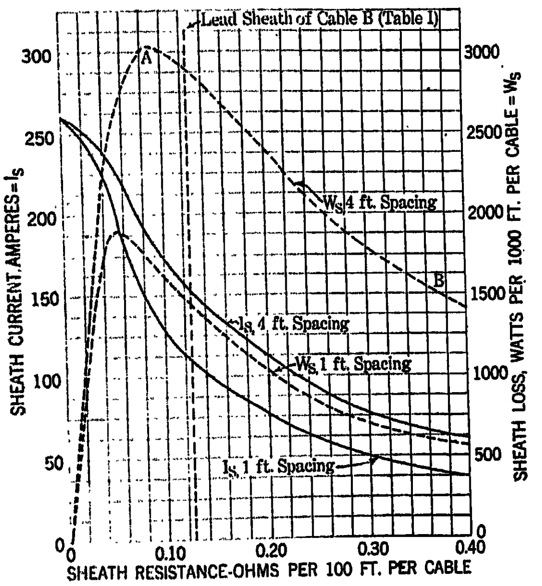


Fig. 2—Sheath Current and Sheath Loss *vs.* Sheath Resistance

Single-phase circuit with 1-ft. and 4-ft. spacing. No armor
Cable B, dimensions given in Table I, except that sheath resistance is considered variable
Sheath bonded as in Fig. 1
Conductor current 260 amperes at 60 cycles per second
All values calculated

the sheath current falls with increasing sheath resistance, the sheath loss first rises to a maximum, and then falls, again approaching zero at infinite sheath resistance. Attention is called to the fact that a particular sheath design for each cable and for each spacing gives a maximum-sheath circulating-current loss. Values of sheath resistance higher or lower than this critical value (at point A for 4 ft. spacing in Fig. 2) will give lower circulating-current losses than those for the critical sheath resistance. For the types of cable usually encountered in practise, and at moderate spacings, the sheath resistance is above the critical value. Attention is also called to the fact, indicated in Fig. 2, that the critical sheath resistance for a given cable is diminished when the spacing between going and return cables is reduced.

A particular 350,000-circular mil unarmored lead-sheath cable B, as specified in Table I, when used in a single-phase going and return circuit at 4 ft. center spacing between cables and when carrying 260 amperes conductor-current at 60 cycles, will according to Fig. 2 have a lead-sheath circulating-current of 150 amperes or about 58 per cent of the conductor-current when the sheaths are bonded at more than one point. The corresponding loss due to lead-sheath circulating-current is 2880 watts per 1000 ft. or 1.4 times the conductor loss. Thus it is seen that the sheath circulating-current for this cable without armor introduces a very considerable loss equal to 140 per cent of the conductor loss, at 4 ft. spacing. Any increase in sheath resistance will reduce the loss as seen in Fig. 2. If the sheath resistance is lowered, a reduction of sheath circulating-current loss will not be obtained until the sheath resistance is made less than half in this particular case.

The simple case of cables with a single non-magnetic circulating-current path has been reviewed in so far as the characteristics of circulating-current magnitudes and losses are concerned, because these characteristics will now be used in a discussion of armored cables to indicate the effect of armor design on total circulating-current losses.

## Armored Lead-Covered Cables

*Total Circulating-Current Losses.* When an armor is added to an ordinary lead-covered cable, and when in a cable circuit both armor and lead sheath are bonded at 2 or more points, so that the bond at each point connects the sheath to the armor of the same cable and also connects from cable to cable, the armor is in multiple with the lead sheath. Thus the addition of the armor is at least approximately equivalent to a lowering of the sheath resistance in so far as the sum total of circulating-current losses is concerned, neglecting the difference between sheath and armor inductances. Hence, if the lead-sheath resistance itself were not higher than the critical value (as *A* in Fig. 2) the addition of the armor would actually tend to reduce the total circulating-current losses. These considerations, of course, do not include the extra eddy-current and hysteresis losses in a magnetic armor (to be discussed later).

The effect of the addition of an armor on the total of circulating-current losses will be illustrated by an example: Cable B in a single-phase 2-wire circuit will again be considered. Its sheath resistance is 0.128 ohm per 1000 ft. per cable. The sheath circulating current and sheath loss at 260-ampere conductor-current at 60 cycles and 4 ft. spacing for this unarmored cable are 150 amperes and 2880 watts per 1000 ft. per cable, respectively, from Fig. 2, as already mentioned. If an armor of 0.12 ohm effective resistance* per 1000 ft. is added to this cable and bonded to the sheath, we may consider, as a rough approximation for estimating the total circulating-current losses, that the armor and sheath are in multiple having a combined resistance of 0.062 ohm. From the upper loss curve in Fig. 2, the

*The armor in question is the steel-wire armor specified for cable A in Table I.

combined circulating-current losses in sheath and armor would be 2840 watts per 1000 ft. for a resistance of 0.062 ohm. By actual test of a 50-ft. length of the cable in question the sum of armor and sheath circulating-current losses is 3100 watts per 1000 ft. per cable in a single-phase circuit at 4 ft. spacing. (See item 4 Table II.) The difference between the two values for total loss is seen to be less than 10 per cent, the higher value by test being caused by the additional inductance due to the magnetic armor, this factor (which is appreciable) not having been allowed for in the rough calculation.

The figures indicate that the use of an armor of only moderately low resistance at the 4-ft. spacing between

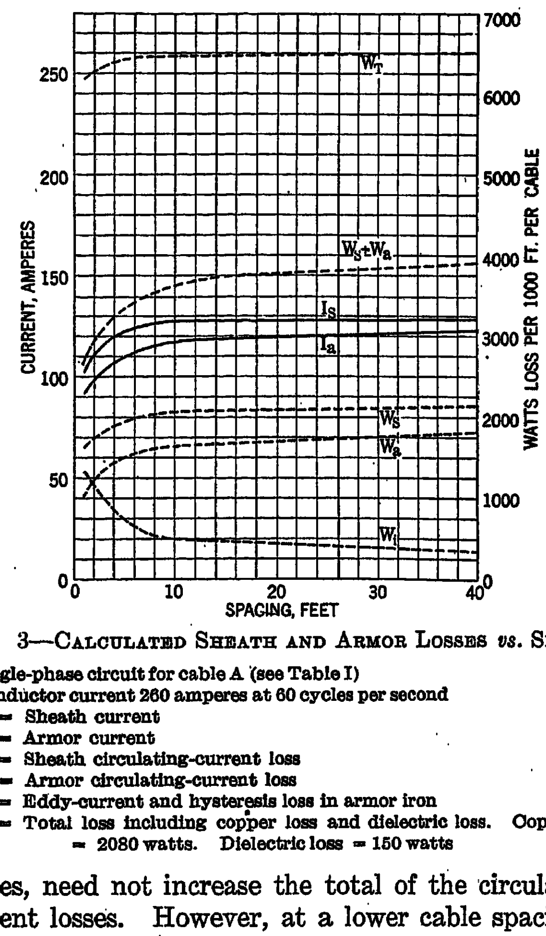


FIG. 3—CALCULATED SHEATH AND ARMOR LOSSES *vs.* SPACING

Single-phase circuit for cable A (see Table I)
Conductor current 260 amperes at 60 cycles per second
$I_s$ = Sheath current
$I_a$ = Armor current
$W_s$ = Sheath circulating-current loss
$W_a$ = Armor circulating-current loss
$W_i$ = Eddy-current and hysteresis loss in armor iron
$W_T$ = Total loss including copper loss and dielectric loss. Copper loss = 2080 watts. Dielectric loss = 150 watts

cables, need not increase the total of the circulating-current losses. However, at a lower cable spacing, a considerably lower armor resistance would have to be used in order that the sum of the circulating-current losses in the armored cable may be less than those in the unarmored cable.

The approximate calculation just given is not recommended for estimating losses in armored cables, because it does not take into account such factors as the difference between the inductances of armor and sheath circuits, and the effect of a magnetic armor on inductance values. Also this procedure would not give the values of the circulating currents in sheath and armor, which currents must be used in determining the extra eddy-current and hysteresis losses in magnetic armors.

*To summarize:* To lead-sheath single-conductor cables (without armor) whose sheaths are bonded or grounded at 2 or more points, a lowering of the sheath resistance at constant cable spacing and constant conductor current, results in a greater circulating current, but will give a lower circulating-current loss if the lead sheath resistances considered are not higher than a certain critical value (see Fig. 2). If, however, the sheath resistance exceeds this critical value, a further increase in sheath resistance will give a lower circulating loss. For the type of cables usually encountered in practise (both armored and unarmored) the sheath resistance is above the critical value.

The addition of an armor bonded at both ends to the lead sheath is roughly equivalent to a lowering of the sheath resistance in so far as the sum of circulating-current losses is concerned. Thus it is even possible by the use of an armor of sufficiently low resistance to obtain a total circulating-current loss materially lower than that for the lead-covered cable without an armor, if the cable spacing be sufficiently large.

## EFFECT OF CABLE SPACING ON CIRCULATING-CURRENT LOSSES

Fig. 3 indicates how the circulating currents and circulating-current losses are affected by varying the spacing between cables from 1 to 40 ft. in a single-phase

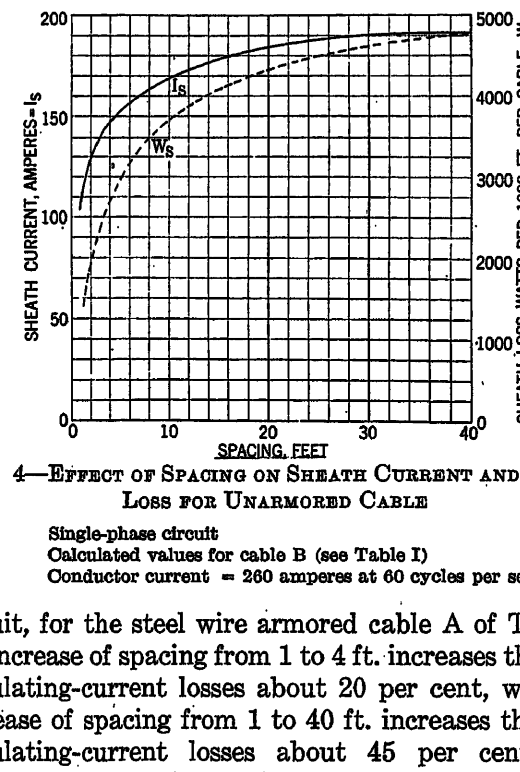


FIG. 4—EFFECT OF SPACING ON SHEATH CURRENT AND SHEATH LOSS FOR UNARMORED CABLE

Single-phase circuit
Calculated values for cable B (see Table I)
Conductor current = 260 amperes at 60 cycles per second

circuit, for the steel wire armored cable A of Table I. An increase of spacing from 1 to 4 ft. increases the total circulating-current losses about 20 per cent, while an increase of spacing from 1 to 40 ft. increases the total circulating-current losses about 45 per cent. For the copper-armored cable C the effect of spacing is very much less, the increase in total circulating-current losses being about 3 per cent when the cable spacing is increased from 1 to 40 ft.

In the case of the unarmored cable B of Table I, the increase of circulating-current loss with spacing is larger than in either of the above cases, as indicated in Fig. 4.

The increase in circulating-current losses with increased spacing is due to the increase in the circulating currents themselves, since the inductances of sheath and armor circuits are increased. In the case of the copper armor the small increase of circulating currents with spacing is due to the almost complete demagnetization produced by the circulating currents.

*The effect of spacing on total losses* is much less than that on the circulating-current losses alone, because the eddy-current and hysteresis losses decrease with increased spacing. Thus for cable A, the increase in total loss, with an increase of spacing from 1 to 40 ft. is only 5 per cent as seen from the upper curve for Fig. 3.

The effects of various cable designs and cable operating conditions on losses will now be considered with the aid of results based on test data and on calculations.

## Losses for Different Kinds of Cables

The kinds of cable to be compared in regard to circulating currents and losses are:

A. Steel-wire armored cable
B. Cable without armor
C. Copper-wire armored cable
D. Cable with iron-pipe armor
E. Steel-tape armored cable

Their constants are given in Table I. Circulating currents and losses are given in Table II. Loss values are given per 1000 ft. of cable in a single-phase circuit of two similar parallel conductors carrying equal and opposite currents at 60 cycles per second. The values are based on test results (see description of tests below) and on calculations as well.

*The ordinary lead-covered cable without armor*, (cable B) is seen in Table II to have a lead-sheath circulating-current loss 1.4 times the conductor loss at 4 ft. spacing between cables, making the sum of the losses (exclusive of dielectric loss) 2.4 times the conductor loss, the sheath circulating current being 150 amperes or 58 per cent of the conductor current. If the cable were operated without bonding the lead sheaths or without grounding at more than one point, the entire circulating-current loss would of course be eliminated and the total loss would be substantially equal to the conductor loss. Hence, operating the plain lead-covered cable with the sheath circuit open clearly gives a saving of more than 50 per cent in total losses, in comparison with the losses occurring when the sheaths are short circuited through bonds of negligible impedance.

If now the same cable is equipped with a particular *steel-wire armor*, (*cable A*), its operation with sheath and armor open circuited is much less attractive, in respect to losses, than the plain unarmored cable with sheath open circuited, because of the armor eddy-current and hysteresis losses, which bring the total loss to 2.3 times the conductor loss (omitting dielectric losses).

The highest total loss for the steel-wire armored cable is obtained when the sheath alone is short circuited, for which case the total loss is 3.6 times the conductor loss at 4 ft. spacing. The armor eddy-current and hysteresis losses have dropped to less than half the value obtained in the previous case (with sheath and armor open) because of the demagnetizing effect, upon the armor, of the sheath current; but a large sheath circulating-current loss, of more than double the conductor loss, is caused by the sheath-circulating current of 185 amperes (71 per cent of the conductor current).

If the armor of this cable is shorted, with the sheath left open, (item 3, Table II) the demagnetizing action of the circulating current (in the armor) is less than in the

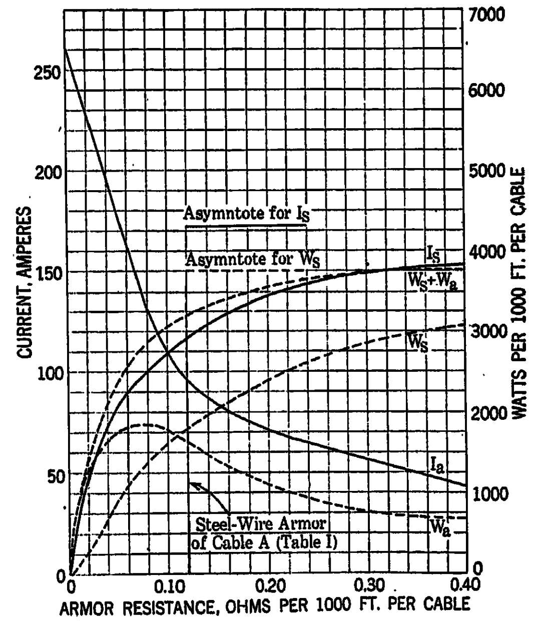


Fig. 5—Calculated Values of Sheath and Armor Currents and Losses vs. Armor Resistance

Single-phase circuit with 4-ft. spacing
For cable A (Table I) except that armor resistance is considered variable from 0 to 0.40 ohm per 1000 ft. per cable
Conductor current 260 amperes at 60 cycles per second
Average circumferential permeability at armor wires assumed constant and equal to 10 throughout the entire range of armor resistance
$I_a$ = armor circulating current
$I_s$ = sheath circulating current
$W_a$ = armor circulating current loss
$W_s$ = sheath circulating current loss

preceding case (item 2, Table II), the circulating current being smaller. Hence the iron armor eddy-current and hysteresis losses are higher than in the preceding case. The armor circulating-current loss is about equal to the conductor loss or about half the value of the sheath circulating-current loss of the case of item 2. Thus the overall losses are 2.8 times the conductor loss when the armor alone is shorted, as against 3.6 times the conductor loss when the sheath alone is shorted, for a cable spacing of 4 ft.

Finally if both sheath and armor of cable A are shorted by bonds of low impedance, as is usually the case in practise, the sheath and the armor carry lower currents and have smaller circulating-current losses than those occurring in the cases of items 2 and 3

respectively. Moreover the total demagnetizing effect upon the armor is sufficiently increased, so that the armor eddy-current and hysteresis losses are lower than in any of the preceding cases, thus making the total losses about 5800 watts per 1000 ft. per cable (or 5.8 watts per ft.) at 260 amperes conductor current, the armor eddy-current and hysteresis losses being only about 10 per cent of the total losses. The ratio of total loss (exclusive of dielectric loss) to conductor loss is 2.8 for this steel-wire armored cable at 4 ft. spacing with both sheath and armor bonded, while the corresponding ratio is 2.4 for the unarmored cable whose sheath is bonded or grounded at 2 or more points. Hence the steel-wire armored cable in question has a total loss less than 20 per cent in excess of the total loss of a similar but unarmored cable, when both cables are operated with low-impedance bonds.

Since various armor designs permit of a wide range of the value of armor resistance, it is of interest to show the effect of armor resistance on currents and circulating-current losses. Fig. 5 has been prepared for this purpose. The range of armor resistance covered is from 0.40 ohm per 1000 ft. per cable down to very low values. The circulating-current losses and current values in sheath and armor for the particular steel-wire armored cable A (see Table I) are shown at the ordinate drawn for 0.12 ohm. As the armor resistance is diminished below this value (at constant average circumferential permeability at the armor wires in Fig. 5) the combined circulating-current losses decrease more and more rapidly although the armor circulating current increases sharply.

*For the copper-wire armored cable C the combined sheath and armor losses* (see Table II) are only about 30 per cent of the conductor loss, on account of the low armor resistance (9 per cent of the steel-wire effective armor resistance at 100 amperes and 60 cycles). The armor current itself is 92 per cent of the conductor current at the 4 ft. spacing. Hence the overall losses for this cable, with sheath and armor bonded and grounded at several points are only 1.3 times the conductor loss or only half as much as the overall losses for a similar cable with the steel-wire armor described.

TABLE I

DIMENSIONS AND CONSTANTS OF CABLES

| | Cable A steel-wire armored | Cable B unarmored | Cable C copper armored | Cable D in iron pipe | Cable E steel tape armored |
|---|---|---|---|---|---|
| Voltage rating (line-to-line kilovolts) | 44 kv. | 44 kv. | 44 kv. | .... | .... |
| Diameter of copper conductor—inches | 0.682 | 0.682 | 0.682 | 0.682 | 0.528 |
| Copper conductor cross-section in circular mils | 350,000 | 350,000 | 350,000 | 350,000 | 211,600 |
| Resistance of copper conductor per 1000 ft. at 25 deg. cent. in milliohms | 30.8 | 30.8 | 30.8 | 30.8 | 50.9 |
| Inside diameter of lead sheath | 1.888 | 1.888 | 1.888 | no lead sheath* | 0.688 |
| Thickness of lead sheath | 0.13 | 0.13 | 0.13 | no lead sheath* | 0.086 |
| Diameter over lead sheath | 2.148 | 2.148 | 2.148 | | 0.86 |
| Armor type and description | single layer steel wire armor | no armor | single layer copper wire armor | 2-in. iron pipe | double layer steel tape armor† |
| No. of armor wires (bands) | 32 | .. | 32 | .... | 2 bands each 1 in. wide |
| Diameter of armor wires (thickness of bands) | 0.18 in. | .. | 0.18 in. | .... | 0.035 in. thick |
| Lay of armor wires (bands) | 10 times the armor dia. | .. | 10 times the armor dia. | .... | 1 in. |
| √ d-c. ohms/cm.³ for armor | $15.4 \times 10^{-6}$ | .. | $1.76 \times 10^{-6}$ | $13.8 \times 10^{-6}$ | .... |
| μ max. by d-c. for armor | 770 at B = 7500 gausses | .. | 1.0 | 1260 at B = 6300 gausses | .... |
| Diameter under armor, in. | 2.398 | .. | 2.39 | 2.0 | 1.03 |
| Diameter over armor, in. | 2.758 | .. | 2.758 | 2.375 | 1.17 |
| Lead sheath resistance per 1000 ft. at 25° C, in milliohms | 128 | 128 | 128 | 68.5* | 505 |
| Effective armor resistance per 1000 ft. in milliohms | 120 at 100 amp. induced armor current | .. | 10.72 | .... | .... |
| Calculated armor d-c resistance per 1000 ft. 25 deg. cent. in milliohms | 94 | .. | 10.72 | 50.5 | .... |

*Copper secondary conductor representing the lead sheath consisting of two rectangular conductors in multiple, each 0.485 × 0 125 in., placed alongside the primary cable within the iron pipe

†The two tape layers wound in opposite directions.

*The cable enclosed in an iron pipe* (cable C), when tested with both the copper-wire secondary circuit (representing the sheath) and the pipe armor short circuited, had a total loss of about 4 times* the conductor loss. While the total loss in this case is higher than the corresponding values for the other cables considered so far—on account of the closed magnetic circuit surrounding the conductor—attention is called

*The losses for a similar cable *with a lead sheath* enclosed in the iron pipe will be materially larger than 4 times the conductor loss, because the usual lead sheath will have a considerably higher resistance than that of the copper wire secondary used in place of the sheath in cable D. The low-resistance copper secondary circuit (resistance about half of that of sheath for cable A) was used to show the effect of a higher than normal amount of demagnetization on the losses in the pipe armor.

to the very considerable reduction in iron pipe losses brought about by the demagnetizing action of the circulating currents, as shown in Table IV: When both circulating-current paths were open, the heavy eddy-current and hysteresis losses in the iron pipe raised the total loss to 25 times the conductor loss, while the demagnetizing action of the circulating currents in the sheath circuit and pipe armor reduced the eddy-

TABLE II

Circulating currents and loss data per 1000 ft. of cable in single-phase circuit, for various kinds of cable operated both with and without bonding (or grounding) of sheath and armor. Conductor current = 260 amperes (except cable E which had 225 ampere) at 60 cycles per sec. Dielectric loss is not included. Bonds are considered to be of negligible impedance. Cable constants given in Table I.

| Item | Cable | Test condition | Circulating currents amps. Lead sheath | Circulating currents amps. Armor | Conductor loss watts | Sheath circul. current loss watts | Armor circul. current loss watts | Armor eddy current & hyst. loss watts | Sum of losses watts | Ratio Total loss* / Conductor loss | Cable spacing ft. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1* | A steel wire armor | Sheath & armor open no circulating currents | 0 | 0 | 2080 | 0 | 0 | 2740 | 4820 | 2.3 | 4 |
| 2* | A | Lead sheath shorted armor open | 185 | 0 | 2080 | 4380 | 0 | 1040 | 7500 | 3.6 | 4 |
| 3* | A | Lead sheath open armor shorted | 0 | 136 | 2080 | 0 | 2220 | 1460 | 5760 | 2.8 | 4 |
| 4* | A | Both lead sheath and armor shorted | 117 | 106 | 2080 | 1750 | 1350 | 600 | 5780 | 2.8 | 4 |
| 4a† | A | Both lead sheath and armor shorted | 123 | 110 | 2080 | 1930 | 1460 | 550 | 6020 | 2.9 | 4 |
| 5† | B Cable without armor | Lead sheath shorted | 150 | 0 | 2080 | 2880 | 0 | 0 | 4960 | 2.4 | 4 |
| 6† | C Copper wire armor | Lead sheath and armor shorted | 23 | 240 | 2080 | 68 | 620 | 0 | 2768 | 1.3 | 4 |
| 7* | D iron pipe armor | Copper wire secondary circuit & pipe circuit shorted | 195‡ | 42 | 2100 | 2680‡ | 3460 | | 8240 | 3.9 | 4.5 |
| | | | Conductor current 225 amperes at 60 cycles per sec. From tests by W. L. Middleton and E. W. Davis. *Elec. Wld.* Nov. 18, 1916, p. 1003 | | | | | | | | |
| 8 | E Steel tape armor | Lead sheath and armor shorted | 168 | 22 | 2580 | 14,250 | 15,000 | | 31,820 | 12.3 | 4 |
| 8a† | E | Lead sheath and armor shorted | 185 | 21 | 2580 | 16,450 | 1220 | 7680 | 27,930 | 10.8 | 4 |

*Values based on test data adjusted for 25 deg. cent. temperature and corrected for resistance and reactance of bonds. The results for the cases having circulating currents thus apply to cables having bonds of negligible impedance.

†Values for items 4a, 5, 6, and 8a were calculated.

‡In copper wire secondary representing the sheath.

*Not including dielectric loss.

TABLE III

Circulating currents and losses by test for steel-wire armored 350,000-cir. mil cable
For cable specifications see Table I cable A.
Single-phase circuit, 23 ft. loop with 50 ft. length of cable, 4 ft. spacing.
Conductor current 260 ampere, at 60 cycles. Tests at room temp. (Aver. 20° C.)

| | Test condition | Circulating currents, amp. Sheath | Circulating currents, amp. Armor | Conductor loss watts | Sheath circul. current loss watts | Armor circul. current loss watts | Armor eddy current & hyst. loss watts | Sum of losses watts |
|---|---|---|---|---|---|---|---|---|
| a | Sheath and armor open | 0 | 0 | 101 | 0 | 0 | 137 | 238 |
| b | Sheath shorted armor open | 175 | 0 | 101 | 200 | 0 | 52 | 353 |
| c | Sheath open armor shorted | 0 | 128 | 101 | 0 | 111 | 73 | 285 |
| d | Both sheath and armor shorted | 110 | 100 | 101 | 79 | 65 | 30 | 275 |

Resistance values by test for 50 ft. length of cable:

| | |
|---|---|
| Conductor resistance | = 1.5 milliohms for 50 ft. |
| Lead sheath resistance | = 6.25 milliohms for 50 ft. |
| Resistance of leaf sheath plus bond | = 6.51 milliohms for 50 ft. |
| Effective armor resistance at 100 amp. | = 6.0 milliohms for 50 ft. |
| Effective armor resistance at 128 amp. | = 6.25 milliohms for 50 ft. |
| Resistance of bond in armor circuit | = 0.5 milliohms. |

TABLE IV

Circulating current and losses from test data for 350,000-cir. mil copper cable enclosed in iron pipe with secondary copper conductor* to represent lead sheath.‡ Values in table are for a 50-ft. cable enclosed in 50 ft. of iron pipe, and are based on best results obtained for the following: Single phase circuit; 21 ft. loop with 46 ft. cable, 4.5 ft. spacing; Iron pipe armor covered only 5.2 ft. of cable on each side of loop about half way along the loop. Conductor current 260 amp. 60 cycle, test at room temperature.

| | Test condition | Circulating currents amps. | | Conductor loss watts | Secondary copper conductor circul. curr. loss watts | Pipe circul. curr. loss watts | Pipe eddy curr. & hyst. loss watts | Sum of losses watts | Ratio of total loss conductor loss |
|---|---|---|---|---|---|---|---|---|---|
| | | Secondary copper conductor | Iron pipe | | | | | | |
| a | No circulating currents | 0 | 0 | 105 | 0 | 0 | 2500 | 2600 | 25. |
| b | Copper secondary conductor shorted pipe circuit open | 211 | 0 | 105 | 157 | 0 | 153 | 415 | 4. |
| c | Pipe circuit shorted copper secondary conductor open | 0 | 184 | 105 | 0 | ←1580†→ | | 1685 | 16. |
| d | Both copper secondary circuit and pipe circuit shorted | 195 | 42 | 105 | 134 | ←173†→ | | 412 | 3.9 |

*Secondary copper conductor placed along side of primary cable conductor inside of pipe: the secondary conductor representing the lead sheath in so far as circulating currents are concerned.

†Total pipe loss including circulating current loss.

‡For cable dimensions see Cable D in Table I.

current and hysteresis losses in the iron pipe sufficiently to bring the total loss down to 4 times the conductor loss, in spite of the addition of the circulating-current losses. These results again bring out the great effect of the circulating currents in sheath and armor towards reducing the eddy-current and hysteresis losses in a magnetic armor.

*Cables with steel-tape armor* have been tested by Middleton and Davis and by others (see Bibliography). The results of tests by Middleton and Davis[21] on a 211,600-circular mil steel-tape armored cable (cable E Table I) are given in Table II item 8. One feature of the results is the high armor loss of 5.8 times the conductor loss. The other interesting feature is the high sheath loss, due to the high sheath current passing through the sheath circuit of relatively large resistance. Both sheath and armor losses are of a materially larger order than in the steel-wire armored cable type, similar to cable A, because the tape armor offers a magnetic path of relatively low reluctance around the cable, thus tending to increase the induced voltages in sheath and armor, and to produce high eddy-current and hysteresis losses in the iron.

*Eddy-Current and Hysteresis Losses.* The preceding data and discussions indicate that the overall losses in steel-armored cables with sheath and armor circuits bonded and grounded at more than one point can be kept within reasonable limits—*i. e.*, within values that are not far in excess of those for unarmored cables likewise bonded and grounded—provided the armor eddy-current and hysteresis losses are kept low. The principal means to this end are indicated by the data of Tables II and IV, and are the following: (1) an armor design of individual wires wound with a large lay so that the magnetic path around the circumference is broken up by many non-magnetic spaces, and the component of magnetizing force along the armor wires is a small fraction of the resultant circumferential magnetizing force. Thus the magnetizing force at the armor due to the several currents is rendered relatively ineffective in producing eddy-current and hysteresis losses; (2) low-resistance circulating-current circuits, so as to keep the resultant magnetizing force itself (at the armor) low by a large demagnetizing effect due to circulating currents.

If both of the above features are incorporated in the cable design, as has been aimed at in the case of the steel-wire armored cable A, item 4 in Table II, the extra armor eddy-current and hysteresis losses will represent only a small fraction of the overall losses, (say 10 per cent at the 4 ft. spacing for this type of cable), and the overall losses themselves will not be very considerably in excess of those for a similar cable without armor, (cable A, item 4 having overall losses about 20 per cent higher than those of cable B item 5 in Table II).

A very material reduction in the permeability of the armor will also tend, as a rule, to reduce the armor eddy-current and hysteresis losses for given dimensions and otherwise fixed constants. However, a general rule for the permeability and resistivity requirements of armor material—with a view to keeping the overall losses low—cannot be given, because changes in the values of armor permeability and resistivity have many-fold effects (*i. e.*, on armor resistance, sheath and armor inductance values, circulating currents in sheath and armor circuits and the resulting losses, and armor eddy-current and hysteresis losses). The choice of the best permeability and resistivity values in any particular case must be determined by analysis of the individual losses for the cable spacing to be used.

*Particular Values of Combined Eddy-Current and Hysteresis Losses for the Steel-Wire Armor of Cable A.* In the discussion of the eddy-current and hysteresis losses in a steel-wire armor, as in the case of cable A, reference must be made to the armor resistance, because the effective armor resistance offered to the flow of the armor circulating current includes certain iron losses—namely those due to the flux component passing

at right angles through the armor wires, as discussed below under "armor resistance." If, then, the armor circulating-current loss is obtained as the product of this armor resistance times the square of the armor-circulating current, the extra eddy-current and hysteresis losses remaining after the armor circulating-current loss is allowed for will be the iron losses due to the flux passing lengthwise through the armor wires.

In the tests of the steel-wire armored cable A, the extra iron losses in the armor were obtained (1) with sheath and armor short circuited, by subtracting from the total measured loss (exclusive of dielectric loss) the conductor loss and the circulating-current losses in the sheath and armor, and (2) with armor only short circuited (the sheath being open), by subtracting from the total measured loss the conductor loss and the armor circulating-current loss. These tests were made for a variety of induced circulating currents in the armor and gave curve C in Fig. 6, in which the extra eddy-current and hysteresis losses (combined) are plotted against $H$, the resultant r. m. s. field intensity acting circumferentially at the center of the armor wires assuming a non-magnetic armor. The field intensity $H$ was computed for the vector sum of the conductor current, sheath current, and half the armor current. The lower part of the curve rises somewhat more rapidly than the square law, due to the increase in permeability. It should be mentioned that the actual magnetic intensity in the iron for any point on the curve is less than the average $H$ value in Fig. 6, because of the air gaps in the circumferential flux path and because of the lay of the armor wires. The data in Fig. 6 apply of course only to the steel-wire armored cable specified under $A$ in Table I, at 60 cycles per sec.

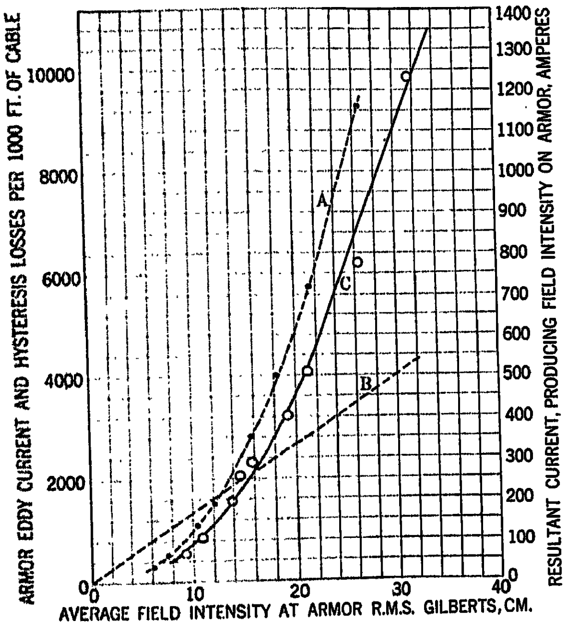


FIG. 6—DATA ON EDDY-CURRENT AND HYSTERESIS LOSSES IN IRON-WIRE ARMOR

A. Total open-circuit eddy-current and hysteresis losses combined in armor for 1000 ft. of cable A in Table I against circumferential magnetizing force $H$ at the armor; values based on tests with armor open circuited

B. $H$ values r. m. s. gilberts per cm. against resultant current acting on the armor. $H$ is the resultant field intensity acting circumferentially at the center of the wires, assuming no magnetic armor present. The curves are based on actual test values

C. Extra eddy-current and hysteresis losses in steel-wire armor for cable A, obtained in the presence of circulating currents in the armor

Frequency 60 cycles per second

A curve of open-circuit armor loss (*i. e.*, when the steel-wire armor is open circuited) is also shown in Fig. 6, curve $A$. These losses average about 25 per cent higher than those of curve $C$, for reasons already explained. The open-circuit armor loss (curve $A$) is not used in the analysis of the losses of cables operating with sheath and armor bonded.

## TESTS

Tests were made for measuring the circulating currents and the various loss components (1) for a steel-wire armored 350,000-circular mil lead-covered cable (cable A in Table I) and (2) for a plain copper 350,000-circular mil cable in an iron pipe (cable D in Table I). Attention is also called to the data in Table II on a 211,600-circular mil single-conductor cable with steel-tape armor, reported by Middleton and Davis, *Electrical World*, November 18, 1916, p. 1003.

The tests reported in the present paper are of the following kinds:

*Steel-Armored Cable A in Single-Phase Circuit.*

(a) Loss measurements with both sheath and armor open (no circulating currents), giving data on total armor eddy-current and hysteresis losses at different conductor currents.

(b) Loss and circulating-current data with lead sheath short circuited and armor open.

(c) Loss and circulating-current data with lead sheath open and armor short circuited.

(d) Loss and circulating-current data with both lead sheath and armor short circuited.

(e) Measurements of steel-wire armor effective resistance at different currents, both in the presence and absence of currents flowing in the conductor within.

(f) Resistivity and permeability data on samples of steel wire from the armor (given in Table I).

(g) Impedance measurements both in the presence and in the absence of circulating currents (reported below under a separate heading).

*Cable D enclosed in iron pipe* with secondary copper conductor to represent lead sheath:

(a) Loss measurements without circulating currents.

(b) Loss and circulating-current data with secondary conductor short circuited and pipe armor open.

(c) Loss and circulating-current data with secondary conductor open and pipe armor short circuited.

(d) Loss and circulating-current data with both secondary conductor and pipe armor short circuited.

The test method and equipment are indicated in Fig. 7. The test cable itself was 50 ft. long and was set

up in a loop of parallel sides at 4 ft. center spacing. The bonds were carefully soldered.

Circulating currents were measured by means of a Rogowski coil* with a thermocouple-type milliammeter, the complete unit being capable of ready calibration when the coil was placed around a conductor carrying a known current. The Rogowski coil was used in order that the extra impedance due to the current-measuring device be negligible in circulating-current measurements.

Losses were measured by an astatic reflecting dynamometer wattmeter. All tests were made at 60 cycles.

*Test Results for Steel-Wire Armored Cable.* The results of the tests in respect to loss values and circulating currents for the steel-wire armored cable are

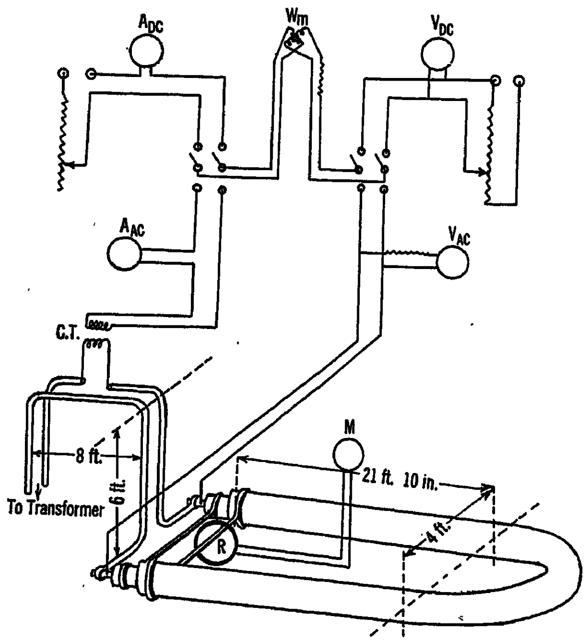


FIG. 7—SCHEMATIC DIAGRAM OF TEST SET-UP AND TEST CIRCUITS

$A_{DC}$ = D-c. ammeter
$V_{DC}$ = D-c. voltmeter
$A_{AC}$ = A-c. ammeter
$V_{AC}$ = A-c. voltmeter
$Wm$ = Wattmeter
$M$ = Thermocouple type milliammeter
$R$ = Rogowski coil
$C. T.$ = Current transformer

given in Table III. At the bottom of the table, the resistance values for the various circuits are given, including the bond resistances. In the table, the values for "sum of losses" were obtained from wattmeter readings, while the values of conductor loss, sheath loss, and armor circulating-current loss were obtained by multiplying the respective resistances by the square of the corresponding test values of current. The difference between the measured total loss and the combined conductor loss and circulating-current losses gave the extra armor eddy-current and hysteresis loss values as tabulated. The resistances for the conductor and for the sheath were measured directly, while the effective armor resistance was based on loss measurements described below.

*An air-core current transformer looped around the particular current to be measured. See Rogowski, W. S., Steinhaus, W., "Measuring the Magnetomotive Force," *Arch. für Elek.*, 1912-1913, pp. 141 and 511.

Attention is called to the fact that the circulating currents in this test were somewhat affected by the resistance and reactance of the end bonds because of the short loop length of 23 ft. In a similar cable circuit 1000 ft. long, the effect of the bonds would be negligible, and the circulating currents obtained would therefore be somewhat higher than the test values given in Table III. The amount of this increase in the circulating currents has been calculated to be about 6 per cent. Thus circulating-current data and loss data per 1000 ft. of cable at 4 ft. spacing are obtained from the test values in Table III by increasing the measured circulating-current values by 6 per cent and then calculating the $I^2 R$ product for the new current values and for the sheath and armor resistance values per 1000 ft. omitting the resistance of bonds. The extra eddy-current and hysteresis losses were also adjusted to the slightly increased circulating-current values in accordance with the average resultant field intensity $H$ at the armor, and with the aid of the law of losses indicated in Fig. 6. Following this procedure the data in Table II, already discussed, were obtained.

*Effective Resistance of Steel-Wire Armor to Circulating Currents.* It is desired to determine the effective armor resistance offered to the armor circulating currents in the presence of the fluxes (in the armor) due to the conductor current and the circulating currents in sheath and armor. This resistance under exactly the conditions stated was not measured in these tests. Instead, the following approximate value of effective armor resistance was measured: Current at 60 cycles per second was passed through the armor and was returned through the copper conductor within, in an opposite direction and the total input into the complete series circuit was measured. From this total was deducted the conductor loss. The extra losses in the iron, *i. e.*, those due to the flux component passing lengthwise through the armor wires, (which extra losses were a relatively small quantity in these measurements) were also determined according to curve $C$, Fig. 6, and deducted. The resulting value divided by the square of the current gave the effective armor resistance as plotted in curve $A$, Fig. 8. Hence this resistance takes into account the losses due to all currents (including eddy currents) flowing lengthwise in the armor wires, but is not exactly the value desired, because in the test from which it was derived, the relative magnitudes of armor and conductor currents and their phase relationship were only roughly representative of those in a cable operated with sheath and armor short circuited. However, the resistance so obtained is considered to be a closer approach to the desired value than the resistance (plotted in curve $B$ Fig. 8) based on the

armor loss measured across the armor terminals when the armor only carries current, because the resistance of curve $B$ is obtained for a current distribution in the armor in the absence of the flux due to the currents in conductor and lead sheath.

The resistance of curve $A$, Fig. 8, was used in the equations for sheath and armor circulating-current calculations (see Appendix for formulas) and gave current values within 5 per cent of the test values for the steel-wire armored cable.

The resistance of curve $A$ is seen to range from 6 to 7.5 milliohms for 50 ft. per cable for cable A (at 60-cycle armor currents from 100 to 200 amperes), these resistance values being from 1.3 to 1.6 times the measured d-c. resistance of the 32 armor wires in multiple. The calculated effective 60-cycle resistance of a single armor wire from cable A, Table I, was 1.15 times the measured d-c. resistance at 100 amperes.

The circulating-current loss in the armor calculated as the product of effective armor resistance (curve $A$, Fig. 8) times the square of the armor-circulating current will of course not include the iron losses due

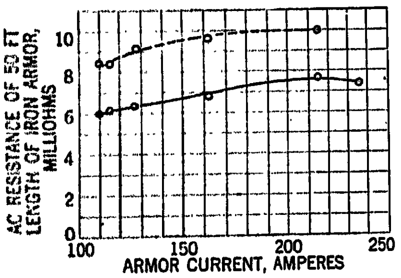


FIG. 8—MEASURED A-C. RESISTANCE OF 50-FT. LENGTH OF STEEL-WIRE ARMOR FOR CABLE A (SEE TABLE I) AT 60 CYCLES PER SECOND

A. Effective armor resistance to armor circulating current; derived from total loss measurement with armor and conductor in series carrying opposite currents, after subtracting copper loss. The small amount of extra iron loss has also been deducted.

B. Armor resistance derived from a-c. loss measurement with current carried by armor only (conductor and sheath open)

D-c. armor resistance by test = 4.7 milliohms for 50 ft. of cable

to the flux component passing lengthwise along the armor wires, and these extra iron losses will have appreciable magnitudes in the steel-wire armored cable even though operating with sheath and armor short circuited, as indicated in Table II item 4 and in curve $C$ of Fig. 6 already discussed.

*Cable Enclosed in Iron Pipe.* In order to obtain some further data on the effect of circulating currents on armor losses, an insulated 350,000-circular mil cable without a lead sheath but passing through iron pipe was tested, the results obtained being those in Table IV. To simulate lead-sheath circulating currents a secondary copper conductor was passed through the pipe and was provided with bonding connections. Measurements were made by the methods already indicated. The results are given in Table IV for a 50 ft. length of cable enclosed in iron pipe, while the test circuit had only 10.4 ft. of cable enclosed in iron pipe. Consequently the results obtained in the presence of circulating currents are only approximately applicable to large lengths of armored cable, because of the resistance and reactance of end connections. However, the sensitiveness of circulating currents to changes of resistance in sheath or armor circuits is less for the cable with iron pipe armor than for the one with steel wire armor, because of the greater inductance due to the magnetic pipe. The results of the tests on the pipe-armored cable have not been corrected for the effects of resistance and reactance of bonding connections, since the purpose of the tests was to show merely the approximate order of magnitude of the losses in the pipe armor.

## CALCULATION OF LOSSES

The calculation of the losses in sheath and armor for steel-armored single-conductor cables may be subdivided into the following three problems:

(1) Establishing formulas for the circulating currents in sheath and armor, these currents to be expressed in terms of certain reactances and resistances pertaining to the cables.

(2) The evaluation of the constants required in (1) for numerical calculation of the circulating currents.

(3) The computation of the circulating-current losses in sheath and in armor and of the extra armor eddy-current and hysteresis losses with the aid of known values of the circulating currents.

These three problems will be discussed separately.

*Method Used in Derived Formulas for the Circulating Currents in Sheath and in Armor for Single-Conductor Armored Cables in a Single-Phase Circuit.* The formulas for the circulating currents are derived in the appendix. The method used may be briefly outlined as follows:

First the fundamental Kirchhoff equations for voltage drops around sheath circuit and around the armor circuit were written. These equations were developed for a long single-phase circuit carrying equal and opposite currents in the two cables. The sheath-circuit equation, for instance, expresses that the sum of the voltages induced in the sheath due to the three currents plus the $R\,I$ drop in the sheath circuit is zero. The armor circuit equation is similar. The induced voltages themselves were expressed in terms of flux linkages.

In working out the final formulas for sheath and armor currents the terms involving flux linkages were converted into corresponding terms involving reactances, such that each reactance term in the equations for current is defined by definite flux linkage terms, partial flux linkages being taken into account by integration. The other values contained in the equations are the resistances, for sheath and armor respectively, and the conductor current.

In computing flux linkage terms, skin effect and proximity effect were neglected—but skin effect was taken into account in the evaluation of the resistance

constants. Due allowance was made for the increase in flux linkages on account of the presence of the magnetic armor. The shielding action of the armor in linkage calculations was also analyzed. For the steel-wire armored cable considered in this paper (cable A in Table I) this shielding action was found to be negligible in so far as mutual-inductance flux linkages are concerned.

For cables in which the extra armor-iron losses are a material factor, exact circulating-current calculations would require taking into account the effect (on flux linkages) of the component of the conductor current consumed in supplying the extra iron losses; *i. e.*, the losses not included in the $R\,I^2$ losses due to the armor circulating current. The factor in question was omitted from the equations derived in the appendix, and therefore the results based on them will be approximate for a cable whose armor has the extra iron losses, the errors increasing with the magnitude of the extra iron losses. The current supplying the extra iron losses could be considered mathematically as a third circulating current flowing in a circuit having a definite resistance and inducing definite voltages in the various circuits. An alternative to carrying out this procedure would be to make the first solution of sheath and armor currents with the equations given, to obtain an approximate value for the extra losses in the iron with these currents, and then to recalculate the circulating currents in terms of a new conductor current adjusted for the current consumed by the extra iron losses. Either of these procedures would materially complicate the calculations. In the calculations for the circulating currents for the steel-wire armored cable A, the currents were not more than 5 per cent higher than those based on tests, although the current consumed by the extra iron losses was neglected in the calculations. For the steel-tape armored cable E one of the circulating currents, similarly calculated, was 10 per cent too high. In other cases, the errors may be still larger, according to the magnitude of the extra iron losses.

*Evaluation of Constants.* The reactance constants required in the calculation of circulating currents are defined by flux-linkage terms tabulated in the appendix. In calculating flux linkages due to a magnetic armor the average circumferential permeability at the armor is required. Data for several armor designs have been published by Harvey and Busby.[2] The three resistance constants are the effective conductor resistance, the effective lead sheath resistance, and the effective armor resistance. The conductor resistance is equal to the d-c. value for the smaller sizes, but for the larger sizes, skin effect must often be considered, by methods well established. The effective lead sheath resistance offered to circulating currents may be taken as equal to the d-c. value without appreciable error.

The effective armor resistance to circulating currents, in the case of magnetic armors, must be calculated with due consideration of permeability and resistivity values or obtained by test. No general rule applicable to resistance calculations for different kinds of armors can be given because the methods will differ according to the shape and bulk of armor material, the arrangement of the armor conductors, and the effective current penetration. Exact calculations would be rather difficult. However, rough calculations can often be made with a fair degree of accuracy using such approximations as fit the particular case without resorting to new researches. If a sample of the cable is available, the armor resistance may be measured approximately, as indicated in the test described.

For the steel-wire armored cable tested, (cable A in Table I) the effective armor resistance at 60 cycles per second was found to be from 30 to 60 per cent higher than the d-c. resistance, depending on the current (see Fig. 8).

For steel-tape armored cables the authors have no test data giving the armor resistance.

*Calculation of Sheath and Armor Losses When the Circulating Currents and Circuit Constants are Known.* When the armor and sheath currents are determined and the resistances of sheath and armor circuits are known, the circulating-current losses in the sheath and in the armor are, of course, simply calculated as the $R\,I^2$ products, as already mentioned.

*The calculation of the extra iron losses in armor* can be made approximately with the aid of empirical data derived by tests on a cable having a similar type of armor, and with the aid of the resultant field intensity at the armor dependent on the conductor current and on the circulating currents (see data in Fig. 6 derived from tests on a steel-wire armored cable). Corresponding data for other types of iron armor have not been obtained.

In the absence of empirical iron-loss data, the extra iron losses in the armor can often be roughly determined—with less reliability—with the aid of suitable approximations carefully chosen for particular cases. Results so obtained will frequently be sufficiently good to predict at least the relative merits of different designs. Further tests would have to be made to establish for general use such methods of calculation for single-conductor cables with magnetic armor.

*Results of calculations of losses* for steel-armored cables are given in Table II, items 4a and 8a, for the steel-wire armored cable A and for the steel-tape armored cable E. The circulating currents were obtained by the formulas in the Appendix, using calculated reactance values. For cable A, the steel-wire armored type, the values of armor resistance and of extra iron losses used were those based on the test curves of Figs. 8 and 6 respectively, while for cable E, all values were calculated.

For cables A and E the calculated current values are seen to be within 5 per cent and 10 per cent, re-

spectively, of the test values, and the agreement between calculated and test values of overall losses is within 5 per cent and 15 per cent respectively. For the steel-tape armored cable the predicted values of armor eddy-current and hysteresis losses are considerably in error, indicating that in cases where the iron losses are a larger part of the overall losses, the calculations may be rather uncertain.

*Other Losses. The conductor loss* (copper loss) will not be discussed because its calculation is well understood, except for the proximity effect (due to neighboring conductors), which effect will be somewhat reduced by the shielding action of the sheath and armor around the conductor. The combined effects due to proximity and shielding will generally be small and of little significance in so far as the conductor loss is concerned.

*Eddy-current losses in the lead sheath* may be of two kinds: (a) those set up in the lead by the concentric fluxes, due to the current in the copper conductor within and the sheath current itself, and (b) those due to the fluxes from neighboring cables cutting through the sheath. The losses of the first kind are negligible for lead sheaths of the customary dimensions, because of the high resistivity and because of the relatively thin wall. The losses of the second kind also will be negligible when the sheaths and armors of the cables are bonded by low impedance bonds, but may become appreciable in the absence of circulating currents, for large cables at close spacings.[20] Additional losses in the sheath due to the longitudinal flux set up by the armor current will generally be negligible, especially when the lay of the armor wires is large.

*Dielectric losses* will not be discussed here. The reader is referred to the extensive literature on the subject.

## Reactance of Armored Cables

Test results on reactance of armored cables are given in Table V. The purpose of the table is to show values of cable reactance, effective resistance, and impedance, per 1000 ft. per cable, in single-phase circuits at 4 ft. spacing between cables, for

steel-wire armored cable A (see Table I for specifications)

bare copper cable of the same copper diameter as that of cable A, but without sheath or armor

steel-tape armored cable E.

In the steel-wire armored cable, the reactance is highest for the condition when armor and sheath are both open circuited, such that the demagnetizing effect of circulating currents is absent (item 1 in Table V). As this demagnetizing action is increased—by shorting the armor alone (item 3), then by shorting the sheath alone (item 2), and finally by shorting both sheath and armor (item 4)—the reactance per cable drops steadily, till in the last case with sheath and armor both carrying circulating currents, the reactance has dropped to nearly 1/3 of the value obtained with both sheath and armor open, at 4 ft. cable spacing. It is also of interest to point out that the reactance for the steel-wire armored cable with both sheath and armor short circuited through low-impedance connections, has a reactance of less than half of that for the plain copper conductor alone without sheath or armor, the spacing between conductors being 4 ft. in all cases.

The relative values of effective cable resistance, based on overall loss values, exclusive of dielectric losses, need no further mention in view of the discussion of losses already given.

Calculations of conductor reactance for single-conductor steel-armored cables carrying circulating currents may readily be made, with the aid of the expressions for flux linkages and reactances given in the Appendix and with the aid of additional flux linkages inside of the sheath. The effect of the magnetic armor on reactance values and on circulating currents must, of course, be considered as indicated in the appendix.

## Economic Considerations

A submarine cable is usually furnished with an armor for mechanical protection; the purpose of the armor is not so much to prevent a small sharp object from damaging the lead sheath, as is the case in an armored cable intended for burying in the earth, but rather

TABLE V

Reactances and effective cable resistances per 1000 ft. per cable in a single-phase circuit for various kinds of cables operated both with and without bonding (or grounding) of sheath and armor. Conductor current = 260 amperes (except cable E which had 225 amperes) at 60 cycles per second. Values based on test data, except item 5

| Item | Cable | Test condition | D-c. cable resistance milliohms | Eff. cable resistance milliohms | Cable impedance milliohms | Cable reactance milliohms | Center spacing between cables. Ft. |
|---|---|---|---|---|---|---|---|
| 1 | A | Sheath and armor open. No circulating currents | 30.8 | 71.5 | 176 | 161 | 4 |
| 2 | A | Lead sheath shorted armor open | 30.8 | 111 | 148 | 98 | 4 |
| 3 | A | Lead sheath open armor shorted | 30.8 | 85 | 141 | 113 | 4 |
| 4 | A | Both lead sheath and armor shorted | 30.8 | 86 | 103 | 57 | 4 |
| 5 | bare copper cable | Bare 350,000-cir. mil cable. No lead sheath no armor | 30.8 | 30.8 | 127 | 123 | 4 |
| 6* | E steel tape armor | Lead sheath and armor shorted | 50.9 | 628 | 674 | 245 | 4 |

*Results taken from test by Middleton and Davis, Bibliography reference No. 21.

to prevent an undue strain on the sheath while the cable is being laid and after it is placed in service. Such a strain might be caused by the tension which occurs in the process of laying, by the action of tides and currents in causing the cable to more down stream, or possibly by a boat with a dragging anchor. For these reasons the wire armor is used rather than a band steel armor (which has inherent high losses); and it is usually possible to obtain all the mechanical strength necessary with a copper armor.

If a portion of the armored cable rises above the water level or runs in ducts which are not at all times completely submerged, then any additional losses due to the armor may seriously affect the rating of the line as a whole. If, however, the armor is terminated at a point which is always below water level and the land section of the cable is completely unarmored, then the losses in the armored cable will not affect the rating of the line appreciably, because the armor throughout its length will be in contact with the water, which is capable of removing very rapidly the heat produced.

It is thus apparent that under practical conditions of operation of submarine single-conductor cables, neither the mechanical strength of the higher-loss steel armor nor the negligible gain in rating which would be obtained by the use of the low-loss copper armor is really an important factor in deciding which type of armor should be used. The choice rests primarily on the economics of the situation.

To illustrate this, Fig. 9 has been prepared. The total annual cost of an armored cable line has been plotted against load factor. This total annual cost is based on a single-phase or symmetrically-spaced three-phase circuit, with a 4-ft. spacing between conductors and is expressed as a percentage of the total annual cost at 100 per cent load factor of a similar unarmored circuit, carrying the same load and arranged in the same way. Since the cost of the copper-armored cable is considerably higher than that of the steel-armored cable, and since the losses of the copper-armored cable are considerably lower than the losses in the steel-armored cable, it is evident that at low load factors the steel-armored cable will tend to be the more economical, while at high load factors, the copper-armored cable tends to be more economical. This is clearly brought out in Fig. 9. The figure indicates that at 58 per cent load factor the economy of the two types of cable tends to be about the same. At higher load factors, the copper-armored cable has the advantage, whereas at lower load factors the steel-armored cable appears to be the more economical. Since the effect of cable spacing on overall losses is quite small for a wide range of spacings (above, say, 2 ft. for the cables in question) the economy curves given for 4 ft. spacing will also apply approximately over the wider range of spacings.

The values used were obtained by averaging economic data furnished by several operating companies. If the fixed charges were exceptionally high or the cost of losses were exceptionally low, the steel-armored cable would probably show up to better advantage, whereas, if the converse conditions held, the copper-armored cable would tend to be favored.

It should be particularly noted that the present economic calculation is based on the assumption that the same load may be carried by either the steel-armored or the copper-armored cable. This assumption is perfectly valid so long as the armored portion of the cable line is at all times completely immersed, since under these conditions the sheath and armor losses are very thoroughly dissipated by the surrounding water and produce only a very small temperature rise. If, however, a portion of the armored cable is above water level either continuously or for a portion of the time during which it is in operation, the losses in the lead sheath and in the armor of that part of the cable will tend to produce much greater temperature rises than they would under water and the current capacity of the cable will be thereby greatly reduced. Since the losses in the copper-armored cable are so much lower than those in the steel-armored cable, the current-carrying capacity of the copper-armored cable under these conditions will be considerably higher than that of steel-armored cable, and the economic study should probably be based on the assumption that the two cables are loaded to the maximum capacity that heating will permit. If this is done, the economic comparison will be more favorable to the copper-armored cable, especially at the higher load factors.

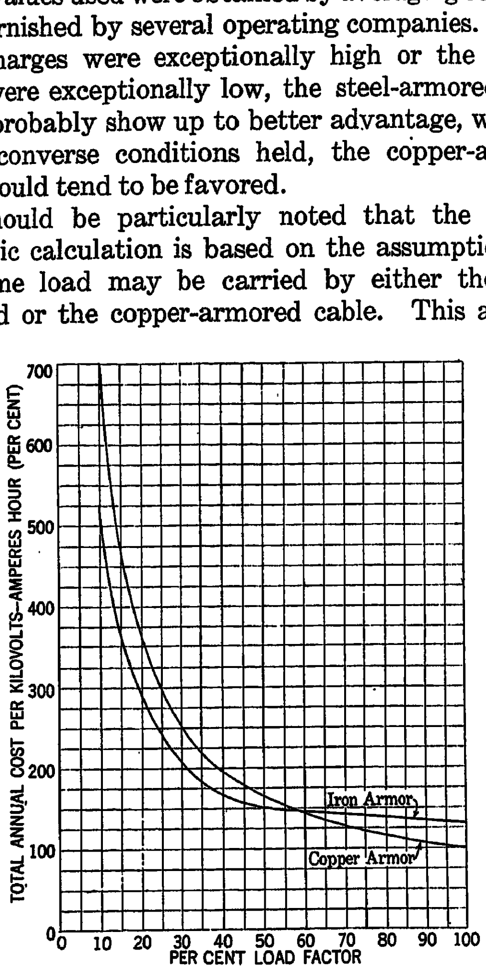


FIG. 9—RELATIVE ANNUAL COSTS PER KV-A-HOUR AT DIFFERENT LOAD FACTORS FOR STEEL-WIRE ARMORED CABLE (CABLE A IN TABLE I) AND FOR A SIMILAR CABLE WITH COPPER-WIRE ARMOR (CABLE C IN TABLE I)

## Appendix

### Derivation of Formulas for Calculating the Induced Circulating Currents in Lead Sheath and Armor

Consider the case of a single-phase circuit, going-and-return cable running parallel to each other, the spacing between center to center of the two cables being $D$. Lead sheath and armor of both cables at the near end and at the far end of the cable circuit are assumed to terminate in heavy cross bonds of negligible impedance through which the induced armor and lead-sheath circulating currents will flow, these currents being induced by the flux passing between the two cables.

### Assumptions on Which the Calculations are Based

(I) That there is no circumferential saturation in the armor, if the latter is magnetic. Values of circumferential permeability in the space occupied by the armor have been given by Harvey and Busby[2] for certain cable armors.

(II) That there is no decrease in the reluctance of the path of a line of force due to cable A (Fig. 10) passing through the armor of cable B if that armor is magnetic. If the cables are close together, and the circumferential permeability is high, this assumption might lead to considerable error.

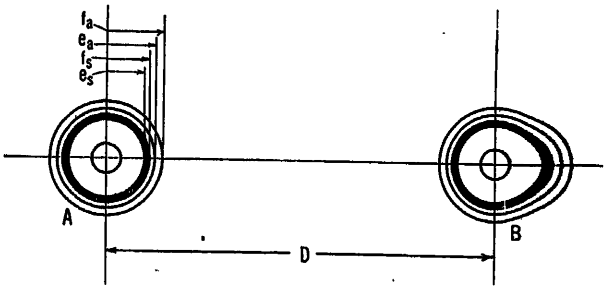


Fig. 10—Schematic Cross Section through a Single-Phase Cable Circuit

Subscripts $s$ and $a$ apply to the lead sheath and armor respectively

(III) That sheath and armor currents are uniformly distributed, in computing the linkages.

The above assumption is not made in determining the resistance of the armor. The effective a-c. resistance is used.

(IV) That the currents in cable B, its sheath, and its armor, are concentrated at the centers of the conductors when considering the effects of cable B on $A$. This assumption is justified, if proximity effect is neglected.

(V) When writing the voltage equations for sheath and armor, the induced voltages due to the currents in conductor, sheath, and armor are included together with the resistance drop. When extra iron losses are present in the armor in addition to the circulating-current losses, the extra iron losses may be considered as circulating-current losses in a third circulating-current circuit, separate from the armor circulating-current circuit. Naturally in a mathematical analysis the current in this third circulating-current circuit will have to be taken into account as contributing to the induced voltages in sheath and armor. In the derivations given below the third circulating current, representing the extra iron losses in armor, and its effect on induced voltages are neglected to simplify the calculations. When the extra iron losses are of a considerable order of magnitude, the approximation in question will, of course, introduce material errors, as already discussed.

### List of Symbols

*General.*

$e_s$ = inside radius of lead sheath, inches
$f_s$ = outside radius of lead sheath, inches
$e_a$ = inside radius of armor, inches
$f_a$ = outside radius of armor, inches
$f$ = frequency, cycles per second
$r_s$ = resistance of lead sheath, ohms
$r_a$ = resistance of armor, ohms
$D$ = spacing between cables, center to center, inches
$\mu$ = circumferential permeability of armor, that is, permeability in the circumferential direction
$\omega = 2\,\pi f$
$R$ = a large distance tending toward infinity, and compared to which $D$ is negligible.

*Currents.*

$I_c$ = current in cable conductor, taken as reference vector
  $I_{ca}$ = current in conductor of cable $A$
  $I_{cb}$ = current in conductor of cable $B$
$I_s$ = current in lead sheath, vector value
  $I_{sa}$ = current in sheath of $A$
  $I_{sb}$ = current in sheath of $B$
$I_a$ = current in armor, vector value
  $I_{aa}$ = current in armor of $A$
  $I_{ab}$ = current in armor of $B$

*Magnetic Flux Linkages* (Refer to Fig. 10)

$\lambda_{cs}$ = partial linkages with sheath of $A$ due to $I_c$ integrated from $e_s$ to $f_s$
$\lambda_{ss}$ = partial linkages with sheath of $A$ due to $I_s$ integrated from $e_s$ to $f_s$
$\lambda_{cs}'$ = total linkages with sheath of $A$ due to $(I_c + I_s)$ integrated from $f_s$ to $e_a$
$\lambda_{cs}''$ = total linkages with sheath of $A$ due to $(I_c + I_s)$ integrated from $e_a$ to $f_a$
$\lambda_{as}$ = total linkages with sheath of $A$ due to $I_a$ in armor of $A$ integrated from $e_a$ to $f_a$
$\lambda_{As}$ = total linkages with sheath of $A$ due to $(I_c + I_s + I_a)$ of $A$ integrated from $f_a$ to $R$
$\lambda_{Bs}'$ = partial linkages with sheath of $A$ due to $(I_c + I_s + I_a)$ in conductor, sheath, and armor of $B$, integrated from $D - f_a$ to $D + f_a$
$\lambda_{Bs}''$ = total linkages with sheath of $A$ due to $(I_c + I_s + I_a)$ in conductor, sheath, and armor of $B$, integrated from $D + f_a$ to $R$

$\lambda_{ca}$ = partial linkages with armor of $A$ due to $(I_c + I_s)$ in conductor and sheath of $A$, integrated from $e_a$ to $f_a$

$\lambda_{aa}$ = partial linkages with armor of $A$ due to $(I_a)$ in armor of $A$ integrated from $e_a$ to $f_a$

$\lambda_{Aa}$ = total linkages with armor of $A$ due to $(I_c + I_s + I_a)$ of $A$ integrated from $f_a$ to $R$

$\lambda_{A_a} = \lambda_{A_s}$

$\lambda_{Ba}'$ = partial linkages with armor of $A$ due to $(I_c + I_s + I_a)$ in armor, sheath, and conductor of $B$, integrated from $D - f_a$ to $D + f_a$

$\lambda_{Ba}''$ = total linkages with armor of $A$ due to $(I_c + I_s + I_a)$ in conductor, sheath, and armor of $B$, integrated from $D + f_a$ to $R$

$\lambda_i$ = partial internal linkages within the iron wires with the current $I_a$, and linkages with that current of longitudinal flux produced by the spiraling of the armor wires

$\lambda_{AS}$ = sum of all linkages with the lead sheath of $A$

$\lambda_{AS} = \lambda_{cs} + \lambda_{ss} + \lambda_{cs}' + \lambda_{cs}'' + \lambda_{as} + \lambda_{As} + \lambda_{Bs}' + \lambda_{Bs}''$

$\lambda_{AA}$ = sum of all linkages with the armor of $A$

$\lambda_{AA} = \lambda_{ca} + \lambda_{aa} + \lambda_{Aa} + \lambda_{Ba}' + \lambda_{Ba}'' + \lambda_i$

*Inductance Coefficients.*

The expressions of the flux linkages, as shown below, consist in each case of the product of a term containing a current, or the vector sum of several currents, and a factor depending on the size and the configuration of the sheath and armor circuits. This latter term is the coefficient of inductance. In general

$$\lambda = I\,L$$

*Reactance Coefficients.*

The reactance coefficient is in each case equal to $2\,\pi\,f$ times the inductance coefficient. The reactance terms used in the final formulas correspond to the following flux linkages:

$X_{cs}$ corresponding to $\lambda_{cs}$
$X_{ss}$ corresponding to $\lambda_{ss}$
$X_{cs}'$ corresponding to $\lambda_{cs}' + \lambda_{cs}''$
$X_{ca}$ corresponding to $\begin{cases}\lambda_{ca}\\ \lambda_{as}\end{cases}$

because $L_{as}$ and $L_{ca}$ are identical

$X_{As}$ corresponding to $\lambda_{As}$
$X_{Bs}'$ corresponding to $\lambda_{Bs}'$
$X_{Bs}''$ corresponding to $\lambda_{Bs}''$
$X_{aa}$ corresponding to $\lambda_{aa}$
$X_{Ba}'$ corresponding to $\lambda_{Ba}'$
$X_{Ba}''$ corresponding to $\lambda_{Ba}''$
$X_i$ corresponding to $\lambda_i$
$X_m$ corresponding to $\begin{cases}\lambda_{As} + \lambda_{Bs}' + \lambda_{Bs}''\\ \lambda_{Aa} + \lambda_{Ba}' + \lambda_{Ba}''\end{cases}$

because $L_{As} + L_{Bs}' + L_{Bs}'' = L_{Aa} + L_{Ba}' + L_{Ba}''$

$X_{AS}$ corresponding to $\lambda_{AS}$
$X_{AA}$ corresponding to $\lambda_{AA}$

## Computation of the Flux Linkages

The general procedure outlined by Alexander Russell[22] and later used in this connection by Clark and Shanklin[12,10] was adopted for the computation of flux linkages. This procedure obviously applies directly, except that it must be modified in the calculation of all linkages within the armor of, say, cable $A$ (Fig. 10), due to currents in the conductor, sheath, and armor of that cable, by the introduction of a factor $\mu$ to take care of the circumferential permeability. Also it is necessary, in some cases, to consider the magnetic shielding effect of the armor in computing $\lambda_{Bs}'$ and $\lambda_{Bs}''$.

It should be pointed out that the circumferential permeability of a steel-wire armor is low, probably varying from 5 to 15 for a single wire armor as shown by tests made by Harvey and Busby.[2] Under these conditions its shielding effect is small, probably not exceeding 25 per cent for $\mu = 10$, as shown by Professor DuBois.[23] Since these components of linkage are not large, it seemed to be a reasonably good approximation to neglect the shielding effect altogether. A calculation based on 100 per cent shielding does not differ materially from one based on zero shielding.

In accordance with the above, the flux-linkage formulas were derived allowing for the circumferential permeability due to the armor and neglecting the shielding effect. The results are given in the tabulation below, where the currents are expressed in amperes and the inductances in henrys per 1000 ft. of cable.

$$\lambda_{cs} = I_c\left(1 - \frac{2\,e_s^2}{F_s^2 - e_s^2}\log_e\frac{f_s}{e_s}\right)0.305 \times 10^{-4} = I_c\,L_{cs} \tag{1}$$

$$\lambda_{ss} = I_s\left[2\left(\frac{e_s^2}{f_s^2 - e_s^2}\right)^2\log_e\frac{f_s}{e_s} + \frac{1}{2}\,\frac{f_s^2 - 3\,e_s^2}{f_s^2 - e_s^2}\right]0.305 \times 10^{-4} = I_s\,L_{ss} \tag{2}$$

$$(\lambda_{cs}' + \lambda_{cs}'') = (I_c + I_s)\,2\left[\log_e\frac{e_a}{f_s} + \mu\log_e\frac{f_a}{e_a}\right]0.305 \times 10^{-4} = (I_c + I_s)\,L_{cs}' \tag{3}$$

$$\lambda_{as} = I_a\,\mu\left(1 - \frac{2\,e_a^2}{f_a^2 - e_a^2}\log_e\frac{f_a}{e_a}\right)0.305\times 10^{-4} = I_a\,L_{as} \tag{4}$$

$$\lambda_{As} = 2\,(I_c + I_s + I_a)\left[\log_e\frac{R}{f_a}\right]0.305 \times 10^{-4} = (I_c + I_s + I_a)\,L_{As} \tag{5}$$

$$(\lambda_{Bs}' + \lambda_{Bs}'') = -2(I_c + I_s + I_a)\left[\log_e\frac{R}{D}\right]0.305\times 10^{-4} = -\,(I_c + I_s + I_a)\,(L_{Bs}' + L_{Bs}'') \tag{6}$$

$$\lambda_{ca} = (I_c + I_s)\,\mu\left[1 - \frac{2\,e_a^2}{f_a^2 - e_a^2}\log_e\frac{f_a}{e_a}\right]0.305 \times 10^{-4} = (I_c + I_s)\,L_{ca} \tag{7}$$

$$\lambda_{aa} = I_a\,\mu\left[2\left(\frac{e_a^2}{f_a^2 - e_a^2}\right)^2 \log_e \frac{f_a}{e_a} + \frac{1}{2}\,\frac{f_a^2 - 3\,e_a^2}{f_a^2 - e_a^2}\right] 0.305 \times 10^{-4} = I_a\,L_{aa} \qquad (8)$$

$$(\lambda_{Ba}' + \lambda_{Ba}'') = (\lambda_{Bs}' + \lambda_{Bs}'')$$

$$= -\,2\,(I_c + I_s + I_a)\left[\log_e \frac{R}{D}\right] 0.305 \times 10^{-4}$$

$$= -\,(I_c + I_s + I_a)\,(L_{Bs}' + L_{Bs}'') \qquad (9)$$

$$(\lambda_{As} + \lambda_{Bs}' + \lambda_{Bs}'') = (\lambda_{Aa} + \lambda_{Ba}' + \lambda_{Ba}'')$$

$$= (I_c + I_s + I_a)\left[\log_e \frac{D}{F_a}\right] 0.305 \times 10^{-4}$$

$$= (I_c + I_s + I_a)\,L_m \qquad (10)$$

$\lambda_i$ has not been computed. Tests indicated that it is negligible for wire armors with large lay (Cable A in Table I).

$$\lambda_{AS} = \lambda_{cs} + \lambda_{ss} + (\lambda_{cs}' + \lambda_{cs}'') + \lambda_{as} + (\lambda_{As} + \lambda_{Bs}' + \lambda_{Bs}'') \qquad (11)$$

$$\lambda_{AA} = \lambda_{ca} + \lambda_{aa} + (\lambda_{Aa} + \lambda_{Ba}' + \lambda_{Ba}'') + \lambda_i \qquad (12)$$

## Derivation of Circulating Current Formulas

The total flux $\lambda_{As}$ linking the lead sheath of Cable A, being produced by the currents in the conductor of $A$, the sheath of $A$ and the armor of $A$ and also by the currents in the conductor, sheath, and armor of $B$, induces a voltage $E_{AS}$ on the lead sheath of cable A. This voltage

$$E_{AS} = -\,\frac{d}{d\,t}\,\lambda_{AS} \qquad (13)$$

Applying Kirchhoff's law to the lead sheath and armor of Cable A the following voltage expressions are obtained:

$$\left.\begin{aligned} I_{sa}\,r_s &= -\,\frac{d}{d\,t}\,\lambda_{AS} \\ I_{aa}\,r_a &= -\,\frac{d}{d\,t}\,\lambda_{AA} \end{aligned}\right\} \qquad (14)$$

where $r_s$ is the lead sheath resistance and $r_a$ is the armor resistance of cable A.

From the flux linkage Equations (11) and (12) replacing $\lambda_{AS}$ and $\lambda_{AA}$ by their respective components and assuming the currents to be sinusoidal, the voltage Equations (14) become

$$I_{sa}\,r_s = -\,j\,\omega\,[I_c\,L_{cs} + I_s\,L_{ss} + (I_c + I_s)\,L_{cs}' + I_a\,L_{as} + (I_c + I_s + I_a)\,L_m] \qquad (15)$$

$$I_{aa}\,r_a = -\,j\,\omega\,[(I_c + I_s)\,L_{ca} + I_a\,L_{aa} + (I_c + I_s + I_a)\,L_m + I_a\,L_i] \qquad (16)$$

From the symmetry of the cable circuit it follows that

$$I_{sa} = -\,I_{sb} = I_s$$
$$I_{aa} = -\,I_{ab} = I_a$$

Replacing $\omega\,L$ by $X$ in Equations (15) and (16)

$$I_s\,r_s = -\,j\,[I_c\,X_{cs} + I_s\,X_{ss} + (I_c + I_s)\,X_{cs}' + I_a\,X_{as} + (I_c + I_s + I_a)\,X_m] \qquad (17)$$

$$I_a\,r_a = -\,j\,[(I_c + I_s)\,X_{ca} + I_a\,X_{aa} + (I_c + I_s + I_a)\,X_m + I_a\,X_i] \qquad (18)$$

Dividing Equations (17) and (18) through by $I_c$ and rearranging these equations

$$\frac{I_s}{I_c} = \frac{-\,j\,[(X_{cs} + X_{cs}' + X_m) + \dfrac{I_a}{I_c}\,(X_{ca} + X_m)]}{r_s + j\,(X_{ss} + X_{cs}' + X_m)} \qquad (19)$$

$$\frac{I_a}{I_c} = \frac{-\,j\,[(X_{ca} + X_m) + \dfrac{I_s}{I_c}\,(X_{ca} + X_m)]}{r_a + j\,(X_{aa} + X_m + X_i)} \qquad (20)$$

Solving the two Equations (19) and (20), assuming $X_i = 0$, the final equations for the circulating currents in lead sheath and armor are obtained.

$$\frac{I_s}{I_c} = \frac{-\,[(X_{ca} + X_m)^2 - (X_{cs} + X_{cs}' + X_m)\,(X_{aa} + X_m) + j\,r_a\,(X_{cs} + X_{cs}' + X_m)]}{r_s\,r_a + (X_{ca} + X_m)^2 - (X_{aa} + X_m)\,(X_{ss} + X_{cs}' + X_m) + j\,[r_s\,(X_{aa} + X_m) + r_a\,(X_{ss} + X_{cs}' + X_m)]} \qquad (21)$$

$$\frac{I_a}{I_c} = \frac{-\,[(X_{ca} + X_m)\,(X_{cs} - X_{ss}) + j\,r_s\,(X_{ca} + X_m)]}{r_s\,r_a + (X_{ca} + X_m)^2 - (X_{aa} + X_m)\,(X_{ss} + X_{cs}' + X_m) + j\,[r_s\,(X_{aa} + X_m) + r_a\,(X_{ss} + X_{cs}' + X_m)]} \qquad (22)$$

## Non-Magnetic Armor

The same formulas (21) and (22) for the circulating currents may be used. When computing $X_{cs}'$, $X_{ca}$, and $X_{aa}$, however, $\mu$ should be put equal to 1.

## Three-Phase Circuit

The calculation of the three-phase circuit is rather long, and has been omitted here. It is not difficult, however, to show that in the case of a three-phase equilaterally-spaced circuit, the same formulas hold as for the single-phase circuit, if the fact that $\Sigma\,I_c = \Sigma\,I_s = \Sigma\,I_a = 0$ is used.

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## Discussion

**E. W. Davis and W. N. Eddy:** The test results on the authors' steel-armor-wire cable (Cable *A* in the paper) give a sheath and armor loss at 260 amperes and 4-ft. spacing of 3940 watts per 1000 ft. of cable with both sheath and armor bonded. Tests made in our laboratory on a cable very similar to the authors' (4/0 instead of 350,000 cir. mil, No. 4 armor wire instead of No. 7, and an 18-in. armor wire lay instead of 22-in.) gave a sheath and armor loss of 4600 watts per 1000 ft. of cable under the same testing conditions. It is believed that the difference in loss between the two cables can be entirely accounted for by the slight differences given in their dimensions, all of which would tend to make the losses greater on the second cable.

Tests on the No. 4/0 cable submerged in sea water with both sheath and armor bonded showed the sheath and armor losses to be only 75 per cent of the same losses in air, because of the more complete bonding provided by the sea water. The losses in clean fresh water were found to be the same as those in air.

The authors mention the demagnetizing action of the armor and sheath circulating currents. This can be shown very clearly by actual measurement of the flux in the sheath and armor. For instance, such measurements on a 6-in. piece of the sheath and armor of the No. 4/0 cable mentioned before gave, at 250 amperes conductor current, 0.45 kiloline of flux with no bond, 0.40 kiloline with 62.5 amperes bond current, and 0.31 kiloline with 135 amperes bond current.

These measurements also bring out the greater flux density found in the low-voltage steel-tape-armored cables. The authors' steel-tape cable *E* (Middleton and Davis) gave at 250 amperes conductor current, 7.9 kilolines with no bond and a corresponding decrease with increasing bond current. That is, the flux on this cable was *eighteen times* that on the No. 4/0 high-voltage steel-wire-armored cable at the same current.

Fig. 1 is submitted herewith as an interesting comparison of some of the loss measurements available. It indicates a consistent influence of the iron-circuit circumferential reluctance on the losses. It also shows an interesting comparison between the bonded and not-bonded losses. At low flux densities, whether caused by low conductor current or high circumferential reluctance, the not-bonded losses are greatest while the reverse is true at higher flux densities. One reason why the new high-voltage armored cables show such relatively low losses is that because of the additional insulation, the cable diameter is relatively large and the circumferential reluctance of the magnetic circuit correspondingly smaller.

The authors' calculation of the circulating-current losses is based on the value of the sheath resistance used. As described in the paper it was measured by passing current through the conductor and the sheath in series. It seems doubtful that such a resistance is always equal to the true resistance of the sheath. The circulating current is induced in the steel armor wires by the flux from the conductor current. As this flux is directly proportional to the circumferential reluctance of the armor it must be maximum at the center line of the armor wires, decreasing to a very low value outside and inside of this line, because of the rapid increase of air gap and decrease of iron in the magnetic circuit (circumferential to the cable). Since the circulating current is induced by the flux it would seem that its density should be greatest where the flux is strongest and least where the flux is weakest. Any change in conductor current should change the flux distribution and result in at least some change in the true resistance of the armor.

On the tenth page of the paper the authors, using their measured sheath resistance, offer an explanation of the increase in circulating-current losses due to the addition of steel armor wire to a lead-covered cable that gives reasonable agreement with test values. Yet it would seem even more reasonable to expect that the true effective resistance of the armor might be considerably higher than their value (which would tend to give lower losses) but that the flux density in the iron was so much greater than in the lead that the induced current density was greater and the losses greater. The combination of these effects might easily account for the test results obtained.

**O. R. Schurig:** The data on armored cables in the discussion by Messrs. Davis and Eddy are a valuable contribution to this subject. Their test results on the effect of water on losses in submarine cables are particularly interesting. The data are consistent with those given in the paper and need no further comments.

In their discussion it is stated, with reference to the measured value of armor resistance used in the paper, that "it seems doubtful that such a resistance is always equal to the true resistance."

Attention is called to the fact that no such claim is made in the paper. On the contrary, a full paragraph on effective resistance of steel-wire armor to circulating currents is devoted to pointing out the nature of the approximations made, the complications involved in measuring the true armor resistance, and the reasons why the resistance measured "is not exactly the value desired" (to quote from the paper). Nevertheless the results of loss calculations with the values from Fig. 8, curve $A$, differ by not more than 10 per cent from the test data, and the calculated circulating-current values are within 5 per cent correct. These results amply justify the use of the test values of armor resistance for performance calculations.

In further discussing the armor resistance, Messrs. Davis and Eddy refer to the distribution of flux in the armor wires, to the effect that the flux "must be a maximum at the center line of the armor wires." There is no doubt that the air-gap flux between the armor wires is densest at the point where the gap is smallest. However, this condition does not by any means determine the distribution of flux in the armor wires themselves, the latter being conditioned upon skin effect and proximity effects. That is, the flux passing across the armor wires (and also the current flowing lengthwise along the armor wires) will tend to crowd toward the upper side of the armor wires or toward the under side of the armor wires, according to whether the armor alone carries current or whether the armor current is induced, as in normal operation bonded, by the resultant flux from conductor and sheath currents. In other words, Messrs. Davis and Eddy are not correct in stating that the flux and the circulating current in the armor are a maximum at the center line of the armor wires, and conclusions drawn from this reasoning cannot be expected to be reliable.

In the last paragraph Messrs. Davis and Eddy discuss the effect of steel-wire armor resistance upon circulating current losses and suggest that the effective armor resistance might be considerably higher than the values used in the paper. All the evidence in the paper indicates, however, that the armor resistance values used in the calculations are rather too high than too low. A higher armor resistance would give still higher losses and greater errors of calculation in the case of cable $A$ (see calculated results in item 4a, Table II and loss curves with changing armor resistance in Fig. 5). Moreover, an analysis of the current distribution in the armor under operating conditions and under the conditions of the test for armor resistance (see Fig. 8, curve $A$) also suggests that the armor resistance values used may be slightly high rather than too low.

In regard to the explanation of the increase in circulating current losses due to the addition of steel armor wires to a lead-covered cable, we are in agreement with Messrs. Davis and Eddy on the point that a magnetic armor will tend to give greater circulating-current losses than a non-magnetic armor having the save effective resistance, because of the additional inductance caused by the magnetic armor.

# Lightning

## Progress in Lightning Research in the Field and in the Laboratory

BY F. W. PEEK, Jr.*

Fellow, A. I. E. E.

### I. INTRODUCTION

RECENT progress in the mastery of lightning problems through combined research in the laboratory and field has been so rapid that it seems important at this time to make a review of the present status of the various phases of the subject. While there is still much to learn, lightning may be said to be now at least on an engineering basis since it is expressed numerically in volts and amperes. It has been removed from the realm of the "medicine man."

The following indicate how rapid the progress has been: The wave shape of lightning has been pictured by the cathode ray oscillograph; the time required for a cloud to discharge has been measured by the cathode ray oscillograph; the attenuation of lightning waves traveling on a transmission line has been determined; natural lightning waves have been reproduced in the laboratory where their effects on transmission lines, insulators, insulation, transformer and protective apparatus have been studied at will; a lightning generator producing over 3,600,000 volts has been constructed and waves from this generator have been sent over transmission lines to test full size tranformers and other apparatus to determine how to make them highly resistant to lightning; scientific work on the time lag of gaps and insulation has been extended, etc. The above list is not complete but will serve to indicate how much progress has been made. The important phases will now be discussed in detail.

### II. LABORATORY RESEARCH

*The Lightning Generator.* Up to the early part of 1927 the laboratory lightning work had progressed so far,[1] that it seemed important to double the 2,000,000 volts available at that time. This high voltage was desirable so that full size apparatus could be tested and results obtained without extrapolation. A 3,600,000-volt generator was built and is in satisfactory operation; and an extension is now available so that about 5,000,000 volts is obtainable. Double the directly generated voltages due to reflection have been measured at the ends of transmission lines.

A radically new method was devised by the author to obtain the very high voltages.[2] The effect is of adding two, three, four, or more of the original generators in series at the proper instant so that all of the respective impulse voltages add together. No rectifiers are used. The a-c. voltage is applied directly to each unit generator. At that instant on the crest of the wave when each unit is fully charged, gap sparkovers take place that connect the generators in series and the impulse occurs. The connections are shown in Fig. 1A. The condensers of the three generators $C_1$, $C_2$, $C_3$ are charged from the transformer to a crest voltage corresponding approximately to the $G_1$ gap setting. Sparkover occurs on $G_1$ followed immediately by sparks on $G_2$ and $G_3$. Resistance $R_a$, $R_b$, and $R_c$, $R_1$ and $R_2$ permit the small 60-cycle changing current to flow but are, in effect, infinite to the very high impulse current. The result is as shown in Fig. 1B. Only three gaps are in series on 3,600,000 kv. and four on 5,000,000 kv. One, two, three, four, or more steps can be used. The wave shape is determined by $R$, $L$, and $C$. Waves varying in

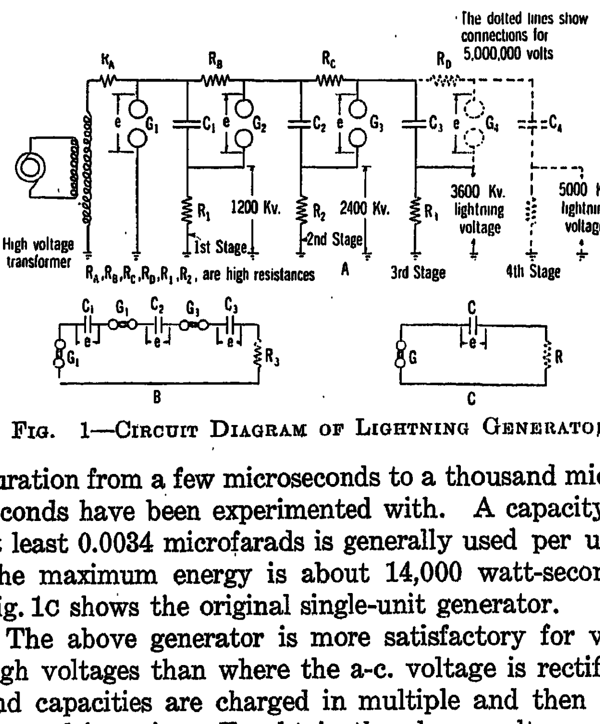


FIG. 1—CIRCUIT DIAGRAM OF LIGHTNING GENERATOR

duration from a few microseconds to a thousand microseconds have been experimented with. A capacity of at least 0.0034 microfarads is generally used per unit. The maximum energy is about 14,000 watt-seconds. Fig. 1C shows the original single-unit generator.

The above generator is more satisfactory for very high voltages than where the a-c. voltage is rectified, and capacities are charged in multiple and then discharged in series. To obtain the above voltage with the Marx connection,[3] 35 to 50 units with 35 to 50 spark-gaps would be required. However, the Marx arrangement is satisfactory for low or moderate voltages.

Fig. 2 shows a 3,600,000-volt flash from the lightning generator. The maximum sparkover distance possible with such a voltage depends upon what wave shape of surge the lightning generator is adjusted to give. With a surge of a very short duration, a sparkover of

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(On January 25, 1929, after the original writing of this paper, the voltage of the lightning generator was increased to 5,000,000 volts, and laboratory lightning research started at that potential).

1. See bibliography for all references.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

only 9 ft. can be secured at 3,600,000 volts crest. Longer distances can be broken down with long waves, as will be explained later, as much as 20 ft. being possible with a 1000-microsecond front. The lightning generator condenser units are shown in Fig. 3.

(*a*) *Measurements at very high voltages*

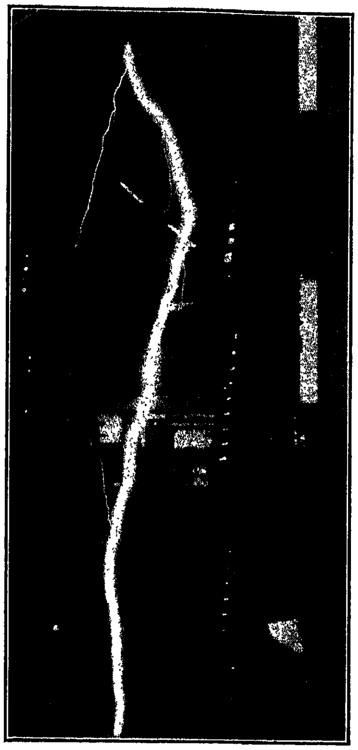

Fig. 2—3,600,000-Volt Artificial Lightning Stroke

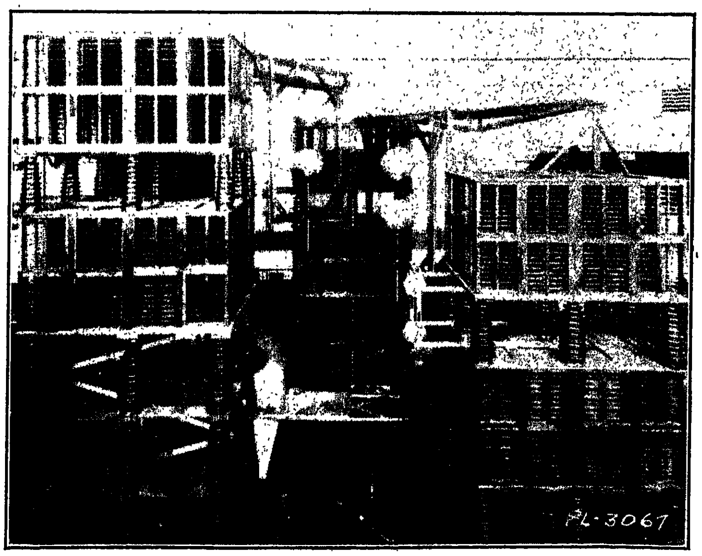


Fig. 3—Lightning Generator

*Voltage.* Considerable skill and experience is necsary to measure high lightning voltages of very short duration. During the pioneer work, it was necessary to check the voltage measurements in a number of ways. The voltages were first calculated from the circuit arrangement; a check by 100-cm. sphere-gaps up to 1500 kv. was made; readings were taken with a surge voltage recorder[4] or klydonograph;[5] as a final method measurements up to the maximum voltage were made with a capacity potentiometer. Surge-voltage recorder records of 5,000,000 volts from the lightning generator are shown in Fig. 4.

*Wave Shape.* In the first studies of transients, wave shapes could not be pictured directly; it was necessary to calculate them. The cathode ray oscillograph[6] now affords a means by which oscillograms can be taken readily. It is interesting that these oscillograms measuring time in microseconds check the early work.[2] Fig. 5 shows a typical oscillogram. This particular wave reaches its crest in a fraction of a microsecond and then decays to half value in five microseconds. Surges are frequently measured along a high-frequency timing wave as in Fig. 6.

As a matter of convenience, such waves will be

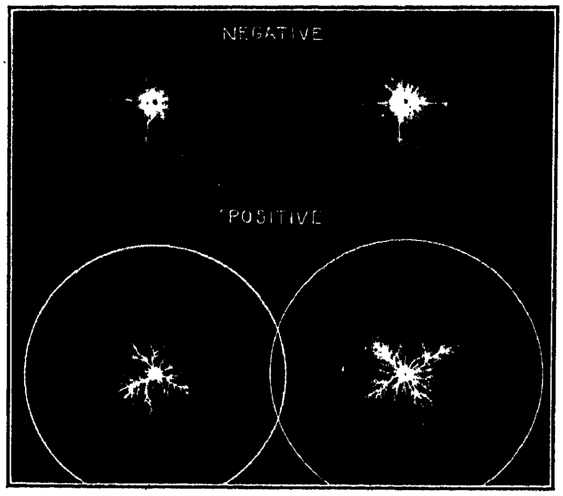


Fig. 4—Surge-Voltage Recorder Records of 5,000,000-Volt Artificial Lightning

reduced to rectangular coordinates as in Fig. 7. The Fig. 7 wave has been used extensively in sparkover tests and is believed to approximate the effects of actual lightning.

(*b*) *Sparkover of spheres, points, and insulators*

The full line curves in Figs. 8, 9, and 11 give the sparkover voltages for different gaps with the Pittsfield high voltage laboratory "standard wave" and other waves. In making these curves, the full effect of the wave was used; that is, the impulse for a given gap setting was increased until sparkover just did occur. In addition curves are given for sparkover on the crests of the waves. Breakdown takes place on the crests of the wave with applied crest voltages considerably higher than those just necessary to cause sparkover. Fig. 12 shows that the lightning sparkover depends upon the length of the string or the spacing of the units. Five and three-fourths in. seems to be approximately the right spacing for 10-in. diameter disks.

(*c*) *The effect of wave shape on the lightning sparkover. Polarity*

With the exception of gaps between electrodes

producing a uniform field the lightning or impulse sparkover voltage is always appreciably higher than the 60-cycle sparkover voltage. The steeper the wave or the shorter the duration of the transient, the higher the crest sparkover voltage. With an exceedingly steep or short wave there may even be a measurable increase for spheres. The lightning breakdown voltage will thus vary because lightning surges vary. The ratio of the lightning to the 60-cycle crest sparkover voltage is

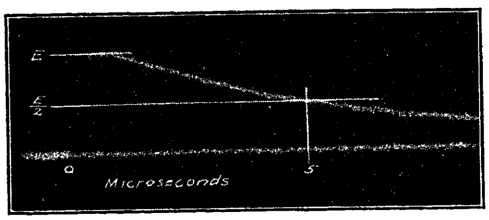


Fig. 5—Typical Five-Microsecond Wave of Lightning Generator

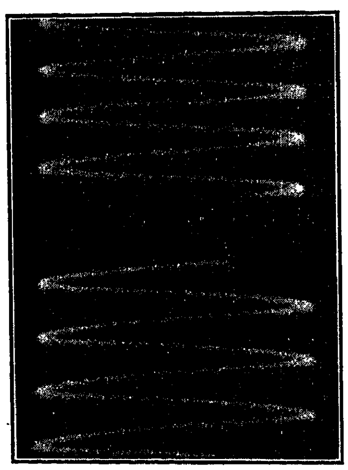

Fig. 6—Wave Measured with Oscillatory Sweep on Cathode Ray Oscillograph

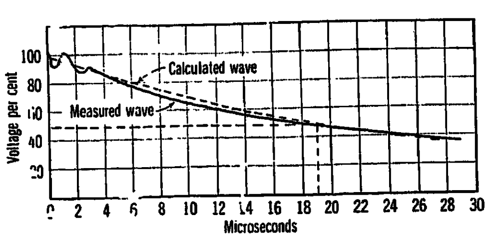


Fig. 7—Lightning Wave of 3,600,000-Volt Lightning Generator Calculated and Wave as Measured by the Cathode Ray Oscillograph

always greater than unity. Some years ago, this was termed the impulse ratio.[2] Under the usual severe lightning conditions in practise, insulator sparkover voltages give an impulse ratio of two. This has been well established by comparing the lightning sparkover voltages of insulators as measured in the field by the surge voltage recorder and the klydonograph with the 60-cycle sparkover voltage. The impulse ratio is thus an indication of the effective duration of the wave. The wave in Fig. 7 gives approximately an impulse ratio of two. Points measured on transmission lines are indicated on one curve of Fig. 11. The standard high-voltage laboratory wave was established before the field data were available. The laboratory lightning thus corresponds to the wave causing sparkover and damage in practise, for if the actual lightning wave were of effectively longer duration the impulse voltages could not be so high.

Whether the front or the tail of the wave is the

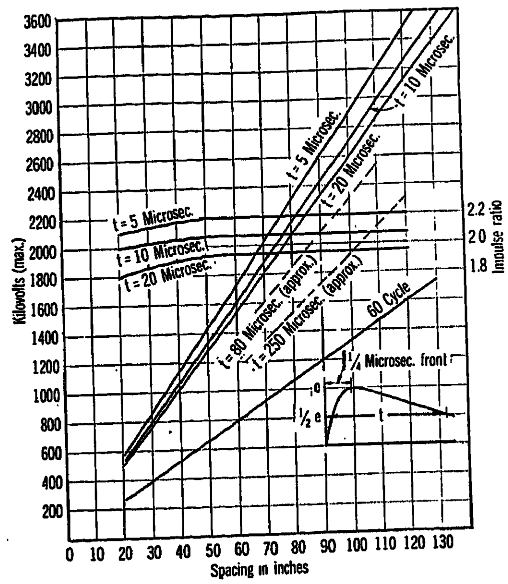


Fig. 8—Point-Gap Sparkover for Different Lightning Waves

See Fig. 5 for oscillogram of five-microsecond wave.

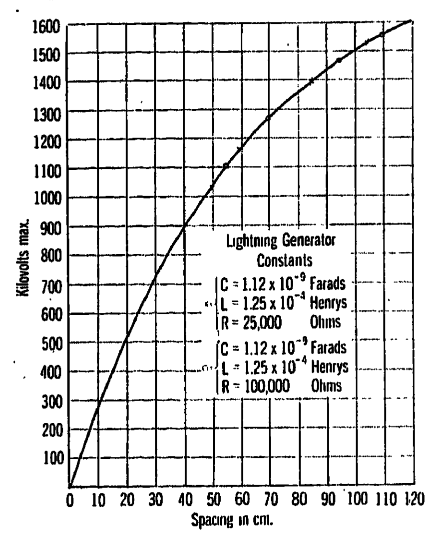


Fig. 9—Sphere-Gap Sparkover

100-cm. diameter spheres—one sphere grounded.

controlling factor in determining the lightning sparkover depends upon the voltage applied. This is well illustrated in the oscillograms of Fig. 13 representing actual test records on a 19.9-cm. point-gap. The same wave shape was used throughout these tests. In the first tests the voltage was increased until sparkover occurred at 50 per cent of the applied impulses. An impulse wave with a crest voltage of 175 kv. was required for

breakdown while the 60-cycle crest sparkover value for the same distance was 125 kv. The impulse ratio was accordingly 1.40. The actual sparking points on the wave are indicated by the crosses. An interesting

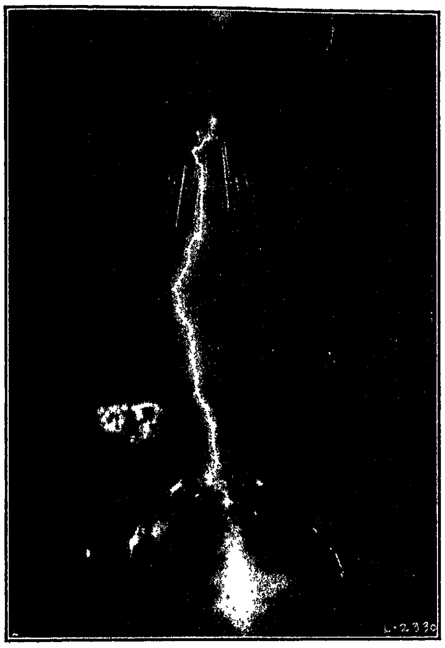

FIG. 10—LIGHTNING SPARKOVER OF 100-CM. SPHERES FOR WAVE OF VERY SHORT DURATION

fact was found here probably for the first time—namely, that sparkover actually took place after the tail of the wave had decreased below the 60-cycle

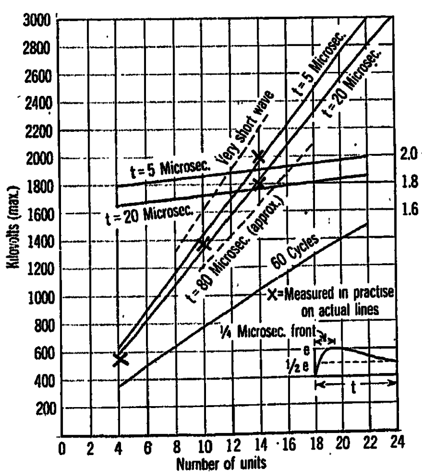


FIG. 11—LIGHTNING AND 60-CYCLE SPARKOVER CURVES OF SUSPENSION INSULATORS FOR DIFFERENT WAVES

See Fig. 5 for oscillogram of five-microsecond wave

value. Apparently the breakdown effect, once started by the overvoltage, continues, so that the spark actually forms after the wave has fallen below the minimum 60-cycle crest sparkover voltage. This is of great theoretical interest, but space does not permit further discussion here. A wave 57.5 per cent in excess of the minimum impulse sparkover voltage was next applied to the gap. As can be seen from Fig. 13, sparkover still took place on the tail but at a higher value. Breakdown of the gap occurred on every applied impulse. With 130 per cent excess voltage, sparkover took place practically on the crest of the wave, while at 178 per

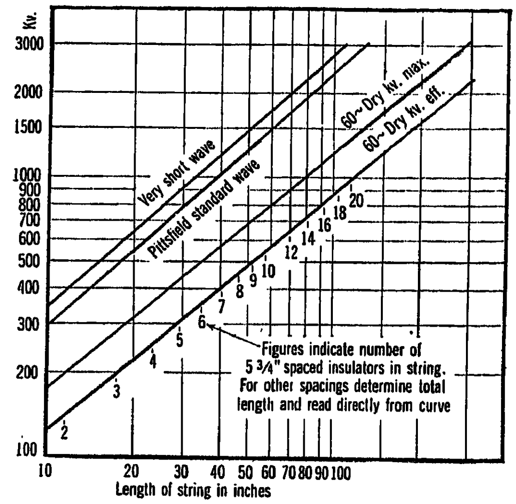


FIG. 12—LIGHTNING AND 60-CYCLE SPARKOVER FOR STANDARD 10-IN. DIAMETER SUSPENSION TYPE INSULATOR

Length of string and actual physical length between center lines of bolt holes; not actual arcing distance or creepage distance

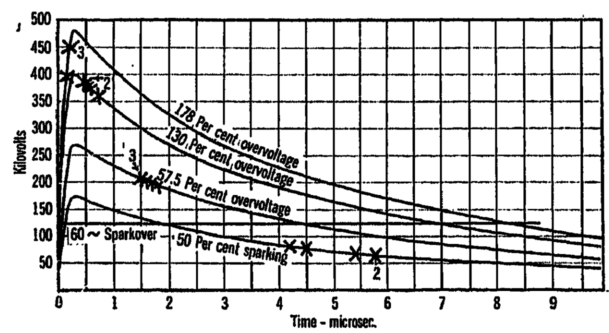


FIG. 13—SPARKOVER OF A POINT-GAP WITH A CONSTANT WAVE SHAPE AT VARIOUS APPLIED VOLTAGES

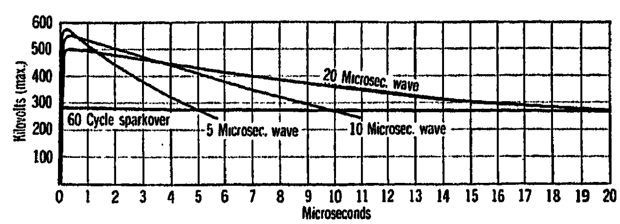


FIG. 14—VARIATION OF SPARKOVER VOLTAGE FOR WAVES OF 5, 10, AND 20 MICROSECONDS DURATION

50-cm. needle-gap—sparking 50 per cent of applied impulses

cent overvoltage, it occurred on the front of the wave. The corresponding impulse ratios for this short gap cover a range from 1.4 to 3.50. Fig. 14 shows the variation in sparkover voltage with waves of the same front but with 5, 10, and 20 microseconds durations above the 60-cycle sparkover voltage.

Tests were also made with waves rising more or less uniformly at various rates and with breakdowns always occurring on the fronts. The results are shown in Fig. 15. For the gap used in Fig. 15, the impulse sparkover voltage approximately equals the 60-cycle sparkover when the time from application of voltage to complete breakdown is approximately 1000 microseconds. This shows the effect of wave-front steepness increasing the sparkover voltage as previously noted.

Standard tests in the laboratory are made by gradu-

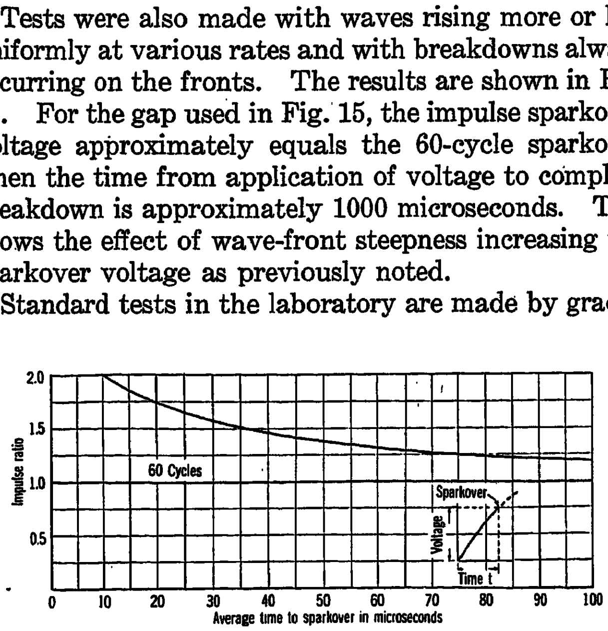


FIG. 15—SPARKOVER VOLTAGE FOR WAVES RISING UNIFORMLY AT VARIOUS RATES

ally increasing the impulse voltage until sparkover occurs on 50 per cent of the applications. The instantaneous breakdown voltage when the front is relatively short then depends largely upon the duration of the tail. This is illustrated in Fig. 16 where the crest voltage is the same for the three waves.

If the impulse ratio is the same the results on solid and liquid insulation are approximately the same whether the standard wave, the lightning wave, or a

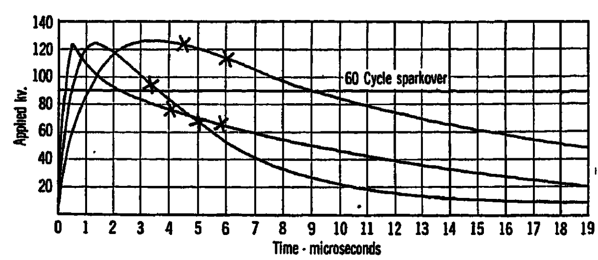


FIG. 16—EFFECT OF WAVE FRONT ON SPARKOVER

Sparkover of 13-cm. needle-gap with given impulse ratio and different waves

Crosses indicate average sparking range

wave with a slowly rising front is used. The three waves are illustrated in Fig. 17. Any of these is equally effective for testing purposes.

A gap between spheres has very little time lag if the spacing is not greater than the diameter of the spheres. In general, therefore, the impulse ratio is practically unity, and the sphere-gap indicates the voltage at the crest of the wave. However, because it is generally desirable to know the effective duration of the wave as well as its crest, in making tests a "time-gap" is necessary. The suspension insulator is a very good gap for this purpose. An example will best illustrate the use of such a gap. Assume that it is desired to compare the lightning sparkover voltage of two entirely different types of bushings, but that it is not possible to do this in the same laboratory with exactly the same waves. A sphere-gap measurement would give the crest of the wave, but equal sparkover voltages would not indicate equivalent bushings unless the shapes of the waves were known. However, a very good comparison can be obtained by the insulator "time-gap" even if the waves differ considerably. This can be done by placing an insulator string in parallel with the bushing, applying impulses, and adding or removing units from the string until 50 per cent of the sparks occurs on each. The equivalent breakdown strength of the bushing is thus obtained in terms of line insulators. Since the impulse ratio of bushings and insulators vary together up and down over a wide range with varying wave shapes, the effect of variations due to such differences is eliminated and a good comparison is obtained. The lightning sparkover voltage of the bushing for any particular wave can then be determined from the lightning sparkover curve of the insulator string. The insulator time-gap also offers a convenient method for

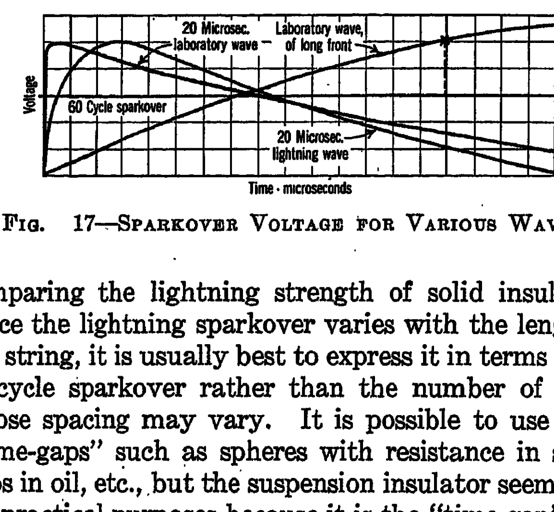


FIG. 17—SPARKOVER VOLTAGE FOR VARIOUS WAVES

comparing the lightning strength of solid insulation. Since the lightning sparkover varies with the length of the string, it is usually best to express it in terms of the 60-cycle sparkover rather than the number of units, whose spacing may vary. It is possible to use other "time-gaps" such as spheres with resistance in series, gaps in oil, etc., but the suspension insulator seems best for practical purposes because it is the "time-gap" that limits the voltage on lines.[1]

When the maximum voltage of the lightning impulse causing an insulator sparkover is measured by a sphere, surge-voltage recorder, or klydonograph and the 60-cycle crest flashover voltage is known, the effective duration of the wave is also obtained. For example, the lightning sparkover of insulator strings measured on the 220-kv. lines of the Pennsylvania Power & Light Company were found to average about 2000 kv. For these insulators the 60-cycle sparkover was about 1000 kv. This indicates an average impulse ratio of 2.0. The usual impulse ratios of natural lightning varies between 1.8 and 2. The crosses in Fig. 11 for four, ten, and fourteen unit insulator strings are flashover voltages due to natural lightning as measured by surge voltage recorders. In a few cases, impulse ratios as high as 2.7 were obtained. These

impulse ratios show that the effective duration varied from 1 to 20 microseconds, where the effective duration is the time that the voltage is above half voltage, or approximately the time above the 60-cycle sparkover. Such waves are illustrated in Figs. 5 and 7 and were actually measured by the cathode ray oscillo-

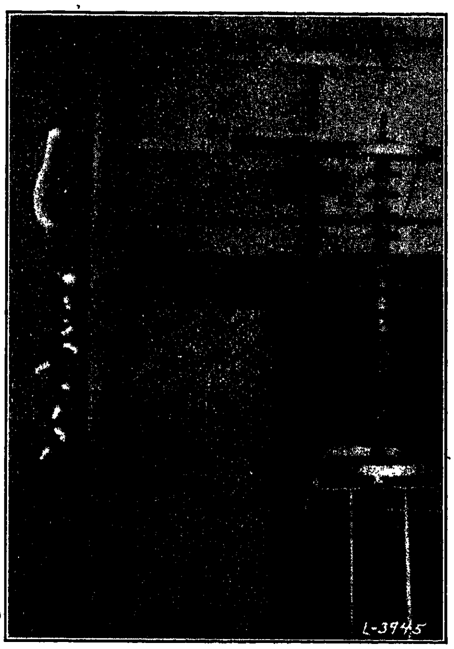


Fig. 18—Lightning Applied to Shielded and Non-Shielded Insulator Strings in Parallel. Sparkover on Non-Shielded String.—80-Microsecond Wave

graph. Thus a wave giving an impulse ratio beween 1.8 and 2 on line insulation represents the average severe field conditions, and the standard laboratory wave, established long before measurements were available, is confirmed as simulating practical conditions. The lightning wave secured on the Pennsylvania Power & Light Company line this last summer had a duration above half voltage of about 20 microseconds.

In the early work[2] on transients it was found that where dissimilar electrodes were used, lower voltage sparkovers occurred with the smaller electrodes positive. This fact has been checked in these recent tests, and the greatest differences in voltage result between a point and a plane. With long insulator strings the polarity effect is not appreciable.

(*d*) *The grading shield*

An important development is the grading shield for insulators.[7] The grading shield bears about the same relation to the insulator string as the ground wire does to the line. An important function of the grading shield is to cause even distribution along the string. This strengthens considerably the path along the insulator surfaces to lightning and forces the arc to take place between the rings which may be set for a lightning sparkover voltage higher than that of the non-shielded string. Destructive cascading is thus prevented. In this way the gain in voltage may be as much as 10 per cent to 12 per cent, and can be checked by comparing the lightning sparkover of the non-shielded string with the needle-gap lightning sparkover of the distance between rings. For instance, the sparkover voltage of a 16-unit non-shielded string from Fig. 11 is 2050 kv. at 20 microseconds. For 85 in. (from Fig. 8) between rings, when the sparkover occurs on a shielded string, it is 2200 kv. For the 20-microsecond wave this is usually over 10 per cent for long strings. For very steep waves it may be more. That there is considerable advantage in voltage for the shielded string is illustrated in Fig. 18 where an impulse of 80 microseconds duration above the 60-cycle sparkover voltage is symmetrically applied to the two strings connected in parallel. The flashover occurs on the non-shielded string. The difference in sparkover voltage is not appreciable with longer waves. Fig. 19 shows a shielded and non-shielded string. Careful adjustment to prevent cascading must be made at the steeper waves because less time is available for allowing corona and surface discharges to distribute properly the voltage stress over the units. Sufficient time is available with slower voltage applications. The gain in voltage with the shield is incidental as its important function is to prevent cascading.

While the sparkover voltage to lightning waves may be increased by the shield, the 60-cycle sparkover voltage may be lowered. This is not a handicap because lightning surges having an impulse ratio of unity and thus corresponding to 60-cycle waves have never been observed in practise. The dry 60-cycle shielded spark-

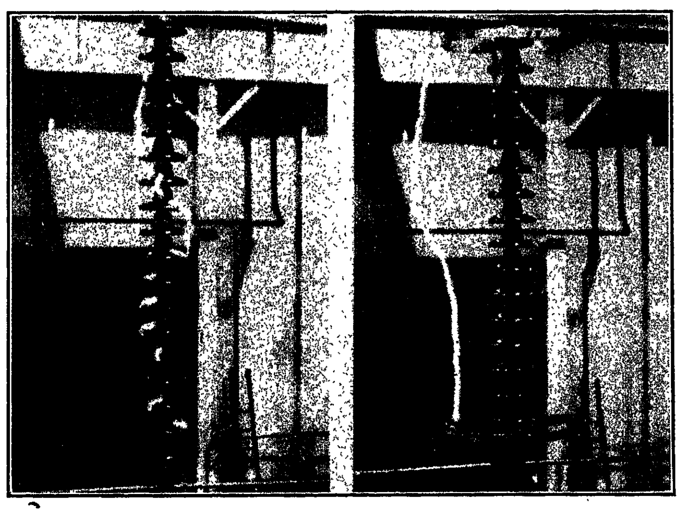

Fig. 19—Typical Lightning Arcs on Shielded and Non-Shielded Insulator Strings

over voltage might be somewhat increased by using *very* large shield surfaces free from sharp ends or points. However, there can be no gain in practise in this way because the large surfaces would be reduced to equivalent "points" in 60-cycle voltages when wet by the first raindrop. Lightning sparkover voltage is not affected by rain.

The oscillogram in Fig. 20 shows the successive cascading of the units in a non-shielded string. That shields prevent deterioration of the units in a string through improved distribution of voltage stresses is forcibly illustrated in tests. After a few lightning sparkovers, insulator units fail in the non-shielded strings while there are no failures in the shielded strings.

In addition to the actual increase in lightning sparkover voltage discussed above, there is also an apparent increase which is probably of more importance. When the energy of the lightning generator is limited, it is necessary to supply a higher voltage to a shielded string to cause sparkover. This apparent increase in sparkover voltage may be of a higher order than the actual increase. The extra voltage must be generated because of the energy dissipated by the "barrel" of corona between the edges of the rings. (See Fig. 21.) The gain has been observed when the energy available approximated that in an average span and should be an approximate measure of the effect in practise since there is one shield for each line per span. This energy dissipating effect by corona has been made use of by purposely designing grading rings of flat strap material in place of smooth surfaced pipes.

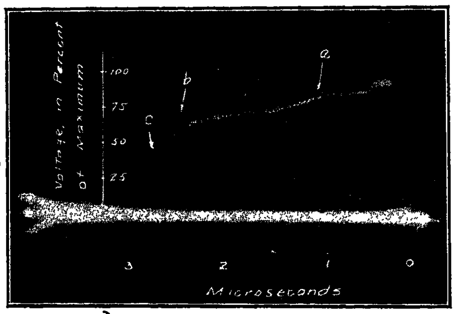


FIG. 20—CATHODE RAY OSCILLOGRAM OF SUCCESSIVE BREAKDOWN OF UNITS IN NON-SHIELDED INSULATOR STRING OF THREE UNITS

(a) Line unit cascade
(b) Second unit cascade
(c) Complete string flashover

The results of lightning sparkover tests with the strings excited at normal 60-cycle voltages were not different from tests on non-excited strings.

From the above it can be seen that a successful shield must grade and increase the strength along the string so that sparkover is forced to occur between rings rather than over the surface of the insulators with the shield at the same time maintaining a high 60-cycle flashover voltage; that the design must be such as to dissipate the maximum energy by corona and thus have the effect of increasing the impulse sparkover voltage; that single sharp points or sudden surface changes are undesirable; that no practical gain results from large rounded surfaces.

From the standpoint of clearing the dynamic arc, complete round or oval rings are highly desirable as a track for the arc when blown by the wind. Anchor points at the ends of a sectionalized shield may cause it to wrap around the string.

Horns cannot prevent cascading without a serious reduction in voltage because they do not properly grade the string. They must be adjusted for a lightning sparkover voltage lower than that of the weak non-shielded string.

(*e*) *Wood poles*

The insulating value of a wood pole to lightning voltages has been measured up to 3,600,000 volts. The measurements show that the strength of wood poles of such varying degrees of wetness and dryness as might occur in practise, range from 100 to 300 kv./ft. A good average value is 180 kv./ft. Thus, a pole 35 ft. high, with a 5-ft. crossarm, would have a lightning sparkover voltage of $40 \times 180 = 7200$ kv. The insulator would add very little to a pole of this length. However, when the length of wood in series with the insulator is not over 10 ft., from 75 to 100 per cent of the insulator flashover voltage may be considered as added to that of the wood to comprise the total pole insulation. A practical example of this is a pole made conducting by a lightning rod to prevent splitting, with the insulation depending upon the insulator and crossarm. In a case of this kind, part of the insulation of the pole could be utilized and protection from splitting afforded at the same time by placing a gap in series with the lightning rod. To prevent splitting, the 6-3-1 ratio shown in Fig. 22 should be used.

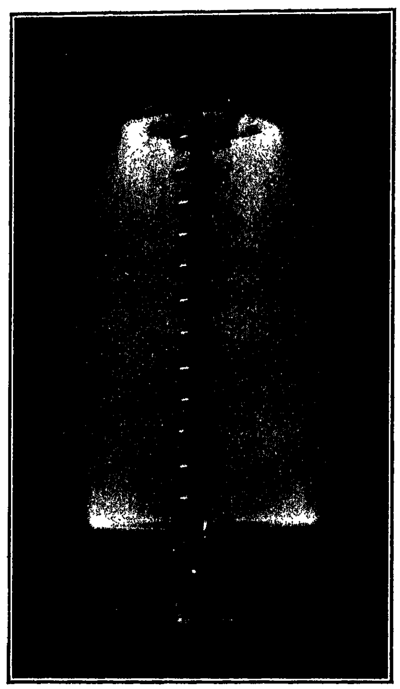

FIG. 21—DISSIPATION OF LIGHTNING ENERGY BY CORONA ON INSULATOR SHIELDS

Whether a pole is wet or dry makes very little

difference on the lightning voltage necessary to cause complete flashover. However, when a pole is quite wet, incipient sparks will take place over the insulator string at voltages approximately equivalent to the lightning sparkover voltage of the insulator string above.

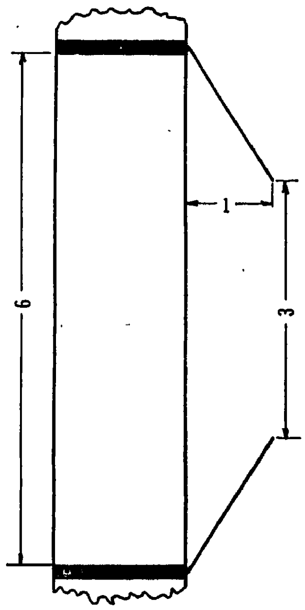


FIG. 22—PROPORTIONS FOR WOOD POLE PROTECTIVE GAP

While wood poles without rods or ground wires may have very high lightning sparkover voltages, there is always a danger of long delays due to split and burned poles that could not be tolerated on important lines. The porcelain insulator is more reliable. However, by use of gaps as illustrated above, part of the insulating value of the wood may be used to advantage on certain secondary lines. Fig. 23 shows a lightning flashover test being made on a 20-ft. pole.

(*f*) *Effect of bus structures on lightning voltages—tower structures*

Tests made on models in the laboratory show that the bus structures of outdoor stations should be of material assistance in reducing transient voltages. There are several effects that help. The grounded steel work acts as a very effective ground wire system which may reduce induced voltages very considerably. Tests on line models built to scale often show as low as one-third voltage when bus structures are added. Full effect of this is not obtained in practise due to the limited physical length of the structures compared to the cloud. A wave traveling to the bus structure would be reduced in voltage due to the reduction in surge impedance. The massed capacity effect of the bus would prevent high voltage reflection. The effects in practise should be quite effective for waves chopped short by insulator arc-overs. Several extra ground wires of a half mile or more in length extending out from a station, should, because of reduction in surge impedance, be very effective in reducing the voltage of incoming waves. On the other hand, tests show that the omission or reduction in the ground wires at the station causes a rise in voltage.

Tests on models have been very useful in determining the best arrangement of ground wires, the effect of high towers at river crossings, etc. Tests are also under way to determine the practicability of protecting towers from direct strokes by rods.

(*g*) *Transformers and Transformer Insulation.*

The new lightning generator has made possible invaluable studies on full size transformers and insulation arrangements. It has long been recognized that the internal insulation of a transformer should be stronger than the bushing while the bushing in turn should be stronger than the adjacent line insulation. Research on transformers has been made by applying lightning waves over a line insulated in the usual way. The general method is to apply gradually increasing impulses until the insulators spark over. Insulator units are then added until either the bushing sparks over or the internal insulation fails. If failures occur internally, the weak points are then strengthened until the flashovers occur on the bushings. The insulator is ideal as a voltage limiting gap for such tests because it performs the same function in practise, limiting the surges in duration as well as magnitude.

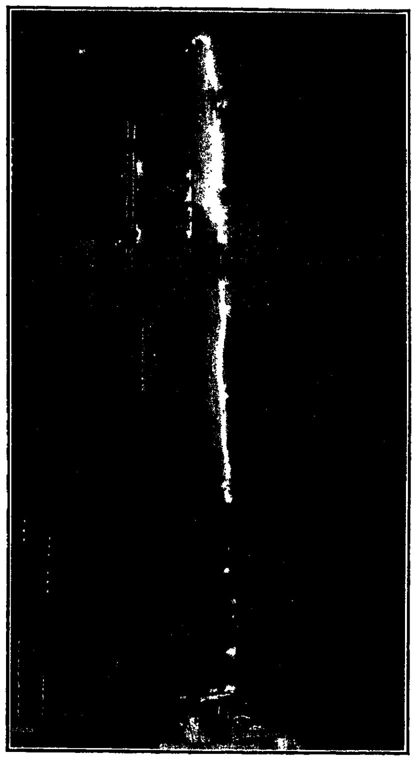

FIG. 23—LIGHTNING SPARKOVER OF 20-FT. WOOD POLE

Cathode ray oscillograph records are taken and the voltage distribution is measured throughout the winding. This last measurement is of extreme importance since it shows that in the usual transformer the voltage

distribution is not constant but varies with steepness and duration of the impulse or the frequency of the transient. High frequencies and steep impulses may cause excessive voltages at any part of the winding. The ideal transformer would be one in which the voltage distribution was the same for all frequencies and wave shapes. Fortunately it has been possible to accomplish these results by the shielded design, which is an entirely new type. It will not be necessary to go into details here as this transformer is described elsewhere.[8]

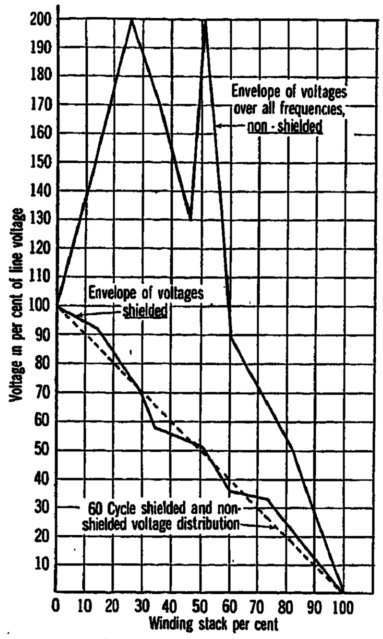


FIG. 24—VOLTAGE DISTRIBUTION AT ALL FREQUENCIES OF SHIELDED AND NON-SHIELDED TRANSFORMERS

Fig. 24 shows the results of tests on an actual transformer. In the shielded transformer the impulse and high-frequency distribution is shown to be practically the same as the 60-cycle distribution. The transient distribution curve, which is the envelope of voltages for all high-frequency transients and impulses, is quite startling. It shows that the shield reduces local transient voltages as much as 80 to 1 and that lightning failure in a non-shielded winding may occur anywhere in the winding depending upon the wave. In the shielded winding free from localized stresses under all waves, breakdown is as definite as an insulator flashover. The above lightning tests are, of course, design tests and not intended for commercial testing. This follows because dismantling every transformer in the routine factory tests to detect possible internal failures is impractical. However, the shield removes the necessity for lightning tests since the impulse distribution becomes the same as that at 60 cycles.

Briefly the reason for the varying distribution of voltage in a non-shielded transformer is as follows: The initial lightning distribution is determined by the distribution of the capacity in the windings and the 60-cycle or long duration voltage distribution by inductance. If the voltage distribution as determined by these factors is not the same an oscillation results until the distribution corresponds to that of the inductance. The shields make the capacity and inductance distribution correspond. The action of the capacity is instantaneous and there is no oscillation.

*Insulation*

The new lightning generator has made possible extensive research on solid and liquid insulation. Fig. 25 gives a curve of typical results obtained.

## III. Field Research

Considerable research work is being done with natural lightning. This work may be divided into two classes:

(a) A study of lightning as it appears on transmission lines either by direct hits or induction.

(b) A direct study of lightning strokes, the clouds producing them, and the effects of the strokes on rods, etc.

*Research on Transmission Lines*

During the past few years a number of the operating companies in collaboration with the manufacturing companies have obtained some very important measurements, particularly of lightning voltages on transmission lines.[9,10] These measurements were obtained with the klydonograph or surge voltage recorder connected at various points along the transmission lines.

*(a) Voltage, polarity, wave shape, limitation of voltage.* The surge-voltage recorder measures the maximum of the wave and indicates the polarity. By comparing the insulator sparkover voltage resulting

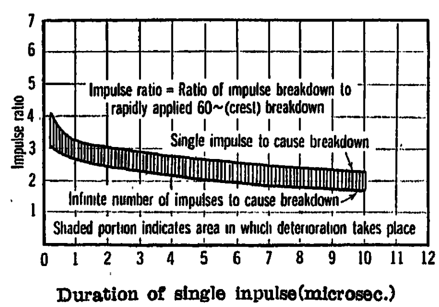


FIG. 25—LIGHTNING VOLTAGE. TIME STRENGTH OF SOLID INSULATION

Typical curve—actual values will vary for different kinds of insulations

from lightning with the 60-cycle sparkover voltage the impulse ratio is obtained. The impulse ratio gives a good indication of the effective duration of the wave. The lightning voltages causing sparkover on transmission lines in various sections of the country give an impulse ratio of the order of two. Waves giving such ratios are shown in Figs. 8, 11, 12. The maximum impulse ratio observed was approximately 2.8.

In the many measurements made, the very low voltages, necessarily induced, were mostly positive, indicating a negative cloud. Most of the excessively high voltages,—probably direct strokes,—were negative, also indicating negative clouds. A few high positive and

low negative voltage surges indicated that some clouds are probably positive. The general indications were that the waves were non-oscillatory.

Measurements on different lines in various parts of the country and on the 220-kv. lines of the Pennsylvania Power & Light Company with 14 and 4 unit insulation show that the maximum lightning voltage on transmission lines is limited to the lightning sparkover of the insulators. Measured points are shown on the laboratory curve, Fig. 11.

(b) *Attenuation.* Successive surge voltage readings

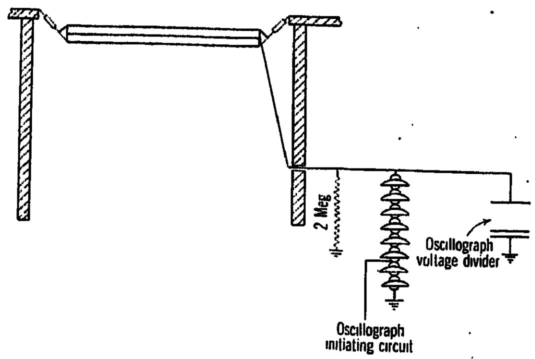


Fig. 26—Method Employed in Measuring Natural Lightning Voltages

of a given lightning wave traveling on transmission lines have shown how the waves attenuate or decrease in voltage. The attenuation or loss in voltage per mile was found to vary as the square of the crest voltage. Thus really high voltages cannot travel far.

*Measurement of Lightning Waves and Time Required for a Cloud to Discharge with the Cathode Ray Oscillograph*

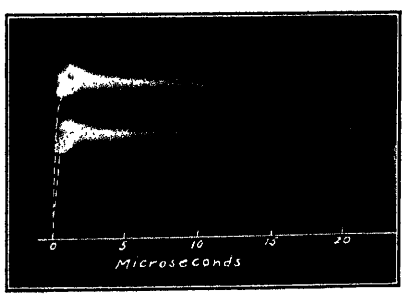


Fig. 27—Oscillogram of Natural Lightning Made on Short Lines—Pittsfield 1928. Drawn Dotted Lines Indicate Approximate Fronts

A portable cathode ray oscillograph, of the Dufour type developed by Mr. Lee and his staff of the General Engineering Laboratory[6] of the General Electric Co., made possible during the past summer the measurement of wave shapes of actual lightning. In order to make proper use of the oscillograph, it was necessary to devise a means of establishing the cathode beam and the sweeping circuit and to have the complete set-up connected to the line as the lightning wave reached it. With the equivalent "switching circuit" developed for this, the complete operation was accomplished in about one microsecond—that is, one-millionth of a second. Part of the wave front would accordingly be lost unless special means were taken to prevent it. One way to accomplish this is to side track the wave around a loop about 1000 ft. long, requiring about one microsecond to travel it, and take it coming back. However, the results so far indicate that this refinement will usually not be necessary. The actuation of the oscillograph is done by means of spark "switches" controlled by the lightning surge. Connection to the line was made by an insulator potentiometer.

Measurements were made on short horizontal antennas and on actual transmission lines. The measurements of the antennas were made at Pittsfield while the transmission line measurements were made on the 220-kv. lines of the Pennsylvania Power & Light Company in cooperation with their engineers.

(a) *Pittsfield measurements.* In the Pittsfield measurements the antennas consisted of three parallel wires 120 ft. long and 40 ft. above ground. The wires were

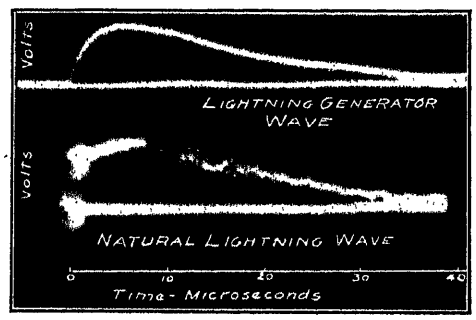


Fig. 28—Comparison of Natural Lightning Wave Measured on Transmission Lines with Cathode Ray Oscillograph with an Artificial Lightning Wave Measured in the Same Way

grounded through a 2,000,000-ohm resistance and connection to the oscillograph was made as in Fig. 26. With this arrangement the lines assume a potential opposite to that of the cloud when the lightning discharge takes place. Since the charge cannot move along this short line, but must be dissipated by leakage, the potential of the conductors rises at a rate and to a magnitude dependent upon the collapse of the cloud field. The time for this conductor voltage wave to reach maximum is thus a measure of the time required for the cloud to discharge. Fig. 27 shows two of the four antennas waves obtained. The wave fronts are of the order of one to two microseconds. The induced voltage crests on the antennas were from 50 to 75 kv. The storm clouds in each case were at least a mile away. Such wave measurements, indicative of actual cloud discharges, are very helpful in determining the shape of surges themselves on transmission lines. A mathematical analysis to correlate these field results with theory is now being undertaken.

(b) *Pennsylvania measurements.* A very good wave

of natural lightning obtained on the 220-kv. line is shown in Fig. 28. The front of this wave practically reaches its maximum in 5 microseconds, decreases to half value in 20 microseconds, and reaches zero in 40 microseconds. The oscillating ripple is apparently due to a local flashover[12] and is not really part of the original wave. A reproduction of this wave by the laboratory lightning generator is also shown in Fig. 28. The effects of the wave are very similar to the standard wave of Fig. 7 and the impulse ratio for insulator sparkover corresponds to those determined by the surge recorder or klydonograph readings. This oscillographic study of lightning surges on actual lines is to be continued on a greater scale next year.

*Study of Direct Strokes*

Fig. 29 is an illustration of the method used in studying the effects of direct strokes. Photographs showed certain office buildings struck. An examination was made to see if any of these hit fell within the protective cone of other buildings as established by former laboratory tests.[1] It was found that they were in agreement with tests on models.

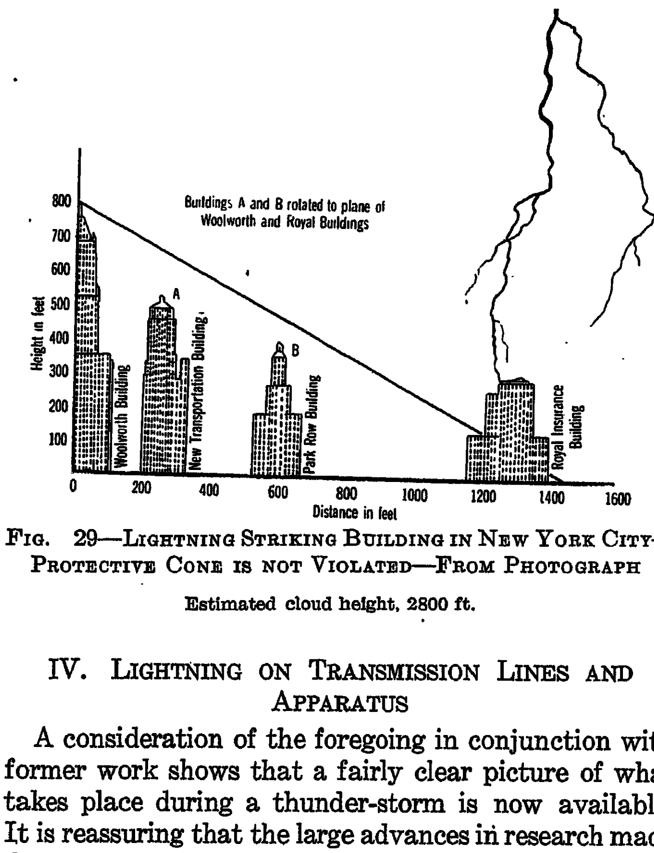


FIG. 29—LIGHTNING STRIKING BUILDING IN NEW YORK CITY—PROTECTIVE CONE IS NOT VIOLATED—FROM PHOTOGRAPH

Estimated cloud height, 2800 ft.

## IV. LIGHTNING ON TRANSMISSION LINES AND APPARATUS

A consideration of the foregoing in conjunction with former work shows that a fairly clear picture of what takes place during a thunder-storm is now available. It is reassuring that the large advances in research made during the past year tend to confirm and clarify what had already been done. Because of this advance in research it seems desirable at this time to re-examine present transmission practise and see if changes are desirable.

*Voltage*

The available data still confirm the rule that the maximum induced voltage on a transmission line depends upon the height of the conductors above ground or

$$V = h\,g\,\alpha$$

Where $V$ = volts, $h$ = average height of conductor in feet, and $\alpha$ depends upon rate of cloud discharge and the initial distribution of bound charge on the line. Calculations indicate that under the usual assumed bound charge conditions, an induced voltage wave high enough to cause insulator flashover can result only from a rapid collapse of the electrostatic field of the cloud. Accordingly this must involve a traveling wave of steep front or of short effective duration. With a slow rate of discharge the bound charge has an opportunity to spread out over the line as it is released, thereby resulting in a lower voltage to ground. A steep wave front would also occur with a surge imposed on a conductor by a direct stroke to it. This would also apply in case of a direct stroke to the tower or ground wire where the conductor did not become involved. From the lightning standpoint low towers with horizontal conductor spacings are desirable. The maximum voltage wave that can travel on the line and reach the apparatus is determined by the line insulation. The waves in practise are generally non-oscillatory and have a wide variety of shapes. However, the waves usually causing insulator sparkover give an impulse ratio of the order of two (2.0), and indicate an effective duration of 1 to 20 microseconds above the 60-cycle flashover value.

*Loss in Voltage as the Lightning Wave Travels on the Line—Attenuation*

The distance that lightning can travel at high voltage on a transmission line is of extreme practical importance. The following table of data from the curve in Mr. Lewis' paper[10] bears this out.

| Lightning voltage kv. | Distance of travel for reduction to half voltage (mi.) | Distance of travel for reduction to 0.80 voltage (mi.) |
|---|---|---|
| 4000 | 1.5 | 0.4 |
| 3000 | 2.1 | 0.5 |
| 2000 | 3.0 | 0.7 |
| 1000 | 6.3 | 1.6 |

For example, 4000 kv. is reduced to half voltage or 2000 kv. in 1.5 miles. This indicates that the badly exposed section of a line normally insulated for 2000 kv. could be highly over-insulated without much danger of subjecting the normally insulated section to excessive voltages. It is only necessary to extend the highly insulated line several spans beyond the exposed section. It also indicates that moderate reduction of insulation at a substation, for instance, should not appreciably increase the outages. For example, 2000 kv. is reduced 20 per cent in traveling less than a mile and 50 per cent in three miles. Any outages within the reduced insulation section must be attributed to local storms and would not be due to line surges coming into the station from outside normally or abnormally insu-

lated sections. Further data on the attenuation factor on other lines are of course desirable.

*The Ground Wire*

Statistical data still confirm the value of the ground wire. These data indicate that outages due to insulator arc-overs are reduced from one-half to one-tenth or more after the installation of the ground wire. On one 220-kv. line the installation of the ground wire over two sections did not seem to have any decided effect on the lightning arc-over voltages. The reason for this seems to be that, because of the peculiar location of the line, the arc-overs are caused mostly by direct hits. Where the spans are long, where the ground wires are near the line conductors, or where the ground resistances are high, a direct stroke is likely to involve the line conductor. This suggests taking special precaution in design of towers, ground wires, or protecting rods to prevent direct strokes side flashing to conductors where the lines are in badly exposed places and subject to direct hits.

The laboratory tests point out that a very important function of the ground wire in reducing voltages is generally overlooked. The lightning corona loss is materially increased by ground wires which greatly limits voltage. The action is the same as on the grading shield in Fig. 21.

The surge impedance of the lines is reduced by the addition of ground wires. A reduction in the surge impedance reduces the voltage of traveling waves. It is thus important to bring all ground wires up to the steel bus structure or to the station. Extra ground wires extending about one-half mile from the station should be of great value for station protection. Voltages caused by local storms would be reduced and waves from distant storms would be lowered to harmless values by attenuation and reduction in surge impedance.

*Lightning Proof Transmission Lines and Coordination of Transformer Insulation and Line Insulation*

It is interesting to consider again if a lightning proof transmission line is possible or practicable. It has been shown that the lightning voltage is independent of the operating voltage and depends upon the height of the line; that the ground wire greatly reduces the lightning voltages; and that the lightning flashover voltage of insulation and the breakdown voltage of apparatus is known. A consideration of these factors shows that a line of moderate height, protected with ground wires and properly insulated, could usually be made lightning proof against induced voltages at a reasonable cost. This comparison is made by calculating the maximum induced voltage from the formulas given above, making proper allowance for the ground wire, and using insulator strength in excess of these values as determined by Figs. 11 and 12. In order to make a line immune from direct strokes the necessity of ground wires above the line is almost obvious because, no matter what the insulation, the limit will be the sparking distance from line to ground. With ground wires the stroke would usually take place to the wires and then along the wires to the tower, preventing insulation arc-over. However, where the line is badly exposed to direct strokes special precaution should be taken in the design of the tower so that side flashes are not likely to take place to the conductor. Rods might be used and special precautions taken as to length of span, ground resistance, distance from conductor to ground wire, etc. Extra ground wires in sections may be necessary to assure immunity against direct strokes.

The limit of the voltage in any line is the lightning flashover voltage of the insulator. It is very important, therefore, when designing a system, so to proportion the insulation that the transformer lightning breakdown voltage is higher than the lightning breakdown voltage of the line insulators in the vicinity of the station.[11] It is obviously not good engineering to make the transformer the weakest link in the insulation chain. The insulators on the rest of the line may be as strong as desired.

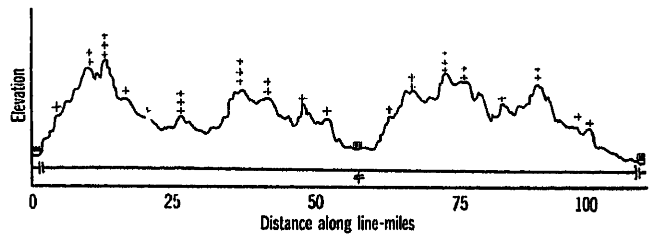


Fig. 30—Distribution of Flashovers along a Transmission Line

+ = Insulator flashovers

The grading shield is as important to the insulator string as the ground wire is to the line. It not only reduces the maximum stress but increases the arc-over voltage and causes the arc to clear the string. The horn and similar arcing devices cannot cause the arc to clear without a serious reduction of the flashover voltage.

In addition to the above factors, the location of the line is very important. See Fig. 30, which suggests extra insulators and ground wires on the exposed sections. The ideal line would thus be as low in height as practicable, be protected by one or more ground wires, and be well insulated with insulators protected by grading shields. The transformer insulation should be somewhat stronger than the bushing flashover voltage, which in turn should be higher than the flashover voltage of the insulators in the immediate vicinity of the line. By immediate vicinity is meant that the coordinated insulation should start within 75 ft. of the apparatus and preferably extend out at least one-half mile. As pointed out above, such arrangement of the insulation should not increase outages. As a precaution, extra ground wires may be added on the coordinated section to provide against local dis-

turbances. With extra ground wires the lightning voltages can be reduced in proportion to the insulator strength. This coordination of insulation has been in effect now for several years on one system with results as anticipated. It is not intended to take the place of the lightning arrester, however, for good lightning arresters are to be recommended as in the past.

V. SUMMARY OF CHARACTERISTICS OF LIGHTNING

Voltage — order of 100,000,000
Current — order of 100,000 amperes
Energy — order of 4 kw-hr.
Power — order of thousand billion hp.
Time — order of a few microseconds
Gradient at breakdown 100 kv/ft.

Time for cloud to discharge from one to ten microseconds.

Discharge generally non-oscillatory. Total energy dissipated in world by lightning 1,200,000 kw. continuously. (Very approximate.)

*Lightning on Lines*

High-voltage waves of few microseconds front and effective duration of 1 to 20 microseconds.

Low-voltage waves of much greater duration. Voltage either by induction or direct strokes.

Current in line for high voltage 2000 to 5000 amperes. Voltages non-oscillatory. Attenuation very rapid for higher voltages.

Breakdown voltages higher than 60-cycle and impulse ratio for average high voltages, two.

Voltage increased directly as height of line—usual ground wires reduce to about one-half. Maximum voltage $V = g \alpha h$ but limited by lightning arc-over of insulator.

Transformer insulation should be stronger than adjacent line insulation.

Approximate lightning proof line seems feasible.

Laboratory—Voltages higher than ever observed on lines have been obtained and design tests made on full size apparatus.

ACKNOWLEDGMENTS

The laboratory cathode ray oscillographic work described here was carried out by Messrs. H. L. Rorden and J. C. Dowell of the laboratory staff. The transmission line field work with the oscillograph was done by Messrs. C. M. Foust, E. A. Evans, and N. Rohats. To Mr. G. D. Heye is due credit for the assembling and plotting of the impulse flashover results obtained with the lightning generator.

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## Discussion

For discussion of this paper see page 468.

# Theoretical and Field Investigations of Lightning

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Fellow, A. I. E. E. Member, A. I. E. E. Associate, A. I. E. E.

*Synopsis.—This paper gives a review of some recent developments in the methods of studying lightning phenomena. The Norinder form of cathode ray oscillograph and its application in Tennessee are discussed, together with the information secured.*

*The second part gives the theory of traveling waves along transmission lines. Reflections at open and grounded ends are considered. A mirror scheme of an infinite series of waves on a double infinite line equivalent to actual waves along a finite line is developed.*

*The third part discusses the manner in which surges are actually produced on lines by lightning and the effect of ground resistance on the protection afforded by ground wires, both with respect to induced and direct strokes.*

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*The Importance of Lightning Research.* During the last few years the increasing importance of the solution of the lightning problem in electrical systems, particularly in long transmission circuits, has resulted in a tremendous increase in the amount of attention given the problem all over the world. The problem has changed from the earliest status of mere scientific interest and the status of perhaps ten years ago, that of convenience, to the present status of very definite necessity. Electrical systems are being reconstructed with the idea of generation in large plants at the most economic point and interconnection between such plants to take advantage of diversified load factors. This requires long distance transmission of large amounts of power between these plants and to the load centers, which, added to the natural growth, involves expenditures in the central station industry of nearly a billion dollars a year. Of this, possibly 150 million dollars a year is expended for transmission circuits. Under the present conditions which demand substantially complete reliability, lightning has been found to be the operating condition which limits the design of transmission circuits. The electrical industry must accept the fact that these enormous expenditures give full warrant for whatever research expenditure and effort may be necessary to provide the solution to the lightning problem.

## I. RECENT DEVELOPMENTS IN FIELD INVESTIGATIONS OF LIGHTNING

*Status of the Work.* The situation has been recognized by many workers in the industry and widespread efforts toward the solution of the problem have been made. In Germany there is a society set up for the purpose of studying the nature of these voltages in transmission circuits and this society has been active for the past few years in a broad program of apparatus development and field study. This work is supplemented by some individual effort on a smaller scale in other European countries. In this country the use of the klydonograph has been widespread and the interest has extended to practically the entire industry. Notwithstanding the great advances in the understanding and knowledge brought about by these studies it has been shown that even the very complete use of these instruments has not given all that is necessary and the indications are clear that continued work along the same lines will not solve the problem. The need has been recognized for new instruments and these have been developed and given preliminary trial in actual use. These new instruments and the work which has been done with them will be described.

*Application of Cathode Ray Oscillograph.* Since the first introduction of cathode ray oscillographs into laboratory study of transient phenomena it has been realized that this instrument would be ideal for determination of the character of lightning voltages in exposed circuits. There have, however, been barriers to this use of the cathode ray oscillograph which seemed at the start to be insurmountable. That the instrument itself is large and is not well suited to use outside of good laboratory conditions, and that it requires rather elaborate accessory equipment, including vacuum pumps of high quality, are disadvantages but these are not insurmountable. The major difficulty has been in providing means for having the oscillograph in operation on the arrival of the transient. If the electron jet is permitted to strike the photographic film or plate in one spot for more than one ten-thousandth of a second or so, a large part of the film becomes so badly fogged as to prevent any decipherable records. It is essential to the solution of the problem to secure records of the entire duration of the abnormal voltages and particularly of the beginning of the wave fronts. Thus, if means are devised to anticipate a transient which may occur at any time, it is necessary that the oscillograph be started by the transient itself, and with zero delay. This is a problem of no small magnitude.

The earliest approach was made some years ago in Germany[2] by use of a Kipp relay. With this arrangement the cathode tube is continuously energized but the jet is deflected to one side until the transient arrives. The transient voltage is led directly to the relay which by an ingenious arrangement brings the

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1. Of the Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

2. D. Gabor, (*Archiv für Elektrotech.*, Band 18, Heft 1)—(Fortschritte im Oscillographieren von Wanderwellen.)

jet quickly to the edge of the film and then more slowly sweeps it across the film providing the time scale. The transient voltage is led to the voltage deflecting plates of the oscillograph through a delaying cable to compensate for the time required to bring the jet to the film.

A second approach was made by G. F. Harrington in July 1926. In this scheme the source of high unidirectional potential for the electron jet is blocked by means of a three-element vacuum tube, the initial

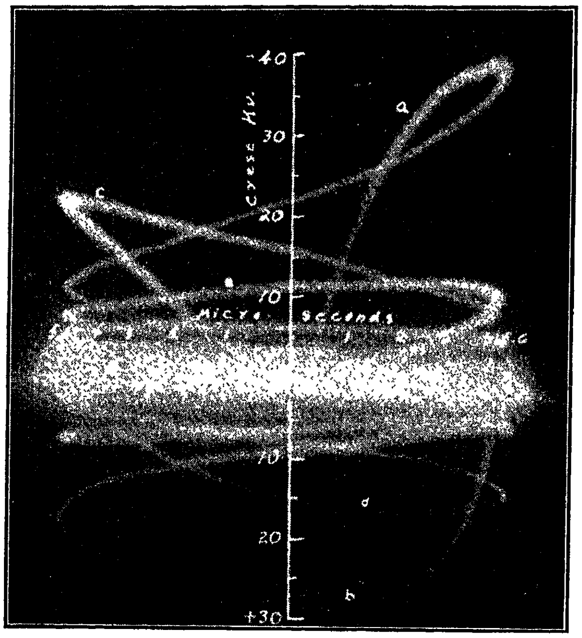


Fig. 1—Oscillogram Taken at Open End of Rankin-Wilmerding Line

grid potential of which is neutralized by the transient itself. Thus, the oscillograph is ready to operate but the electron jet is prevented from forming by the blocking effect of the tube. When the transient arrives and neutralizes the initial grid potential the electron jet is formed. The time required to form the jet may be compensated for by use of a delaying cable as with the Kipp relay.

These schemes appear promising but the results which have been made public thus far, as well as our experience, indicate that before records can be obtained of the whole wave front some further developments will be necessary. Our experience indicates that there is an irregularity in the time intervals to bring the jet to the film after the arrival of the transient, which is comparable to the time of building up of the wave fronts to be measured.

*Early Field Trials.* The early work in the laboratory with the Harrington scheme had indicated the need for further trials on a circuit of considerable length in order to evaluate this deficiency and attempt to find ways around it. In order to provide for such work and to carry on other investigations beyond the limitations of concentrated laboratory equipment, permission was obtained in 1926 from the Duquesne Light Company for two years use of a six-wire circuit extending from Rankin to Wilmerding, a distance of about five miles, close to East Pittsburgh, Pennsylvania. A surge generator of conventional type was set up at the Rankin end of the line and measuring equipment, including klydonographs and a Dufour type oscillograph, was provided at the Wilmerding end. Transients were introduced into the line by direct discharge of the surge generator at the Rankin end and also by the use of an artificial cloud arrangement by which a half mile of one or more conductors over the Rankin end of the circuit were charged and discharged inducing and releasing bound charges in the continuous conductors. The transients thus introduced were observed with the oscillograph at the Wilmerding end and by means of the klydonograph at an intermediate point.

Representative records from this test are shown in Figs. 1, 2, and 3, which illustrates that the major benefit of this kind of work is additional quantitative determination of the line constants affecting surge propagation. Observations were made illustrating the control of surge current by surge impedance of the line, reflec-

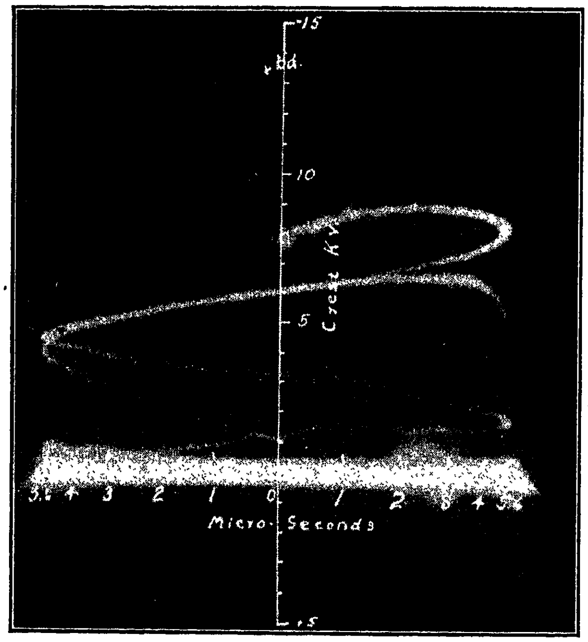


Fig. 2—Oscillogram Taken at End of Line with Lightning Arrester, Rankin Wilmerding Line

tions from open and grounded terminals, attenuation with travel on the line, partial reflection by choke coils, and reduction of voltage by various kinds of protective equipment.

In securing this information use was made of a control system between the two stations by which the surge was timed to precede the beginning of operation of the oscillograph by a definite amount. This scheme is illustrated in Fig. 4. Attempts to make the oscillograph wholly automatic by means of devices such as described above, gave no further benefit than to demon-

strate the need for some improvement to make them dependable in securing the first part of the transient. Various methods were tried to overcome these difficulties but none proved successful.

*The Norinder Cathode Ray Oscillograph.* During these tests it came to our attention that successful work had been done by Dr. Harald Norinder of the Swedish Royal Board of Waterfalls, Upsala, Sweden, in making cathode ray oscillograms of actual lightning voltages. This indicated the availability of a means for the solution of the problem and accordingly arrangements were made for Dr. Norinder to visit this country to discuss his work and the work being done here.[3] It proved that Dr. Norinder's results had been secured by means of an ingenious improvement over the previously known forms of cathode ray oscillographs by which operation is made entirely automatic on the arrival of the transient. Fig. 5 illustrates the scheme invented by Dr. Norinder.

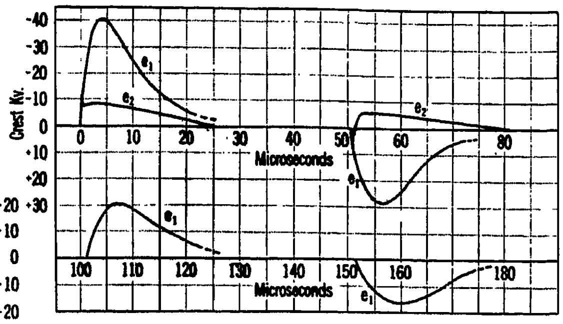


FIG. 3—WAVES RECORDED AT END OF RANKIN-WILMERDING LINE, REPRODUCED FROM OSCILLOGRAMS FIGS. 1 AND 2

$e_1$ Waves are successive arrivals of surge with line open

$e_2$ Waves are successive arrivals of same surge but with a lightning arrester at the end of the line

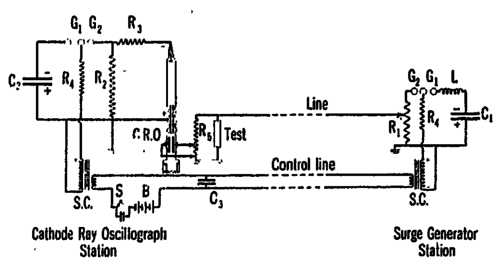


FIG. 4—CONTROL SCHEME USED IN RANKIN-WILMERDING LINE TEST

When it is desired to prepare to make a record of an anticipated disturbance with this instrument, the necessary potential is applied between the anode and cathode resulting in the formation of the electron jet, the useful portion of which passes through the opening in the anode and, since no deflecting potential is applied to the upper deflecting plates, the electrons strike the target and are stopped. Stray electrons are prevented from reaching and fogging the film by the lower diaphram, the orifice of which is wholly shielded by the target. When the transient arrives a proportionate deflecting potential from a potentiometer is applied to the upper deflecting plates and the beam is caused to move off the target. Below the target the beam is again bent back by the reversed potential. By proper proportioning of the length and spacing of the two pairs of deflecting plates the beam is caused to pass through the orifice in the diaphragm for all values of applied potential. The relay function is thus performed by the oscillograph itself and a complete record is made of any transient whenever it may occur. With this scheme it is possible to have the oscillograph in operating condition for a period of two hours or so without any appreciable fogging of the film so that the operation, technique, and field studies reduce to the mere attention necessary to maintain the proper vacuum, etc. However, during the development and laboratory trials of the instrument it was found necessary to take precaution by minor design details against various difficulties, mostly due to the effect of normal line voltage on stray electrons, in order to make the full possibilities of the instrument available in studies on operating circuits. Fig. 6 shows the first oscillograph of this type made for use in this country.

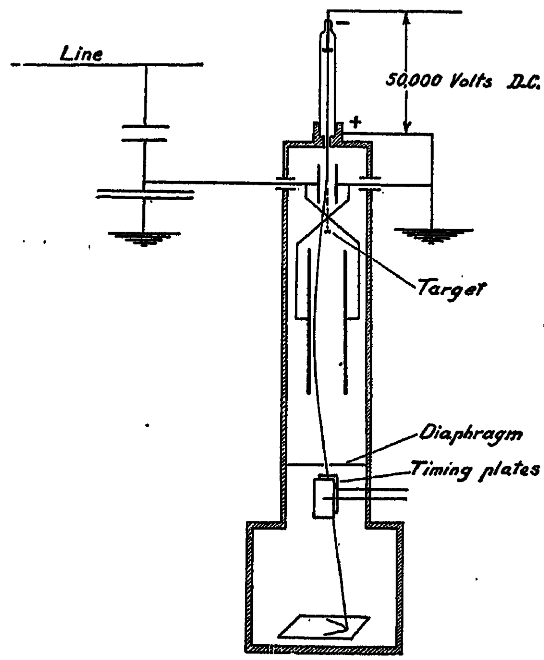


FIG. 5—DIAGRAMATIC SCHEME OF NORINDER OSCILLOGRAPH

Instruments and potentiometers for the purpose of recording transient voltages on operating lines are designed and used so that the normal line voltage does not deflect the beam beyond the limits of the target.

3. Dr. Harald Norinder, "Some Electrophysical Conditions Determining Lightning Surges," *Journal Franklin Institute*, June 1928.

Dr. Harald Norinder, *The Cathode-Ray Oscillograph as Used in the Study of Lightning and Other Surges on Transmission Lines*, A. I. E. E. Quarterly TRANS., Vol. 47, No. 2, p. 446.

The record of transient voltage thus begins at slightly over the crest value of normal line voltage and extends to the limit of the instrument. It is possible to make records of voltages less than line voltage by filtering schemes using capacitors and resistors. In general, however, the interest is limited to those voltages which exceed the line voltage by a considerable margin. For this reason such schemes have not been used here.

In order to provide the time scale the film may be mounted on a drum and rotated at a high speed. The mechanical limitations prevent speeds in excess of approximately 200 ft. per second. With such an arrangement 100 microseconds would appear as approximately one-quarter inch so that the records show only the magnitude and general shape of the transient. More detailed records can be secured by sweeping the beam electrically across the film or by oscillating the beam at right angles to the direction of voltage deflection. With these schemes any desired time scale can be secured. The deflecting plates used for either the sweeping or oscillating scheme are placed below the target so that they are ready to operate at all times and become effective as soon as the beam is deflected around the target by the transient.

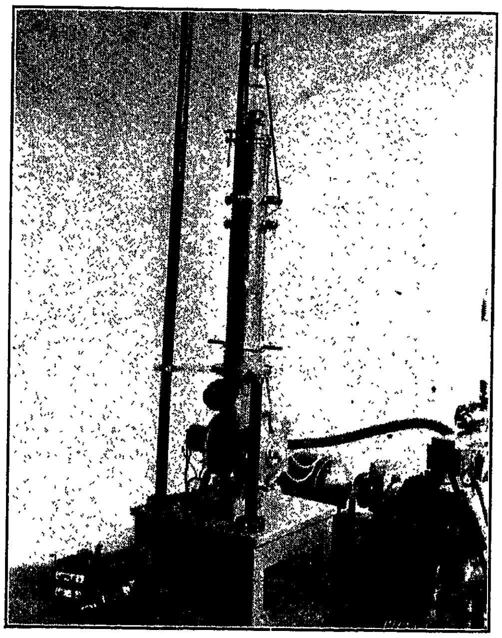

Fig. 6—Norinder Type Cathode Ray Oscillograph

*Use of Norinder Oscillographs in Field Studies.* Laboratory trials substantiated the experience of Dr. Norinder in establishing the usefulness of the instrument. It was recognized, however, that the complete solution of the lightning problem will require a rather extended use of such devices and that the expense and effort necessary to install and operate the required number of instruments will be great. It was felt to be necessary, therefore, to get final assurance by a trial installation in the field that the instrument will give all the information which is needed. It was further felt to be worth while to secure such information as could be obtained, even with the small number of installations which could be made in the short time available, during the 1928 lightning season. Arrangements were accordingly made for an installation of two instruments in Tennessee and a second installation of a single instrument in Illinois.

The Tennessee location was chosen as it met the requirements of a test for this purpose—a highly insulated line in a territory where lightning storms are very frequent. A detailed survey of the operating record and of the location showed that the most suitable point on the line would be where it passes over the western end of the Chilhowee ridge of the Great Smoky Mountains. Through the cooperation of the Knoxville Power Company and the Aluminum Company of America, arrangements were made for locating two stations, one on each side of the Chilhowee ridge.

Each of the stations was designed to give as complete information as is possible. Each oscillograph, which gives the voltage record on a single conductor, was supported by a series of three klydonographs each for three-phase records, one located alongside the oscillograph and one approximately a mile away in each direction along the line. It was felt to be highly desirable, if not entirely necessary, to get a fairly accurate record of the location of the cloud discharge relative to the line and test station. Accordingly, provision was made to measure and record the difference in time of arrival of the electrical and audible disturbances by the use of a microphone, antenna, and Osiso. The microphones were located about a mile on each side of the test station and the electrical impulses due to the noise of the thunder were brought into the station over circuits provided for the purpose. The Osiso was arranged to start automatically upon occurrence of any disturbance and to record the impulses from the microphone and those originating in the antenna at the station. The difference in time between the record for the antenna and for the microphone provides a measure of the distance from each microphone location to the cloud discharge. By plotting circles with radii corresponding to this distance and with centers at the microphone locations the position of the cloud discharge is determined to be at one of two points where the circles intersect. These points are on opposite sides of the line but are equidistant from the line and from the point where the oscillograph is connected so that there is no advantage in distinguishing between them. The general arrangement of the test stations is indicated by the diagram of Fig. 7 and by the photographs of Figs. 8, 9, 10, and 11.

In order to supplement the electrical records with direct photographs of the lightning flashes without requiring too much attention of the operators, special cameras were provided which photograph the entire sky on a single plate. These cameras which make use of the principle of the "fish-eye camera," but which

secure the results by the use of lenses, were developed by Mr. R. Hill in England.[4] The lens scheme is shown in Fig. 12.

The oscillographs and accessories were designed and built during the period between January and about the

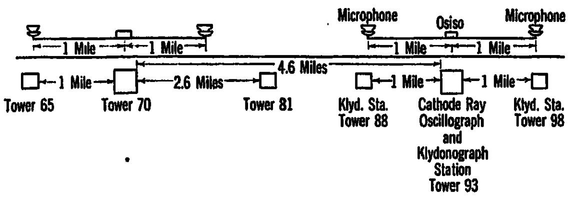


Fig. 7—Layout of Test at Chilhowee, Tenn.

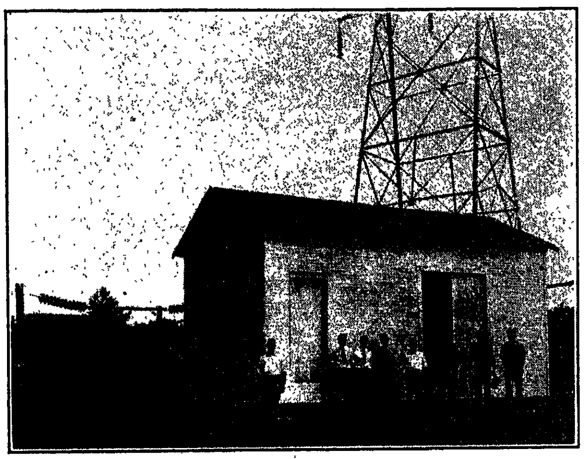

Fig. 8—Test House at Tower 70, Cheoah-Alcoa Line

Fig. 9—Oscillographic Apparatus at Tower 93, Cheoah-Alcoa Line

first of June, 1928. Installations in the field were completed by the early part of July and from this time the stations were ready to record any lightning which occurred.

The original plan was to provide a third station in Illinois through the cooperation of the Public Service Company of Northern Illinois, but the preliminary work of building the instruments and making the field installation was not completed in time to warrant beginning the test until the lightning season of 1929.

It was planned to make the first few records using the rotating film in order to get a general picture of the nature of the surges and then to take later records using the oscillator for the time scale deflections.

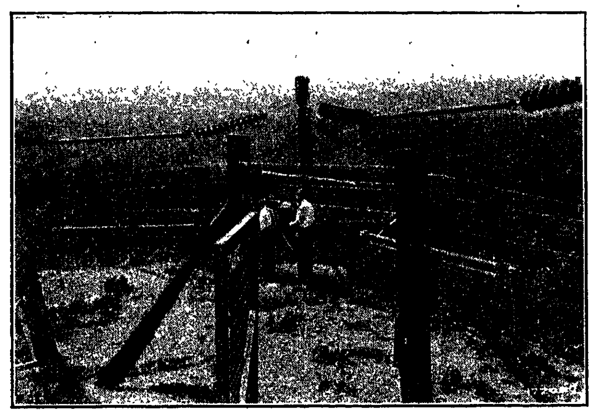

Fig. 10—Three-Phase Klydonographs Potentiometers

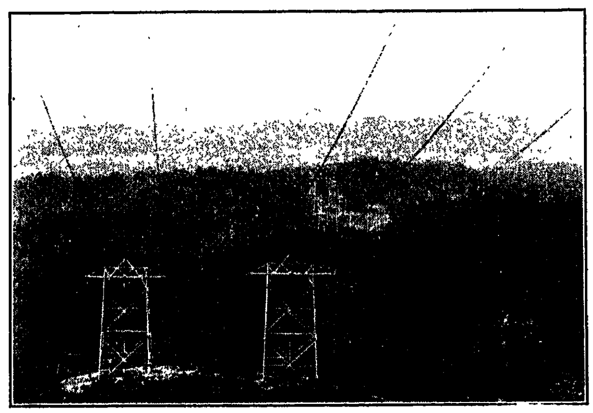

Fig. 11—Looking North from Chilhowee Mountain, Showing Tower 81

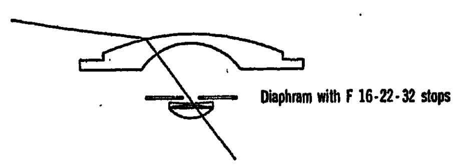


Fig. 12—Lens Arrangement of Hill Camera

Unfortunately, however, only a single disturbance of appreciable magnitude appeared on the line during the period between the time when the apparatus was set up until November 4 when the test was discontinued for the year. This record is shown in Fig. 13. The corresponding klydonograms at the same station are shown in Figs. 14, 15, and 16. No voltage was recorded on either the klydonographs or the oscillographs at the other station, four and one-half miles away.

The microphone and Osiso arrangements for determining the location of the lightning disturbance were

4. R. Hill, "A Lens for Whole Sky Photography," *Proc. of the Optical Convention,* 1926, Part II.

not in service at the time of this record. The storm conditions were, however, observed by the operators and the following general ideas developed. There were many discharges between clouds apparently directly over or nearly over the line without causing any line disturbances of sufficient magnitude to be recorded by either klydonograph or oscillograph. Strokes to earth were observed in the near vicinity of

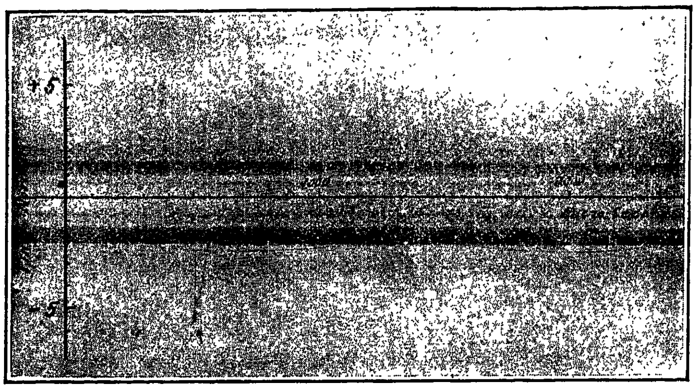

FIG. 13—OSCILLOGRAM OF LIGHTNING SURGE ON TRANSMISSION LINE. 750 KV.

the line without causing any voltage of sufficient magnitude to make either klydonograph or oscillograph records. The particular stroke which gave the records

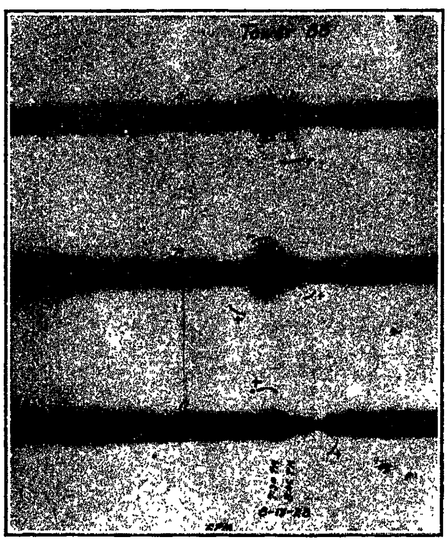

FIG. 14—KLYDONOGRAPH RECORD AT TOWER 88, CORRESPONDING TO RECORD OF FIG. 13

shown in Figs. 13 to 16 was observed by one of the operators to be extremely close to the station but later examination did not show the exact place where it struck the earth.

*Accomplishments of the First Year's Work.* Perhaps the accomplishment of greatest importance resulting from the work during this summer is the demonstration that complete oscillograms of lightning voltages can be secured by the use of the Norinder form of cathode ray oscillograph. The klydonograph record shows that only one voltage of appreciable magnitude occurred on the line at an oscillograph station during the test. A complete oscillogram of this surge was secured. It shows the beginning, the entire wave front, the crest, and the entire duration of the transient.

A second finding of major importance is that any

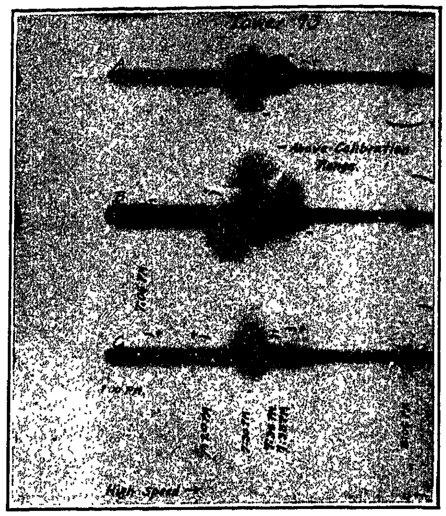

FIG. 15—KLYDONOGRAPH RECORD AT TOWER 93, CORRESPONDING TO RECORD OF FIG. 13

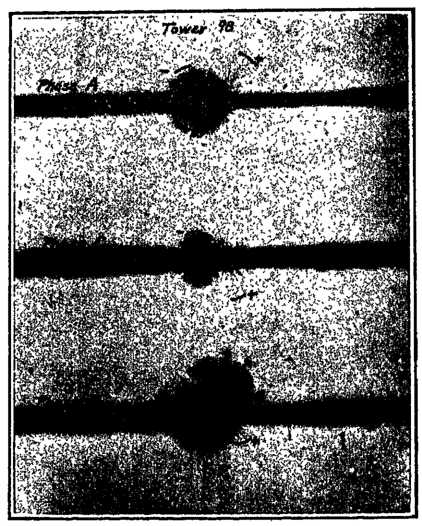

FIG. 16—KLYDONOGRAPH RECORD AT TOWER 98, CORRESPONDING TO RECORD OF FIG. 13

large number of serious overvoltages at any one point on a line during a single season is not to be anticipated. This fact has also been indicated by the previous extensive work with the klydonograph. Its significance in connection with lightning research is that it will be necessary to make a considerable number of installations and to operate them for several years in order to secure records covering the entire range of lightning voltages and operating conditions for various territories.

To be certain that any plans for control or protection are adequate it will be necessary to know the maximum rate of rise of voltage, the maximum crest value which might be reached, and the maximum duration. To determine the extent of protective measures which will be economically justified it will probably be necessary also to learn the relation between these maximum values and the average or ordinary values.

It is indicated by the klydonograph work of the past and also by observation of service results that all transmission lines, regardless of the insulation used, are liable to lightning flashover at times. We may conclude from this that the maximum value of lightning voltages exceeds any insulation which has been used thus far. It is possible, even probable, that the margin between the maximum lightning voltage and the present maximum insulation strength is considerable and before adequate control or protection can be provided this margin must be determined. Thus, in order to determine the crest value of lightning voltage which may be reached and to determine the average or ordinary value at the time of flashover it will be necessary to provide in some way for measurements on a line overinsulated beyond the present practicable values. This might be done by providing in some of the installations for possibly doubling the number of insulator units used in a section of the line a few miles long adjacent to the oscillograph. As an alternative it might be done, as has been proposed, by building a special line a few miles long securing a very high value of overinsulation by the use of some such material as wood or rope which, while not satisfactory for continuous service, would serve under transient voltage conditions.

*Need for Coordination of Effort.* It is the plan to continue the tests in Tennessee and in Illinois and probably several more instruments will be installed during the next season at other locations. With the proposed program we could in course of time obtain sufficient data to determine the characteristics of lightning waves so that their effect on transmission lines and connected apparatus could be predicted and improvements in design and construction would result. The time required, due to the necessarily limited scope of our work or that of any other similar group, to carry out the necessary program would, however, be more than the electrical power industry would care to countenance for an investigation so vital to the future of power development.. Since in all probability the major advantages that will result from carrying out this program will be in transmission line design and construction, in which the manufacturers of electrical power apparatus do not participate, we feel that it would be to the advantage of the electrical power industry as a whole to speed up this investigation by taking an active part in it, so that many more observation stations could be installed and the necessary data obtained in a reasonable time. While the financial benefits that will accrue from the complete determination of the characteristics of lightning surges cannot be predicted with assurance, it is the experience of American industry in all lines that once a problem of major importance is accurately defined, a satisfactory solution is promptly secured. When we consider that the annual expenditure of the industry in transmission is of the order of 150 million dollars, it seems clear that there is ample justification for spending the million dollars or so necessary for adequate lightning research.

There is general recognition that a solution of the lightning problem is necessary if the full possibilities of the development of the electrical power industry are to be realized. The work reported here demonstrates that the means necessary for obtaining a complete solution of the first part of the lightning problem are available. Similar work being conducted by other groups along somewhat different lines will doubtless result in availability of equally satisfactory means. The problem should not be confined to the manufacturers nor to the operators but should be participated in by the whole industry. It cannot be solved by either the manufacturers or operators working alone nor properly by any small group. The solution requires the financial and technical support of the whole industry, and now that the equipment is available it is only necessary to work out the necessary organization of effort.

## II. Theoretical Discussion of the Propagation of Lightning Surges

*Waves on Wires.* The fundamental equations for current and voltage are:

$$-\frac{\partial v}{\partial x} = R\,i + L\,\frac{d\,i}{d\,t} \tag{1}$$

$$-\frac{\partial i}{\partial x} = K\,v + C\,\frac{d\,v}{d\,t} \tag{2}$$

From (1) we have

$$-\frac{\partial^2 v}{\partial x^2} = \left(R + L\frac{d}{d\,t}\right)\frac{\partial i}{\partial x}$$

Substituting from (2) for $\frac{\partial i}{\partial x}$ we have

$$\frac{\partial^2 v}{\partial x^2} = \left(R + L\,\frac{d}{d\,t}\right)\left(K + C\,\frac{d}{d\,t}\right)v$$

$$\frac{\partial^2 v}{\partial x^2} = L\,C\,\frac{d^2 v}{d\,t^2} + (K\,L + R\,C)\,\frac{d\,v}{d\,t} + K\,R\,V$$

Dividing by $L\,C$ and denoting $\frac{1}{L\,C}$ by $\alpha^2$

$$\frac{\alpha^2\,\partial^2 v}{\partial x^2} = \frac{d^2 v}{d\,t^2} + \left(\frac{K}{C} + \frac{R}{L}\right)\frac{d\,v}{d\,t} + \frac{K\,R}{L\,C}\cdot V \tag{3}$$

Take as a tentative solution

$$V = A\,e^{\lambda t + \gamma x}$$

Substituting this in (3) we have

$$\lambda^2 + \lambda\left(\frac{K}{C} + \frac{R}{L}\right) + \frac{K\,R}{L\,C} = \alpha^2 \cdot \gamma^2$$

Solving for $\lambda$ we obtain

$$\lambda = -\left(\frac{K}{2\,C} + \frac{R}{2\,L}\right) \pm \sqrt{\alpha^2\,\gamma^2 + \left(\frac{K}{2\,C} + \frac{R}{2\,L}\right)^2 - \frac{K\,R}{L\,C}}$$

Let $\gamma = \pm j\,p$

$$\lambda = -\left(\frac{K}{2\,C} + \frac{R}{2\,L}\right) \pm \sqrt{\left(\frac{K}{2\,C} - \frac{R}{2\,L}\right)^2 - \alpha^2 p^2}$$

$$= -\left(\frac{K}{2\,C} + \frac{R}{2\,L}\right) \pm j\,p\,\alpha\sqrt{1 - \frac{1}{\alpha^2\,p^2}\left(\frac{K}{2\,C} - \frac{R}{2\,L}\right)^2}$$

The solution will therefore have the form

$$V = \sum e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}\left[\left\{\frac{A+j\,B}{2}\,e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} + \frac{A-j\,B}{2}\,e^{-jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} + \left\{\frac{C+i\,D}{2}\,e^{jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} + \frac{C-j\,D}{2}\,e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\}\right] \quad (4)$$

In the above solution $e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}$ is called the attenuation factor and

$$\sqrt{1 - \frac{1}{\alpha^2\,p^2}\left(\frac{K}{2\,C} - \frac{R}{2\,L}\right)^2}$$

is called the distortion factor. It will be noticed that the latter is unity, denoting no distortion, when $\frac{K}{2\,C} = \frac{R}{2\,L}$. This simple principle is the theoretical basis for the Pupin coil which by increasing $L$ reduces the ratio $\frac{R}{2\,L}$ until it is equal to $\frac{K}{2\,C}$. It will further be noted that there are two pairs of conjugate functions which combined form two real functions which are the real solution of the differential equation.

Let us now consider the value of the first wave represented by the terms in the first pair of brackets of Equation (4), at a point $x = x'$ when $t = 0$. For the sake of clearness we shall consider the attenuation and distortion factors to be both unity. The value is given by

$$|V|_{\substack{x=x'\\t=0}} = \sum\left(\frac{A+j\,B}{2}\,e^{jpx'} + \frac{A-j\,B}{2}\,e^{-jpx'}\right)$$

Now consider a point on the line at the end of time $t = t'$, let the value of $x$ for the point be $x = \alpha\,t' + x'$. Then, substituting these values in Equation (4), making the attenuation factor and the distortion factor both unity, we shall have:

$$|V|_{\substack{x=\alpha t'+x'\\t=t'}} = \sum\left(\frac{A+j\,B}{2}\,e^{jpx'} + \frac{A-j\,B}{2}\,e^{-jpx'}\right)$$

the same value as before for $x = x'$ and $t = 0$. Consequently, the first bracketed term of Equation (4) represents a positively traveling wave having velocity $\alpha$. Similarly, the second bracketed term represents a negatively traveling wave having the same velocity.

It will be noted that the distortion factor apparently reduces the velocity of propagation and that the amount of reduction varies with the frequency, being higher the lower the frequency. It is important that this apparent velocity be not confused with the velocity of propagation which remains unchanged and is always $\alpha$ or $1/\sqrt{L\,C}$. It will be clear, after considering the matter, that the only way in which the original $\psi\,(x)$ can change during propagation is by a progressive change in the relative phase positions of its constituents, this is the effect produced by the distortion factor. The discontinuities on the wave will be propagated at the velocity $\alpha$, as will be apparent due to the fact that the distortion factor approached unity for large values of $p$, and these determine the phases of the discontinuities.

It will be noted that Equation (4) is a perfectly general solution for any system of waves on an infinite line whose value can be expressed, when $t = 0$ by a function of $x$. It will not be necessary nor advisable to maintain this generality throughout this discussion. It will be assumed that for $t = 0$ the wave is given in the neighborhood of $x = 0$ by a charge distributed on the line between limits $+\,a$ and $-\,a$ and that the potential between these limits due to this charge is given by $2\,\varphi\,(x)$ at the time $t = 0$. In which case the charge will divide into two equal charges moving in opposite directions. So that in Equation (4) $B$ must be equal to $A$ and $D$ must be equal to $-\,B$.

On this assumption Equation (4) may be expressed in the form

$$V = \sum e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}\left[\left\{A\cos p\left(x - \alpha\,t\sqrt{1 - \frac{1}{\alpha^2\,p^2}\left(\frac{K}{2\,C} - \frac{R}{2\,L}\right)^2}\right) - B\sin p\left(x - \alpha\,t\sqrt{1 - \frac{1}{\alpha^2\,p^2}\left(\frac{K}{2\,C} - \frac{R}{2\,L}\right)^2}\right)\right\}\right.$$

$$+\left\{A\cos p\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)\right.$$

$$\left.\left.+B\sin p\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)\right\}\right] \tag{4a}$$

This form of solution is appropriate for some analytical work but is not well suited for expressing a non-periodic function of $(x)$. To do this we must use Fourier integrals. If we have a function $\varphi(x')$ which represents a non-periodic function $\Psi(x)$ between the limits $-a$ and $a$, and if the value of the non-periodic function is zero for all values of the independent variable outside these limits, the non-periodic function may be expressed by the infinite definite integral.

$$\psi(x)=\frac{1}{\pi}\int_0^{\infty}d\,p\int_{-a}^{+a}\varphi(x')\cos p\,(x-x')\,d\,x' \tag{5}$$

where $(\varphi(x))^{+a}_{-a}+(0)^{\infty}_{+a}+(0)^{-\infty}_{-a}=\psi(x)$.
The usual statement of the integral is

$$\psi(x)=\frac{1}{\pi}\int_0^{\infty}d\,p\int_{-\infty}^{+\infty}\psi(x')\cos p\,(x-x')\,d\,x' \tag{6}$$

The statement given in (5) is appropriate for the problem as usually presented. Where $\psi(x')$ between limit $-a$ and $+a$ can be represented only by several functions of the form $\varphi(x)$ the integration between $-a$ and $a$ must be divided to correspond with the limits imposed on the several functions. The solution appropriate for an infinite line in which a single charge of definite finite distribution is induced by lightning is, therefore

$$V=e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}\frac{1}{\pi}\int_0^{\infty}d\,p\int_{-\infty}^{+\infty}\varphi(x')$$

$$\left\{\cos p\left(x-x'-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\Big)\frac{K}{2C}-\frac{R}{2L}\Big)^2}\right)\right.$$

$$\left.+\cos p\left(x-x'+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)\right\}d\,x' \tag{7}$$

Since $t$ does not enter into the integration, the form of the integral is given when $t=0$ and is

$$V_0=e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}\frac{1}{\pi}\int_0^{\infty}dp\int_{-a}^{+a}\varphi(x')\left\{\cos p\,(x-x')\right.$$

$$\left.+\cos p\,(x-x')\right\}d\,x' \tag{8}$$

where it should be noted that the wave must be considered as consisting of two equal parts, one of which becomes the positive and the other the negative traveling wave.

The above solution is appropriate for lines of infinite length subject to a single lightning stroke and may be used for investigation of surges over long lines where only the attenuation of incoming surges are of interest. For short lines the end effects become of major importance. In looking into these problems the form of solution given by (4) which is analytically the equivalent of (7) will be found most convenient and we shall use it in the form

$$V=\sum_{p=0}^{p=\infty}e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}(A+j\,B)$$

$$\left\{e^{jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right.$$

$$\left.+e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \tag{9}$$

which is the appropriate form for the vector representation of a single non-periodic induced surge traveling over the line. The true solution will be the real part of the solution obtained by (9). It will be found later on that such a surge will become periodic as the result of the boundary conditions set up by the finite line. It will be noticed that in Equation (5) the coefficient of the sine term of the negative wave is of opposite sign to the coefficient of the sine term of the positive wave. This is the necessary condition on an infinite line for the positively traveling wave and the negatively traveling wave to be exact counterparts of each other. Similarly in vector representation in the negative traveling wave the exponential $e^{-p(x+\alpha t)}$ corresponds to the exponential $e^{p(x-\alpha t)}$ since time is always positive whereas $x$ is positive for the positive wave and negative for the negative wave.

From (2) we obtain the symbolic expression for $i$, namely

$$i=-\frac{K+C\,D_t}{D_x}V \tag{10}$$

where $D_t=\frac{d}{d\,t}$ and $D_x=\frac{\partial}{\partial\,x}$. Applying this to Equation (9) we have

$$I=\sum_{p=0}^{p=\infty}e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}(A+j\,B)\sqrt{\frac{C}{L}}$$

$$\left\{\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}+\frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right)\right\}$$

$$\left\{e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right.$$

$$\left. - e^{-jp\left(x + \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right\} \qquad (11)$$

the real solution being the real part of the above equation as in the case of the potential. The quantity $\sqrt{\frac{C}{L}}$ is called the surge admittance, $\sqrt{\frac{L}{C}}$ being the surge impedance. It will be seen that if the distortion factor is close to unity the current is equal to the potential multiplied by the surge admittance. If distortion is present the current will be out of phase with the potential and will have a somewhat different form.

Equations (**9**) and (**11**) refer only to an infinite line over which a single surge caused by a released bound charge is traveling.

The type of line referred to above may be called a doubly infinite line. Let us now consider a singly infinite line open at one end and let the value of $x$ at this end be $l_1$. Let us consider Equation (**11**). It will be obvious that in the form given it is not general enough for this particular problem as the initial condition imposed when $t = 0$ makes the negatively traveling wave identically zero for $x = > l_1$. To generalize this equation we shall retain the original coefficient of the negatively traveling wave but add another term $A'$ whose value must be consistent with the conditions $V_0 = 2\,\varphi\,(x)$ when $t = 0$. The equation so altered is still a solution of the general equation and is given for $x = l_1$ by

$$I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} \sqrt{\frac{C}{L}}$$

$$\left\{ \sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2} + \frac{j}{\alpha p}\left(\frac{K}{2C} - \frac{R}{2L}\right) \right\}$$

$$\left\{ (A + jB)\, e^{jp\left(l_1 - \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} - A'\, e^{-jp\left(l_1 + \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right\} \qquad (12)$$

where, since for values of greater than $\alpha_1$ the original negative traveling wave is zero it does not appear in the equation. Now the condition imposed by the open end is that $I$ shall be identically zero. This condition will be satisfied only if we make

$$A' = (A + jB)\, e^{j2pl_1} \qquad (13)$$

The solution for a singly infinite line open at $x = l_1$ is therefore

$$I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} (A + jB) \sqrt{\frac{C}{L}}$$

$$\left\{ \sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2} + \frac{j}{\alpha p}\left(\frac{K}{2C} - \frac{R}{2L}\right) \right\}$$

$$\left\{ e^{jp\left(x - \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} - (1 + e^{j2pl_1})\, e^{-jp\left(x + \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right\} \qquad (14)$$

An examination of this equation shows that when $t = 0$ the negatively traveling wave given by the term having coefficient $(A + jB)\, e^{2l_1}$ will be zero at every point within the singly infinite line. The solution for the potential is obtained by reversing the operation (**10**) but it may be obtained directly from (**14**) by inspection. It is given by

$$V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} (A + jB)$$

$$\left\{ e^{jp\left(x - \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} + (1 + e^{j2pl_1})\, e^{-jp\left(x + \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right\} \qquad (15)$$

This equation shows that at the end of the singly infinite line the potential reaches twice the value obtained by the wave traveling in any other part of the line. In a similar manner it may be shown for a singly infinite line open at $x = -l_2$, the solution for current and e. m. f. are:

$$I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} (A + jB) \sqrt{\frac{C}{L}}$$

$$\left\{ \sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2} + \frac{j}{\alpha p}\left(\frac{K}{2C} - \frac{R}{2L}\right) \right\}$$

$$\left\{ (1 + e^{j2pl_2})\, e^{jp\left(x - \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} - e^{-jp\left(x + \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right\} \qquad (16)$$

$$V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} (A + jB)$$

$$\left\{ (1 + e^{j2pl_2})\, e^{jp\left(x - \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right.$$

$$+\, e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\Bigg\} \tag{17}$$

The waves having the factors $e^{j2pl_1}$ and $e^{j2pl_2}$ are reflected waves and it will be noticed that at the positive end of the line the positively traveling wave is reflected into a negative traveling wave of opposite sign in the case of the current, and of the same sign in the case of the potential. The multiplying factor $e^{j2pl_1}$ is introduced in the case of reflection of a positive traveling wave into a negative traveling wave and the factor $e^{j2pl_2}$ in case of reflection of a negative traveling wave into a positive traveling wave. We may now write down the solution of the finite line having ends at $x = l_1$ and $x = -l_2$; the total length of the line is $l = l_1 + l_2$. By taking into account the successive reflection we arrive at the factors for the positive and negative waves; they are:

Positive: $(1 + e^{j2pl_2})\,(1 + e^{j2pl} + e^{j4pl} + \dots e^{j2plr})$

Negative: $(1 + e^{j2pl_1})\,(1 + e^{j2pl} + e^{j4pl} + \dots e^{j2plr})$ (18)

In this manner, starting with the idea of a lightning wave induced in a doubly infinite line we arrive at a solution for a finite line. The solution is:

$$\begin{aligned} I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}\,(A+jB)\,(1+e^{j2pl} &+ e^{j4pl}+\dots)\sqrt{\frac{C}{L}}\left\{\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}+\frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right)\right\} \\ &\left\{(1+e^{j2pl_2})\,e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} - (1+e^{j2pl_1})\,e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \end{aligned} \tag{19}$$

$$\begin{aligned} V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}\,&(A+jB)\,(1+e^{j2pl}+e^{j4pl}+\dots) \\ &\left\{(1+e^{j2pl_2})\,e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} + (1+e^{j2pl_1})\,e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \end{aligned} \tag{20}$$

These solutions have been obtained by considering the terminal conditions separately and they show that the result is an oscillatory solution, but as a result of (19) and (20) we can combine the three terminal conditions and obtain a Fourier expansion for $2\,\varphi(x)$, when $t = 0$, which will give us the solution direct in its simplest form. We shall deal with this later on after we have taken up the singly infinite line grounded at one and the finite line grounded at both ends.

In the singly infinite line grounded at the positive end $x = l_1$ the potential at the end must be identically zero; we may therefore, following the same principles as in the case of the singly infinite line, write down the solution. It is

$$\begin{aligned} V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}\,&(A+jB) \\ &\left\{e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} + (1-e^{j2pl_1})\,e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \end{aligned} \tag{21}$$

$$\begin{aligned} I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}\,&(A+jB)\sqrt{\frac{C}{L}}\left\{\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}+\frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right)\right\} \\ &\left\{e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)} - (1-e^{j2pl_1})\,e^{-jp\left(x+\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right\} \end{aligned} \tag{22}$$

with similar expression for a singly infinite line grounded at the negative side. In the case of the finite line grounded at both ends we have the following factors for the positive and negative waves:

Positive: $(1 - e^{j2pl_2})\,(1 + e^{j2pl} + e^{j4pl} + \dots)$

Negative: $(1 - e^{j2pl_1})\,(1 + e^{j2pl} + e^{j4pl} + \dots)$

The solution for this case is:

$$\begin{aligned} I = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t}\,(A+jB)\,(1+e^{j2pl} &+ e^{j4pl}+\dots)\sqrt{\frac{C}{L}}\left\{\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}+\frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right)\right\} \\ &\left\{(1-e^{j2pl_2})\,e^{jp\left(x-\alpha t\sqrt{1-\frac{1}{\alpha^2p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2}\right)}\right. \end{aligned}$$

$$\left.- (1 - e^{j2pl_1})\, e^{-jp\left(x + \alpha t\sqrt{-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)}\right\} \quad (23)$$

$$V = \sum_{p=0}^{p=\infty} e^{-\left(\frac{K}{2C} + \frac{R}{2L}\right)t} (A + j\,B)\,(1 + e^{j2pl} + e^{j4pl} + \ldots)$$

$$\left\{ (1 - e^{j2pl_2})\, e^{jp\left(x - \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right.$$

$$\left.- (1 - e^{j2pl_1})\, e^{-jp\left(x + \alpha t\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)}\right\} \quad (24)$$

Let us now examine Equation (20) at $t = 0$. A term of the Fourier expansion for the positive traveling wave is:

$(A + j\,B)\,(1 + e^{j2pl} + e^{j4pl} + \ldots)\,(1 + e^{j2pl_2})\,e^{jpx}$
or $(A + j\,B)\,(1 + e^{j2pl} + e^{j4pl} + \ldots)\,(e^{jpx} + e^{jp(x+2l_2)})$
Now it will be observed that in the case of the variable $(e^{jpx} + e^{jp(x+2l_2)})$ the second term will have the same value as the first term if we take the value of $x$ for it is equal to $x - 2\,l_2$. The second term therefore might be expressed as arising from the reflection of the original wave at $t = 0$ in a mirror placed at the end $x = -\,l_2$ of the line. The factor $(1 + e^{j2pl} + e^{j4pl} + \ldots)$ shows that all the positive waves are derived by repeating the original wave and its reflection, so that from the beginning of one to the beginning of the next the distance is $2\,l$ or twice the length of the line. If we take the negative traveling wave we find that it arrives from a similar system of waves which may be ascribed to the reflected image of the original wave in a mirror at the positive end of the line, which wave and its reflected image are repeated every $2\,l$ length in the positive direction.

Let us now add to the succession of positive waves to the left of the line a succession of positive waves corresponding to the negative waves to the right of the line and vice versa. It will be evident that the positive waves to the right and the negative waves to the left will produce no effect on the line at any time but will remain entirely imaginary as far as the line is concerned. The result will, however, have the effect of making the system of waves harmonic from $-\,\infty$ to $+\,\infty$. If we imagine a mirror, placed at the two ends of the line and the original static wave reflected in the two mirrors the result will be a double infinite set of images in the two mirrors, which will be an exact representation of the system of waves required to give the solution of Equation (20); as the wave divides into positive and negative traveling waves these images will also divide into positive traveling waves, which will be the system of waves as given by the solution, Equation (20).

Since the solution at $t = 0$ is given by a doubly infinite periodic function of $x$ it may be represented by a Fourier series of integral harmonics of the wave length. In this case, since the system of waves is unidirectional, the actual wave length will be $2\,l$ and in addition to even harmonics there will be a constant term in the expansion.

Examining Equation (24) we find that the original static wave may be replaced by a doubly infinite wave of wave length $2\,l$ and which may be obtained by considering a mirror which reflects an inverted image to be placed at each end of the line. See Fig. (17). Similarly, if we have to do with a finite line grounded at one end and open at the other, the system of images obtained by placing an ordinary mirror at the open end and a mirror reflecting an inverted image at the other end will give the appropriate expansion for $\varphi\,(x)$ when $t = 0$. In this case the wavelength will be $4\,l$. Fig. 17.

In obtaining the appropriate Fourier expansion in any case of a finite line with like ends we need consider only the position of the system between $-\,2\,l_2$ and $2\,l_1$ as the rest of the system simply repeats this portion periodically, the fundamental wave length being $2\,l$. Both Fourier expansions will in general have even as well as odd harmonics and in the case of the positive reflection there will be a non-harmonic term. Where the surge starts at the middle of the line, the wave length for the positive reflection becomes $l$ and there are even as well as odd harmonics; in the case of the negative reflection the wavelength remains $2\,l$ but there are no even harmonics.

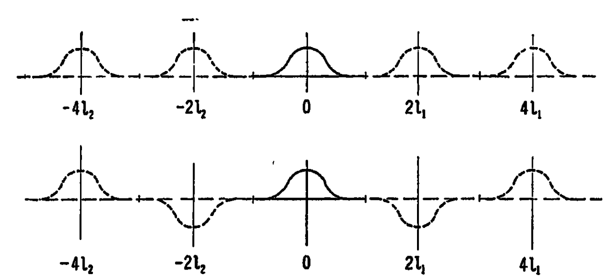


Fig. 17

If we denote the function of $x$ when $t = 0$ obtained in this manner by $2\,\psi\,(x)$, so that $\psi\,(x)$ represents the static function from which the positive and negative waves are propagated, we shall have for the open finite line

$$\psi\,(x) = \text{The real part of } A_0 + \sum_1^\infty (A_n + j\,B_n)\, e^{jn\frac{\pi}{l}x} \quad (25)$$

And for the line grounded at both ends

$$\psi\,(x) = \text{The real part of } \sum_1^\infty (A_n + j\,B_n)\, e^{jn\frac{\pi}{l}x} \quad (26)$$

where

$$A_0 = \frac{1}{2l} \int_{-2l_2}^{+2l_1} \psi(x)\, d\, x \tag{27}$$

$$A_n = \frac{1}{l} \int_{-2l_2}^{+2l_1} \psi(x) \cos \frac{\pi n}{l} x\, d\, x \tag{28}$$

$$B_n = \frac{1}{l} \int_{-2l_2}^{+2l_1} \psi(x) \sin \frac{\pi n}{l} x\, d\, x \tag{29}$$

The solution for the finite line open at both ends may therefore be stated as follows:

$$V = \sum_{n=0}^{n=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} \Bigg[ A_0 + (A_n + j\, B_n) \left\{ e^{j\frac{n\pi}{l}\left(x - \alpha t \sqrt{1 - \frac{l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} + e^{-j\frac{n\pi}{l}\left(x + \alpha t \sqrt{1 - \frac{l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{C} - \frac{R}{2L}\right)^2}\right)} \right\} \Bigg] \tag{30}$$

The expression for the current is obtained by changing the sign of the second exponential term and multiplying the whole quantity within the summation sign by

$$\sqrt{\frac{C}{L}} \left\{ \sqrt{1 - \frac{l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2} + \frac{j\, l}{\alpha\, n\, \pi}\left(\frac{K}{2C} - \frac{R}{2L}\right) \right\} \tag{31}$$

which is the true surge admittance.

For the finite line grounded at both ends the potential will be

$$V = \sum_{n=0}^{n=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A_n + j\, B_n) \left\{ e^{j\frac{n\pi}{l}\left(x - \alpha t \sqrt{1 - \frac{l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} + e^{-j\frac{n\pi}{l}\left(x + \alpha t \sqrt{1 - \frac{l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right\} \tag{32}$$

and the current may be obtained by the same procedure used in the case of the open finite line. In each case the actual solution is the real part of the summation.

In the case of a finite line open at one end and grounded at the other the wave length is $4\, l$ as can be ascertained by means of the mirror scheme which was described above and the expression for the potential will be:

$$V = \sum_{n=0}^{n=\infty} e^{-\left(\frac{K}{2C}+\frac{R}{2L}\right)t} (A_n + j\, B_n) \left\{ e^{j\frac{n\pi}{2l}\left(x - \alpha t \sqrt{1 - \frac{4l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} + e^{-j\frac{n\pi}{2l}\left(x + \alpha t \sqrt{1 - \frac{4l^2}{\alpha^2 n^2 \pi^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2}\right)} \right\} \tag{33}$$

and the current may be obtained by changing the sign of the second exponential term and multiplying the whole quantity within the summation sign by (31).

In the above solutions the value of $\frac{n\,\pi}{l}$ and $\frac{n\,\pi}{2\,l}$ is the same as the coefficient $p$ in the theory of doubly infinite and singly infinite lines.

The principle of conservation of energy affords a general means for introducing into the general solution of the differential equation the effect of any physical discontinuity. This principle states that the total energy of the system together with that dissipated must be constant.

The principle shows us at once why the initial wave when $t = 0$ divides into two equal waves flowing in opposite directions for in order to satisfy this principle the total energy must remain constant and equal to the total energy in the system at time $t = 0$ less the dissipated energy. The kinetic and potential energy in the traveling wave at any point of the line are equal. Therefore, the original wave must divide into two equal waves, traveling in opposite directions and having one-half its crest value except for the effect of the energy dissipated which has the effect of reducing this value with time. Each wave has now one-half the energy of the original wave less the dissipated energy and in each wave one-half the energy is electrostatic energy, or potential, and the other half is electromagnetic energy, or current. Thus, the system satisfies the condition imposed by the principle of conservation of energy.

Again, considering the terminal condition at an open end, the electro-kinetic energy must be zero as all the energy at the terminal must become potential energy. This condition is satisfied when two equal potential waves traveling in opposite directions are coincident in value at the point under consideration. Thus the potential wave is reflected positively while the current wave is reflected negatively. Similarly for a closed end the potential energy must be zero and therefore the potential wave is reflected negatively while the current wave is reflected positively.

These specific conditions were of course imposed upon the equation when the terminal conditions $i = 0$ or $V = 0$ were introduced. Where the discontinuity is not complete as in those two cases, it becomes necessary

to introduce specifically the principle of energy into the boundary condition at the point of discontinuity.

In the case of the finite line we perceive that at the terminals there must be a periodic recurrence of the phenomena due to successive reflection as the energy must remain in the system until dissipated, and we perceive at once that this condition may be exactly represented by an initial distribution periodic with respect to $x$ in an infinite line which resolves into two traveling waves of equal value traveling in opposite direction. Two of the nodal points of this system are at the terminals of the finite line, and as would be expected with like terminal conditions at either end the wave length of the initial distribution would be twice the length of the line and in the case of opposite condition at either terminal it would be four times the length of the line.

We shall now direct our attention to a singly infinite line connected at the point $x = l_1$ to a similar line having different transmission constants.

We shall from now on confine ourselves to the solution of the specific differential equation of the electric circuit. We shall for brevity designate

$$\frac{K}{2C} + \frac{R}{2L} \text{ by } u$$

$$\sqrt{1 - \frac{1}{\alpha^2 p^2}\left(\frac{K}{2C} - \frac{R}{2L}\right)^2} \text{ by } v$$

$$\sqrt{\frac{C}{L}} \left\{ \sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2} + \frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right) \right\} \text{ by } Y$$

The reciprocal of this or the surge impedance

$$\sqrt{\frac{L}{C}} \left\{ \sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2} - \frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right) \right\} \text{ by } Z$$

Then we have for the doubly infinite line

$$V = e^{-ut} \sum_{p=0}^{p=\infty} (A + jB) \left\{ e^{jp(x-vt)} + e^{-jp(x+vt)} \right\} \tag{32}$$

$$I = e^{-ut} \sum_{p=0}^{p=\infty} Y(A + jB) \left\{ e^{jp(x-vt)} - e^{-jp(x+vt)} \right\} \tag{33}$$

The actual values being the real part of these vector quantities. It should be noted that the surge impedance being equal to

$$\sqrt{\frac{L}{C}} \left\{ \sqrt{1-\frac{1}{\alpha^2 p^2}\left(\frac{K}{2C}-\frac{R}{2L}\right)^2} - \frac{j}{\alpha p}\left(\frac{K}{2C}-\frac{R}{2L}\right) \right\}$$

the radical sign over $\frac{L}{C}$ implies the signs $\pm$. The positive sign applies to positive traveling waves and the negative sign applies to negative traveling waves. This accounts for the change in sign of the negative traveling current wave as compared with the negative traveling potential wave.

Let us now consider both lines as having the same attenuation and distortion factors.

At the junction point of two lines the conditions imposed are

(a) The current shall be continuous.

(b) The potentials at the junction point on either side shall be the same.

Let the junction point of the two lines be at the point $x = l_1$ of the first line. It will be obvious that if the original static potential distribution $2\varphi(x)$ from which the wave on the first line originated had the value zero at the point only the positive traveling wave will be involved at the junction but since the discontinuity at the junction may give rise to a reflected wave which will travel in a negative direction, we shall assume for $V_1$ the value

$$V_1 = V_{+1} + V_{-1} \tag{34}$$

There will be no negative traveling wave in the second line and therefore

$$V_2 = V_{+2} \tag{35}$$

We have, therefore

$$V_{+2} - V_{-1} = V_{+1} \tag{36}$$

$$Y_2 V_{+2} + Y_1 V_{-1} = Y_1 V_{+1} \tag{37}$$

We have, therefore

$$V_{+2} = \frac{2Y_1}{Y_1 + Y_2} V_{+1} = \frac{2Z_2}{Z_1 + Z_2} V_{+1} \tag{38}$$

$$V_{-1} = \frac{Y_1 - Y_2}{Y_1 + Y_2} V_{+1} = \frac{Z_2 - Z_1}{Z_1 + Z_2} V_{+1} \tag{39}$$

No particular advantage is obtained by considering this case on the basis of reflected images as the expansion cannot be effected in terms of a Fourier series. We shall therefore confine ourselves to the statement of the solution, as we did in the case of the singly infinite lines. The potential and current in the first line are given by

$$V_1 = e^{-ut} \sum_{p=0}^{p=\infty} (A_1 + jB_1) \left\{ e^{jp(x-vt)} + \left(1 + \frac{Y_1 - Y_2}{Y_1 + Y_2} e^{j2pl_1}\right) e^{-jp(x+vt)} \right\} \tag{40}$$

$$I_1 = e^{-ut} \sum_{p=0}^{p=\infty} (A_1 + j\,B_1)\,Y_1$$

$$\left\{ e^{jp(x-vt)} - \left(1 + \frac{Y_1 - Y_2}{Y_1 + Y_2}\,e^{j2pl_1}\right) e^{-jp(x+vt)} \right\} \qquad (41)$$

The potential and current in the second line are given by

$$V_2 = e^{-ut} \sum_{p=0}^{p=\infty} (A_1 + j\,B_1)\,\frac{2\,Y_1}{Y_1 + Y_2}\,e^{jp(x-vt)} \Bigg\} \qquad (42)$$

$$I_2 = e^{-ut} \sum_{p=0}^{p=\infty} Y_2\,(A_1 + j\,B_1)\,\frac{2\,Y_1}{Y_1 + Y_2}\,e^{jp(x-vt)} \Bigg\} \qquad (43)$$

For the finite line of length $l$ connected on either end to infinite lines of the same constants having different surge impedances from the first line but the same attenuation and distortion we may write

$$\frac{Y_1 - Y_2}{Y_1 + Y_2} = a$$

and taking into account successive reflections, we have

$$V_1 = e^{-ut} \sum_{0}^{\infty} \Big[ (A_1 + j\,B_1)\,(1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)$$

$$\left\{ (1 + a\,e^{j2pl_2})\,e^{jp(x-vt)} + (1 + a\,e^{j2pl_1})\,e^{-jp(x+vt)} \right\} \Big] \qquad (44)$$

$$I_1 = e^{-ut} \sum_{p=0}^{p=\infty} \Big[ (A_1 + j\,B_1)(1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)\,Y_1$$

$$\left\{ (1 + a\,e^{j2pl_2})\,e^{jp(x-vt)} - (1 + a\,e^{j2pl_1})\,e^{-jp(x+vt)} \right\} \Big] \qquad (45)$$

The potential and current in the second line will be obviously

$$V_2 = e^{-ut} \sum_{p=0}^{p=\infty} \left\{ \frac{2\,Y_1}{Y_1 + Y_2}\,(A_1 + j\,B_1) \right.$$

$$\left. (1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)\,(1 + a\,e^{j2pl_2})\,e^{jp(x-vt)} \right\} \qquad (46)$$

$$I_2 = e^{-ut} \sum_{p=0}^{p=\infty} \frac{2\,Y_1\,Y_2}{Y_1 + Y_2}\,(A_1 + j\,B_1)$$

$$(1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)\,(1 + a\,e^{j2pl_2})\,e^{jp(x-vt)} \Bigg\} \qquad (47)$$

The potential and current in the third line, since $Y_3 = Y_2$ is

$$V_3 = e^{-ut} \sum_{p=0}^{p=\infty} \left\{ \frac{2\,Y_1}{Y_1 + Y_2}\,(A_1 + j\,B_1) \right.$$

$$\left. (1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)\,(1 + a\,e^{j2pl_1})\,e^{-jp(x+vt)} \right\} \qquad (48)$$

$$I_3 = e^{-ut} \sum_{p=0}^{p=\infty} \left\{ -\frac{2\,Y_1\,Y_2}{Y_1 + Y_2}\,(A_1 + j\,B_1) \right.$$

$$\left. (1 + a^2\,e^{j2pl} + a^4\,e^{j4pl} + \ldots)\,(1 + a\,e^{j2pl_1})\,e^{-jp(x+vt)} \right\} \qquad (49)$$

The solutions given above are not of much value for analytic work but are useful when plotting the potentials graphically. The analytic solution would depend upon an infinite series of images of the initial wave of decreasing magnitude and can only be expressed as a Fourier integral but the physical picture is of benefit when making a graphical analysis.

An important case is that in which two transmission lines feed into a common line and surges are released on the two lines having different magnitudes. We shall consider the two in which the surges are propagated as No. 1 and No. 2, the other line being No. 3. The propagated surge in No. 1 and No. 2 will be $V_{+1}$ and $V_{+2}$ and the transmitted surge in No. 3 will be $V_{+3}$. We shall assume the value of the surge admittance of No. 1 and No. 2 to be both $Y$ and of No. 3 to be $Y_3$. Then assuming all lines to have the same attenuation and distortion

$$V_3 = V_{+1} + V_{-1} \qquad (50)$$

$$V_3 = V_{+2} + V_{-2} \qquad (51)$$

$$Y_3\,V_3 = Y\,(V_{+1} + V_{+2}) - Y\,(V_{-1} + V_{-2}) \qquad (52)$$

In these equations $V_{+1}$ and $V_{+2}$ are supposed to be known, $V_3$, $V_{-1}$, and $V_{-2}$ are unknown.

We have by substituting from (50) in (52)

$$(Y + Y_3)\,V_{-1} + Y\,V_{-2} = (Y - Y_3)\,V_{+1} + Y\,V_{+2},$$

and by substituting from (51) in (52)

$$Y\,V_{-1} + (Y + Y_3)\,V_{-2} = Y\,V_{+1} + (Y - Y_3)\,V_{+2}$$

Solving for $V_{-1}$ and $V_{-2}$ we have

$$V_{-1} = \frac{2\,Y\,Y_3\,V_{+2} - Y_3^2\,V_{+1}}{2\,Y\,Y_3 + Y_3^2} \qquad (53)$$

$$V_{-2} = \frac{2\,Y\,Y_3\,V_{+1} - Y_3^2\,V_{+2}}{2\,Y\,Y_3 + Y_3^2} \qquad (54)$$

and

$$V_3 = \frac{2\,Y\,Y_3\,(V_{+1} + V_{+2})}{2\,Y\,Y_3 + Y_3^2} \qquad (55)$$

Still another important case is where a line of surge

admittance $Y_0$ terminates in two lines of surge admittances $Y_1$ and $Y_2$. In this case, by similar methods

$$V_1 = \frac{2\,Y_0}{Y_0 + Y_1 + Y_2} V_0 \qquad (56)$$

Extending this still further to the case of a line of surge admittance $Y_0$ terminating in three lines of surge admittances $Y_1$, $Y_2$, and $Y_3$, the equation for the transmitted voltage is:

$$V_1 = \frac{2\,Y_0}{Y_0 + Y_1 + Y_2 + Y_3} V_0 \qquad (57)$$

A resistance between the junction and ground acts the same as an additional infinite line having the same surge admittance.

## III. The Effect of Ground Resistance of Overhead Ground Wire Protection of Transmission Lines

*Overhead Ground Wires as Protection from Induced Waves.* Equations (40) and (41) are of great importance for estimating the actual protection afforded by overhead ground wires to transmission lines when the value of the ground resistance is taken into account. In this problem the ground resistance may be regarded as an infinite line having a surge impedance equal to the ground resistance. The two adjacent sections of the overhead ground wire both feed into the common ground and may be regarded as in multiple. Since what we desire is a quantitative relation between a perfect ground and grounds having various values of resistance, it will serve our purpose to consider the potential to extend over three spans and to be constant over this whole length. The spans being 1000 ft. in length this would mean that the line surge is a square topped wave of about three microseconds wavelength. Assuming a typical 220-kv. line with two ¾-in. ground wires spaced 30 ft. apart the surge impedance will be 290 ohms or the surge admittance 0.00344 for one pair of ground wires. For two pair feeding with a common ground it will be 0.00688. If the ground resistance is 80 ohms it will be equivalent to an infinite line having admittance 0.0125. We have then

$$a = \frac{Y_1 - Y_2}{Y_1 + Y_2} = \frac{-0.00562}{0.01938} = -0.290$$

$Y_1$ being the surge admittance of the ground wires in parallel and $Y_2$ the admittance of the ground resistance. Let the mean height of the transmission line be 50 ft. and of the ground wires 60 ft. Then if the gradient due to the thunder cloud is 50 kv. per foot, the induced potential will be 2500 kv. and on the ground wires 3000 kv. We may assume that the ground wires when at zero potential would reduce the potential on the transmission lines 50 per cent. When the potential of the ground wire is not zero a first approximation will be to add to the 50 per cent voltage on the transmission line the same percentage of 50 per cent as the ground wire potential at that point is of its full voltage, due regard being paid to signs.

Fig. 18 shows the potentials of ground wires and potentials of line on this basis for several reflections with a ground resistance of 80 ohms.

Fig. 19 shows the potentials of ground wires, line wires, and difference of potential at the tower for ground resistance of 20 ohms.

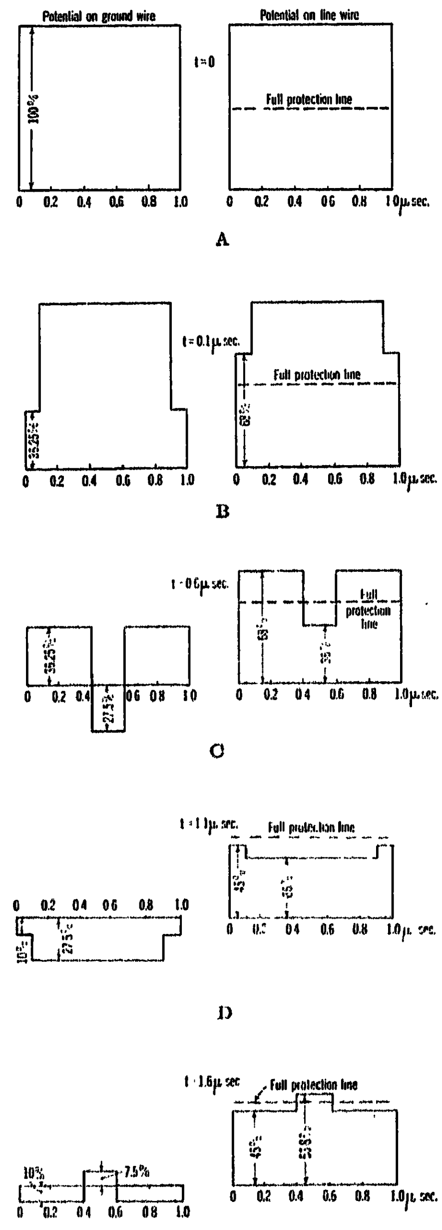


Fig. 18—Progress of Wave on Transmission Line and Ground Wire After the Release of a Bound Charge

Resistance of tower and ground = 80 ohms

Figs. 20 and 21 show the same voltages at the tower for resistances of 80 and 200 ohms respectively.

It will be noted that for an interval of two microseconds the lower resistance gives the lower voltage with respect to ground. The curves of difference of potential

between line conductor and ground wires, which is the same as the tower top, seems to favor the higher ground resistance. However, the tendency to arc over an insulator string depends not only on the potential to tower top but also on the potential to ground. Furthermore, the performance of ground wires during

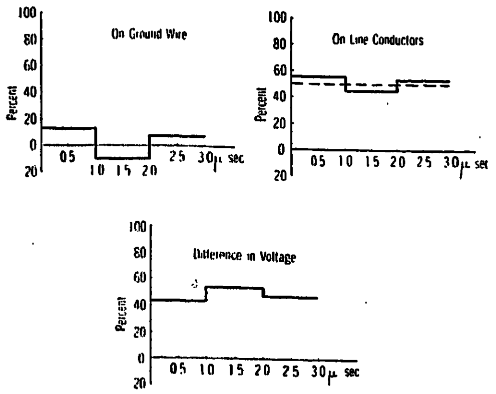


FIG. 19—INDUCED VOLTAGES ON LINES WITH GROUND WIRES

Voltages at the towers—based on equal induced voltages on ground and line wires ground resistance $R$ = 20 ohms

direct strokes is more important and the following discussion will show that for direct strokes a low ground resistance is essential.

*Overhead Ground-Wires as Protection against Direct Strokes.* A second important practical application for these formulas is for the case of a direct stroke of lightning. We shall first take up the case where the lightning stroke hits the middle of the span.

The phenomenon of lightning discharge has been clarified to a large extent by the experiments of Torok[5]

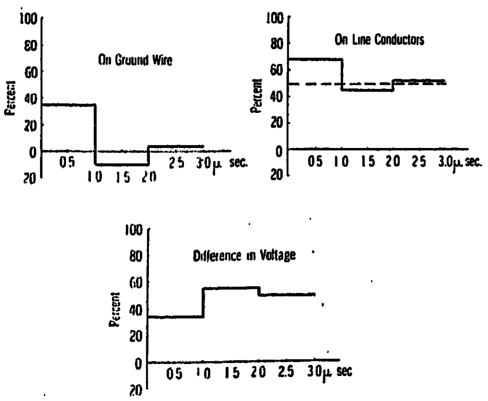


FIG. 20—INDUCED VOLTAGES ON LINES WITH GROUND WIRES

Voltages at the tower—based on equal induced voltages on ground and line wires ground resistance $R$ = 80 ohms

in suppressed discharges between sphere gaps. This phenomenon of suppressed discharge is very frequently observed in thunderstorms and the laboratory duplication of it has helped a great deal in the understanding of lightning discharges.

[5] J. J. Torok, "Surge Impulse Breakdown of Air," TRANSACTIONS A. I. E. E., Vol. 47, No. 2, April 1928, p. 349.

As a result of the study of Torok's suppressed discharges we have formed the following picture of what takes place just before and during a lightning discharge. The thunder cloud has first built up sufficient potential to produce local ionization by collision; as a result, space charges are formed in the neighborhood of the cloud which still further increase local ionization. The ionization begins to take place along preferred paths and the space charges become most dense in the neighborhood of these paths. Finally the density of the ionization paths becomes so great that the temperature along these paths becomes sufficient to cause thermionic emission of electrons from the molecules. This results in an intense stream of electrons which progressively produce other electrons by collision followed by thermal ionization. The result is that the front of the charged space rapidly approaches the earth with accelerated motion until a complete conducting circuit is formed

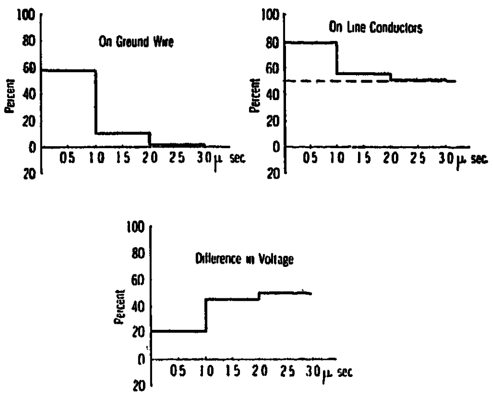


FIG. 21—INDUCED VOLTAGES ON LINES WITH GROUND WIRES

Voltages at the towers—based on equal induced voltages on ground and line wires ground resistance $R$ = 200 ohms

by the ionized paths from cloud to earth, which culminates in a lightning stroke.

It should be noted that the front of the charged space is not at the same potential as the cloud at any time. The effect of the space charge is to produce a drop of potential from cloud to front as the front progresses; but the result of this progression of the space charge is the increase of the intensity of stress in the air path from its front to the earth, and further ionization of the air path, unless the cloud is not able to support the additional charge required of it and maintain potential sufficient to complete the flashover. In the latter case the flashover is suppressed.

It is obvious that the intensity of a lightning stroke varies widely depending upon the height and nature of the cloud, which determines the quantity of charge that can be dissipated in a single stroke. Many strokes never reach the earth as frequently the progress of the streamer is stopped in mid-air due to insufficient energy to support and urge it forward. Others barely reach the earth and do so with little energy. Still others, of course, are intense in varying degrees.

In the initial stage before the streamer or ionized paths have reached the earth, the front of the ionized and charged space may be propagated at an average velocity of from one-twentieth to one one-hundredth the velocity of light. The first figure is the approximate value of this velocity as obtained between spheres, the second is the order of this velocity as obtained along strings of insulators. In other words, the preliminary or setting up stage of the flashover from a cloud may take from 25 to 125 microseconds, and during this time low potential surges flow from the ends of the line to the point of influence. Obviously, during this stage the field stress near the earth directly under the streamer point increases and beyond a short distance away it decreases. Thus, if a stroke takes place at some distance from a line the bound charges can travel away in both directions faster than the decrease of the field gradient. This is why a stroke must be very close to a line in order to induce a surge of appreciable magnitude. In the case of a direct stroke when the ionized path is complete there is propagated at once into the transmission line a steep wave front surge, the crest value of which may last for two or more microseconds, depending upon the potential of the cloud and how much of it is drained off over the discharge path.

Measurements made in connection with suppressed discharges between sphere gaps show that during the initiation period there is a very great flow of current between the two spheres before the flashover is established. Consequently, it is probable that a very large portion of the charge from the cloud is dissipated in the streamer before the actual flashover takes place, amounting to a large proportion of the total charge that passes from the cloud. This helps to lower the crest value of the wave when the flash hits the transmission line.

When the flashover takes place the cloud is discharged over the ionized path in the form of a surge traveling like any other traveling wave over a conducting path and since the surge impedance of the path which it must follow decreases progressively as it approaches the earth, the crest of the propagated wave will decrease in potential progressively as it approaches earth so that although the potential of the cloud may be something like one hundred million volts, the propagated wave when it reaches a transmission line may not have a crest of more than four or five million volts. This crest of the wave of the lightning stroke when it hits the line is further reduced by reason of the fact that its surge impedance is very much less at the striking point than at the cloud so that the actual wave propagated from the cloud as it approaches the earth along the ionized path is reduced in crest value. It may be assumed that when it strikes the transmission line it has a surge impedance of the same order as that of three transmission line conductors in parallel and, since it divides into two waves at this point, the crest value of each of these waves depends upon the surge impedances of the conductors struck and the lightning path, and the voltage of the lightning as it enters the line. We may assume that the maximum crest from the stroke lasts two or more microseconds. This wave may enter the ground wires, the transmission lines, or both, and it will be reflected at the towers in both the earth and other conductors involved. There is no foundation for the opinion that all lightning strokes are so severe that they involve everything in the vicinity. It is quite likely that many strokes strike a conductor of a transmission line and travel in both directions with no flashover, or flash over the insulators on that conductor without involving the other conductors. It is also likely that ground wires may be struck and conduct the energy away without involving the conductors. Evidence which indicates this has been obtained in klydonograph tests. If all conductors are short circuited by the lightning stroke, the potential of all conductors should be the same. However, the highest records obtained have usually indicated that the surge on one conductor was considerably the highest indicating that it had received the direct stroke and that the potentials on the other phases were induced.

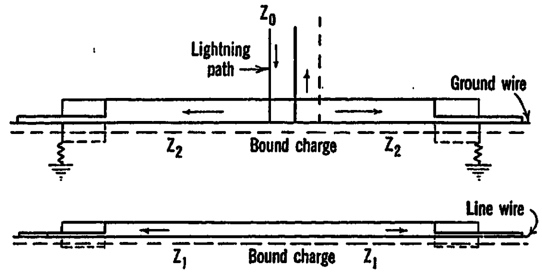


Fig. 22—Diagram of Direct Stroke to Ground Wire in Middle of Span Showing Voltage Waves

Undoubtedly, ground wires give a large measure of protection against direct strokes. Being placed above the conductors they receive the majority of the strokes and, except for the more intense ones, conduct the surge to ground without a flashover of the line insulators. Complete protection is not obtained by ground wires because some strokes are so intense that the potentials of the ground wire and tower top are raised to where they flash to the conductor, and also the factors determining the path of a lightning stroke are so varied that line conductors are sometimes struck even though there are ground wires above them. It is obvious that ground wires will be most effective if they are made of good conducting material and if the tower ground resistance is low. A steel ground wire is likely to be melted in two by the heat generated in carrying off the lightning surge and become involved in the line conductors. Therefore, it is important that a conductor having at least a surface of good conducting material be used as ground wires.

Figs. 22 and 23 show the nature of the resulting voltage waves when a stroke hits the ground wires in the

center of a span and at the tower respectively. The waves traveling in each direction from the point hit may be calculated by Equations (56) and (57). Assuming that the stroke strikes the ground wire only, there is a potential induced on the line conductors by the traveling wave but, since the line conductor is insulated and since the wave travels in both directions from a point there can be no current in the line conductor and therefore no mutual reaction on the ground wire. Thus, the line wire does not influence the waves on the ground wire and the voltage on the line wire may be calculated by the equation;[6]

$$V_1 = \frac{z}{Z_2} V_2 .$$

In the first case, that where the lightning strikes the

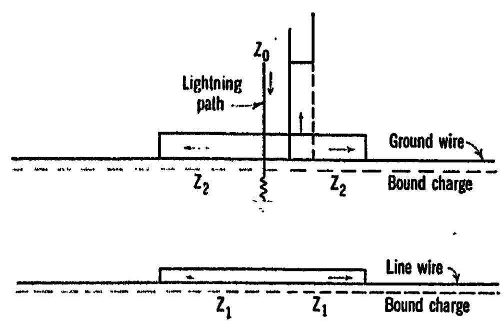


FIG. 23—DIAGRAM OF DIRECT STROKE TO TOWER WITH GROUND WIRES SHOWING VOLTAGE WAVES

middle of the span, there is a reflection when the wave reaches the tower. The voltages at this point are:[5]

$$V_{1t} = V_1 - \frac{z}{Z_2 + 2\,R} V_2$$

and
$$V_{2t} = \frac{2\,R}{Z_2 + 2\,R}\, V_2$$

where $V_{1t}$ is the voltage on the line conductor at the tower, $V_{2t}$ the voltage of the ground wire, and hence the tower top; $V_1$ and $V_2$ the oncoming voltages on the line wire and ground wire respectively; $Z_2$ the surge impedance of the ground wire, $z$ the mutual surge impedance, and $R$ the tower and ground resistance.

Assuming the same typical 200-kv. line chosen above, the surge impedance of the two ¾-in. ground wires in parallel $Z_2$, is 290 ohms, that of three one-in. line conductors in parallel, $Z_1$, is 205 ohms, and the mutual surge impedance $z$, is 96 ohms. The surge impedance of the lightning path at the line height will be assumed to be, $Z_0 = 200$ ohms. The surge admittances then are $Y_0 = 0.005$, $Y_1 = 0.00488$, $Y_2 = 0.00344$. The surge impedance of one line conductor is 470 ohms and the surge admittance 0.00222. With these constants the voltages at the tower were calculated in terms of the lightning voltage $V_0$ at the point of contact for various ground resistances and the results plotted in Fig. 24. In addition to curves for $V_1$ and $V_2$, a curve of $V_2 - V_1$, or the potential across

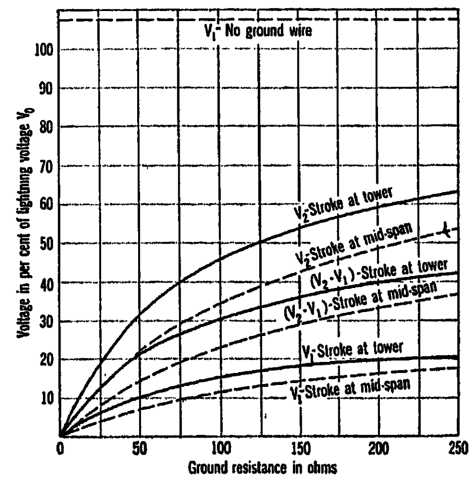


FIG. 24—SURGE VOLTAGE ON GROUND WIRE $V_2$, ON LINE WIRE $V_1$ AND DIFFERENCE OF VOLTAGE AS A FUNCTION OF GROUND RESISTANCE IN OHMS IN TERMS OF THE LIGHTNING VOLTAGE AT THE LINE $V_0$

the insulators, has been plotted. Although based on an assumed surge impedance of lightning path, $V_0$, these curves illustrate the importance of low ground

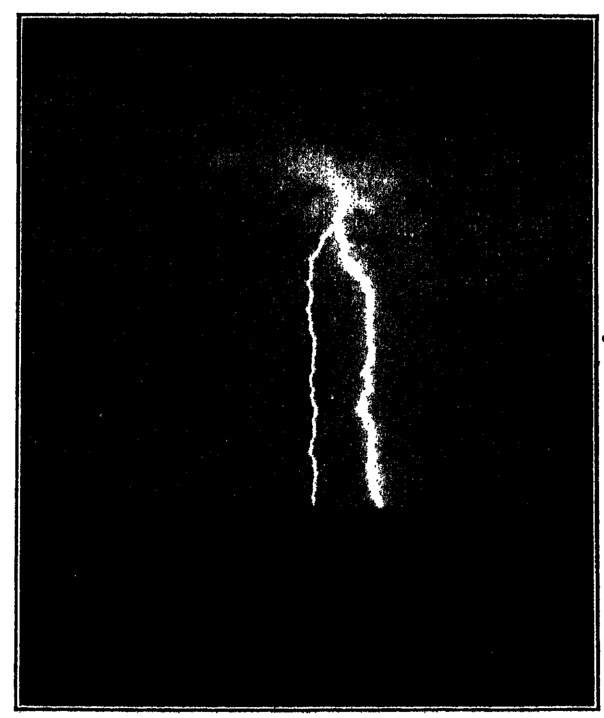

FIG. 25—ILLUSTRATION SHOWING LIGHTNING STROKE NEAR CHEOH-ALCOA LINE

resistance in obtaining the highest measure of protection from ground wires against direct strokes. The value given for $V_1$ with no ground wires was calculated on the basis that the lightning stroke hit only one

6. J. H. Cox and J. Slepian, "Effect of Ground Wires on Traveling Waves," *Elec. Wld.*, September 22, 1928.

conductor. This illustrates that a large measure of protection is obtained from ground wires even with a high ground resistance.

ACKNOWLEDGMENT

In preparing this paper we had the assistance of several of our colleagues in the Engineering Department of the Westinghouse Company who took part in the various activities in connection with the lightning investigation. In particular the contributions to the success of this development were made by Mr. E. Beck and Mr. O. Ackerman, who carried out the actual design work under the direction of Doctor Norinder and Mr. A. L. Atherton. In the technique of handling the oscillograph in the field, a great deal of credit is due Mr. A. M. Opsahl whose skill and experience in cathode ray oscillographs was of great value in the field investigation. The observation stations were in charge of Mr. M. E. Gainder and Mr. Robert Sparks who carried out the work very creditably.

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# Discussion

### THEORETICAL AND FIELD INVESTIGATION OF LIGHTNING

(FORTESCUE, ATHERTON, AND COX)

### FIELD AND THE LABORATORY

(PEEK)

NEW YORK, N. Y., JANUARY 29, 1929.

**K. B. McEachron:** It is a matter of considerable interest to me that just a year ago at the Mid-Winter Convention in discussing Dr. Harald Norinder's lecture I stated that we had succeeded in finding a means of automatically setting the cathode ray oscillograph in operation when lightning came along. Since that time we have seen four or five installations, and a certain amount of data has been secured.

The study of the effects of lightning on electrical systems is naturally divided into two parts. One is to find out what kind of surges natural lightning produces both for indirect and direct strokes. The other part is to apply to lines impulses similar to natural lightning and observe the effects not only on apparatus but the effect of lines and apparatus on the waves traveling in the conductors.

During the interval since the last Winter Convention, the lightning-arrester research laboratory of the General Electric Company has begun a study of artificial lightning on the Turners Falls transmission line, which extends from Pittsfield to the Connecticut River, almost 40 mi. away.

A portable impulse generator which gives an open-circuit voltage of nearly half a million has been constructed. At the Pittsfield end a cathode ray oscillograph has been set up. By the use of this equipment it is possible to apply to the transmission line voltages of any desired wave shape and to record what happens to those waves when they strike apparatus in the station. This can be repeated as often as desired and thus the results may be checked accurately.

The problem of attenuation is easily solved by this method because we are able to apply to the transmission line not only full waves but chopped waves which correspond to what happens when lightning flashes over insulation, so that the effect of the various factors which influence attenuation may be determined.

This method of investigation is particularly valuable in studying the effects of protective devices. Considerable effort has been expended in the past two or three years on one system in particular to secure quantitative data on the protective value of lightning arresters when functioning under natural lightning discharges. Because of the fact that we are unable to have the arrester there and not have it there for the same lightning surge, the data have been inconclusive, but now we are able to go to the transmission line and apply a definite wave both with and without the arrester and find out what the degree of protection is.

This work has brought out an interesting side line. Until a year ago we were quite satisfied to make all our calculations on transmission lines in terms of perpendicular waves. Once we began to put on these lines waves which were not perpendicular, we were no longer satisfied with calculations of perpendicular waves, and it became necessary to develop means of calculating waves which are not perpendicular and which do not have perpendicular tails. This has been quite successfully worked out, and in a forthcoming article in the *General Electric Review* will be given a method which I believe is new for calculating the

effect of capacity and inductance, and combinations of capacity and inductance and resistance at the terminals of lines.

**J. H. Cox:** As in the past, Mr. Peek's paper has given us some new and valuable information on the subject of Lightning. However, I feel that the note of definiteness and finality in many of his statements and deductions is not warranted by the available facts, and coming from one prominent in the field of lightning research, is to be regretted, as it is likely to be misleading to those not so familiar with the actual situation. One would be inclined, after reading Mr. Peek's paper, to believe that the whole story about lightning is known, and to feel that any more work along this line is unnecessary. Such a feeling would, indeed, be unfortunate in view of the vital need of more lightning data, and of the fact that we are now only fairly started on the most difficult and expensive part of the lightning research program. As Messrs. Fortescue and Atherton and I attempted to bring out in our paper presented at this meeting, lightning still is the limiting factor, preventing the realization of the possible advantages of transmission systems. We do not have lightning-proof lines, we do not know enough about lightning, and we need more data.

True, the klydonograph has yielded much valuable information, but it has definite limitations. Data from it indicate rather definitely the magnitude of lightning surges as limited by line insulation, the number of surges occurring at one point, the attenuation along the line, and the polarity of surges. They indicate only approximately the steepness of the wave front, practically nothing of the duration and the magnitude to which lightning surges would go if not limited by line insulation, and nothing of the shape of the tail of the waves. Mr. Peek states that surges recorded by the klydonograph indicate that insulation has impulse ratios of from 1.8 to 2.8 to lightning surges. The surges indicating these values probably flashed over on the front of the wave and therefore indicate something of the wave front but nothing about the intended duration of the native lightning surge. The only klydonograph records that indicate anything about the duration of lightning surges are those of surges which did not flash over the line. In our rather wide experience with this instrument we have only recorded one surge indicating a high impulse ratio when there was not a simultaneous line flashover, and we have a great number of data which indicate that it is possible for a surge to flash over a line without a power follow and a trip out. Also, one such record is far too few on which to draw any conclusions, in that direction, particularly in view of the limitations of station logs. The conclusion indicated by the klydonograph records is that line insulation is not capable of withstanding any surges much above the 60-cycle crest flashover voltage for their entire duration, and that the impulse ratio for lightning surges is very little greater than one. From laboratory data this indicates that the usual duration of lightning surges is not less than 20 microseconds, but something over 20 microseconds, although this cannot be conclusive.

All of the above merely emphasizes that, although the klydonograph has greatly enriched our knowledge of lightning, there is crying need for further information. This can be obtained with the present forms of the cathode ray oscillograph. At the present time we have the sum total of two authentic oscillograms of lightning on transmission lines in the United States, one obtained by the Westinghouse Company and one by the General Electric Company. In addition to these there are those obtained on low-voltage lines in Europe, as discussed by Dr. Norinder before the Institute at the 1928 Winter Convention. These are far too few on which to base conclusions and none of them is all that could be desired. However, the experience in obtaining them was sufficient to assure that entirely satisfactory oscillograms can be secured. Undoubtedly, they will be secured, but it will take a great deal of effort and all possible support is needed. In view of this, it would be most unfortunate to delay the program of this work by misleading those from whom support is needed, to believe that the lightning surge problem is solved.

**J. J. Torok:** Mr. Peek has mentioned these phases of "Transient Phenomena" that are most interesting to the operating engineer. He has dealt with the characteristics of transients as they appear upon the transmission line including some of their effects upon apparatus. The conclusions drawn are in general taken from data obtained in the laboratory. The laboratory experiments are in turn justified, chiefly by the close agreement of the impulse ratio of insulators as determined by klydonograph studies in the field and by more exact methods in the laboratory.

In the study of insulator flashover, it happens that the boundary conditions are of cardinal importance, since they occur rather frequently. The waves corresponding to low impulse ratios are of low magnitude and long duration, thus simulating a 60-cycle wave. To reduce the number of flashovers incurred from such waves, it is only necessary to arrange the insulators and auxiliary equipment to give a high 60-cycle breakdown value. In general, the increase in 60-cycle breakdown is accomplished by creating a more homogeneous field with large-surface electrodes. Hence, grading rings and arcing rings should have large surfaces to prevent low voltage surges, that are not detrimental to the transformers, from flashing over the insulators and causing outages.

Large-surface electrodes possess very desirable characteristics under rapidly rising surges of high magnitude. They set up a more homogeneous field in which the breakdown process is far more rapid once it is started. As a result they have a low impulse ratio. The deteriorative effect of the surges upon solid insulation is a factor that increases with the voltage, thereby branding any protective device giving high impulse ratios as detrimental.

Thus it is seen that large-surface electrodes have the most desirable characteristics for all types of surges. Simple laboratory tests have shown that rain drops do not change the characteristics of large-surface rings to any appreciable extent.

Mr. Peek has tried to show the process of insulator flashover by means of a volt-time cathode ray oscillogram. Much can be learned from such records. However, a study of sphere-gap breakdown has shown that only a part of the story is told in volt-time oscillograms, since a kink in a wave may come from other causes than breakdowns. Dr. J. Slepian suggested that the streamers developing during the process of breakdown might draw an appreciable current. Accordingly various tests were carried out on sphere-gaps as well as insulators. In homogeneous fields, streamer currents of over 3000 amperes have been recorded. In a non-uniform field these currents are considerably lower. In the latter case the streamer currents become appreciable only after the ionization process has been carried to the final stages.

Recent work on insulator strings equipped with sharp-edged arcing rings has shown quite conclusively that the purely corona currents shown in Fig. 21 of Mr. Peek's paper are too small to detect with an oscillograph sensitive to a current of 5 amperes. The observations were made up until 8 microseconds after the potential had exceeded the 60-cycle flashover value. The wave had an abrupt front and a flat top maintained at 1.25 times the 60-cycle breakdown value. The current did not become appreciable until the streamers or actual breakdown had started, after which flashover is inevitable. With currents as small as those due to corona between rings only, it would take at least a 100-mi. line of such construction to reduce a 900-kv. surge to 700 kv., which was the 60-cycle flashover value of the arrangement tested. In view of these results, we would be interested to have Mr. Peek explain more fully how enough energy to prevent a flashover can be absorbed on one insulator string.

**P. H. McAuley:** In the Marx surge generator which Mr. Peek criticizes, the supply voltage is rectified and the condensers are charged in parallel and discharged in series. The group

voltage is limited by the a-c. voltage available and by the voltage rating of the rectifiers. Standard equipment is used throughout.

With Mr. Peek's system the charging period of the condensers necessarily is limited to 1/240 sec. Furthermore, unless the polarity of the generated surge is left to chance, a synchronous timing switch or a tripping device must be used. To obtain a small number of gaps in the discharge circuit, which is of little practical advantage as a matter of fact, it is necessary to use special high-voltage condensers of large dimensions and of limited capacity. An increase in voltage or capacity is complicated compared to the other connection.

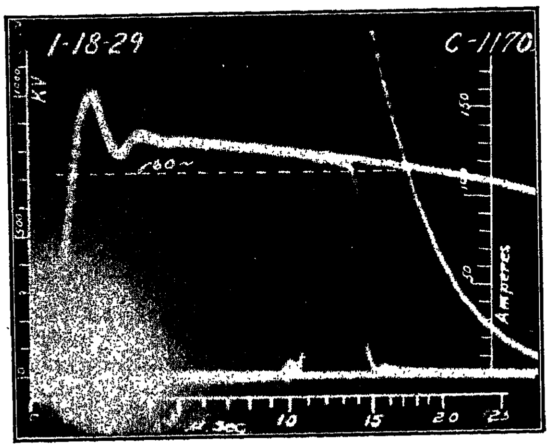



FIG. 1

Perhaps the main advantage of the Marx-connected generator is its flexibility and ease of handling. This is very well illustrated in connection with cathode ray oscillograph work. Fig. 1 herewith shows an oscillogram made with 3 successive applications of voltage to an insulator string.

1. Full wave without flashover
2. Flashover at 14 microseconds
3. Current on flashover

Note the perfect synchronization of the two voltage waves up to the time of flashover. The same perfection is obtained with a time scale many times as large. The completeness of detail compared to Figs. 5 and 6 of Mr. Peek's paper is very noticeable.

**C. F. Harding:** The papers at this session confirm the belief even more positively than the many previously presented upon this subject, that the cathode ray oscillograph, equipped for field use with adequate accessories and automatic control devices, is the only instrument suitable for the study of lightning transients. Although it may be supplemented by the klydonograph for determining approximately the attenuation of the transient, only the cathode ray oscillograph, automatically operated by the transient itself, will provide a permanent record of the magnitude, steepness of wave front, shape, and duration of the lightning surge.

The first step, therefore, looking toward the solution of the problem of adequate lightning protection, is to secure several such oscillographs, obtain simultaneously the maximum number of records upon transmission lines, upon exposed primary distribution systems at various voltages throughout the country and even upon secondary distribution systems. Secondly, such transients as have been found to exist should be reproduced by means of lightning generators in the laboratory and the cathode ray oscillograph used to check the type and magnitude of the transient used and its effect upon the equivalent line or lightning-protective equipment.

Such an oscillograph has been designed, constructed, and used by R. H. George, Research Associate of the Purdue University Engineering Experiment Station at LaFayette, Ind.[1]

Its advantages, particularly for field use, as demonstrated last summer upon the 140-kv. transmission lines of the Consumers Power Company of Jackson, Michigan are as follows:

(1) A sharply focused electron jet of high photographic intensity is available at all times, without fogging the film. Such a beam has been in readiness for hours at a time without film fogging.

(2) The transient itself initiates the exposure, whether it be positive or negative, and reproduces all of the surge after the first one-fourth or one-half microsecond. Advantage is taken of the voltage, proportional to the rate of change of leading current in the condensive circuit of an air-core, close-coupled transformer to affect the grid potential of one of two vacuum tubes, the plate voltage of which controls the electron-jet release of the oscillograph.

(3) The jet is readily focused to a sharp image upon the film by means of a new electrostatic focusing control operated from outside the vacuum. This process is not sensitive to gas pressure within the vacuum chamber.

(4) This oscillograph, which is of the hot-cathode type, provides many advantages, such as satisfactory operation with the wide range of gas pressure likely to exist within the oscillograph chamber in a field installation and with a variety of voltages ranging from 500 to 20,000 volts as an accelerating potential for the electron jet. Any gas pressure below 30 microns may be employed. The deflecting plates may be adjusted from outside the vacuum chamber.

This oscillograph is portable, entirely self-contained, and operated from any 110-volt, 60-cycle source with less than 1 kw. of input. The transient deflecting potential and the transient initiating voltages are supplied from simple aerials or capacitance potentiometers placed near the line upon which the surge is to be measured.

(5) A film holder, of the daylight-loading type, provides from 75 to 100 exposures, each 3 by 5 in. in area, with one loading. The vacuum is not broken during this period and the unexposed area of film may be moved into place for exposure in a predetermined sequence by manual or automatic operation.

(6) A timing wave from an oscillator, ranging from a 10,000- to a 500,000-cycle sine wave, is provided to spread out the transient which is easily recorded with a 10-kv. beam when traveling across the film at 125 mi. per sec.

It is hoped that many parallel investigations may be made during the next two or three lightning seasons in order that the interruptions to service may be reduced to a minimum.

**Herman Halperin:** From reading the papers, I get the impression there are various degrees of confidence in knowledge of lightning and its effects. It seems to me that considerable investigation is still necessary. For instance, there are only a few oscillograms on wild lightning.

It would be well to extend these studies to distribution circuits ranging from 115 volts up to, say, 15,000 volts, since the operation on such circuits affects the customers directly.

All of the oscillograms on wild lightning show single strokes although meteorologists say that frequently several strokes occur very close together in time and space.

Perhaps the effects of such multiple flashes would be different in line apparatus as compared to the effects of single flashes.

**G. D. Floyd:** The Hydro-Electric Power Commission placed in operation October 1, last year, a single circuit of 220-kv. line, extending from Ottawa to Toronto, a distance of about 200 mi.

In going over the literature that has been accumulated on lightning surges and correlated data, we found that apparently

1. This instrument is described in *A New Type Hot-Cathode Oscillograph*, by R. R. George, which will appear in the A. I. E. E. Quarterly TRANS., July 1929.

the greatest number of data had been accumulated on the magnitude of the lightning surge on the transmission line itself and a relatively small number on the surge that might appear at the terminal apparatus.

I feel that there seems to be a difference of opinion among investigators on this subject, so I want to make a plea for those engineers who have made close and comprehensive study of the question of insulation strength and insulation surges, if it is possible, to get together. In the situation that exists, the buyer of apparatus doesn't know where he is.

I do think that Mr. Peek's paper was a little optimistic. There is a remarkable agreement between the actual lightning surge that he recorded and the one that he produced. I think one is very optimistic indeed if he would say from the result of ten records obtained in actual surges that the agreement would be maintained in all cases. I think that even a thousand records would be necessary before one could conclusively say that the actual surge was alike in all cases. It may be that if a hundred were taken, ten of them would appear like the record Mr. Peek has obtained and the other ninety would be altogether different.

**C. E. Ambelang:** Referring to Mr. Peek's paper, we have at present seven lines of over 200 mi. total of single circuit on double-circuit towers, with provision for two ground wires, one ground wire installed above each circuit, half way between the center of the tower and the conductor.

Last year we had three lightning interruptions in these seven lines, two of which were, we believe, direct strokes, one being over all three phases, and the second one involving 12- and 33-kv. lines on wood poles that were underneath the 132-kv. line.

Our insulator strings have a lightning flashover value, according to Mr. Peek, of 1350 kv. Using a 60 per cent reduction due to ground wire we have an induced lightning voltage of 3375 kv., which gives us a potential gradient for an average height of a conductor, 72 ft., of 46.8 kv. per ft.

After applying this to a wood-pole line, a 35-ft. pole would give us 1638 kv. induced voltage on our wood-pole lines. Mr. Peek in his paper gives an average insulating value of wood poles of 180 kv. per ft., which would make a 35-ft. pole with a 5-ft. crossarm have a lightning sparkover value of 7200 kv. That is a potential gradient of 206 kv., which is four times that on the tower lines in the same territory.

Citing from our records, there are three pole lines operating in the same vicinity. These lines are without ground wires.

Line *A*—12-kv. line, 17 mi. in length, had 33 cases of insulator flashover due to lightning last year. Eighteen were on guyed structures and 15 on unguyed structures.

Line *B*—33-kv., 17½ mi. long, 15 cases of flashover, 8 on guyed and 7 on unguyed poles.

Line *C*—33-kv., 19 mi. long, had 11 cases of flashover, 4 on guyed poles and 7 on unguyed poles.

It is these unguyed structures that we are worrying about because we ought to have some way of reducing the flashovers, especially if that insulating value of the wood pole is anywhere near correct. We should like to ask Mr. Peek what his explanation would be of the large number of cases of lightning flashovers on the wood-pole lines.

Mr. Peek states, "When a pole is quite wet, incipient sparks will take place over the insulator string at voltages approximately equivalent to the lightning sparkover voltage of the insulator string."

Could it be possible that such sparks could be responsible for the trouble which we have experienced, these sparks occurring within the limit of the voltage induced by lightning during ordinary storms?

**E. S. Lee:** An oscillogram of a lightning wave taken last summer is shown in the accompanying illustration Fig. 2. This was taken July 27, 1928 on the 220-kv. Wallenpaupack-Siegfried transmission line of the Pennsylvania Power and Light Company by means of a General Electric cathode ray oscillograph. The record shows that the surge voltage was positive in polarity and uni-directional, reaching its maximum in 5 microseconds, and decreasing to half value in 20 microseconds, and to zero in 40 microseconds. Superimposed on the surge at the peak was a highly damped oscillation of 2,000,000 cycles probably caused by induction from the insulator flashover which occurred on an adjacent conductor.

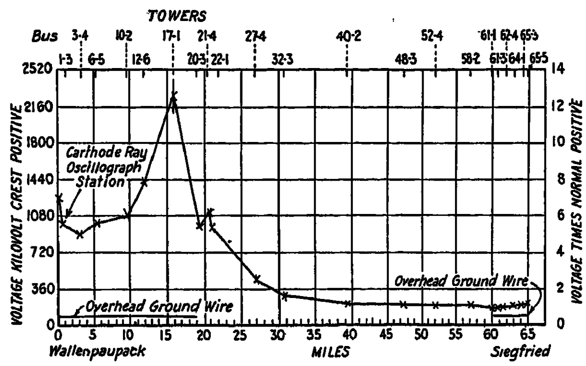



FIG. 2—VALUES OF VOLTAGE RECORDED BY SURGE VOLTAGE RECORDERS ALONG THE LINE

Surge voltage recorders connected to Y conductor Wallenpaupack-Siegfried 220-kv. line. Pennsylvania Power and Light Co.

As far as is known, this is the first cathode ray oscillogram of a surge voltage due to lightning obtained on a high-voltage power transmission line in the United States. It is remarkably similar in time characteristics to the waves which have been used in the laboratory. Fig. 28 in Mr. Peek's paper shows the almost exact similarity between the wave form of this surge and one used in his laboratory for experimental work. Also the cathode ray oscillogram shown by Messrs. Fortescue, Atherton, and Cox, in their Fig. 13, shows the wave form of a surge voltage with essentially these same time characteristics, if I interpret their oscillogram correctly.

When this oscillogram was made, simultaneous records of voltage were made by surge recorders connected to the same conductor (the West conductor) at various points along the line. These are shown graphically in Fig. 3 herewith. The maximum recorded voltage was at tower 17-1, the value being between

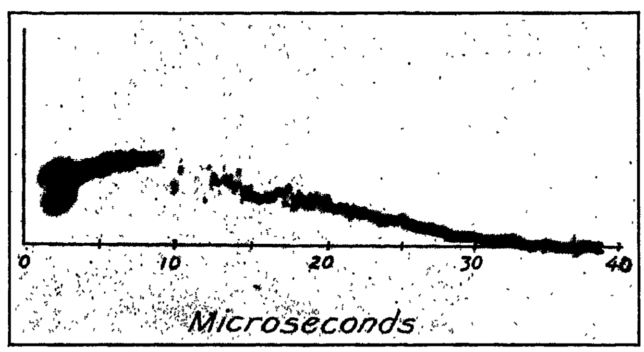



FIG. 3—FIRST CATHODE RAY OSCILLOGRAM OF LIGHTNING SURGE ON TRANSMISSION LINE IN AMERICA

2100 kv. and 2450 kv. At tower 1-3 near this cathode ray oscillograph station the voltage measured was 1000 kv. At the end of the line the voltage was 1260 kv., showing the characteristic increase at the line terminal. Towards the other end of the line the voltages measured decreased rapidly in value.

Line inspection following this surge showed that at tower 16-3 three insulators were broken on the middle conductor and practi-

cally all the insulators were flashed on the east conductor. A surge voltage recorder connected to this conductor at tower 1-3 recorded 1660 kv. The insulator string on the middle conductor at this tower flashed over. At this tower the spacing between the shielding rings had been reduced to provide protection for the terminal apparatus.

**P. H. Thomas:** I want to raise one point in connection with the impulse ratio and the flashover strength that can be counted upon from insulator string. Consider a short length of transmission line, an antenna, if you please, which is open at the ends. Upon a lightning discharge, the first result of the flash is to leave a free, unbound charge on the transmission line which raises its potential to a high steady value. As long as the line is insulated, this potential will remain. The insulators supporting the line must withstand this potential on its so-called 60-cycle insulating value. Is this not the condition we have on actual transmission lines except for the fact that after the flash, current or charge can run off in one direction or the other along the line itself?

If this statement is true, and I don't see how we can escape it, and we assume an exposure in which there is a large section of line which has been raised to this high potential, will there not almost inevitably be time enough before the central portion of the exposed section can be discharged by the melting away from the ends, to allow the strength of insulators to be reduced nearly to the 60-cycle value? The only assumption on which we can conclude that a high impulse ratio can be counted upon is that the section of line affected is very short. This apparently is usually the actual fact.

Instead of picturing actual lightning as a flash between a large flat level metal plate and the earth which is another plane surface, let us picture the charged cloud as a highly irregular upper plate with probably a funnel shaped portion pointing downward toward the earth from which the discharge actually occurs. Then it would seem very reasonable to assume that the only portion of a transmission line which has the maximum induced potential would be of very short length. Whether this would be less than one span or more than two or three spans, I don't know. In such a case it is easy to believe that a charge once accumulated and suddenly freed by the flash will actually begin to flow off even before the discharge is completed. In such a case we will get a wave form for the tail of the surge in the line, similar to the forms which we see on the oscillograph. It is only on this assumption of very short line exposures, as I see it, that we can explain such tails to the surge.

May we not conclude that in a surge on a transmission line, the rising part of a lightning surge, the 1 or 2 or 5 microseconds portion, the steep front is the part whose form is due to the lightning and the tail part, which is from the crest down, is not due to the lightning but merely depends upon the amount of line affected by the induction and the facility with which it flows off at the end.

There is one other matter I should like to mention briefly and that is the effect of resistance in the ground connection of towers. A great deal of importance has been put upon this matter of ohmic ground resistance. I am not convinced that ohmic resistance is important. If you ground both ends of a long transmission line and pass alternating current through it at 60 cycles and measure the total inductance and resistance of the loop, you will find that there is a very large and predominating amount of reactance in the ground return.

The net result is this: That we must consider even in lightning, I am sure, the effect of the reactance of the path of the current in the ground as well as its resistance.

In measurements of ground resistance and reactance at 60 cycles, the reactance is greater than the resistance. At lightning frequencies the reactance of that circuit is multiplied 100,000 times, while the resistance does not change except for skin effect and is much less than the increase in reactance. From this it would seem to follow that the resistance is relatively unimportant in lightning surges.

**J. R. Eaton:** The Consumers Power Company cooperating with Purdue University has been studying the effects of lightning on transmission lines in a manner very similar to that described in the paper by Messrs. Fortescue, Atherton, and Cox. The cathode ray oscillograph used in the study is of a type recently developed by R. H. George of Purdue University as mentioned in C. F. Harding's discussion.

Fig. 4 herewith is an oscillogram of a laboratory wave made with the instrument operating exactly as when set to record lightning transients. From this record it is seen that the instrument gets into operation in less than ¼ microsecond, and records the complete wave shape thereafter.

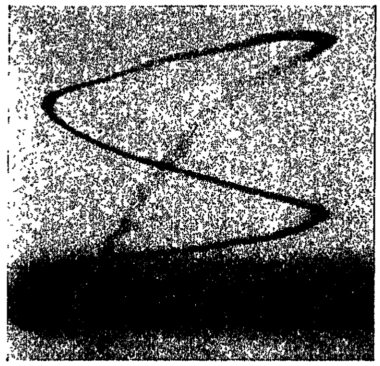


Fig. 4

The instrument was connected to a 140-kv. line through a capacitance potentiometer quite similar to the one used by the Westinghouse Company as was described by Mr. Cox.

During the period of operation no lightning discharges took place in the immediate neighborhood of the oscillograph station. Storms on distant sections of the system produced flashover which resulted in general system disturbances. Four cathode ray oscillograms were recorded as the result of such surges but the wave shapes obtained were probably the record of secondary disturbances rather than of the lightning itself.

The results of the past summer's work has demonstrated to our satisfaction that this oscillograph is well suited for recording the wave shape of lightning surge voltages.

In the early spring the investigations will be continued with the set in operation in the middle of a 100-mi. line. This point has been chosen with a view to eliminating wave distortion due to reflections at the end of the line and to lightning arrester operation.

**Edward Beck:** One of the most interesting aspects of Mr. Peek's paper is the confirmation of the high-speed cathode ray oscillograph as a tool for lightning research. This instrument now gives us knowledge where before there has been much conjecture. For example, it was formerly believed among certain investigators that lightning disturbances were exceedingly steep,—fronts of a few hundredths of a microsecond were frequently mentioned. Before the cathode ray oscillograph was available it was thought that waves of such steepness were easily reproduced in the laboratory by circuits whose calculated constants would lead to the expectation of waves with fronts of this order. The oscillograph has shown that factors not always considered previously, enter into the calculations of impulse generators, particularly where steep-front transients are concerned, and that exceedingly steep waves are not generally procured except by very special methods. On the other hand, it was found that the agreement between calculations and actual measurement for waves with longer fronts is excellent.

We are led to believe by oscillographic studies that the fronts of natural lightning transients are not as steep as formerly believed. Both of the oscillograms of transients on high-voltage transmission lines presented before this convention are apparently in agreement that the fronts are of the order of five microseconds. Of course the data so far collected are too meager to draw definite conclusions at this time, and no one would be rash enough to say that the problems have been solved. With the tools now at our command, we are on the threshold of complete knowledge, but the work has but little more than begun. One of the oscillograms shown today is complete, showing the entire front. There is still much information to be collected concerning the various transients apt to occur, such as their coordination with the location of strokes, rate of cloud discharge, and details regarding the transient fronts. But it has been established that these things can now be measured, and with oscillographs that make it possible to obtain complete records, the information will not be long forthcoming, particularly with concerted effort. The two oscillograms published, while they agree as to front, indicate already a variation in duration. Whereas the Pennsylvania transient indicates a duration of 20 microseconds above line voltage, the Tennessee record shows a duration of 50 microseconds above normal voltage. On this evidence I must disagree with Mr. Peek's conclusions that the duration of lightning transients may be 1 to 20 microseconds only. The similarity of the fronts lead us to believe that a wave with a quarter microsecond front is a very steep one, whereas formerly this was considered relatively slow and much more rapid ones were thought frequent.

I should like to ask Mr. Peek what is the crest voltage of the lightning transient shown in Fig. 28 of his paper, also whether the time lag of the oscillograph introduced by the external switching arrangement is sufficiently definite and constant so that the missing parts of the wave fronts can be interpolated with accuracy. Our laboratory investigations show that irregularities of several microseconds occur.

**Philip Sporn:** In connection with Mr. Floyd's discussion of Mr. Peek's paper, it might be of interest if I mention some experience we had in putting surge voltage recorders on equipment at terminal points. For quite a number of years we have made a practise of installing permanent recorders at all the large transformer installations and treating them as part of the equipment. We did that particularly on complicated transformer designs. Over a period of three years, we have obtained a considerable number of data that we believe has helped us in our design and operating problems.

**H. L. Wallau:** (communicated after adjournment) In Mr. Peek's paper reference is made to Mr. Lewis' work in connection with the attenuation of lightning surges along a transmission line.

The information usually desired is the distance along a transmission line which a surge will travel before the initial voltage $E\,o$ is reduced to some given value $E$.

The formula as given there by Mr. Lewis' is not in convenient form for this solution but may be converted to that below.

$$L = 6200\ Y / E\,o$$

in which

$L$ = distance in miles

$6200 = 1/K = 1/0.00016$

$Y = (1 - x)/x$

where $x$ is a decimal fraction such that $x\ E\,o = E$.

It will be seen that for a decrease to one-half of the original value of surge voltage $Y$ becomes unity and $L = 6200/E\,o$.

For other values of $E$ the value of $L$ so obtained is increased or decreased by the corresponding value of $Y$ as shown below.

| Value of $E$ in per cent of $E\,o$ | Multiplying factor to be applied to $L = 6200/E\,o$ |
|---|---|
| 90 | 1/9 or 0.11 |
| 80 | 2/8 0.25 |
| 70 | 3/7 0.43 |
| 60 | 4/6 0.67 |
| 50 | 5/5 1.00 |
| 40 | 6/4 1.50 |
| 30 | 7/3 2.33 |
| 20 | 8/2 4.00 |
| 10 | 9/1 9.00 |

**H. L. Melvin:** (communicated after adjournment) Referring to the section of Mr. Peek's paper on the subject of wood poles, I am assuming that the data and information were taken from the tests made during the past year for several of the operating companies under the general supervision of the company by which I am employed. These tests were undertaken for the purpose of obtaining some fundamental data on the impulse insulation characteristics of wood-pole lines constructed for 66-kv. to 132-kv. operation. It is hoped that sufficient actual operating experience will be obtained by the close of the coming lightning season on the several transmission lines, on which the results of the tests are being tried in an experimental way utilizing the insulating value of wood in varying ways and degrees, to check the information obtained in the laboratory. It may also be possible to obtain some idea of the value of increased impulse insulation on the performance of transmission lines.

As Mr. Peek has stated the species of wood, amount of moisture or contamination seems to have little influence on the impulse sparkover values. Quite a variation was observed between the samples tested, but this is as would be expected since ordinary wood should not have uniform insulation characteristics. A value of 170 kv. per ft. plus or minus 20 per cent is being used for wood poles and 190 kv. per ft. plus or minus 20 per cent for wood crossarms and timbers of smaller cross sections suitable for long wood guy insulators. These values are for tail of wave sparkover and the specific test wave used. They also include practically all the test points.

It might be stated that the 6-to-3-to-1 ratio of wood to air gap to clearance of the gap from the wood for designing by-pass horn gaps to protect wood poles from damage would be critical for an average specimen and the particular test wave. To provide a factor of safety for the average pole and provide protection for poles as low as 20 per cent under the average a ratio of 10-to-3.6-to-1.75 is being used. These relations cannot be used for crossarm protection as the voltage appearing on the arm is a function of the relation between the type and amount of porcelain and the crossarm length. Whether by-pass air gaps employing these dimensions will protect poles under actual lightning conditions will require field experience.

To obtain sparkover values for combinations of porcelain insulation and wood it was necessary to make tests on the combinations by varying the amounts of porcelain, crossarms, and pole through the ranges which might be used in practise. The sparkover values of the component parts cannot be added directly, which is to be expected, and it is also impracticable as yet to give any rule which can be applied for combining the individual values to get the overall sparkover voltages.

If the results of these tests are confirmed by field experience and a considerable improvement in operating performance can be expected from lines with high impulse insulation strength, having available complete test data of this character should make it possible to improve the conventional wood-pole designs and develop modified designs which will utilize the wood to a much greater degree and more effectively. This may not be

practicable in all situations, however, on account of the possibility of crossarm and pole fires.

Possibly it would be well to caution against the application of these very incomplete data as there are many other factors to be taken into account and additional data required in a co-ordinated design. This discussion should simply be taken as a brief outline of the tests and their possible application.

**Harald Norinder:** (communicated after adjournment) The paper presented by Messrs. Fortescue, Atherton, and Cox gives a full and complete description of a field research station for investigation of lightning transients on transmission lines. This paper has been completed by a theoretical discussion of the propagation of lightning surges on transmission lines and of the effect of ground resistance on the protection afforded by overhead ground wires. The theoretical part of the paper completes in a very happy manner the existing literature, particularly as the authors' treatment is in such a form that it is easily applied to actual problems on transmission lines. The effect of ground-wire protection has been treated by the authors in an original manner which will have considerable importance in all experimental work on ground-wire protection.

It is with great satisfaction that we can state that several fully equipped stations for the investigation of lightning surges have been erected in locations in the United States where the expected frequency and violence of thunderstorms is high. There is thus a good opportunity for securing valuable records in a relatively short time. It is unfortunate that only one surge of large magnitude has occurred in these locations. However, this does not diminish the general and future value of the station. For a number of years, I have had the opportunity to observe the occurrence of lightning transients on lines in Sweden. Sometimes a whole group of thunderstorms will pass in the vicinity of the transmission lines without producing any lightning surges at all. On the other hand, the lines have carried whole sets of disturbances in other storms. There are many accidental circumstances which govern the production of surges on electrical systems. Hence I am not astonished that more records were not secured last summer.

In this connection it will be of interest to recapitulate the lightning researches which have been carried on in the Laboratory of the Swedish Royal Board of Waterfalls near Uppsala. Ten years ago the Board of Waterfalls decided to proceed with a systematic research of the general field conditions during thunderstorms. The motive for this work was a desire to obtain records of field changes produced by lightning discharges and thus to have data for studying the development of lightning surges on transmission lines under full load conditions. This extensive research in Sweden was suggested by the Chief Director of the Swedish Royal Board of Waterfalls, W. Borgquist. His opinion was that researches along these lines were a practical necessity. He believed that it was quite necessary to revise our ideas on the overvoltage problem and that this revision could be undertaken only with the aid of extensive experimental work. The same point of view was later taken also in other countries. At the present time the necessity for such work is clear to everyone who is concerned with the possibility of service of long transmission lines. At the beginning of the researches, much of the literature failed to disclose any information on the field forces or general field condition during thunderstorms. Opinions were divided and the conclusions arrived at were sometimes not in accord with existing physical facts and laws. Therefore it was necessary to begin at the beginning.

The first steps in the research were to study the field conditions by extensive experimental work. In the existing literature it was assumed that the field forces were of the order of 3 kv. per ft. Observations at three or four stations soon showed field forces at least ten times higher. These results have later on been fully confirmed by the extensive investigations carried on in the United States by means of klydonographs.

From these higher field forces we could derive conclusions regarding the distribution of charges that were dissipated by lightning strokes. With high field forces, the regions must be relatively small and their radii sometimes cannot have been more than approximately 1000 ft. Simultaneous records from three stations operating at considerable distances from each other fully confirm these conclusions. This relatively small extent of the discharged regions made it possible to explain why a transmission line often does not show the expected frequency of occurrence of lightning surges during the storm. The field changes were sometimes extended over such a limited region that only an inconsiderable part of the transmission line was influenced. Data thus obtained were of great value in estimating the order of magnitude of surges on transmission lines, but it was not possible, by this method of observation, to secure any records of the character of the wave front and duration of surges.

Our next problem was to analyze the variations with time of the gradient in free atmosphere during the lightning disturbance. Such observations should give valuable indication as to how to take up the most important problem, namely, direct records of transients on transmission lines operating under normal conditions.

For this purpose I constructed a cathode ray oscillograph consisting of a closed cathode tube, a photographic lens system, and a revolving drum. By this arrangement, it was possible to record variations of 0.0001 sec. The oscillograph was connected to a suitable antenna circuit so balanced that it followed the lightning variation without appreciable distortion. The first results were obtained during the summer of 1921. They showed that the lightning discharges were unidirectional or quasi-periodic. It was sometimes quite evident that it took considerable time to develop the discharging process. In 1923 the cathode ray tube was connected to a transmission line. Thus I was the first to have opportunity to observe visually lightning transients on a transmission line. These direct visual observations were of value for the later construction of a special cathode ray oscillograph for recording these transients.

The construction of such a special instrument was begun in 1922 and during the season of 1925 we had the good fortune to take the first records of transients on the transmission line. Since then the instrument has been improved in many respects. In its present form it is identical with the cathode ray oscillograph described by the authors of the paper. With this oscillograph I have recorded numerous lightning transients on a 20-kv. transmission line during normal operation. The results I presented during the 1928 Winter Convention of the A. I. E. E.[2]

The cathode ray oscillograph necessitates several accessory pieces of apparatus, the most important of which is the high-vacuum pump arrangement. We have put forth considerable effort in the development of an effective high-vacuum pump. Another important adjunct is an indicator for the transient. The transient may be observed visually on a separate cathode tube with a fluorescent screen. This tube is connected in parallel with the cathode ray oscillograph. Hence it is possible by visual observation of the spot on the tube to follow the records, their amplitude, and their polarity.

Comparison of my records obtained at the Uppsala Station where I had used a method of coupling to the line analogous to the one used in the Tennessee location shows full agreement in the general shape and structure of the surges. In the immediate future, we shall take up particular studies of the wave-front variations. For that purpose we have need of several cathode ray oscillographs working simultaneously. Although in general the records secured more or less agree, it is dangerous to conclude from this that we will be able to discover some one typical kind of lightning surge. One must not forget that the primary phenomena which occur during thunderstorms are too shifting to allow regular forms of transients on the circuits. This

2. A. I. E. E. Quarterly TRANS., Vol. 47, April 1928, p. 446.

conclusion applies particularly to disturbances from direct strokes. The most recent studies of the character of lightning discharges show the most variable discharge forms. I will illustrate this by some typical records of lightning discharges at the Uppsala Station. These records which are reproduced in Fig. 5 herewith were secured during the thunderstorm season of 1928.

During these researches we connected the cathode ray oscillograph to an antenna circuit with a time constant of a few microseconds. The circuit would thus follow the field variations without distortion. The method of investigation has been described by me in the *Franklin Inst. Jl.*, Vol. 205, June 1928. Only one oscillograph could be devoted to these researches. Hence it was not possible to study the variations at more than one location. In order to save time it was necessary to record several lightning strokes on the same film. By means of the simultaneous indications of the visually observed cathode tube, it was easily possible to separate the records on the film.

In Fig. 5 there are reproduced 8 separate lightning strokes with high amplitude. Besides these there are some smaller records produced by more distant strokes, sometimes of a pronounced slow moving characteristic. The remoteness of the 8 strokes cannot be given with full certainty in all cases. The

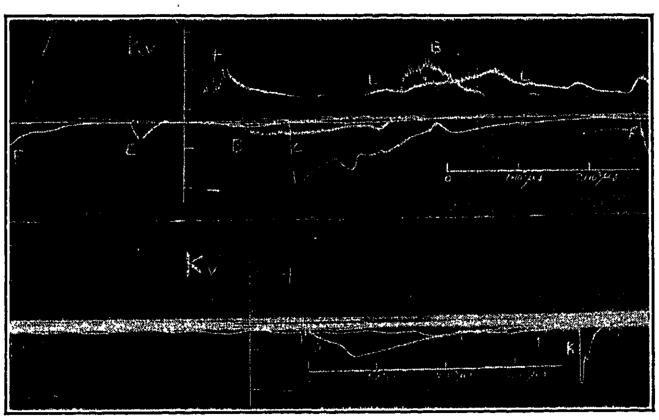


Fig. 5—Lightning Discharges Recorded with Cathode Ray Oscillograph at the Uppsala Station, Sweden, During the Summer of 1928

strokes marked with *K* and *I* were at a distance of either 3 or 3.2 kilometers. *B* was 3.9 kilometers away. The stroke marked *A* was 1.4 kilometers away and *C* and *F* either 2.8 or 2.1 kilometers. In Fig. 5 we note only two, namely *A* and *L*, which showed pronounced positive charge in the antenna circuit. *B* was quasiperiodic with changing charge in the circuit. The remaining ones showed negative charge in the antenna circuit.

Comparing, for instance, *B*, *C*, and *I*, we note essential differences in their general form of variation with time. This is physically well explained by the very shifting field structures in a thunderstorm atmosphere. In only a few instances do uniform conditions exist. It is possible that the discharge of *C*, *F*, and *E* are produced one after another with no significant differences in the general structure of the field.

The time to reach a crest value is very variable. We need only to compare *K* with *I* to observe this. In one case the crest value is reached in a few microseconds, in the other case we estimate the time to reach crest value to be about 800 microseconds. Of particular interest are the superimposed high-frequency voltage variations with amplitudes reaching 40 per cent of the voltage variation of the main discharge. These variations are very pronounced in *B* and *L*. In the future we shall take another time scale to analyze more completely the superimposed high-frequency variations. Using only one record station I have not found it possible to get a physical explanation of these superimposed fluctuations. As I have already pointed out in the *Franklin Inst. Jl.*, they are of interest with regard to the severe atmospherics in radio receivers.

The phenomena of lightning discharges were discussed before the Franklin Institute. In that discussion I based my opinion on researches made before the season of 1928. The results of the past season are in full agreement with earlier ones. The discharge process of a lightning stroke must, as a conclusion from our records, be preceded by partial streamer discharges which penetrate the dielectric and thus produce an ionization over increasing regions. The phenomenon is thus similar to the suppressed partial discharges that have been observed by Torok between sphere-gaps and described by him before the Institute.[3] Messrs. Fortescue, Atherton, and Cox have in their paper given a picture of what takes place just before and during a lightning discharge in the atmosphere. A close examination of the records of Fig. 5 show that their conclusions are quite true.

We have observed in some of the records that such discharges with the above mentioned suppressed discharges are pronounced. Such a one is *K* in Fig. 5. The crest value was in this case reached within a few microseconds. Careful examination of the records, however, shows that the stroke was prepared by a set of streamers of short individual duration. The total number of the streamers in this case has been estimated to be about 50 during a time of 1000 microseconds. Thus it has taken quite a long time to build up an ionization process sufficient to permit the main stroke to occur.

Not in all cases have we been able to study the discharge process as clearly as in this case. The record obtained depends somewhat on the location of lightning discharge with respect to the measuring circuit. In the above mentioned case the orientation must have been a very favorable one.

A further problem of interest is the general variation in form of lightning strokes when the strokes constitute a connection between cloud and earth. At least one and probably two such strokes were recorded in Fig. 5. Either *C* or *F* constitutes the general form of a lightning stroke between cloud and earth. At the moment of one of these two discharges a lightning stroke was observed to hit a wet pole of a 3000-volt line. This pole was about 2 kilometers away from the station. The 3-kv. line was connected to the 20-kv. line through a transformer. This line went out of service through its protective apparatus. It is thus evident that one of these two records *C* or *F* was caused by a stroke between cloud and earth. I cannot definitely say which one of them it was. The other stroke caused a heavy surge on another 20-kv. transmission line at some hundred meters distance from the one mentioned above. In the case of this surge, however, we have no certain observation of a direct stroke on the line.

We can conclude that discharges *C* and *F* represent types which occur when the stroke occurs between cloud and earth. Neither the lines nor the transformers were injured by this direct stroke, a circumstance that may be considered a little astonishing at first thought. We must remember, however, that there is pronounced difference between different lightning strokes.

There is no doubt that we must calculate with lightning strokes having energy varying in amount from 50 or 100 coulombs down to a fraction of a coulomb. It is obvious that in general direct strokes can pass into the poles and transmission lines without any hazard at all.

It is not possible to draw general conclusions from some few records. It is dangerous to make generalizations. The problem of lightning surges on transmission lines can be cleared up only by extensive and intensive research. My results secured by field studies are proof that such work is possible.

In our work in the study of lightning surges, we have had at

3. *Surge Impulse Breakdown of Air*, by J. J. Torok, A. I. E. E. Quarterly Trans., Vol. 47, April 1928, p. 349.

our disposal only one oscillograph. This has brought about a deplorable limitation in our work. For instance it has not been possible to make simultaneous records of surges at different points on a transmission line at the same time. Our efforts for future work are directed to the equipment of our station with several oscillographs which may work simultaneously. Visitors in our laboratory have sometimes expressed the opinion that "A cathode ray oscillograph is too complicated an affair." This is true, but requires qualification. The real fact of the matter is that lightning or a lightning disturbance is a complicated phenomenon and it is rather remarkable that research on such complicated phenomena can be carried on with apparatus not more complicated than our cathode ray oscillograph.

Objections have been made that it will always be difficult to engage people skillful enough to carry out such work. This is not the case as has been borne out by experience with the Tennessee station. There it has been possible to build and put into operation two fully equipped stations within a short time. Moreover, it will always be possible to find young scientists and engineers who have interest enough in the problem to surmount all difficulties which may exist. It is especially fortunate that the results of such research on lightning are of the utmost practical importance, besides being of great scientific interest. Such a happy coincidence will always be stimulating. It should be pointed out that it is not only for protection of transmission lines that the research on lightning is important. The protection of buildings and oil tanks and the elimination of atmospherics at radio receiving stations are other phases of the problem which await exact solution.

When discussing the future of lightning studies among interested people, I have always predicted that the United States will probably constitute the most fitting region for extended lightning and thunderstorm studies. My reasons for stating this are three: relatively high frequency and the violence of lightning storms in America, the great development and importance of transmission lines and other structures requiring protection, and last but not least, the traditions from the days of the venerable Franklin, whose keen experiments are worthy of the highest admiration.

**C. L. Fortescue:** (communicated after adjournment) I agree with Mr. Peek's remarks to the effect that lightning investigation has reached a stage where it may be said to be on an engineering basis. However, I wish to emphasize the fact that as Mr. Peek remarks, there is a great deal more to learn about lightning before we can confidently say we know enough to predict how a given transmission line will behave under lightning conditions.

I was very much interested in Mr. Peek's description of his new lightning generator which is apparently a modification of the Marx scheme which we are using in our high-voltage laboratory. The difference is that in Peek's generator the system of condensers is charged direct from the a-c. system, which means that the charge must be built up in the condensers in one-quarter cycle, and since the charging must be effected through high resistances, this appears to me to be a serious limitation. The limitation of the Marx scheme, which uses direct current for charging, is not in the matter of charging the condensers but in the limit of the rectifier tubes which seems to be about 100,000 volts direct current. Consequently, with a d-c. scheme many more sections are required. On the other hand there is no limit to the time of charging. However, I do not agree with Mr. Peek that the Marx scheme using direct current is only applicable to low-voltage surges. In our Trafford Laboratory we have a 2,000,000-volt generator which has been extremely satisfactory and which could, if necessary, with equal reliability, be extended to much higher voltages.

The gratifying progress that has been made within the last few years in the study of lightning phenomena naturally leads one to be somewhat optimistic in regard to what is known about lightning surges at the present time. My impression on first reading Mr. Peek's paper was that he presents too rosy a view of what has been achieved up to the present time in regard to lightning investigation and the same impression might be conveyed to those unfamiliar with the difficulties of the investigation and the immense amount of work that has yet to be done before we can claim to have a real working knowledge of lightning and its effects on transmission lines. A further reading of the paper indicates to me that Mr. Peek's reaction was that of a man who has been working for years on a more or less speculative program and begins to see the path broadening out and as a result many of his speculations are being confirmed by actual observation and he naturally remarks "lightning has been removed from the realm of the 'medicine man' and brought to more or less of an engineering basis."

As regards the wave form of lightning, I am inclined to disagree with Mr. Peek in the impression he conveys in his paper that it has a very definite wave form which is closely approximated by his laboratory wave; for example, where impulse ratios of 2 and larger are obtained, with the wave form Mr. Peek uses, the same impulse ratios could be obtained on lightning waves building up at a much slower rate but causing flashover before the crest is reached. Thus for such a wave building up at the rate of 4000 or 5000 kv. per microsecond, an impulse ratio of two or more may be obtained on a 14-unit string. I gather that this is illustrated in Fig. 17 of Mr. Peek's paper, where he shows three waves. One of these is a 20-microsecond laboratory wave, the second an intermediate wave, and the third one having a very long front; all three of them give the same impulse ratio when flashing over an insulator string. Mr. Peek's laboratory wave has a certain advantage from a laboratory point of view and this is that with a very high crest value impressed, where the flashover takes place at the front, he has an extremely steep wave, much steeper than I believe is possible due to lightning.

It is difficult on purely theoretical grounds to deduce how steep a wave front may be obtained from a direct stroke of lightning to the line; possibly some day we may be able to approximate the value from calculations but at the present time we do not know enough about the behavior of lightning, the volume of the cloud that is discharged, etc., to be able to tell how much of the line is affected due to a direct stroke.

I am inclined to think that Mr. Peek is over-optimistic in his estimate of the rate of rise of the lightning wave which he shows. Our experience is that the time required to initiate the cathode stream may vary considerably. so that the steepness of the wave front of the natural lightning wave which he shows in Fig. 28 may be a great deal slower than he supposes. I should like Mr. Peek to give in his discussion the details of his initiating scheme and such tests as he has made to determine its speed of response. However, I take it that Mr. Peek thinks that any wave which will give an impulse factor on a string of insulators, as high as that obtained with lightning, is satisfactory and he does not insist that his laboratory wave is necessarily the wave representative of actual lightning. In fact, one might say that the effect on insulator strings of 10 or more units, of the most severe lightning waves as recorded on the klydonograph up to date is the same as the effect of a wave having a slope of 4000 to 5000 kv. per microsecond and having a crest value such that flashover takes place on the front or just at the crest.

Regarding Part IV, I agree with Mr. Peek except in the following statement which he makes: "However, the waves usually causing insulator sparkover give an impulse ratio of the order of two (2.0) and indicate an effective duration of one to 20 microseconds above the 60-cycle flashover value." As I have remarked in the last paragraph, this might just as well be caused by a long-front wave which, if the insulator did not flash over, might give a much higher value than when the flashover takes place on the front. I agree with Mr. Peek that lightning corona loss may very materially decrease the voltage due to lightning

when ground wires are employed. I do not agree that corona from grading rings will have an appreciable effect in preventing flashover of insulator strings.

In regard to the question of lightning-proof transmission lines and coordination of transformer insulation and line insulation, I agree in general with Mr. Peek's conclusions but it is necessary to be very cautious in regard to such phrases as "lightning-proof transmission lines." As I have stated in our paper, the direct stroke of lightning may vary all the way from a very light stroke to an extremely heavy stroke. We have no information yet as to the possible potential that may be built up in a transmission line due to a heavy stroke of lightning. We do not know therefore whether a system of ground wires could be designed which would adequately protect transmission lines against the most severe strokes. However, I myself am quite optimistic and believe that in the course of time we shall be able to achieve a transmission line which is practically lightning-proof.

Mr. Peek's conclusions from his Pittsfield tests as to the time for a cloud discharge do not satisfy me. The necessary ionization phenomena are much slower than might be inferred from his results. Our work with suppressed discharges indicates a much longer time than he gives; however further data will no doubt enable us to determine the actual time of discharge. It is my belief that the minimum time is considerably more than 10 microseconds and the lower limit given by Mr. Peek of one microsecond is absolutely impossible.

To sum up, I think Mr. Peek has presented in his paper a very interesting summary of our knowledge up to date in regard to lightning phenomena. In some respects it presents perhaps too rosy a picture, which may mislead those who are not familiar with the difficulties to be overcome in obtaining scientific data on these phenomena, but as usual he has presented a very worth while paper.

**J. H. Cox:** The work being carried on in connection with artificial surges on the Turner Falls transmission line described by Mr. McEachron cannot be too highly commended. There is a great deal still unknown regarding the performance of surges on actual lines. A great amount of theoretical work has been done on this subject but the values which have been used for the line characteristics may need modification. It is certain that these can be determined with the use of artificial surges in a small fraction of the time which will be required if use is made only of natural lightning surges.

A start was made on this problem in 1927, in the tests on the Rankin-Wilmerding line described in the authors' paper. These tests were on a line insulated for 33 kv. Mr. McEachron is extending this information with his tests which are on a line insulated for 66 kv. Plans are now being made for a similar test on a 220-kv. line to be performed cooperatively by the Public Service Electric & Gas Company and the Westinghouse Electric & Manufacturing Company. With the tests now in progress information on the performance of surges on transmission lines covering the entire range of lines now in service should soon be available.

It is true, as Mr. Floyd points out, that economics do not permit the study of the effect of surges on actual terminal apparatus to the point of destruction. However, even this study can be made economically feasible by the use of full-size models which present the same effect as actual apparatus but which cost only a fraction of the amount of money. We have done considerable work with apparatus models in the laboratory and the value of this work can be greatly increased by combining it with surge tests on actual lines. Furthermore, the effect of the apparatus on surges can be studied with actual apparatus without particular hazard to that apparatus.

The oscillograph described by Professor Harding has considerable advantage in certain types of work. Where the taking of an oscillogram of natural lightning depends upon getting the oscillo graph into operation after the arrival of a surge, the time delay is cut down and made definite by having a source of ionization available, as with the use of the hot cathode. It has been our experience with the use of the cold cathode, where the electron jet depends upon ionization by collision, that the time required to bring the jet to the film is too long and too irregular to be satisfactory. If Professor Harding is able to obtain a good jet in one-quarter microsecond and operate with the oscillograph evacuated only to 30 microns, the instrument is certainly valuable.

Mr. Halperin has stressed the need for lightning information in connection with the lower-voltage circuits. This accentuates the need for wider spread application of cathode ray oscillographs. With the present number in use and the more urgent need in the higher voltage range there are no instruments available for distribution-line investigation. It is agreed that this information is necessary and in time a sufficient number of oscillograph applications must be made to obtain it.

Mr. Thomas has discussed the nature of a lightning surge. However, his discussion pertains only to the induced stroke. We have come to the conclusion that the induced stroke is relatively unimportant in connection with high-voltage transmission lines. It is true, as he says, that the surge can travel away on a continuous line during the discharge of the cloud. This lengthens the surge as he describes, and we are of the opinion that it lengthens it to a point where its voltage is not important. The relatively slow rate of discharge of the thundercloud is shown by some of Dr. Norinder's records on isolated antenna where the bound charge could not get away. The extremely short line presents the same effect as an antenna but this is not a usual case.

Although we would state the reasons somewhat differently, we agree with Mr. Thomas that the 60-cycle crest voltage is the criterion of the strength of the line insulation. Laboratory tests have shown that the lower limit of impulse flashover of line insulation is very little more than the 60-cycle crest flashover for surges 10 microseconds or more in duration. All evidence indicates that lightning surges, whether induced or direct, are at least this long.

Mr. Thomas is correct in saying that the ground resistance is not the only important factor in the impedance to surge current. Not only reactance but capacity also is involved. The resistance plus the surge impedance of tower footing and ground return must be considered. Probably the reason that more attention has been given to the ground resistance is because very little is known regarding the surge impedance of towers and grounds and there is not much that can be done regarding the surge impedance. This is one of the factors which will be studied in the investigations of surges on transmission lines.

Some comments have been made regarding the disagreement existing between various investigators in this field and the suggestion has been made that these investigators get together and present a united front to the industry. We do not think that this would be particularly desirable since it would be misleading in indicating that the problem was solved when that is actually not the case. The reason there is disagreement is because certain factors are not definitely known and therefore are speculative. Different individuals interpret the existing evidence differently and therefore disagree. So long as this is the case the public has the right to be informed and should not be led to believe that the questionable factors have been determined. However, we feel sure that although there is a certain amount of disagreement as to some of the features of the problem, there is complete accord in the desire for further knowledge on the subject and the conviction that a great amount of further research work is necessary.

**F. W. Peek, Jr.:** I shall confine my closing remarks as closely as possible to those phases of the discussion that have to do with engineering. It is my opinion that most of the differences, possibly because of the short time that the paper has been available, have been due to hasty reading. For instance, Mr. Cox states that after reading my paper "one would be

inclined to believe that the whole story of lightning is known." In the first paragraph I state, "While there is still much to learn, lightning may be said to be at least on an engineering basis." I purposely was brief in the introduction in order to allow space for the numerous data included in the paper. The data at least were carefully obtained and the reader may use them in reaching his own conclusions. The fact that the lightning voltage has been increased to 5,000,000 and that very extensive field and laboratory work is under way indicates that there was no intention on my part of discouraging further work. In fact, I am still doing everything possible to expedite this type of research just as I have done in the past.

I shall answer the discussions of Messrs. Cox, Torok, McAuley, Beck, and Fortescue in a group since their points over-lap to a considerable extent.

Data are given on the sparkover of sphere-gaps, points, insulators, etc., up to 3,600,000 volts and for waves varying from about 5000 microseconds (60 cycles) duration to less than 1 microsecond duration. The whole possible range is covered. Data were also obtained on the front of waves reaching the crest in 1 to 100 microseconds. This wide range was covered for a comparison of all possible results in practise. In making comparative tests several waves have been used as standard. This was necessary for obvious reasons. These standard waves are the ones that give the same lightning flashover voltages as obtained in practise. The crosses in Fig. 11, which show the sparkover voltages measured on 4-, 10-, and 14-unit strings for natural lightning, correspond to laboratory values for the 5 to 20 microsecond range. This time is given above the half-voltage value or while the wave is active in causing sparkover (for impulse ratio of 2). The total length of these waves is thus 10 to 40 microseconds. Impulse ratio for this range is of the order of 2. It is quite probable that lower impulse ratios will be observed in practise, particularly at the lower voltages.

I have not stated that all lightning waves are of the same shape as the one obtained on the Wallenpaupack line; on the contrary, lightning waves, because there are all sizes and conditions of clouds, must occur in a very wide variety of ways. A large range of waves must be expected. However, where the voltages are very high the waves must be steep and due to rapidly discharging clouds. What I did state was this: Measurements with the surge-voltage recorders give the maximum value of the voltage in case of an insulator sparkover by lightning. An approximation of the steepness of the wave or the effective duration is obtained by comparing the sparkover as measured by the surge voltage recorder or klydonograph with the crest 60-cycle sparkover. In the many measurements that have been obtained on high-voltage lines, the lightning, where the voltage was high enough to cause sparkover, approximated twice the 60-cycle value, or the impulse ratio was 2. This means waves of an effective duration or steepness of front as indicated in Figs. 5 and 11 or for effective durations (time above the 60-cycle arcover value) of 5 to 20 microseconds. Of course the actual length of the lightning waves may be very much longer than these values. For instance, a sparkover may occur on the front of the wave reaching crest in 10 microseconds.

It can be seen why the wave fronts are likely to be steep when voltages are high. There are two ways in which high voltages can be obtained—either by direct stroke, which must be applied very suddenly and thus be steep and give a high impulse ratio, or by an induced stroke from a rapidly discharging cloud. The induced voltage upon a line depends upon a charge on a given length. If the charge is spread over twice the length the voltage is reduced to one-half. The induced voltage occurs when a bound charge on a line is released by the discharge of the cloud. If the cloud discharges slowly the charge is released slowly and spreads over a considerable portion of the line since it travels at the rate of about 1000 feet in a microsecond. The induced voltage then cannot be high. On the other hand, if the cloud discharges rapidly the charge does not spread and the induced voltage is very high. For example, assume a 1000-ft. cloud or a 1000-ft. section of charged line and a gradient of 100 kv. per ft. ($V = g\, h\, \alpha$). If the cloud discharges instantly the induced voltage on a long transmission line 40 ft. high would be 4000 kv., for a cloud discharging in 5 microseconds, 1000 kv., and for a cloud discharging in 10 microseconds, 600 kv. These voltages, however, would obtain only with a discharge in the near vicinity of the line. All the above waves would be considered steep. The voltage values show that high induced voltages on transmission lines require a rapidly discharging cloud. Although there must be all kinds of waves during a thunder storm, the highest voltage ones must be fairly steep, and the lower voltage ones less steep. It is because of this that the laboratory tests have covered such a wide range of waves and data over this range have been given in the paper.

I dislike to enter any discussion other than an engineering one on the lightning generator. My original lightning generator operated without a rectifier and is described in my paper *Effect of Transient Voltages on Dielectrics*, A. I. E. E., September 1915, to which I refer you. In this early investigation it was found convenient to use the term "microsecond" as a unit of time. The term "impulse ratio" was also first used in this paper as a convenient method of giving the effect of the lightning voltage. In the wide range of tests in that paper it was necessary to calculate the wave shape and determine the time of the transients in that manner. However, some of these tests have since been repeated and actual measurements by the Dufour oscillograph have checked this early work. This lightning generator contains inductance, capacitance, and resistance. In fact it is similar to all other electrical apparatus in that respect. However, the important point is that it does develop 5,000,000 volts directly (probably approximately 10,000,000 volts by reflection) with considerable energy and produces waves of any shape desired. These waves are readily measured by the cathode ray oscillograph. In fact, I might state that this oscillograph is used in the laboratory to the same extent as a voltmeter or an ammeter.

The tests on the grading shield were made with top and bottom rings. The loss is caused by a multitude of incipient arcs distributed around and between the strap rings. These arcs are much more intense than the corona but not sufficiently developed to permit the dynamic voltage to follow. As much as 25 per cent of the voltage has been dissipated in this manner. The question as to the relative value of the strap ring and a ring of a very large section has risen. There is no particular gain in the use of a ring of large section, since when it becomes wet with the first rain drops its arcover voltage is reduced to the needle-gap value.

The natural lightning waves measured on transmission lines are very accurate and definite. The time for actuating the oscillograph was well established by means of a parallel oscillograph. The method used in making the oscillograph self-actuating employed the three-electrode gap.

I was very much interested in Mr. Thomas' remarks. My discussion on the cloud discharge above applies. The crest, shape, and length of lightning waves depends upon the initial distribution of charge on the line and the rate of discharge. The length of the wave approximately equals the length of the bound charge plus the distance of travel during the time the cloud is discharging. Of course a large part of the tail of the wave has no effect in causing insulator arcovers since it is at a very low voltage. The effective part of the wave is that portion above the 60-cycle arcover of the insulator. The maximum value of voltage is lower the longer it takes the cloud to discharge, and becomes less with decreasing length of the initial charge or with the size of the cloud. The front depends upon the size of the cloud, or rather on the length of the charge on the line and the rate of discharge.

I cannot give Mr. Ambelang a definite reply without further information. However, the incipient arcs over insulators when

the poles are wet, to which I refer in my paper, may cause phase-to-phase arcs at fairly low voltage. Low-voltage arcovers may also result due to the shortened path at the guys. In designing a wood-pole line, to take advantage of the insulating value of the wood, these various factors must be considered.

It has been our goal to devise methods for predetermining lightning voltages on transmission lines, to obtain the lightning strength of insulators, to determine the best methods of protection and of making apparatus highly resistant to lightning, to study the origin, travel, and decay of lightning waves on transmission lines, and to determine the effects of lightning waves on inductance coils and transformers. Of particular interest has been the cathode ray study of what happens inside of a transformer when a lightning impulse strikes it. External waves cause tremendous internal stresses in transformers. The result of this study has been the development of a non-resonating transformer. In this new transformer, localized stresses are reduced as much as 100:1 compared to the ordinary transformer. Our general method has been to make tests in the laboratory on models and full size apparatus, to send lightning waves over lines and determine their effects on insulators and apparatus, to make extensive tests in the field, and to coordinate the results by a mathematical study. I have given the above brief description of our methods in answer to Mr. Floyd's question regarding the effect of lightning on terminal apparatus.

It is quite true, as in the case of all pioneer work, that a certain amount of speculation was necessary in the early stages of the research. Progress is made in that way. However, the necessity for speculation in this work has now become very much reduced due to the definite knowledge that has already been gained.

As to a lightning-proof line, I am quite certain that it is coming. In this connection I might point out that principles developed in research applied to existing lines have reduced outages as much as 10:1. The main problem now is to determine economic methods of preventing outages due to direct strokes.

The operating data given by Mr. Sporn are extremely interesting. Statistical data from the operating companies, if carefully made, would be of great value to us in solving these problems. I might state here that we are already greatly indebted to the engineers of many operating companies, and advancements that have been made are greatly due to their help and cooperation.

# 1927 Lightning Experience on the 132-Kv. Transmission Lines of the American Gas and Electric Company

BY PHILIP SPORN[1]

Member, A. I. E. E.

## INTRODUCTION

WITHIN the past two years a series of papers,[2,3,4] was presented before the Institute on the performance of a number of 132-kv. steel tower lines on the system of the American Gas and Electric Company during the years 1925 and 1926.

It is proposed in this paper to continue and give the 1927 history of the 132-kv. transmission system investigated and described in the previous three papers.

## GENERAL

Fig. 1 shows the 132-kv. transmission network in question. As has been previously pointed out, it comprises approximately 948 mi. of line, which was in service for all or a part of 1927; the total circuit miles being approximately 1266 in service for the same period. Details with regard to the territory traversed, the nature of the country, and details with regard to the generating stations, and the points where they feed into the network have already been described.[3]

## 1927 PERFORMANCE

A brief summary of the principal characteristics of the various lines, together with their 1926 and the 1927 lightning performance, is given in Table I. It should be noted in this connection that while no quantitative data are available, the opinion gathered from all parts of the system indicates that in general, 1927 was a year of less severe and less frequent lightning than 1926.

An examination of the table shows the following with regard to the various lines:

*Glen Lyn-Roanoke.* The number of outages, while still high, was considerably less in 1926 than in 1927. Further, the number of outages, as compared with the performance of the other lines in the same territory, does not show up very much out of line. The number of cases of damage to insulators, conductors, and hardware, in proportion to the number of outages, is unusually high; the line, however, had no arcing protection devices of any sort during that entire year. It is pertinent also that the cases of damage on this line were determined by an actual climbing inspection of every tower on the line. The damage to the insulators varied from light marking of the glaze to the shattering of two or three units in a string; and the damage to the conductor varied all the way from slight pitting and small blistering to the burning of three and four strands in the conductor.

*Glen Lyn-Switchback.* This line was not in service a sufficiently long period during 1927 to furnish any authoritative data.

*Lima-Fostoria.* The number of outages was about half of what it was in 1926. The decreased amount of lightning during 1927 would, however, furnish the reason for this.

It will be seen that the total number of cases of damage exceeds the number of outages, the probable reason being that the cases of damage cited include a

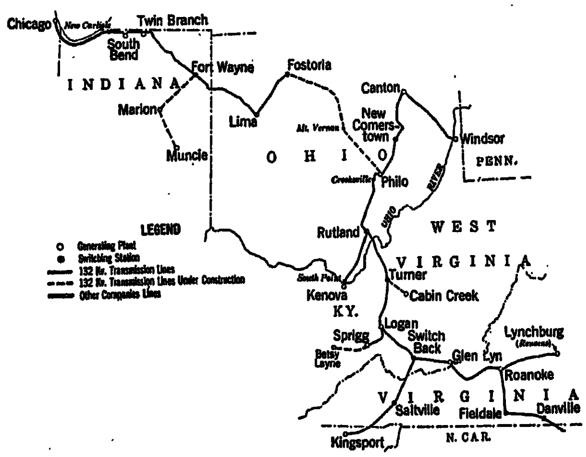



FIG. 1

number of cases accumulated from previous years that had not been detected until 1927.

The damage to the insulators in this case varied from mere burns on as many as three or four units in a string to broken insulators, while the conductor damage varied from pitting and light burning for a few feet from the point of contact, to burning of as many as four strands.

*Lima-Twin Branch.* The 1927 performance of the Lima-Twin Branch line when compared with its performance during 1926, bears about the same relationship as the 1927 performance of the Lima-Fostoria line bears to its 1926 performance. The cases of damage here are definitely known to represent accumulated damage from the previous year. The cases of damage disclosed in every instance but one, only a slight blistering of a considerable number of units

1. Electrical Engineer, Amer. Gas and Elec. Co., New York, N. Y.

2. *Lightning and Other Experience*, Sindeband and Sporn, A. I. E. E. TRANS., Vol. 45, p. 770.

3. *1926 Lightning Experience on 132-Kv. Transmission Lines*, Philip Sporn, A. I. E. E. TRANS., Vol. 47, p. 668.

4. *Surge Voltage Investigation on the 132-Kv. Transmission Lines of Amer. Gas and Elec. Co.*, Philip Sporn, A. I. E. E. Quarterly TRANS., Vol. 47, October 1928, p. 1132.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

TABLE I
CHARACTERISTICS AND PERFORMANCES OF 132-KV. LINES OF AMERICAN GAS AND ELECTRIC COMPANY 1926 AND 1927

| Characteristics / Column No. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Line designation......... | Glen Lyn Roanoke | Glen Lyn Switch-back | Lima Fostoria | Lima Tw. Branch | Logan Sprigg | Philo Canton | Philo Crooks-ville | Philo Turner | Roa-noke Dan-ville | Roa-noke Reusens | Rutland So. Point | Saltville Kingsport | Switchback Logan | Switchback Saltville | Turner Logan | Tw. Branch So. Bend | Windsor Canton | So. Bend Mich City |
| Length of line (miles).... | 65.0 | 30.0 | 45.6 | 128.5 | 21.0 | 73 | 15.4 | 118.7 | 65.0 | 43.0 | 50.3 | 56.0 | 50.0 | 46.0 | 40.2 | 4.9 | 55.0 | 40.0 |
| Number of circuits....... | 2 | 2 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 2 | 2 | 2 | 1 |
| Number of ground wires.. | 1 | 1 | 1 | 1 | None | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | None | 1 | 2 | 1 |
| Rings and horns......... | No | Yes | No | Yes | No | Yes | No | No | No | Yes | No | Yes | Yes | Yes | Yes | Yes | No | No |
| 1926 Lightning Performance | | | | | | 1925 1926 | | | | | | | | | | | | |
| One circuit only out...... | 16 | | 9 | 23 | 3 | 13 | 3 | 11 | 0 | 13 | 3 | | | | 16 | 0 | 9 | .. |
| Both circuits out......... | 4 | | .. | .. | .. | 2 | .. | .. | .. | .. | .. | | | | 2 | 0 | 2 | .. |
| Total outages........... | 20 | | 9 | 23 | 3 | 88 15 | 3 | 11 | 0 | 13 | 3 | | | | 18 | 0 | 11 | .. |
| Circuit outages for 100 mi. of line per yr. 1926..... | 48* | 0† | 19.8 | 17.9 | 31* | 120 20.5 | 19.5 | 10.6* | 0* | 36.6* | 6.3* | 0† | 0† | 0† | 44.8 | 0 | 20 | .. |
| Circuit outages per 100 mi. of circuits per year 1926 | 24* | 0† | 19.8 | 17.9 | 31* | 60 10.3 | 19.5 | 10.6* | 0* | 36.6* | 0.3* | 0† | 0† | 0† | 22.4 | 0 | 10 | .. |
| 1927 Lightning Performance | | | | | | | | | | | | | | | | | | |
| One circuit only out..... | 21‡ | | 4 | 8 | 7 | 3‡ | 1 | 7 | 16 | 16 | 7 | .. | .. | .. | 20 | 1 | 2‡ | 4 |
| Both circuits out......... | 3 | | .. | .. | .. | 1 | .. | .. | .. | .. | .. | .. | .. | .. | 4 | 1 | 0 | .. |
| Total outages........... | 24 | 0 | 4 | 8 | 7 | 4 | 1 | 7 | 16 | 16 | 7 | .. | .. | .. | 24 | 2 | 2 | 4 |
| Circuit outages per 100 mi. of line per yr. 1927..... | 37 | 0* | 8.8 | 6.2 | 33.3 | 5.5 | 6.5 | 5.9 | 24.6 | 37.2 | 13.9 | 0* | 0* | 0* | 57.2 | 41 | 3.6 | 10.0 |
| Circuit outages per 100 mi of circuit per yr. 1927.. | 18.5 | 0* | 8.8 | 6.2 | 33.3 | 2.8 | 6.5 | 5.9 | 24.6 | 37.2 | 13.9 | 0* | 0* | 0* | 28.6 | 20.5 | 1.8 | 10.0 |
| Damage to insulators.... | 17 | 1 | 7■ | 28■ | 0¶ | 0 | 0 | 2 | 14 | 2 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 1 |
| Damage to conductor and hardware............. | 14 | 3 | 5 | 0 | 0¶ | 3 | 0 | 1 | 5 | 7 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 |
| Total cases of damage.... | 20 | 3 | 9 | 28 | 0¶ | 3 | 0 | 2 | 14 | 7 | 0 | 0 | 0 | 0 | 5 | 1 | 0 | 1 |

*Corrected for 1 calendar year (in service only part of year.)
†Not in service.
‡One circuit out of service at the time of F. O. in 1 case.
¶Superficial inspection—Towers not climbed.
■Probably partly accumulation of 2 previous years.

in a string, the number varying from three to all ten in the string. In none of these cases was any conductor damage noted, indicating that these cases of damage had resulted from a cascading arc that did not stay on long enough or did not do enough damage to be detected from a ground inspection. This would explain why they were not discovered, therefore, until a tower climbing inspection was made.

*Logan-Sprigg.* This line, which is a wood pole steel crossarm line unusually well insulated (even without giving any weight to the insulation of the wood itself) showed up during 1927 even worse, from the standpoint of outages, than it showed up in 1926, and except for the Roanoke-Reusens and the Turner-Logan line had the largest number of outages per hundred miles of circuit. The line has no ground wire.

It will be observed that no cases of damage were discovered, but on the other hand, the inspection that the line received was mainly a ground inspection.

*Philo-Canton.* The number of outages on this line was only 30 per cent of the number that occurred in 1926, which is accounted for partly by the mildness of the 1927 lightning previously referred to. It is interesting, however, to compare this with the number of outages in 1925. As will be seen, in 1925, when no ground wire was installed on the line and no protective devices of the type being employed now were utilized, the number of outages per hundred miles of line was more than twenty times as many as occurred in 1927. During 1927, there was no case where damage of any kind to the insulators occurred, although there were cases where the conductor was pitted. In some cases it was actually burned, but no strands burned through. In one case the horn and ring assembly were quite badly burned, and in the other case they were definitely pitted.

*Philo-Crooksville.* The 1927 experience with regard to outages is in line with the experience on the Philo-Canton line. No damage of any kind could be traced to the one outage that took place.

*Philo-Turner.* The number of outages in this case also was about 50 per cent of the number that took place in 1926. Two cases of damage occurred but these were very severe; in one case four insulators were shattered on one phase, and four insulators on another, and eight strands of wire burned through. No arcing protection of any kind was employed at that tower. In another case on a strain tower, where double strings were employed and a standard set of arcing horns (this particular circuit had been equipped with no arcing protective devices of any kind except at dead end points where standard arcing horns were used) one-half of a double string was completely opened by the burning through of one unit at the line end. The same arc chipped the other twelve units in the string and burned the tips of the horns but did not damage the conductor.

*Roanoke-Danville.* There are no data for the previous year with which to compare the 1927 performance of this line, but it will be observed that its 1927 performance is on about a par with the performance of the Glen Lyn-Roanoke line. The number of flashovers per hundred miles of line is moderately high. The number of cases in which damage occurred is here again relatively high as compared to the number of outages, but again the inspection was very thorough, so that it is certain that all cases of damage were noted. The damage in most cases confined itself to pitting, although in many cases this consisted of severe pitting of all or a portion of the insulators in a string, with blistering of wire for as much as five feet from the point of suspension. In some cases, however, insulators were actually broken and strands of wire were burned, although not through. This line, as will be noted, had no arcing protective devices. It is also well to note in connection with the damage that the possible fault current in the case of a flashover is a minimum for this line as compared with all the other 132-kv. lines on the system.

*Roanoke-Reusens.* The 1927 experience practically duplicated the 1926 experience, the number of flashovers being unusually high. At the present time (October 1928) ground resistance measurements are being taken on the line, which it is hoped will give some clue as to the reason for the abnormally high level of outages on the line. However, all cases of damage on this line were again determined by climbing of every tower and are unusually interesting in view of the large number of outages. Of the seven cases of damage, it will be seen that only two represent damage to insulators, and these were very light spots on the glaze. The conductor, too, was pitted only in two instances, whereas in every case burning and sometimes very severe burning was found on either the rings or the horns, and often on both.

*Rutland-South Point.* The number of outages is not unusually high but higher than in 1926. As was previously pointed out, however, such data for 1926 were obtained for a portion of the season only. So far as it is possible to find out, no damage occurred on the lines. It is definitely known that no serious damage occurred, but the line was subject only to ground inspection during this period.

*Saltville-Kingsport, Switchback-Logan, Switchback-Saltville.* These three lines went into service so late in 1927, that the data obtained cannot be used to form any conclusions in regard to their lightning performance during that year.

*Turner-Logan.* The number of outages on this line was even higher in 1927 than in 1926, and with the exception of the Roanoke-Reusens line and the Logan-Sprigg line, had the highest number of outages per hundred miles of circuit. Neither the Turner-Logan nor the Logan-Sprigg line has a ground wire. The Turner-Logan line is, however, a steel tower line. The number of cases of damage is unusually small in view of the large number of flashovers. This must be

qualified, however, by noting that this line again was not subjected to a thorough inspection for its entire length, but was so inspected for only half its distance. Only one case of really severe damage was found, and this was represented by a single broken insulator. All other cases consisted of very slight pitting of insulators or conductor, or both.

*Twin Branch-South Bend.* On a mileage basis, the number of flashovers this line had in 1927 was high, but it is to be noted that this line is only approximately five miles long. No flashovers were reported in 1926. On the basis of the two year average, therefore, the experience lines up very well with what would be expected. The damage that occurred consisted of very slight blistering to a number of insulators but it is possible here, as in the case of the Lima-Fostoria and Lima-Twin Branch lines, that this was a hangover from the previous year.

*Windsor-Canton.* The number of flashovers on this line was unusually low, being only 18 per cent of the number that occurred in 1926. It will be seen that this line had the least number of flashovers of any line on the entire system. The line has the shortest span and is the lowest above ground of any line on the system and is the only line that has two ground wires.

*South Bend-Michigan City.* The number of flashovers on this line is of the same order as those on the Lima-Fostoria, Lima-Twin Branch, and the Twin Branch-South Bend lines. The one case of damage consisted of slight blistering of all insulators on all three phases.

## Discussion of 1927 Experience with Regard to Several Phases of Design

*1. Effect of Ground Wire.* On the two circuit lines with ground wire in service during all of 1927, there occurred 16.2 outages per one hundred miles of line. On the two circuit lines without ground wire in service all of 1927, there occurred 59.2 outages per one hundred miles per line, a ratio of 3.7. In the case of one of the single circuit lines—a single circuit wood pole line—although the insulation is considerably higher than the average, there occurred 33.3 outages per one hundred miles of line. The Windsor-Canton line with two ground wires had only 3.6 outages per one hundred miles per line per year. It is believed that this experience has further and effectively demonstrated the positive value of the ground wire in reducing the lightning voltages on the transmission line and even more effectively reducing the number of outages.

*2. One vs. Two Ground Wires.* The Windsor-Canton line with two ground wires showed the lowest number of outages per one hundred miles of line per year, namely, 3.6. The average for the entire system is 16.4, including single and double circuit lines and also those with and without ground wires. This average, it is to be noted, covers only those lines that were in service the entire year. On the other hand, it must be borne in mind that the Windsor-Canton line is a much shorter span line than all of the other steel tower lines (the Windsor-Canton average is 8.85 towers per mile). However, the employment of two ground wires has undoubtedly contributed to this performance.

*3. Relative Shielding of the 3 Phases by Ground Wires.* In Table II is shown the location of trouble on

TABLE II
LOCATION OF TROUBLE ON INSULATORS AND WIRE
1927

| Line | Total | Top | Middle | Bottom |
|---|---|---|---|---|
| Glen Lyn-Roanoke | 20 | 6 | 10 | 12 |
| Glen Lyn-Switchback | 3 | 1 | 2 | 2 |
| Lima-Fostoria | 9 | 8 | 0 | 1 |
| Lima-Twin Branch | 28 | 27 | 2 | 0 |
| Philo-Canton | 3 | 2 | 1 | 1 |
| Philo-Turner | 2 | 2 | 0 | 1 |
| Roanoke-Danville | 14 | 5 | 3 | 8 |
| Roanoke-Reusens | 7 | 3 | 4 | 4 |
| Turner-Logan | 5 | 3 | 4 | 0 |
| Twin Branch-South Bend | 1 | 1 | 0 | 0 |
| South Bend-Michigan City | 1 | 1 | 1 | 1 |
| Totals | 93 | 59 | 27 | 30 |
| Totals excluding Lima-Fostoria and Lima-Twin Branch | 56 | 24 | 25 | 29 |
| *Totals excluding Lima-Fostoria Lima-Twin Branch & Turner-Logan | 51 | 21 | 21 | 29 |

*Lima-Fostoria and Lime-Twin Branch excluded as damage appears to be accumulation of trouble since line was first built, and not for 1927 only Turner-Logan excluded, being a line without ground wire.

insulators and wire, separated into the top, middle, and bottom conductors. Disregarding the Turner-Logan line (because of the fact that it had no ground wire) and the Lima-Fostoria and Lima-Twin Branch lines (because of the fact that the trouble discovered in 1927 represented, without a doubt, trouble that had accumulated from previous years and that had not previously been noted) we find 21 cases of conductor damage on the top phase, 21 on the middle, and 29 on the bottom. This would indicate that for the case where one ground wire was employed immediately above and centrally with respect to the two top conductors, the shielding furnished by the ground wire to the top and middle conductors is such that the net average lightning voltage on the two, in spite of the difference in height between the two, is the same; but the bottom conductor, in spite of its still lower level and therefore lower induced lightning potentials, receives so much less shielding than the other two that the result is a higher net induced voltage on it than on the upper two conductors. It would seem, therefore, that another ground wire properly placed with respect to the bottom conductor, would more nearly equalize the net induced voltages and therefore the number of flashovers and number of damages on all three phases. This experience is extremely interesting since calculations on its shielding effect made before the installation of the ground wire, based on the work of Mr. Peek, indicated an expected lightning voltage on the top and middle wires of approximately the same value and an approximately 15 per cent higher value on the bottom wire.

The Turner-Logan line, having no ground wire,

had three cases of trouble or damage on the top, four on the middle, and none on the bottom conductors.

The large number of cases of trouble on the top conductor on the Lima-Fostoria and Lima-Twin Branch line, it is confidently believed, is due to cases of trouble accumulated from the time when no ground wire at all was used on the line. It has already been pointed out that the first complete tower inspection by climbing was made in 1927.

*4. Use of Protective Devices.* Assuming that the cases of trouble on the Lima-Twin Branch line date back mostly to the period preceding the use of the ground wire and the arcing protective devices, we find on that line, on the Roanoke-Danville line, on the South Bend-Michigan City line, and on the Glen Lyn-Roanoke line, 67 per cent of the observed cases of trouble, although these lines represent only 31.5 per cent of the total line mileage. As already pointed out in the discussion of the individual lines, the damage to the lines where rings and horns were employed was in general confined to blistering of the wire and a marking or slight blistering of one or two insulators and only very rarely was a strand burned in two. On the other hand, many cases of burning were found on the rings and horns, although these were in no case serious enough to require replacement of the assembly. Where no rings or horns were employed, however, the damage was not only numerically more plentiful, but from a severity standpoint, was far heavier, and in one case actually burned one side of a double string right through.

*5. Ground Resistances.* Table III shows the

TABLE III
TOWER GROUND RESISTANCE AT TOWERS WHERE FLASHOVER OCCURRED

| Lima–Twin Branch Tower | Ohms Gr. resist. | Tower | (Ohms) Gr. resist. |
|---|---|---|---|
| 7 | 1.2 | 176 | 2.4 |
| 111 | 2.5 | 186 | 2.2 |
| 112 | 2.5 | 189 | 2.2 |
| 138 | 2.8 | 221 | Not measured beyond Tower 210 |
| 139 | 2.4 | 222 | |
| 141 | 2.8 | 240 | |
| 142 | 2.2 | 244 | |
| 143 | 2.3 | 252 | |
| 144 | 2.2 | 265 | |
| 152 | 2.0 | 281 | |
| 165 | 1.3 | 288 | |
| 169 | 2.0 | 291 | |
| 170 | 2.0 | 296 | |
| 175 | 2.0 | | |
| Philo–Canton | | | |
| 82 | 28.0 | 160 | 6.9 |
| 83 | 2.4 | | |
| Philo–Turner | | | |
| 3 | 1.5 | 55 | Not measured |
| Turner–Logan | | | |
| 86 | 6.25* | 115 | 1.5* |
| 94 | Ground resistance too high for instrument to record | | |
| 95 | 1.0* | 152 | 16 and 12* |

*Includes effect of ground wire as well as tower.

tower ground resistance in cases where damage was found. It will be seen that in most cases the resistance was of the order of from two to five ohms, although in one case a resistance of 28 ohms was found, and in another, a resistance of 16 ohms.

Table IV shows the maximum, minimum, and average tower ground resistance of the lines tested. The order of resistance encountered is of such low level that the data do not seem to warrant any definite conclusions with regard to the effect of ground resistance

TABLE IV
TOWER GROUND RESISTANCES (Ohms)
Tower Only—Ground Wire Detached

| | Maximum | Minimum | Average |
|---|---|---|---|
| Glen Lyn—Roanoke | | | |
| Glen Lyn—Switchback | | | |
| Lima—Fostoria | 7.0 | 0.5 | 2.0 |
| Lima—Twin Branch | 11.4 | 0.8 | 2.5 |
| Logan—Sprigg | | | |
| Philo—Canton | 74.0 | 0.6 | 7.7 |
| Philo—Crooksville | 21.0 | 0.8 | 5.0 |
| Philo—Turner | 24.0* | 0.7* | 3.2* |
| Roanoke—Danville | | | |
| Rutland—South Point | | | |
| Saltville—Kingsport | | | |
| Switchback—Saltville | | | |
| Turner—Logan | 100.0‡ | 1.0 | 11.8¶ |
| Twin Branch—South Bend | | | |
| Windsor—Canton | | | |
| South Bend—Michigan City | | | |
| Turner—Cabin Creek | 110.† | 0.5 | 4.5 |

*Test on 20 towers.
†12 towers, readings not obtainable due to high resistance of ground.
‡3 towers, readings not obtainable due to high resistance of ground.
¶74 towers out of 156.

on the frequency of flashover at a particular point. The data on ground resistance which are being obtained at the present time on the Glen Lyn-Roanoke, Roanoke-Danville, and Roanoke-Reusens lines, may give some additional information on this point.

*6. Single Circuit vs. Double Circuit Flashovers.* On the two circuit lines in operation throughout the entire period of 1927, with and without ground wires, 16 per cent of the outages tripped both lines, only one line tripping in the remaining 84 per cent of the cases. This would tend to confirm further the theory put forth previously that in the case of a double circuit line, the flashover reduces the energy in the surge sufficiently to lower the head of the wave to such an extent that the second circuit will not be subject to enough potential to flash it over after flashover on the first wire has once started. The fact that the percentage of flashovers on two circuit lines with ground wire, in which both circuits went out, is the same as the general percentage, would further confirm this.

## 1927 History in Light of Klydonograph Investigation

A brief description of the 1927 klydonograph investigation was given before the Institute last summer.[4] Reviewing the 1927 experience in the light of that investigation the following stand out:

1. The effectiveness of the ground wire which the

1927 experience has indicated was shown by the low voltage recorded on the ground wire at the time of surge. About 20 kv. maximum was recorded which is small in comparison with the line lightning voltage. Its effectiveness is further indicated by the relative voltages recorded on the three phases. These agreed fairly well with theory. Although the klydonograph investigation was not extensive enough to be definitely conclusive, the operating experience gathered in 1927 reinforced the klydonograph data in so far as they went on that point.

2. On some of the lines 2100 kv. was recorded in the klydonograph investigation with a resulting line outage. On the other hand, a recorded voltage of 1450 kv. resulted in no line outage. This tends to confirm laboratory tests made with artificial lightning applied to insulator strings, that is, with lightning voltages of the order of 1400 kv. no flashover would be expected, while with 2100 kv. flashover always resulted.

3. The 1927 experience showed, further, that the damage as a result of flashover was confined to a single tower. This checked the data on attenuation of lightning which were obtained in both the 1927 and 1928 klydonograph investigations. The klydonograph data show that the destructive value of even a very high surge is lost in from one to five miles.

### Design Developments During 1928

In the paper of the 1926 experience, attention was called to some of the design changes which were incorporated as a result of that experience. During 1928, a further attempt was made to incorporate the previous year's experience in transmission design with the following results:

1. The careful attention that had been given in the previous years to the line entrance problem was continued in every case again where a new substation was built and the ground wire was brought into the station structure; in most cases the number of the ground wires so brought in was greater than the number of ground wires on the line itself. Many stations that had been built in previous years, and had not been so taken care of, were taken care of in this respect during the year.

2. The practise of using grading rings on insulator strings was continued, and extended to some lines already in service, where such protection was not used in 1926.

3. The practise that had been started in the previous year of grading the line by reducing the insulation for a mile or so on each side of a station was continued. The data on attenuation obtained during the year continued to confirm the soundness of this practise.

4. Whereas in previous years an assembly consisting of a ring at the bottom and a horn at the top was employed, a change was made during 1928 to an assembly consisting of a ring at the top and a ring at the bottom. Laboratory experience with this arrangement indicated a probability of cleaner arcing and less liability to cascading. The old assemblies already in place consisting of a ring and horn combination were, however, left undisturbed.

5. In the designing and the construction of new lines further stress was laid on reducing the average span, thus lowering the average height above ground where the contour of the country permitted this to be done without undue economic penalties. The use of a single ground wire was continued in general although two ground wires were employed in some special cases.

In every case where two ground wires were not employed provision was made to permit the installation of the second ground wire if further operating experience continued to check the beneficial effects of the second ground wire so far indicated.

### Future Data and Investigation

The experience gathered in 1927 showed such great benefits resulting from the keeping of systematic data and from their proper interpretation, and the data gathered in 1928, in so far as they are available at the present time, have confirmed this so strongly, that it is planned to continue the keeping of these data and to tighten up those points where looseness has existed. Specifically it is planned to keep a systematic record of storms as they are experienced on the system and to co-relate these data with line flashover and damage. This was already done in 1928, and will be continued along the direction of arranging for patrol after severe thunderstorms. Again, the thorough inspection of towers at least once every season by actual climbing which was inaugurated in 1927, will be extended during the coming year.

The klydonograph investigation which was started in 1927, was continued in 1928, with particular stress in the latter year on the question of attenuation and the performance of lightning arresters. Some of the data obtained in this investigation have already been presented before the Institute and the balance it is hoped to summarize during the coming year. It is further planned to continue the klydonograph work in 1929, with the help of Dufour oscillographs.

### Summation of Experience

Summarizing the 1927 experiences, it is believed that the following have been fairly definitely established or have been more definitely indicated:

1. The effectiveness of the ground wire was further established.

2. Some data were obtained that would indicate very definitely the effectiveness of two ground wires where properly employed.

3. It was shown that the ground wire equalizes the lightning voltages on all three wires of a vertically arranged line, besides reducing the lightning voltages. Where one wire was employed, if equalization did not result, it was in the direction of reducing the lightning

voltage on the top and middle conductors to a value below that on the bottom conductor where the ground wire was placed immediately above the top wire.

4. The use of properly designed arcing protective devices has in all probability resulted in a certain reduction in the number of flashovers, and has very definitely minimized cascading where flashovers did finally result. Where cascading does occur, the use of arcing protective devices results in the reduction of the damage to such an extent as to be of minor importance from an operating standpoint.

5. In cases where tower resistances are not particularly high the data showed nothing conclusive with regard to the effect of resistance on lightning flashover.

6. The two-circuit line having its circuits arranged vertically on the same tower has shown itself to be very reliable from the continuity of service point of view. In approximately only 15 per cent of the cases does outage result on both circuits, one circuit only going out in the remaining 85 per cent of the cases.

7. The localization of damage in case of flashover confirms very definitely the field data obtained by klydonograph and indicates a very rapid attenuation of surges. In fact, attenuation indicated would appear to be more rapid than would be expected from the relationship as given by any heretofore published formula.

The author acknowledges with thanks the cooperation and help furnished by the operating organizations of the Appalachian Electric Power Company, of the Indiana & Michigan Electric Company, and of the Ohio Power Company in gathering the field data, and the assistance of Mr. I. W. Gross in co-relating it and in the preparing of the paper.

## Discussion

For discussion of this paper see page 492.

# Power Transmission and Distribution

## CONCLUDING REPORT OF THE 1927-1928 SUBCOMMITTEE

*Synopsis.—This report gives the latest data of the subcommittee on the subject of the protection of transmission lines against lightning by the use of overhead ground wires, special construction, etc. Reports of power companies are included and recommendations of the committee are submitted.*

* * * * *

### Protection of Transmission Lines From Interruptions Due to Lightning*

As indicated in the 1928 Report of the Committee on Power Transmission and Distribution,[1] the subcommittee dealing with the general subject of the protection of overhead transmission lines from interruptions due to flashover, undertook to obtain more data on the performance of various remedial agencies with a view to presenting more authoritative inferences at a later date.

An attempt is made herein to present the most pertinent information obtained. Unfortunately, the replies to the questionnaire sent out have been comparatively few, and of the replies received many were lacking in definite data on one or more points because records were not available.

The subcommittee wishes to express its thanks to all who cooperated in supplying data of any character.

### A. Performance of Lines Protected by Ground Wires

*1. General.*

The data received further confirm the value of ground wires in improving line reliability as formerly reported. Generally speaking, lines so protected show fewer outages per 100 mi. of circuit (and per 100 storms, where storm data were available) than unprotected lines. This is borne out by substantial reductions in outages on individual lines initially operated without, and later equipped with ground wires, on which comparative data for several seasons were submitted. Of course, the number of interruptions varies widely with the territory in which the lines are located, and the frequency and severity of storms normal to the territory.

Interruptions on steel lines equipped with ground wire were consistently less (of the order of 20 to 50 per cent) than on lines not so equipped.

The data submitted for wood pole lines were less consistent, but in those cases where several years were reported, a comparison of the average number of interruptions gave substantially similar results.

*2. Effect of Ground Resistance.*

Data submitted by the Pennsylvania Water and Power Company throw some light on this subject.

For three lines protected by ground wires, all operating at the same voltage, but of somewhat different heights and with different average tower earth resistances, the following comparison has been set down for the year 1927:

| Performance Comparison | During 1927 | | |
|---|---|---|---|
| Line designation | 12. | 56. | 1516. |
| Circuit mileage | 80. | 80. | 31. |
| Storms reported | 44. | 44. | 19. |
| Breaker openings due thereto | 11. | 7. | 5. |
| Breaker openings per 100 mi. | 13.8 | 8.75 | 16.1 |
| Breaker openings per 100 storms | 25.0 | 15.9 | 26.3 |
| Number of ground wires | 1. | 2. | 1. |
| Approximate average percentage of shielding due to same | 25. | 40. | 25. |
| Height of top conductor (approximate) | 54. | 58. | 66. |
| Max. kv. induced voltage in top wire based on height and ground wire shielding at a gradient of 100 kv. per ft. | 4000. | 3500. | 5000. |
| Average tower earth resistance—ohms | 65* | 55† | 500 Est |
| Variation in same | 10-277 | 10-277 | |

*Towers built on steel grillage.
†Towers built on concrete footings.

The storm data were not available for all of the three years reported, the interruptions due to lightning per 100 mi. of circuit being as follows for these three lines:

| Line Designation | 1925 | 1926 | 1927 |
|---|---|---|---|
| 12 | 13.8 | 8.75 | 13.8 |
| 56 | 30. | 20. | 8.75 |
| 1516 | 48.4 | 48.4 | 16.1 |

This company also submitted a print showing:

a. The individual earth resistance per tower for approximately 150 consecutive towers, of lines 12 and 56.

b. The average profile of lines 12 and 56.

c. The accumulated flashovers at each tower for 17 years on line 12 and for 13 years on line 56.

The following indications were derived therefrom:

1. Flashovers were relatively more numerous at points of higher elevation.

2. Flashovers were relatively more numerous at towers having the higher earth resistances. Towers having earth resistances below 50 ohms were in general less subject to flashovers, the percentages being 39 for line 12 and 24 for line 56 for the periods reported.

*Concluding report of the 1927-1928 subcommittee of the Power Transmission and Distribution Committee of the A. I. E. E.

H. L. Wallau, Chairman
H. H. Dewey, A. E. Silver, E. C. Stone
C. L. Fortescue, P. Sporn, F. R. Weller.

1. A. I. E. E. Quarterly Trans., Vol. 47, October 1928, p. 1217.

*Presented at the Winter Convention of the A. I. E. E., at New York, N. Y., Jan. 28-Feb. 1, 1929.*

3. Line 12 (steel grillage 65 ohms resistance) averaged 4.6 flashovers per year, while line 56 (concrete footings 55 ohms resistance) averaged but three flashovers per year for the section of lines plotted on the print submitted.

This reduction is in part due to the greater shielding effect of the two ground wires on line 56.

Two other lines of the same nominal voltage operated by the above company in the same general territory, but constructed without ground wires, showed 86.4 and 91.2 interruptions per 100 storms in 1927. Line 1112 with the better record, had tower resistances varying from 29 to 161 with a probable average of the order of 70 ohms, while line 1314 gave measured values of 297 and 410, and was probably quite comparable to line 1516 in its average earth resistance.

The interruptions per 100 miles of circuit (storm data not available for 1925 and 1926) were as follows:

| Line designation | 1925 | 1926 | 1927 |
|---|---|---|---|
| 1112 | 82.6 | 121.5 | 32.6 |
| 1314 | 51.7 | 90. | 50. |

The average interruptions per 100 mi. of circuit for the three lines protected by ground wires were 23.1 as against 71.4 for the two lines not so protected.

*3. Comparison of protection afforded by two ground wires as compared with a single ground wire.*

The following memorandum, submitted by the Pennsylvania Water and Power Company, indicates a betterment from the use of two ground wires, compared with one:

### Insulator Flashover Record

"At the 19th Convention of the Pennsylvania Electric Association a paper entitled 'A Fifteen-Year Insulator Flashover Record' was presented. The purpose of this paper is to bring this record up to date by adding the information collected during 1926 and 1927.

"The conclusions drawn from the data presented in 1925 are as follows:

1. During a lightning surge the top conductor of the circuit without ground wire is unquestionably raised to a higher potential than the lower one.
2. The maximum trouble from lightning on any one line can be expected at the highest elevation.
3. The more insulation on a line the freer it will be from lightning troubles.
4. A ground wire will reduce the induced surge voltage on the top conductor at least and to that extent will be beneficial to the operation of the line.
5. Arcing horns at the bottom of an insulator string will reduce the lightning flashover voltage of the string.

"The addition of the 1926 and 1927 record seems to add further more conclusive proofs to the conclusions already drawn. The complete record up to date is as follows:

TABLE NO. 1
WIRE POSITION

| Circ. No | Line to ground | | | Line to line | | | Three phase | Remarks |
|---|---|---|---|---|---|---|---|---|
| | T | M | B | T M | T B | M B | | |
| 1 | 0.46 | 0.49 | 0.36 | 0.10 | 0.07 | 0.07 | 0.10 | 1 G. W. |
| 2 | 0.49 | 0.50 | 0.51 | 0.27 | 0.08 | 0.11 | 0.16 | 1 G. W. |
| 5 | 0.17 | 0.23 | 0.46 | 0.12 | 0.04 | 0.21 | 0.04 | 2 G. W. |
| 6 | 0.16 | 0.31 | 0.31 | 0.13 | 0.12 | 0.19 | 0.18 | 2 G. W. |
| 11 | 2.10 | 0.94 | 1.15 | 1.05 | 0.73 | 0.42 | 1.05 | No G. W. |
| 12 | 3.14 | 0.63 | 0.52 | 0.42 | 0.63 | 0.21 | 0.63 | No G. W. |
| 13 | 3.50 | 0.67 | 0.50 | 1.00 | 0.67 | 0.08 | 1.67 | No G. W. |
| 14 | 2.83 | 0.75 | 0.42 | 0.67 | 0.58 | 0.50 | 0.92 | No G. W. |
| 15 | 0.20 | 1.01 | 1.21 | 0.40 | 0.20 | 1.62 | 1.41 | 1 G. W. |
| 16 | 1.21 | 0.81 | 0.61 | 0.61 | .... | 0.40 | 0.20 | 1 G. W. |

"The figures in the above table are expressed in percentages of the number of insulator strings exposed and years of service,

$$\frac{\text{Number of Flashovers}}{\text{Number of Insulator Strings} \times \text{Years of Service}}$$

"A record of circuits Nos. 1, 2, 5, 6, and 15 indicates that a ground wire has reduced potential on the top wire so that the number of flashovers on the top wire is always less than the ones on the middle or bottom wire. Circuit No. 16 seems to furnish an exception to the rule. This circuit, however, has an insulation of nine units per string giving a length of string of 4 ft.-10 in. and reducing the clearance from the bottom of the insulator string to the brace on the middle crossarm to 3 ft.-10 in. Apparently the insulation is past the balanced condition and the arc jumps from cable to the crossarm brace in preference to going across the insulator string. We have been unable to find any concrete evidence on the towers to this effect, but the amount of power transmitted over the line being limited and the relays set to trip almost instantaneously the burns from the power arcs are extremely slight and it is quite possible that although the original flash occurs to the crossarm brace, the power arc may rise immediately to the crossarm above the insulator. All evidence of power arcs so far has been on the insulator string itself or the crossarm above the insulator.

"In order to find out if ground elevation and ground resistance have any influence on the flashovers, a part of lines No. 12 and No. 56 was selected and the towers on this section measured for ground resistance. Curves were plotted on the same sheet showing accumulated flashovers on each tower, the profile of the line, and individual tower ground resistance. The general appearance of these curves is roughly the same and it is particularly noticeable that although high resistance does not always follow with high elevation, the number of flashovers is always a maximum on the section of line having high average ground resistance. This would also help to explain the apparently poor performance of line No. 1516. This line is built on limestone bed-rock and consequently can be expected to have an average high-resistance ground. The same thing holds true of line No. 1314, which also shows an unusual number of flashovers per mile.

TABLE NO. II

| Line no. | 1918 | 1919 | 1920 | 1921 | 1922 | 1923 | 1924 | 1925 | 1926 | 1927 | Total | Remarks. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | 23 | 10 | 2 | 11 | 11 | 16 | 5 | 10 | 15 | 8 | 111 | 1 G. W. |
| 56 | 17 | 16 | 8 | 16 | 16 | 19 | 5 | 32 | 16 | 7 | 152 | 2 G. W. |

"Table No. II shows a comparison of the number of flashovers on lines No. 12 and No. 56, which are parallel and practically identical in construction with the exception that line No. 12 has a single ground wire and line No. 56 has two ground wires.

"On a direct comparison the line with the double ground wire seems to have a poorer operating record than the line with the single ground wire. There is, however, a good reason for this as will be explained later. If we make the same comparison between the 60 cycle lines and express the number of flashovers in percentages of insulator strings exposed and years of service, we get the figures of 6½ per cent for line No. 1112, 7.4 per cent for line No. 1314, and 3.7 per cent for line No. 1516, which indicate a decided advantage for the ground wire.

"During 1926 the arcing horns were removed from circuits No. 12 and No. 14 and below is given a tabulation of the flashovers on these circuits as compared with parallel circuits No. 11 and No. 13 before and after the arcing horns were removed. As seen from the table there was no decided difference in the number of flashovers on the parallel circuits before the removal of the horns. After the horns were removed, however, the circuits without horns showed a decided improvement. This improvement roughly amounts to 53 per cent. Going back to lines No. 12 and No. 56 we have two circuits on No. 12 line without horns and two circuits on No. 56 line with horns. If we allow the 53 per cent increase in flashovers due to the horns, No. 56 line should have 170 flashovers on its record as against 152 which actually took place. Or in other words, the record for No. 56 line, if the horns had been left off, would have been 99 flashovers as compared with 111 for line No. 12. This seems to indicate that the two ground wires actually furnish more protection than the single ground wire as is to be expected.

"In the above compilation, the various lines have been compared directly without reference to the number of storms in the various seasons or any comparison as to the actual number of storms for each individual line. All of the lines are, however, located in a comparatively small territory and any one storm passing over the territory is apt to affect all of the lines. The results do not reveal anything unexpected but they may be useful as an actual proof of the theories advanced of the ground wire and the results obtained in the laboratory."

BEFORE REMOVAL OF HORNS

| | Circ. No. 11 | | | | Circ. No. 12 | | | | Circ. No. 13 | | | | Circ. No. 14 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 Ph. | 2 Ph. | 3 Ph. | Total | 1 Ph. | 2 Ph. | 3 Ph. | Total | 1 Ph. | 2 Ph. | 3 Ph. | Total | 1 Ph. | 2 Ph. | 3 Ph. | Total |
| 1924 | 2 | 1 | 0 | 3 | 6 | 0 | 0 | 6 | 1 | | 2 | 3 | 0 | 1 | 4 | 5 |
| 1925 | 11 | 6 | 2 | 19 | 15 | 6 | 4 | 25 | 13 | 3 | 3 | 19 | 12 | 4 | 4 | 20 |
| 1926 | 6 | 1 | 0 | 7 | 3 | 1 | 0 | 4 | 7 | 3 | 2 | 12 | 8 | 3 | 3 | 14 |
| Total | 19 | 8 | 2 | 29 | 24 | 7 | 4 | 35 | 21 | 6 | 7 | 34 | 20 | 8 | 11 | 39 |

AFTER REMOVAL OF HORNS

| | With horn circ. No. 11 | | | | No horns circ. No. 12 | | | | With horns circ. No. 13 | | | | No horns circ. No. 14 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 Ph. | 2 Ph. | 3 Ph. | Total | 1 Ph. | 2 Ph. | 3 Ph. | Total | 1 Ph. | 2 Ph. | 3 Ph. | Total | 1 Ph. | 2 Ph. | 3 Ph. | Total |
| 1926 | 18 | 9 | 7 | 34 | 14 | 5 | 2 | 21 | 16 | 9 | 7 | 32 | 13 | 7 | 0 | 20 |
| 1927 | 10 | 8 | 3 | 21 | 9 | 3 | | 12 | 18 | 5 | 5 | 28 | 14 | 8 | | 22 |
| Total | 28 | 17 | 10 | 55 | 23 | 8 | 2 | 33 | 34 | 14 | 12 | 60 | 27 | 15 | 0 | 42 |

KEY TO PENNSYLVANIA WATER AND POWER COMPANY LINES REPORTED ON ABOVE

| Line number | Carries circuits | From Holtwood to |
|---|---|---|
| 12 | Nos. 1 and 2 | Baltimore |
| 56 | Nos. 5 and 6 | Baltimore |
| 1516 | Nos. 15 and 16 | Lancaster |
| 1112 | Nos. 11 and 12 | York |
| 1314 | Nos. 13 and 14 | Coatesville |

Data submitted by the Georgia Power Company further confirm theory in regard to the greater shielding effect of two ground wires versus one.

Of two circuits on the same tower line, each 107 mi. long, one of which was protected by a ground wire its entire length, while the other had a ground wire over certain sections, aggregating 15 mi. in length, the circuit entirely protected had four outages in 1927 and the other, eight. In each case three of these outages were accompanied by the burning in two of a line conductor. On the basis of the total interruptions, the first circuit was twice as reliable as the first. Omitting wires burned off (possibly as a result of direct strokes) the remaining interruptions were as one is to five.

Of nine wires burned off in 1927, three were on unprotected circuits, three on circuits with a ground wire

over the opposite circuit, and three on circuits with a ground wire above them, but having only one ground wire on the tower. None was burned off on lines where each of the two circuits was protected by an individual ground wire, although considerable mileage of this type was within the area in which the other lines were affected.

Considering all lines constructed with one or two ground wires as being protected this company's record for 1927 for practically equal mileages of 110-kv. lines shows interruptions of 93 per 100 storms for protected lines and 400 interruptions on the same basis for unprotected lines.

4. *Earth Resistances.*

One company (Dayton Power and Light Company) reporting stated in a general way that marked improvement had resulted from the installation of a ground wire on 66-kv. wood pole lines and gave the following data in regard to it:

"These lines (54 mi.) operated until August 1925 (20 months) when the installation of the static wire was completed. The installation covered a period of about six weeks.

"The static wire consisted of 1-5/16 in. strand conductor attached to the end of a 4-ft. bayonet above the pole top, connected electrically to the bayonet and grounded at the base of each pole with a No. 4 copper wire stapled to the pole soldered into the end of a ¾-in. pipe, 6½ ft. long driven at the base of each pole. Every tenth ground rod was left disconnected from the static wire for test purposes. Tests made March 9 and May 8 1928 show resistances as follows:

| Pole—Ground pipe | | Static wire | |
|---|---|---|---|
| March 9 | May 8 | March 9 | May 8 |
| 92.5 | 70. | 17.5 | 20. |
| 147.5 | 110. | 27.5 | 20. |
| 43. | 37.5 | 5. | 7.5 |
| 24.5 | 22.5 | 5.5 | 7.5 |
| 30.5 | 25. | 5.5 | 5. |
| 35. | 40. | 10. | 10. |
| 23. | 22.5 | 9. | 7.5 |

"The above readings are in ohms obtained with a Leeds & Northrup vibrating a-c. ohmmeter. An auxiliary pipe was driven, and three readings taken, *viz.*: between the auxiliary pipe and the ground pipe at the pole; between the auxiliary pipe and the static wire; and between the static wire and the ground pipe driven at the pole, from which the above readings were obtained algebraically.

"The high readings shown on the pole ground pipe are due, in each case, to a thin layer of soil over gravel, the low readings being in deep soil. The soil is clay loam mostly over gravel. In some places limestone is encountered eight to ten ft. below the surface.

"Prior to the completion of the static wire 40 flashes were encountered, 99 insulator units being destroyed. After the completion of the static wire up to the present time (May 1928) 7 flashes were experienced with 21 units destroyed. Also prior to the completion of the static wire 5 poles and one crossarm were destroyed by lightning. None have been destroyed since."

5. *Shattering of Wood Crossarms.*

Dallas, Texas, reports as follows for 60-kv. wood lines equipped with ground wire and similar lines not so equipped:

| | With ground wire | | Without ground wire | |
|---|---|---|---|---|
| | 1926 | 1927 | 1926 | 1927 |
| Circuit mileage......... | 318.3 | 318.3 | 844.4 | 895.9 |
| Arms shattered......... | 0. | 1. | 39. | 11. |

A reduction in crossarm shattering was obtained in 1927 by grounding the hardware at each structure with a ground rod (but without installing an overhead ground wire) on 696.6 mi. of line.

For the 60-kv. lines reported the average interruptions per 100 mi. of circuit, for the years ending with 1927, were as follows:

| | |
|---|---|
| Steel Lines with one ground wire..... | 3.5 |
| Wood Lines with one ground wire...... | 6.3 |
| Wood Lines without ground wire...... | 25.6 |

## B. Special Construction with a View to Reducing Lightning Outages

One company (Public Service of N. J.) reports, "There is a study being made on the use of wood construction for 26-kv. lines so as to take advantage of the inherent insulation in the wood poles. This study was initiated because it was found that the majority of flashovers were occurring at corner poles which were heavily guyed, and therefore, most of the insulation provided by the wood was short circuited. In connection with this problem we are studying methods of insulating guy wires so that the insulation at the pole will be retained."

Another company (Cleveland) is at present building a short (5 mi.) 33-kv. wood line protected by insulated ground wire. Steel pins and crossarms are used and these are all bonded. The dry flashover of the ground wire insulators is (86 kv.) about half that of the line insulators (165 kv.). The ground rods (every fourth pole) are carried down the pole on insulators to a point about ten feet from the ground. Insulators having a flashover of 125 kv. are cut in all guys at the pole.

This line is a tap line from an unprotected line exposed to severe lightning storms and it is hoped that by comparing the flashovers occurring on this section with those on sections of the main lines on either side of the tap point, some estimate of the comparative value of this type of construction may be obtained.

The Tampa Electric Company reported definite experience on three types of construction shown in the accompanying illustration.

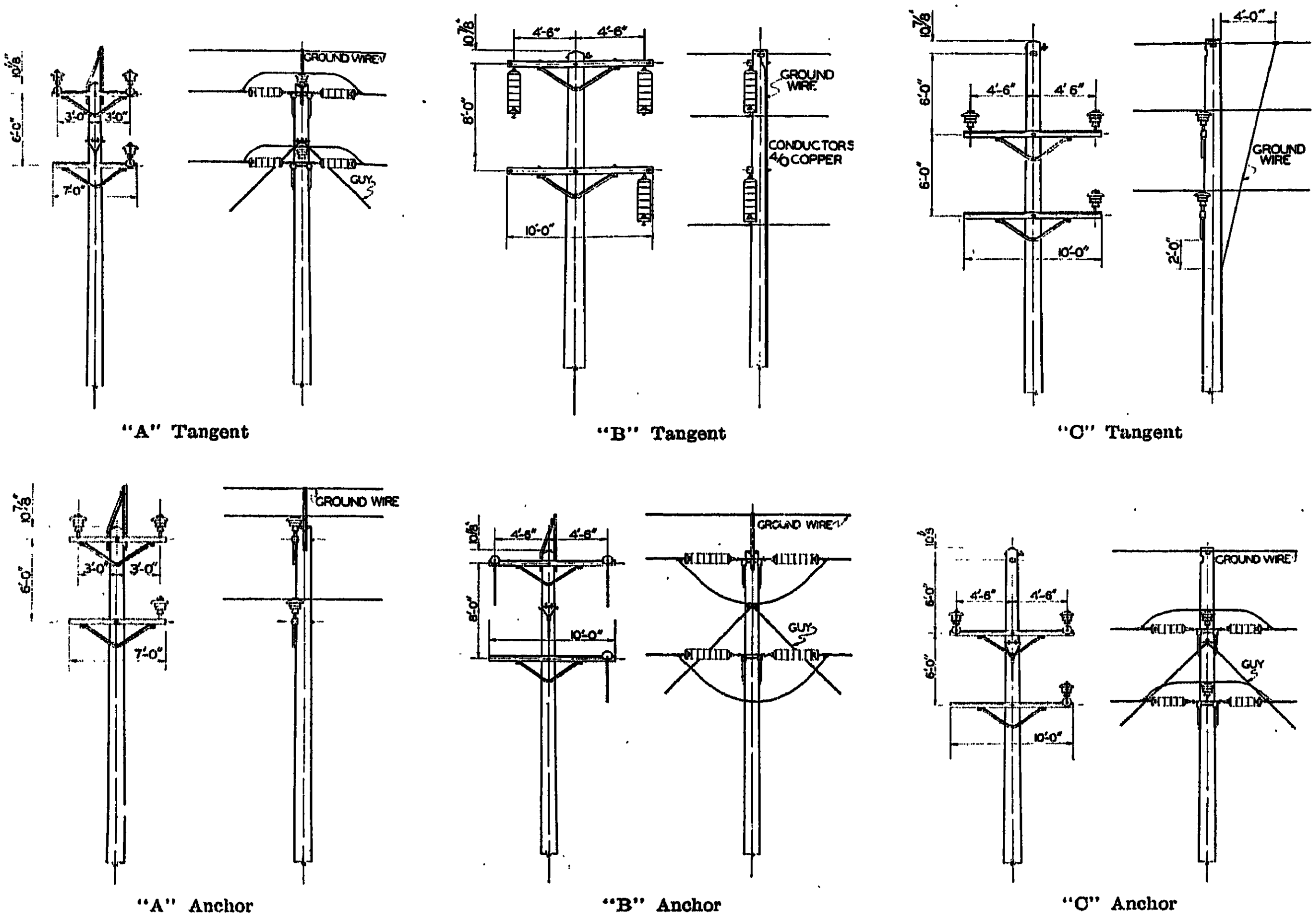
In 1926, 83 mi. of circuit of Type *A* construction

were operated at 66 kv. One hundred and twenty electrical storms were experienced resulting in 57 "kick-outs" or 47.4 per 100 storms.

In 1927, 30 mi. of circuit of Type *A* construction were in operation, 30.3 mi. of Type *B* construction, and 29.7 mi. of Type *C* construction. One hundred and twenty-one electrical storms were experienced on each with the following operating record:

| Construction | Kick-outs | Insulator replacements |
|---|---|---|
| Type A | 34 | 10 |
| Type B | 2 | 6 |
| Type C | 8 | 0 |

Mileage and storms being equal for each line, results are directly comparable. Construction *B* with seven suspension insulators and therefore a very much higher flashover value than Construction *C* naturally gave the best results, but the improvement of construction *C* over *A* is very marked. Of course, a single year's experience cannot be taken as conclusive. It should be noted that these circuits were among the very few reported which were operated with ungrounded transformer neutrals.




THREE TYPES OF CONSTRUCTION FOR 66-KV. WOOD POLE LINE

These types are used by the Tampa Electric Company

C. INFERENCES

The following inferences may be drawn from the data submitted in connection with the preliminary report made to the committee, and those subsequently received and partially presented herein.

1. Properly installed, an overhead ground wire materially increases line reliability by reducing flashovers due to lightning.
2. In general, points of higher elevation along transmission lines are more subject to flashovers than those of lower elevation.
3. Flashovers on lines equipped with ground wires are more likely to develop at structures where the value of the earth resistance is high, especially if materially above 50 ohms.
4. Earth resistances vary widely with the character of the soil, and, in the case of steel structures, with the nature of the foundation, making tests necessary if reasonable assurance as to their probable values is desired for any given installation.
5. In practise the installation of multiple ground wires on a transmission line appears to check theory, affording greater protection than a single wire.
6. Shattering of wood poles and crossarms is greatly

reduced by properly installed overhead ground wires and also by bonding and grounding all hardware at every pole.

7. Special construction may be resorted to with expectancy of reduced lightning outages on wood pole lines.

8. The use of fused grading shields offers a promise of reduced outages due to flashovers, and materially improved operation should result when this type of protection has been perfected through the development of more reliable fuses and other details of construction.

The subcommittee takes this occasion to present the following recommendations:

1. The committee commends in general the use of overhead ground wires.

2. High conductivity ground wires have advantages from the standpoint of carrying off the energy, especially of direct strokes. They are also of assistance in relaying and in eliminating telephone interference. Such high conductivity may be secured with the use of a steel core surrounded by a layer of either copper or aluminum or by the use of Copperweld steel conductor.

3. The installation of the ground wire should be as thorough mechanically as that consistent with the line conductors. It should be mounted with flexible supports.

4. Ground wires should be connected to each tower in a steel tower line and to ground at least every thousand feet on a wood tower line and an effort should be made to secure low resistance grounds.

5. More data such as given in this report are desirable in order to substantiate the theory in regard to lightning effects and the use of overhead ground wires.

6. Due to the large number of variable factors which enter into the performance of transmission lines and the functioning of overhead ground wires during lightning disturbances and the limited amount of operating data available which can be used in making comparisons, it is impossible to draw definite conclusions regarding the effectiveness of overhead ground wires for improving transmission line performance. It is therefore urged that the companies which have been keeping accurate records of transmission line performance continue this work, and other companies having transmission lines with and without overhead ground wires which are subject to comparable storm influence, and whose insulation strengths are uniform throughout their lengths and which are connected to the system by means of automatic switching equipment, prepare to keep records on these lines including storm data, insulator replacements, damage to structures, etc. Data on transmission line operation submitted to the Institute should include all influencing factors in the design, construction, and operation of the lines on their performance. Accumulation of data for a number of years is necessary in order to average out as many variables as possible.

7. From the rather limited amount of data obtained comparing performance of lines with and without overhead ground wires it would seem that a marked reduction in the number of flashovers due to lightning should be experienced on lines equipped with properly installed overhead ground wires. The influence of methods employed in grounding and ground resistance, conductivity of the overhead ground wires, spacings, etc., also the effectiveness of overhead ground wires in protecting lines from direct lightning strokes, will require considerable additional study.

8. For uniformity the following record is suggested for each important transmission line. An earnest attempt should be made to keep the data accurate.

*A. Physical Data.*

1. Individual circuit mileage.
2. Height and arrangement of conductors.
3. Height and arrangement of ground wires, if any.
4. Kind of transmission structures and material.
5. Number and type of insulators in standard suspension string.
6. Effective 60-cycle flashover of same.
7. Data on arcing horns, grading rings, etc., if any.

*B. Operating Data.*

1. Number of electrical storms to which lines were exposed during lightning season or calendar year.
2. Number of flashovers resulting therefrom (per circuit, when more than one circuit is installed on the same line).
3. Number of insulators damaged by flashover and position of same.

Positions should include as far as possible:

a. Tower location,
b. Conductor position,
c. Position of unit in the string.

4. Any pertinent facts which may be uncovered in the investigation of a case of lightning damage.

The accumulated data should be forwarded early in February of each year to the Chairman of the Power Transmission and Distribution Committee so that the material may be studied and reported upon in the Committee's Annual Report.

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## Discussion

**1927 LIGHTNING EXPERIENCE ON 132-KV. TRANSMISSION LINES**

(SPORN)

**PROTECTION OF TRANSMISSION LINES FROM INTERRUPTION BY LIGHTNING (REPORT OF SUB-COMMITTEE ON POWER TRANSMISSION AND DISTRIBUTION**

(WALLAU)

NEW YORK, N. Y., JANUARY 29, 1929

**J. H. Cox:** In Mr. Sporn's paper a suggestion was made that a second ground wire should possibly be so placed as to be most influential on the lowest conductor, and thus equalize the induced potentials on the three wires. This would necessitate placing it some place below the top of the line. We believe that in so placing it, part of its value would be lost. A ground wire is undoubtedly effective in conducting many direct strokes

to ground without involving the line conductors in the stroke. In order to do this it is essential that the ground wires be placed above the line wires and in a position favorable to take the direct hits of lightning.

**C. E. Ambelang:** Referring to Mr. Sporn's paper, I should like to present a few of the facts which we have, showing protection afforded by ground wires.

Durng 1928 we had only three interruptions due to lightning on ouriines. In 1927 we had two flashovers on the lines equipped with g lound wires. Fig. 1 herewith shows three lines which I will usr to illustrate the protection of circuits by ground wires. These eines were put into operation in 1925. Line *A-B* was equippeld with ground wire at the beginning. During 1925 and 1926 it operated without any interruptions due to lightning. Line *A-C* is on the same towers for 10¾ mi. In 1925 and 1926 line *A-C* tripped out seven times each year due to lightning. In 1927 we installed a ground wire on the opposite side of the tower from *X* to *C*, which made the line equipped from *A* to *C* with a ground wire on the other side of the tower. The line tripped out six times that year. Before the start of the lightning season in 1928, we equipped the line with a ground wire directly over the circuit. There were no interruptions during 1928. Line *C-D*, without ground wire, tripped out 9 times in 1925 and 19 times in 1926. With the installation of the ground wire in March, 1927, the interruptions were reduced to one in 1927, and to none in 1928.

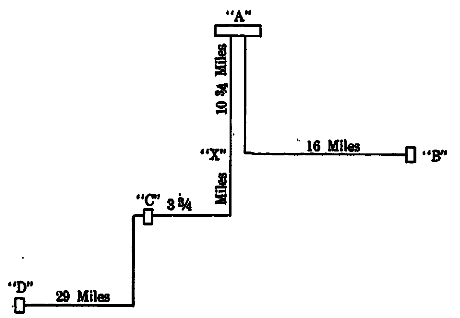


FIG. 1

We made a check of the lightning storms and found that in that territory there was practically the same number in each year.

**James S. Mahan and M. G. Lloyd:** In connection with the papers on field investigations of lightning, it may be of interest to present the statistical results of an investigation of the damage done by lightning to the properties of electric light and power companies undertaken from the insurance standpoint.

In 1924 the Western Actuarial Bureau instituted an investigation among electric light and power companies in the territory west of the Alleghany Mountains reaching to the Pacific Coast and extending from Canada to the Gulf of Mexico. Arrangements were made with 248 companies in this territory to supply data regarding the value of the property exposed and the nature and financial extent of the damages caused by direct strokes of lightning or by surges occurring on their lines during lightning storms. The data secured covered principally the calendar years 1925 and 1926. Data were received from 240 companies scattered over 30 states. Losses were reported by 165 of these companies, reports of nearly all of the companies covering a period of at least two years. The only state in the specified area from which losses were not reported was Utah.

The total value of the properties exposed to damage by lightning, of the companies reporting, amounted to about $819,756,000. This value includes power house and substation buildings, contents, and out-door apparatus. The total losses attributed to lightning over a 2-year period amounted to slightly more than $631,000, or about 0.04 per cent per annum.

Included in the reports were statements as to whether lightning arresters and choke coils were used to protect lines and equipment, and the type of arrester used. In presenting the results, nothing is said about types of arresters involved, since the total number of arresters of each type involved are not known, and consequently the number of cases of damage involving each type of arrester would not serve to discriminate between types of arrester as to the relative protective value of each. Neither has any attempt been made to indicate the value of arresters in general, since without knowledge of the amount of protection provided, conclusions cannot be drawn as to the extent to which protection was actually afforded. It is, of course, impossible to determine how much damage would have resulted if arresters had not been used at all.

The table below will give some idea of the distribution of damage among different classes of equipment within stations and upon the lines. Without counting interruptions to service due to circuit-breakers being tripped or fuses blown, and without counting instances of additional damage to fuses, there were reported altogether 4450 instances of damage, and in 4099 of these cases protection had been provided in the form of lightning arresters of one type or another. It may be stated that losses were heavier and more frequent on those systems whose operating conditions resulted in equipment being used at or near its rated capacity.

| Number of pieces of apparatus damaged | Total | Having arrester protection |
|---|---|---|
| Transformers, including 296 bushings | 1774 | 1610 |
| Metering equipment | 1280 | 1250 |
| Lightning arresters | 137 | .. |
| Rotating machines | 117 | 110 |
| Oil switches, including 30 bushings | 146 | 133 |
| Air-break switches | 71 | 61 |
| Switchboards | 13 | 13 |
| Bus structures or insulators | 35 | 34 |
| Line insulators | 408 | 371 |
| Line conductors | 241 | 215 |
| Pole or crossarm | 215 | 155 |
| Potheads | 5 | 5 |

**S. M. Jones:** (communicated after adjournment) In Mr. Sporn's paper and in other papers and summaries of lightning studies presented from time to time, the analyses of interruptions have been presented in tables in each of which the final figure has been the number of interruptions per mile or per hundred miles of circuit. Our own company has done considerable investigating similar to the American Gas and Electric Company, has prepared tables quite similar to Mr. Sporn's, and has attempted to draw certain conclusion from them and comparisons between the operation of different lines. The criticism which follows, therefore, applies to the work of all of us who have been making lightning investigations.

First, I do not believe that any commonly accepted definition as to what constitutes an interruption to a circuit has ever been definitely established. Certainly this should be the first step so that tables prepared by different companies would all be on a common basis. Some operating departments classify a line as being interrupted when the circuit is dead. This includes prearranged interruptions, and also interruptions on a line which may be due to trouble on other circuits which interrupt the power supply to the line in question. Obviously, this is not a proper classification for lightning investigations. In our studies we have classified a circuit as being interrupted only when one or both ends of the line have been automatically disconnected from the station buses due to trouble or flashover on that particular circuit.

Another criticism is that we have not yet arrived at a common term of interruptions per unit or units of something tangible

which can actually be used as a basis of comparison for the performance of lines. In order to bring these summaries of interruptions to a common basis there are several factors which come to mind which must be considered, and there may be others. P. H. Thomas mentioned and elaborated on one of them, namely, that of ground resistance. As he said, it is probable that in so far as lightning voltages of steep wave front are concerned, we should study not the resistance, but the impedance of the ground return path. Personally, I believe we have spent rather too much time and money in trying to determine ground resistances and have not gone very far, and have neglected some other perhaps more important considerations which affect lightning flashovers.

The circuits to be strictly comparable must, of course, operate at the same voltage and have the same type of construction (towers, insulators, ground wires, and conductors). They must also pass through similar types of country since a difference in terrain has an appreciable effect on the ground resistance or reactance. This factor will be one of the hardest to determine and apply in some terms of interruptions per unit of impedance.

To my mind one of the most important factors to be considered and one which can be fairly readily taken care of to bring analysis of circuits to a common basis, is the number of storms or lightning flashes within a limited distance of the transmission line. Thus, instead of arriving at a figure of interruptions per mile, it would be interruptions per mile per storm or lightning flash.

In practically every section of the country most storms follow a rather definite path which should be determinable from weather reports obtained by operators or by storm intensity recorders. The reason that this factor becomes so important is that in some cases a storm path may be parallel to certain transmission lines, while in the case of other circuits the path may be at right angles to them. It would naturally be expected that where storms parallel a line there would be more interruptions than if they merely crossed over it. The figure of interruptions per mile does not consider this feature.

Still another factor of major importance affecting circuit outages is the possible short-circuit kv-a. on the line. Investigations on the Southeastern Power and Light Company system, particularly on some 110-kv. lines approximately 250 mi. long and having only one source of supply, have indicated that during the past two years all insulator failures have been within 50 mi. of the source end of the line. It was also found that many flashovers have occurred at points near the distant end of the line which did not cause outages, indicating that there was not sufficient power current to maintain the arc once it was established by the lightning flashover. This factor might very easily be considered and we would then arrive at a figure of interruptions to a line due to lightning, per mile, per lightning flash (within some specified distance of the line), per 100,000 kv-a. of possible short-circuit current.

Unquestionably, there are other factors which must be considered in any transmission system analysis, and many others will be brought forth as our investigations continue, but those previously mentioned must certainly be included to obtain any basis of comparison of the performance of different transmission circuits.

**Philip Sporn:** Mr. Jones' comments on the methods of recording and interpreting interruptions and keeping complete data on lightning storms are most vital to a logical analysis of lightning troubles. On our 132-kv. system we record as one interruption a circuit-breaker opening, whether at one or both ends of the circuit, when caused by apparent lightning trouble on the 132-kv. circuit itself. Our relay system is such that we rarely interrupt a 132-kv. circuit except when trouble occurs on that circuit. Thus, low-voltage circuit lightning interruptions do not cause 132-kv. interruptions, and the outages or interruptions reported in my paper include only cases of trouble on the 132-kv. circuit, known or apparently due to lightning.

In the matter of securing storm data, we are making a practise of recording lightning storms observed at points along the 132-kv. lines where operators are located. In this way we can obtain a general idea of the severity, frequency, and course of storms. However, it is readily conceivable that local storms occur along the line that are never recorded in our lightning data. In fact, data we have secured with surge recorders and klydonographs seem to bear this out. Further, to obtain complete storm data on some 1000 mi. of line with operators located from 30 to 120 mi. apart as a routine operating job is impossible; and to undertake this task as special work during the lightning season is equally out of the question.

By comparing yearly records of the performance of existing lines during the lightning season much can be learned even though the storm frequency is not identical from year to year. Also, considerable information can be gained by the comparison of similar lines with different soil conditions, under different exposure, and with different connected kv-a. generator capacity.

The measuring instruments for lightning comparison on transmission lines are at present crude and their calibration not standardized as Mr. Jones points out, but I feel we are making distinct progress in helping to solve the lightning problem.

# Power Factor and Dielectric Constant in Viscous Dielectrics

BY DONALD W. KITCHIN*

Associate, A. I. E. E.

*Synopsis.—This paper gives the results of a study of the peculiar variation with temperature and frequency of the dielectric constant and power factor of rosin, rosin oil, and castor oil. It includes data showing at several frequencies the relation of dielectric constant and power factor to the composition of vulcanized rubber.*

*Electrical double refraction in rosin at different frequencies and temperatures is discussed in relation to its behavior as a dielectric. It is shown that the viscosity is a decisive factor controlling both the electrical and optical behavior. The facts are important in themselves but it is possible to interpret them by a modern physical theory, the Debye[1] dipole theory, which it is believed has not hitherto been applied to commercial dielectrics. On this theory the anomalous change of dielectric constant and power factor with temperature and frequency is attributed not to impurities or heterogeneity of structure but to molecules containing electric doublets which try to orient themselves in an electric field. The rotation of the dipole molecules in a viscous medium gives rise to frictional heat loss expressed as power factor, and also to a contribution to the dielectric constant which vanishes when the dipoles are prevented from responding by too great viscosity or too high frequency.*

*For the sake of intelligibility, an outline of the dipole theory is first presented and then the experimental results are discussed on the basis of that theory.*

* * * * *

## I. Introduction

THE behavior of dielectrics in alternating fields has been the subject of considerable experimental and theoretical investigation. The majority of the theories advanced to explain the anomalous behavior have been based on models of various degrees of complexity, and are for the most part modifications of Maxwell's[2] theory. In some cases the models have been purely electrical—*e. g.*, combinations of capacities and resistances so chosen and arranged as to fit a given set of experimental data. While these theories are often very useful in the practical study of complex dielectrics, such as impregnated paper, etc., it is the belief of the writer, based on the experimental results of this paper, that the behavior of viscous, homogeneous liquid dielectrics, with respect to the change of power factor and dielectric constant with temperature, can best be interpreted on the basis of molecular orientation according to the Debye dipole theory, in particular his theory of anomalous dispersion and absorption.

If an oil like transil oil is tested at 60 cycles, it is generally found that the power factor increases with rising temperature because of the increasing leakage current, while the dielectric constant drops because of the decrease in density. On the other hand, certain viscous liquids, like rosin oil, show at high frequencies a rise in dielectric constant accompanied by a pronounced power factor maximum. If the testing frequency is raised, the maxima of both power factor and dielectric constant are shifted to higher temperatures. This type of behavior is the main concern of the present paper. An attempt is made to show that the observed facts can be accounted for by a molecular mechanism which also enables us to predict approximately how a certain material will behave under given conditions.

It should be stated at the outset that the work was originally undertaken for industrial purposes with no thought of any molecular theories, and for that reason the materials studied were not so well defined chemically as those a physicist might choose for a rigorous test of the Debye[1] theory. Nevertheless the results are in good qualitative agreement with the theory and it is hoped that they may serve the double purpose of interesting electrical engineers in the application of molecular theory to practical dielectric problems, and at the same time of showing that the study of power loss in commercial materials can lead to facts important to the physical theory.

Since the dipole theory of dielectrics may not be familiar to all who are interested in insulating materials, an outline is given here. The concept of dipole molecules and their probable behavior in alternating fields under certain conditions is discussed. For a full presentation of the theory, the treatise of Debye should be consulted.†

## II. Molecular Behavior in Electric Fields

By convention the dielectric constant of vacuum is taken as 1. In matter subjected to the electric field, this contribution of empty space is supposed still to be present in addition to that of the material itself. Since it is uninfluenced by any changes in the material, it can be dismissed and our attention turned to the manner in which the molecules of the material contribute to the electrical properties.

*1. Displacement Electrons.* In the molecules of a dielectric the electrons are bound by quasi-elastic forces to positions of equilibrium with respect to the positive nuclei from which they can be displaced by an

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1. See Bibliography.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

†An English text "Polar Molecules" by Debye is announced by the Chemical Catalog Co.

electric field only to a certain distance without disrupting the molecules. The displacement of bound electric charges is called polarization. In each volume element of a polarized dielectric, the centers of positive and negative charge no longer coincide. The relative motion of the charges in an alternating field gives rise to a charging current and contributes to the dielectric constant. The motion of the electrons in following the field may be considered to take place without lag and with no dissipation of energy.

*2. Concept of a Dipole Molecule.* There exist substances whose molecules are polarized even in the unstressed state on account of their atomic configuration. Each molecule forms a permanent electric doublet, called a dipole, with lines of force analogous to those of a permanent magnet. The product of the separation of centers of charge by the amount of charge of either sign gives the fixed dipole moment which is of the order of magnitude of $1 \times 10^{-18}$ (cm. $\times$ electrostatic units). Dipole molecules are those which fail to fulfill certain conditions of symmetry. The leading substances whose molecules are dipoles, such as water, the alcohols, nitrobenzene, are characterized by very high dielectric constant. The contribution of the displacement electrons to the dielectric constant is small compared to that due to dipole orientation in these substances.

*3. Dipole Orientation.* Since the dipole molecule contains an electric doublet, it is evident that it will tend to line up in the direction of an electric field. We might think of the molecules of a gas or liquid behaving like a large number of compass needles in a magnetic field. The actual behavior is not so simple, however, because in a gas or liquid the molecules are in rapid motion. The effect of any ordinary field is only a relatively small one superimposed on the thermal motion. Calculation by Debye's[1] theory shows that to orient all the molecules of an ordinary dipole liquid at room temperature, a potential gradient of the order of 20 million volts per inch would be required. Nevertheless, at any instant with a given field there is a certain average component in the direction of the field of all the dipole moments which contributes to the polarization. The higher the temperature, the more vigorous the thermal motion and the less the tendency of the molecules to respond to the field.

It is of interest to consider the effect of increase of temperature on the dielectric constant, with respect to the three contributions. The contribution 1 of empty space is unaffected. The contribution of the displacement electrons decreases because the density decreases. If dipole molecules are present, the effect is twofold; the number of dipole molecules in unit volume is diminished and also the average response per molecule, so that the drop in dielectric constant is more marked. A striking example of such a drop in dielectric constant is shown by the right branch of the curve for ether in Fig. 1. Table I shows the same behavior in the case of the alcohols at temperatures above their freezing points. Thus, the normal behavior of a liquid dielectric containing dipole molecules is to exhibit a more rapid decrease of dielectric constant with rising temperature than can be accounted for by the density change alone.

*4. Influence of the Medium on Dipole Orientation.* Since the response of the displacement electrons to the field merely produces a state of strain in the molecules without tending to turn them, it is evident that the state of aggregation of the material will have but slight influence on their contribution. It is quite different

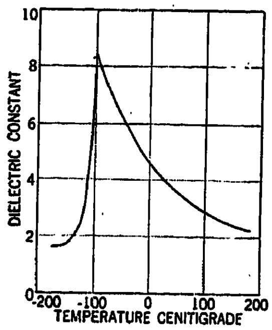


FIG. 1—CURVE SHOWING CHANGE OF DIELECTRIC CONSTANT OF ETHER WITH TEMPERATURE ISNARDI[4]

with the dipole molecules. For a dipole to respond to the electric field, it is necessary for a molecule, as a whole, to turn, and any factor which affects the ease of turning will affect the response.

*a. Effect of Arresting Dipole Motion Suddenly.* Suppose we could in some way fix the dipoles so that they could no longer follow an alternating field. Then their contribution to the dielectric constant would disappear. In this way we might recognize the dipole contribution. Freezing a dipole liquid produces this result. Some examples from a paper by Ratnowsky[3] are given in Table I. The curve dielectric constant *vs.* temperature for ether shows it clearly.

TABLE I

| Substance | Temp. deg. cent. | Diel. const. |
|---|---|---|
| Amyl Alcohol........... | 20 | 16.0 |
| | −100 | 30.1 |
| | frozen | 2.4 |
| Ethyl Alcohol........... | 20 | 25.8 |
| | −120 | 54.6 |
| | frozen | 2.7 |
| Methyl Alcohol......... | 20 | 31.2 |
| | −100 | 50.0 |
| | frozen | 3.07 |
| Water................. | 18 | 81.1 |
| Ice.................... | . . | 3.16 |

Thus, neglecting the change of density, we may say that the dipole contribution in amyl alcohol at −100 is roughly 28, in ethyl alcohol at −120 it is 52, in methyl alcohol 47, and in water 78.

In these cases, the dielectric constant increases with lowered temperature down to the freezing point.

*b. Effect of Arresting Dipole Motion Gradually.* If instead of suddenly arresting the dipole orientation we could apply a gradual braking action, the dielectric constant, instead of dropping abruptly, would fall gradually to the value corresponding to the contribution of displacement electrons and of free space. This gradual slowing down of the dipole motion occurs in liquids which on being cooled exhibit a great increase in viscosity instead of freezing to crystalline solids. Rosin oil, castor oil, and glycerine (see Bock[5]) are good examples of this type of liquid. The dipole orientation is opposed by the viscosity of the liquid and when it becomes very great, the molecules can respond only to a slowly changing field. They are not fixed, as in a frozen liquid, but have only become sluggish. In a one million cycle field, the response of the dipoles finally becomes negligible as the temperature is lowered, because the direction of the field changes too rapidly for them to follow. The dielectric constant for this frequency is then due only to the displacement electrons and to free space. The dipoles can still respond to a less rapidly changing field and the dielectric constant for 60 cycles is higher. A simple analogy would be a bottle half full of molasses. If you invert it, the molasses will take a certain time to flow down. If you turn it over and over rapidly, the response to gravity will be slight and it will behave like a solid. If the molasses is warmed or diluted to decrease the viscosity, it will respond to more rapid turning.

*5. Losses Due to Dipole Friction.* The motion of the dipole molecules turning in a viscous environment gives rise to frictional losses, so that some of the energy of the field is dissipated in the form of heat. The magnitude of the resulting power factor is determined by two factors, namely, the amount of motion, of which the dielectric constant is a function, and the viscosity opposing the motion. At temperatures where the viscosity is low, the dipole power factor (so called to distinguish it from that due to leakage) is small. On the other hand, at viscosities great enough practically to prevent orientation the power factor is again small. The maximum occurs in the intermediate region of temperature where the resultant of motion and high viscosity is greatest. This is the region where the dielectric constant decreases with falling temperature, as shown by the experimental results of the present work. As already pointed out, in viscous liquids, as distinguished from crystalline solids, the dipoles are not fixed and can therefore follow a field which reverses sufficiently slowly. It seems reasonable to suppose that with a given liquid in a definite state, the molecules can follow an alternating field with ease up to some range of frequencies where it becomes difficult for them to respond. We might then speak of a *characteristic frequency* for a given material. By this is roughly meant that the molecules can readily follow a field of lower frequency than the characteristic frequency, but that at higher frequencies the orientation falls off. A more precise conception of the characteristic frequency is given in the following sketch of the Debye[1] theory of anomalous dispersion.

## III. The Debye Theory of Anomalous Dispersion and Absorption

1. A number of liquids which at ordinary frequencies are characterized by unusually high dielectric constants was found by Drude[6] and others to suffer a marked decrease in dielectric constant with increasing frequency in the very high frequency region of the electrical spectrum. This decrease was accompanied by a pronounced energy absorption. Water showed a drop from $\epsilon = 81$ to $\epsilon = 3$. The following table, quoted from Debye[1] shows the behavior of amyl alcohol and glycerine.

TABLE II

| Amyl Alcohol 18 deg. cent. | | Glycerine 15 deg. cent. | |
|---|---|---|---|
| Frequency | Diel. Const. | Frequency | Diel. Const. |
| Low | 16 | $25 \times 10^6 \sim$ | 56.2 |
| $150 \times 10^6 \sim$ | 10.8 | $150 \times 10^6 \sim$ | 39.1 |
| $412 \times 10^6 \sim$ | 4.7 | $400 \times 10^6 \sim$ | 25.4 |
| | | $111 \times 10^8 \sim$ | 2.53 |

Such decrease of dielectric constant with increasing frequency is called "anomalous dispersion," a term derived from Optics where "normal dispersion" means increase of refractive index $n$ with increasing light frequency. In the vicinity of an absorption band the opposite change may occur, and is called "anomalous dispersion." The accompanying pronounced absorption of light energy is called "anomalous absorption." On the Maxwell electromagnetic theory $\epsilon = n^2$ and therefore the change of $\epsilon$ with frequency is also called "dispersion" and is "anomalous" when $\epsilon$ drops with increasing frequency. The energy loss in this region is likewise called "anomalous absorption." Anomalous dispersion and absorption are therefore defined as a decrease in dielectric constant with increasing applied frequency accompanied by a power factor maximum.

As Debye says, it is natural to attribute this anomalous dispersion to the fact that whereas the displacement electrons can respond to alternating fields of any frequency, dipole orientation becomes increasingly difficult with rising frequency. Therefore, anomalous dispersion and absorption should occur only in dipole liquids.

Consider such a liquid between the plates of a condenser. In the unstressed state, the thermal motion of the molecules gives rise to a perfectly random arrangement and the vector sum in any direction of the individual dipole moments is 0. There is no polarization. If an electrostatic field is now applied, there is a tendency for some of the slower moving molecules to line up in the field. The perfectly random arrangement of the molecules in the unstressed liquid changes to a somewhat more orderly one. If the field is removed, the molecules revert to the previous random state.

Whereas the displacement electrons spring back to their unstressed positions in the molecules instantaneously, the dipole molecules require a finite time to return to the original random state. The arrangement produced by the field falls off exponentially, and the time required to drop to the $1/e$th part is called the "*relaxation time,*" $\tau$ ($e = 2.718$, the base of the natural logarithms). The relaxation time is thus a measure of the quickness of the response of the dipole molecules. The reciprocal of the relaxation time $\frac{1}{\tau}$ gives the "*characteristic frequency*" $f_0$. It depends in a given substance on the size of the molecule, the temperature, and the viscosity. If the assumptions are made that the molecules are spheres of radius $r$, and that the frictional resistance opposing their turning obeys the same law as with an ordinary sphere, we have, according to Debye:

$$f_0 = \frac{1}{\tau} = \frac{k\,T}{8\,\pi^2\,\eta\,r^3} = \frac{T}{\eta} \text{ times a constant} \qquad (1)$$

$f_0$ = characteristic frequency
$r$ = radius of sphere equivalent to the actual molecule
$\eta$ = coefficient of viscosity
$k$ = Boltzmann constant, $1.37 \times 10^{-16}$ erg per deg. cent.
$T$ = absolute temperature

This relation is an ideal one which can be expected, when applied to viscous liquids like rosin and castor oil, to give only the correct order of magnitude. Knowledge of $f_0$ shows in what frequency range anomalous dispersion and absorption should occur. As long as the applied frequency $f$ is low compared to $f_0$, the dipoles are able to respond to the field, and the dielectric constant is independent of the frequency. On the other hand, when $f$ is much higher than $f_0$, the field changes faster than the dipoles can turn, so that the dipole contribution to $\epsilon$ disappears. Moreover, since the alternating field produces practically no turning, the frictional loss vanishes and the dipole power factor becomes negligible. The region of $\epsilon$ drop and maximum power factor occurs where $f$ and $f_0$ are comparable.

By means of statistical reasoning, Debye obtains relations which permit the calculation of the power loss from the change $\epsilon_0 - \epsilon_\infty$ where $\epsilon_0$ means the dielectric constant at frequencies much lower than $f_0$ and $\epsilon_\infty$ that at frequencies so high that the dipole response has disappeared. The energy loss is maximum when

$$\frac{f}{f_0} = \frac{\epsilon_\infty + 2}{\epsilon_0 + 2}\sqrt{\frac{\epsilon_0}{\epsilon_\infty}} \qquad (2)$$

For materials like rosin oil in which the change in $\epsilon$ is small, the power factor maximum comes almost exactly at $f = f_0$.

The general nature of the phenomenon of anomalous dispersion and absorption is more readily shown by reproducing the curves calculated by Debye for glycerine, than by giving the mathematical development of the theory. The results which are plotted against $\log \frac{f}{f_0}$ are the refractive index $n$ and the index of absorption $\alpha$. Since they are functions of the dielectric constant and energy loss respectively, their behavior with respect to frequency is similar and it is unnecessary to convert them to the usual electrical quantities.

In the curves showing the experimental results, the general resemblance to these theoretical curves may readily be noted. The direction of the experimental curves is opposite, because the ratio $f/f_0$ is decreasing with rising temperature.

2. *The Frequency Region of Anomalous Dispersion.* In the past it has been difficult to obtain conclusive data to support the theory of anomalous dispersion. The reason lies in the extremely high frequencies necessary to produce the effect. Consider the case of water. At 27 deg. cent. (300 absolute), the viscosity coefficient $\eta = 0.01$. If the radius of the molecule is taken firs

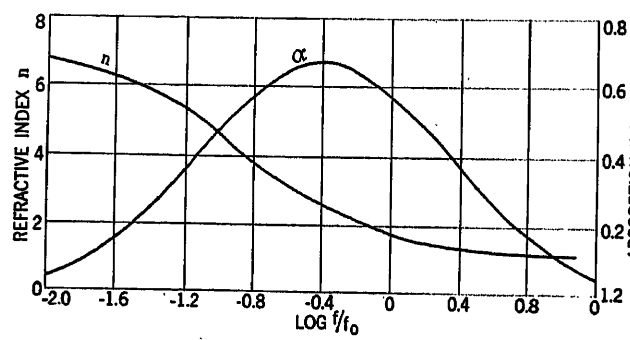


Fig. 2—Theoretical Curves showing Anomalous Dispersion and Absorption in Glycerine [Debye[1]]

as $1 \times 10^{-8}$ cm. for single molecules, and as twice that amount for associated molecules, we obtain by Equation (1) characteristic frequencies of 50 billion and 6 billion cycles. The phenomenon of anomalous dispersion and absorption has been studied in this frequency region by waves obtained from extremely minute Hertz oscillators. Optical methods were used to measure $n$ and $\alpha$. Such waves are difficult to produce and to control, and are highly damped and variable in intensity. The outstanding work of this sort has been done by Nichols and Tear.[7]

If the region of anomalous dispersion could be shifted into a more accessible frequency region, such as that of the radio waves, the measurements would be more reliable and easier to make. Quoting from Debye—"Particular emphasis, at least qualitatively, may be placed on the prediction that (a) substances with large molecules and great viscosity must show noticeable anomalous dispersion and absorption even at relatively long waves, and (b), that the dispersion and particularly the absorption will be very sensitive to temperature, since the coefficient of viscosity is very dependent on

temperature." He points to the behavior of amyl alcohol and glycerine (Table II) in support of the shift to lower frequencies. The experimental results of the present paper are in agreement with both predictions. It is shown that even at frequencies as low as 60 cycles it is possible to obtain anomalous dispersion and absorption. Thus, the range of the phenomena has been extended from the extreme frequency region of billions of cycles to that with which electrical engineers are concerned.

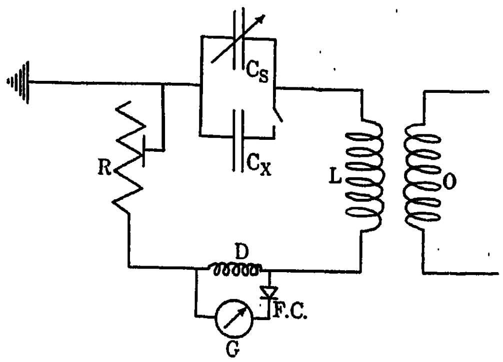


Fig. 3

$C_s$ standard variable condenser
$C_x$ test condenser
$G$ micro-ammeter
$R$ decade resistance
$L$ inductance
$D$ drop coil of a few turns
$F.C.$ crystal rectifier
$O$ coupling to oscillator

## IV. Experimental Method

The apparatus and method were very simple. The first series of experiments was made at one million cycles, the temperature being varied. A simple Hartley oscillator using a UX 201-A tube supplied the high-frequency current. Wavemeters were used to measure the frequency.

The measuring circuit is a simple resonant circuit with the experimental and the standard condenser in parallel. With $C_x$ in and $R = O$, the circuit is tuned to resonance by varying $C_s$. The voltage drop across coil $D$ causes a current, rectified by the crystal, to flow through $G$. The coupling to the oscillator is adjusted to give a suitable galvanometer deflection. Then $C_x$ is removed, the circuit retuned, and $R$ increased to give the same deflection. Then $C_x = C_{s2} - C_{s1}$, the difference in the two readings of $C_s$. The equivalent series resistance of $C_x$ is obtained by the relation:

$$R_x = R\,(C_{s2}/C_x)^2$$

From the measured values, the power factor is obtained by the equation

$$P\,F = 2\,\pi\,n\,C_x\,R_x.$$

For the work at higher frequencies up to 11 million cycles, the same principles were followed. A suitable oscillator and measuring circuit were used. Instead of the decade resistance a set of resistances was made, consisting of short straight lengths of No. 40 cupron wire, soldered to heavy copper lead and sealed into glass tubes. Insertion of these resistors in the circuit caused very little detuning.

The test condensers consisted of coaxial metal cylinders. Tests showed the power factor and the capacity variation with temperature of the empty condensers to be negligible.

## V. Discussion of Experimental Results

It was not until after a number of tests had been made that the molecular interpretation suggested itself. Tested at one million cycles, a sample of a cable compound containing 5 per cent of rosin showed higher power factor at room temperature than at 200 deg. fahr., whereas at 60 cycles the behavior was the reverse. The ingredients were then tested separately. Transil oil tested at one million cycles between 70 deg. and 160 deg. fahr. showed negligible power factor. Light amber petrolatum showed a gradual decrease in power factor from 0.16 per cent at 74 deg. to 0.03 per cent at 210 deg. The dielectric constant decreased with rising temperature as expected. The peculiar behavior of a commercial rosin oil (50 per cent rosin, 50 per cent mineral oil) shown in Fig. 4 gave the first hint of an explanation of the anomalous change in power factor and dielectric constant. Inspection of the viscosity curve shows that the power factor maximum and the drop in dielectric constant occurs in a temperature region where the viscosity increase is very rapid. This suggests the idea that the power factor and a part of the dielectric constant depends on some mode of motion which is greatly influenced by the viscosity of the medium. Dipole orientation discussed above readily accounts for this behavior.

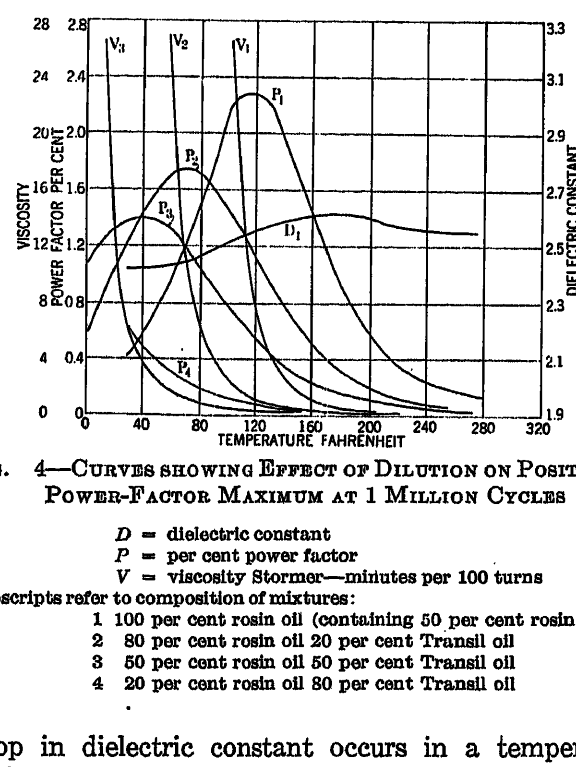


Fig. 4—Curves showing Effect of Dilution on Position of Power-Factor Maximum at 1 Million Cycles

$D$ = dielectric constant
$P$ = per cent power factor
$V$ = viscosity Stormer—minutes per 100 turns
Subscripts refer to composition of mixtures:
1 100 per cent rosin oil (containing 50 per cent rosin)
2 80 per cent rosin oil 20 per cent Transil oil
3 50 per cent rosin oil 50 per cent Transil oil
4 20 per cent rosin oil 80 per cent Transil oil

*1. Effect of Dilution.* If the behavior was due to

some chemical change in the dissolved rosin rather than to the effect of viscosity, dilution with a neutral oil (*i. e.*, one of negligible power factor) like Transil oil, should merely decrease the power factor without shifting the position of the maximum. If, on the other hand, the dipole explanation is correct, dilution with Transil oil to decrease the viscosity should shift both the power factor and dielectric constant curves for constant frequency to lower temperatures. Obviously, since the concentration of dipoles is decreased by dilution, the dielectric constant drop and the maximum power factor are diminished. Fig. 4 shows that the curves for one million cycles shift in the predicted manner. For each concentration of rosin oil, the corresponding curves show the same relative positions. It is readily seen that an enormous increase of viscosity is necessary to cut down the orientation sufficiently to cause a large power factor decrease. This is to be expected, since the great increase in dipole friction with rising viscosity partly counteracts the effect of diminishing orientation. The same reasoning explains the fact that the dielectric constant always starts to fall at lower viscosity, *i. e.*, higher temperature, than where the power factor maximum is reached.

Substitution of the more viscous Nujol for Transil oil in a 50 per cent mix with rosin oil shifted the position of the power factor maximum from 40 deg. to 55 deg. fahr. This is additional proof of the orientation theory, since the mere substitution of one neutral oil for another should have no chemical effect on the rosin content. Replacing the Transil oil with 50 per cent of heavy cylinder oil caused a still greater shift to 75 deg.

An interesting fact was shown by the behavior of a mix of 50 per cent rosin oil in 140 deg. paraffin. At 122 deg. it was apparently solid, yet the power factor curve showed no break. The explanation lies in the fact that the apparent consistency is not a true indication of the actual environment opposing the motion of the dipole molecules. In the stiff, salve-like mixture there is a distributed liquid component in which the dipoles turn freely until this liquid component becomes very viscous. An analogy is electrolytic conduction in a dilute gelatine gel where the ions pass unhindered by the apparent solidity. Of course, the viscosity of the liquid component is affected by the change in composition as the paraffin separates out. This kind of behavior of apparent solids has led some investigators to attribute power factor maxima and dielectric constant drops to change in state of the material. In such cases a change of applied frequency should furnish a criterion. Lower frequency should shift the curves into the solid region, while much higher frequency should cause the whole change to occur in the liquid state.

*2. Effect of Change of Frequency.* Equation (2) shows that when $\epsilon_0 - \epsilon_\infty$ is small, the power factor peak and the drop in $\epsilon$ occur very nearly at $f = f_0$. According to Equation (1), $f_0$ is proportional to $T/\eta$, for a given material. The viscosity curves show the change of viscosity with temperature to be very great, so that a relatively small change in the temperature of rosin, for example, causes an enormous change in the characteristic frequency. It is therefore much more convenient to work at fixed frequency and variable temperature, *i. e.*, $f$ constant and $f_0$ varying greatly, than to keep $f_0$ constant (constant temperature) and employ a correspondingly wide range of applied frequencies. Although the behavior observed when this method is used cannot strictly be called "anomalous dispersion" as defined in Section III, it is obviously due to the same cause, being determined by the ratio $f/f_0$. On this basis, at higher fixed applied frequencies the curves would shift to higher temperatures. Fig. 5 shows the results at 60 cycles, 1 million, and 10 million cycles, obtained with Hercules Wood Rosin *F*. The two higher frequency curves and the viscosity curve are plotted from data obtained by the writer; the data for the 60-cycle curve and the resistivity curve were kindly furnished by the Hercules Powder Co. These curves show some very interesting features with regard to the orientation theory.

*a. Dielectric Constant of Rosin vs. Temperature and Frequency.* The course of the dielectric constant curves is in good agreement with the theory. At 80 deg. fahr. $\epsilon$ is the same for all three frequencies. This value $\epsilon = 2.68$ is due to the displacement electrons alone, as shown by the fact that the power factor has dropped to negligible values. The viscosity of the rosin at this point is so enormous that even the slightest dipole motion would give rise to high frictional losses, so that we may be sure that the contribution of dipole motion to the dielectric constant is practically nil. Thus, when the dipoles are practically fixed—*e. g.*, down to 60 cycles, the dielectric constant is independent of frequency. On the other hand, the $\epsilon$ curves again converge on the high temperature side, because there the viscosity is very low, resulting in a characteristic frequency high compared to the three applied frequencies, and the dipole response is complete. At 180 deg. the dielectric constant for 60 cycles reaches a maximum, while it is still low for the two radio frequencies. At this point the dipoles respond fully to the 60-cycle field, but are too sluggish to follow the higher frequency fields. Above this temperature $\epsilon$ for 60 cycles drops because while the dipole response no longer increases, the density decreases. At 260 deg. and at about 310 deg. the dipole response to the 1 and 10 million cycle fields is complete.

*b. The "V" Curve of Power Factor vs. Temperature.* It may be noted that the 60-cycle power factor curve shows an upward bend above 210 deg. fahr., a behavior not shown by any of the radio frequency curves. Inspection of the resistivity curve gives the explanation. While at 210 deg. the dipole power factor has become very slight, the leakage is rapidly increasing. The leakage current becomes comparable with the 60-cycle charging current. At radio frequencies, on the other

hand, although the leakage current is even greater because of higher temperature, it is negligible compared with the charging current and gives no upward turn to the curves. Power factor minima resembling this part of the curve (commonly called *V* curves) have frequently been observed in tests on cable compounds. Various explanations have been given. Hochstädter[8] attributes the right branch to leakage increasing with temperature, and the left branch to "hysteresis." This explanation seems to be the correct one if "hysteresis" is taken to mean loss due to dipole friction. Dunsheath[9] disagrees with Hochstädter's explanation and attempts to account for the *V* curves on the basis of his $I^2 R$ theory. This theory is based on a model consisting of capacities and resistances. The increase with rising temperature is accounted for by assuming the capacities and resistances in parallel, so that decrease in resistivity causes rising power factor. On the low temperature side of the minimum the capacities are assumed to be in series, so that now increase in resistivity as the temperature falls again causes a rise in power factor. On this basis, the maximum could only be explained by assuming that the parallel arrangement was again restored at the proper point. This theory seems to the writer to be highly artificial, and incapable of explaining either the observed influence of viscosity in the rosin mixtures or anomalous dispersion and absorption in pure liquids containing only a single molecular species.

*c. Relation of Spacing and Width of Power Factor Curves.* The comparative widths of the curves of

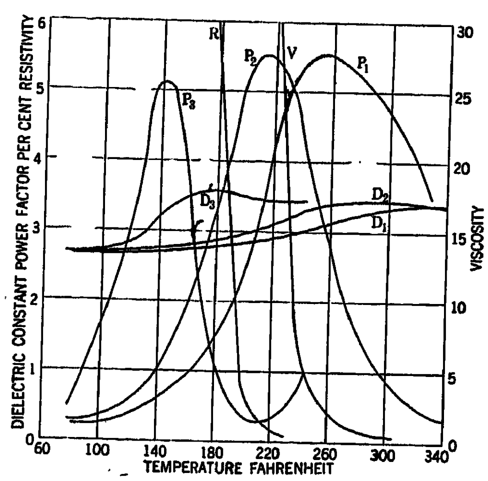


FIG. 5—CURVES SHOWING BEHAVIOR OF WOOD ROSIN AT DIFFERENT FREQUENCIES

*V* viscosity Stormer in minutes per 100 turns
*R* resistivity in ohms $\times 10^{12}$ per cu. cm.
*P* per cent power factor
*D* dielectric constant
Subscripts refer to frequency:
(1) 10 million cycles
(2) 1 million cycles
(3) 60 cycles

power factor and dielectric constant (Fig. 5) and the way they are spaced with respect to temperature are good qualitative evidence of the correctness of the orientation theory. By Equation (1), $f_0$ is proportional to $T/\eta$. Since the change of $\eta$ is very great compared to that of $T$, it is simpler to neglect the direct effect on $f_0$ of change of $T$ and consider only the effect of change of $\eta$. Then on the orientation theory the

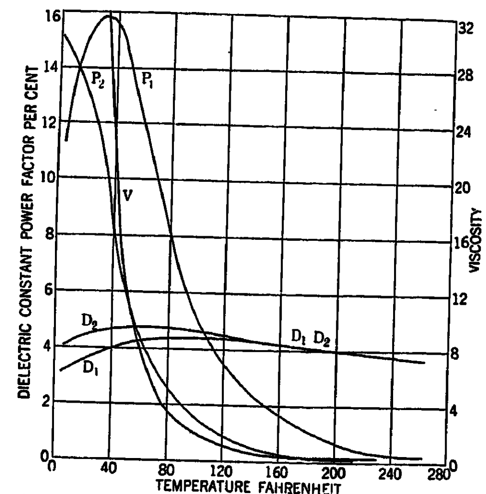


FIG. 6—CURVES SHOWING BEHAVIOR OF CASTOR OIL AT 2 FREQUENCIES

*V* viscosity Stormer in minutes per 100 turns
*P* per cent power factor
*D* dielectric constant
Subscripts refer to frequency:
1 10.9 million cycles
2 1.415 million cycles

anomalous dispersion and absorption curves for equal multiples of frequency would be of equal width and spaced at equal temperature intervals, provided the viscosity changed exponentially with temperature. That is, if the viscosity *vs.* temperature curve were exponential, the spacing of the power factor and dielectric constant curves for 100, 1000, 10,000, etc., cycles would be uniform. But inspection of the viscosity curve for rosin shows that the rate of increase of viscosity with falling temperature does not progress exponentially. Thus, starting at 300 deg. and taking the ratios of successive viscosities at 10 deg. intervals downward, we find 2 to 1, 2.5 to 1, 3.1 to 1, 5.5 to 1.

The result is that the 60-cycle curve, being in a region of viscosity increase which is much more rapid than that in the higher temperature radio frequency region, is narrower than the one million-cycle curve, which in turn is narrower than the ten million-cycle curve. Also, the one and ten million-cycle curves are farther apart relative to the frequency difference than the 60-cycle and one million-cycle curves. The viscosities corresponding to one and ten million-cycle power factor maxima are approximately in the ratio 10 to 1 as they should be.

To test further the spacing of the power factor peaks with respect to viscosity, tests were made at $1.415 \times 10^6$ and at $10^7$ cycles on two other samples, one of commercial rosin oil (50 per cent rosin), the other 50 per cent by weight wood rosin *F* in Transil oil. Since the curves are of the typical shape, they are omitted.

The peaks for the rosin oil occur at 120 deg. and 152 deg. fahr. The viscosities at these temperatures are in the ratio 6 to 1. The peaks for the diluted wood rosin are at 96 deg. and 136 deg. and the corresponding viscosities in the ratio 6.5 to 1. Since the frequency ratio is 1 to 7, the agreement with the prediction is satisfactory in view of the uncertainty of the exact positions of the power factor peaks.

*d. Tests on Castor Oil.* In order that all the tests might not be confined to rosin-containing materials, similar tests were made on a sample of refined castor oil. The curves are shown in Fig. 6. It will be noted that very high frequency is required to bring the power factor maximum above 0 deg. fahr. This shows that much greater viscous friction is required to prevent molecular orientation at a given frequency than in the case of rosin oil. This fact indicates molecules of small dimensions and larger dipole moment. The larger dipole moment is also shown by the higher dielectric constant and the higher power factor at maximum. As low as 130 deg. fahr. the dielectric constant is seen to be independent of frequency up to 10.9 million cycles. The tests were not carried to low enough temperature to make the dielectric constant curves for the two frequencies come together as the rosin curves did, but it is evident that they would. Castor oil thus shows behavior similar to rosin and rosin oil and the results with rosin are not due to any peculiarity other than its dipole nature and high viscosity.

*e. Numerical Check of Equation* (1). It is of interest to substitute in the equation $f_0 = k\,T/8\,\pi^2\,\eta\,r^3$ the values for rosin and castor oil to see if a reasonable figure for the molecular radius results in each case. Since the power factor maximum occurs practically at $f = f_0$, and the temperature and viscosity are known, Equation (1) can be solved for $r$. For rosin when $f = 10^7$, the peak is at $T = 396$ deg. absolute. At this temperature $\eta = 0.63$. Solving for $r$ gives for rosin $r = 4.8 \times 10^{-8}$ cm.

Similar calculation gives for castor oil the value $r = 2 \times 10^{-8}$ cm. The fact that these results are comparable with molecular sizes obtained by entirely different methods (X-rays, spreading of oil films, etc.) gives a striking proof of the fundamental correctness of the orientation theory.

*3. Optical Evidence of Orientation.* Some results of optical tests on rosin are included because of the additional support they give to the orientation theory. The work was done with Professor Hans Müller of the Massachusetts Institute of Technology and is more fully treated in a joint paper.[11]

The phenomenon of electrical double refraction, called the Kerr[12] effect, is well known. Many materials become doubly refracting in a strong electrostatic field. The behavior in liquids is observed by placing the material in a cell between Nicol's prisms set at 45 deg. to the direction of the electric field and at right angles to each other and passing through a strong beam of light. In the unstressed condition the liquid has no influence on the light and the crossed nicols extinguish the beam. If under stress the liquid becomes doubly refracting, the plane polarized light entering the liquid through the first nicol becomes elliptically polarized and some light gets through the second nicol. An image of a slit placed between the first nicol and the source of light may then be observed with a suitable eyepiece.

Since in the unstressed liquid the dipole molecules are arranged in a perfectly random manner, the refractive index is the same in all directions. But in an electric field the dipoles tend to orient, so that the average number of molecules having their dipole axes in line with the field is greater than at right angles. The refractive indices parallel and at right angles to the field are different, giving double refraction. Non-dipole liquids show some Kerr effect but polar liquids show it much more strongly. Since this effect depends on dipole orientation, the time required for the effect to disappear after sudden discharge should equal the relaxation time already defined for electrical polarization. Then, by reasoning similar to that which applies to the change of dielectric constant, the effect should fail to appear in an alternating field of frequency too high for the dipoles to follow. This prediction was confirmed by the behavior of rosin. At room temperature, solid rosin showed a good Kerr effect with a static field of 10,000 volts, but none whatever with a 60-cycle field even though the stress was increased to 75,000 volts. The sudden discharge was produced by short circuiting the cell. A very long relaxation time for the optical effect was noted at room temperature. After discharge, it took at least 30 seconds for the image of the slit to disappear. On the application of 60-cycle stress and gradually rising temperature, a faint image was observed at about 120 deg. fahr., increasing in intensity until at 170 deg. it was as strong as for a static field. The observations were checked several times after the rosin had been kept at constant temperature for 24 hr. and the threshold temperature for the appearance of the effect was found to be about 100 deg. Thus the optical behavior is closely parallel to the change of dielectric constant at 60 cycles in the same temperature region. To produce a similar effect at radio frequency was more difficult. A high voltage field of 1.5 million cycles was produced by a Tesla coil driven by an oscillator using a UX-852 75-watt vacuum tube. The power loss in the liquid rosin caused peculiar convection effects which blurred the image of the slit. However, the disturbance due to convection required about 1 second to appear after the field was applied, so that during the first few instants an image could be observed. The difficulties of observation precluded an accurate determination of the threshold temperature. The Kerr effect could be observed unmistakeably above 240 deg. which is in approximately the correct region. Thus, the optical

behavior of rosin at different applied frequencies gives strong additional support to the orientation theory advanced to explain anomalous power factor and dielectric constant changes.

*4. Electrical Behavior of Rubber-Sulfur Compounds.* Electrical tests at various frequencies on samples of vulcanized rubber containing different amounts of cobined sulphur (Figs. 7 and 8) show very interesting features which may be related to the dipole theory. Curtis, McPherson, and Scott[10] showed the interesting relations of power factor and dielectric constant at 1000 cycles to composition of vulcanized rubber. They found a power factor maximum of 8 per cent at 13 per cent sulphur, and a dielectric constant maximum of 3.75 per cent at 10.5 per cent sulphur. Both properties showed a minimum at 19 per cent. These workers pointed out the fact that the maxima and minima occurred at compositions corresponding to simple formulas and claimed that "compound formation is the fundamental basis for the interpretation of the observed changes." The power factor maximum would correspond to $(C_5H_8)_3S$, and the dielectric constant maximum to $(C_5H_8)_4S$. No explanation is given of the fact that the two maxima occur at different compositions.

Figs. 7 and 8 show that the positions of the maxima can be shifted over a wide range of composition by changing the test frequency and that, therefore, the fact that at 1000 cycles the maxima did coincide with compositions corresponding to simple chemical compounds is without significance.

The suggestion that certain rubber-sulphur chemical combinations are probably dipole molecules is due to Mr. C. R. Boggs. He pointed out the fact that while the untreated rubber molecule is electrically neutral, as is shown by electrical tests, the chemical addition of sulphur in such a manner as to produce an unsymmetrical molecule would produce a dipole. In other words, if more S groups became attached to the rubber molecule on one side of the original center of charge than on the other, a dipole molecule would result. If further S groups were added so that the arrangement became symmetrical, the molecule would again lose its dipole properties. Such a compensating effect is well known in other substances. Methane $CH_4$ is electrically neutral. If a strong dipole-producing group, like the nitro group, is added to form nitro-methane $CH_3NO_2$ by replacing one H, the molecule shows a high dipole moment, as indicated by the dielectric constant 56. The addition of more $NO_2$ groups, however, progressively reduces the dipole moment until with the symmetrical tetra-nitro-methane $C(NO_2)_4$ the dielectric constant falls to the neutral value of 2.1. Many other examples might be given, but the above shows well the progressive compensation.

In the low sulphur range the chances are greater that the compounds formed will be dipoles. As the percentage of sulphur is increased, the probability of compensation increases. Thus, the composition indicated by the per cent of combined sulphur at maximum power factor or dielectric constant would certainly not correspond to the formula for the molecule of maximum dipole moment. Starting with 0 per cent S and increasing the amount of combined sulphur, we should therefore expect to find at first a rise of power factor and dielectric constant to maximum as the number of dipoles increased, followed by a drop in both properties as the further addition of sulphur atoms eliminated the dipoles by compensation. The mechanism of the dipole power loss would here also be one of heat loss due to the mo-

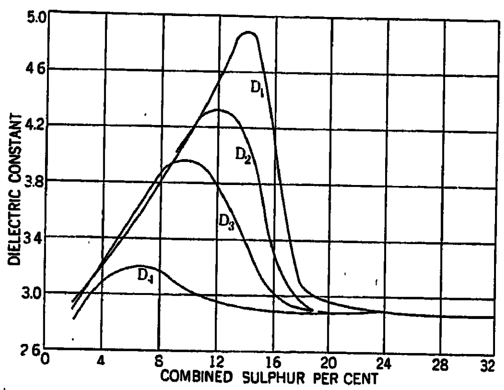


FIG. 7—CURVES SHOWING DIELECTRIC CONSTANT *vs.* PER CENT COMBINED SULPHUR AT 4 FREQUENCIES

Subscripts refer to frequency:
1 0 cycles, *i. e.*, static stress
2 60 cycles
3 1000 cycles
4 380,000 cycles

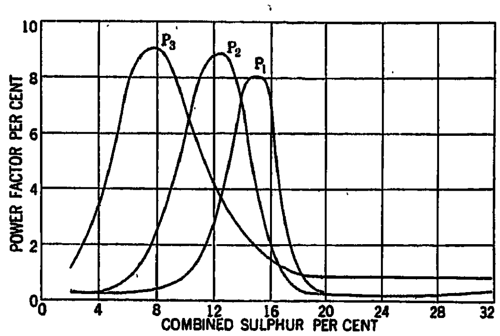


FIG. 8—CURVES SHOWING POWER FACTOR *vs.* PER CENT COMBINED SULPHUR AT 3 FREQUENCIES

Subscripts refer to frequency:
1 60 cycles
2 1000 cycles
3 380,000 cycles

tion. In this case, however, the molecules would probably be held by elastic forces, so that in responding to the changing field they would subject the surrounding medium to rapid stretch and release. Since rubber is an imperfectly elastic material, this internal motion would give rise to frictional losses measured electrically as power loss. The above discussion considers only the rise and decay of dipole effect due to change in number of dipoles. Another important factor is the increasing hardness of the medium. As the sulphur content

increases, the rubber becomes more and more rigid, so that even if the number of dipoles were constant, the amount of motion in a given field would change greatly with changing composition due to the change in opposing forces. Thus, the phenomenon in vulcanized rubber is more complicated than in the liquid materials previously studied, in that simultaneously the number of dipoles and the stiffness of the medium is changing.

However, it is possible to predict qualitatively some electrical features that should be observed if the application of the orientation theory to rubber is permissible. There should be a characteristic frequency for each composition in the region where dipole molecules exist; the softer the rubber, the higher the characteristic frequency. Then the maxima in the dielectric constant and power factor should occur at lower sulphur contents the higher the applied frequency. The curves (Figs. 7 and 8) show this to be true. As in the case of the oils, the maximum dielectric constant occurred at higher temperature—*i. e.*, lower viscosity than the maximum power factor, so in the rubber it should lie on the low sulphur side of the power factor maximum where the rubber is softer. Comparison of the corresponding curves in the two figures shows that this is so. The progressive drop in the values of the dielectric constant maxima for increasing frequency is readily explained. From 0 per cent S to 14 per cent, at least, the number of dipoles is increasing nearly linearly as the d-c. dielectric constant curve shows. But, although there are more dipoles at 10 per cent S than at 6 per cent S, the dielectric constant for 380,000 cycles is greater at 6 per cent because at higher sulphur contents the stiffness becomes too great for dipole response at this frequency. Similar relations hold for the other frequencies. Even the d-c. test probably does not show the composition at which the maximum number of dipoles are present. The speed of orientation may become so low at high sulphur contents that the usual d-c. test does not allow the full orientation. This idea agrees with the very long relaxation time in solid rosin shown by the optical tests.

A thorough study of the electrical properties of vulcanized rubber might throw considerable light on the nature of the vulcanization reactions. The application of the orientation hypothesis to rubber is only tentative and considerable critical experimental work would be needed to place it on as firm a basis as in the case of viscous liquids.

## CONCLUSIONS

1. The peculiar temperature variation of dielectric constant and power factor at different frequencies of rosin, rosin oil, and castor oil, and the anomalous change in electric double refraction of rosin have been shown to be functions of the viscosity.

2. The influence of viscosity on both the electrical and optical properties has been explained on the Debye theory of dipole orientation.

3. The electrical behavior of vulcanized rubber samples of various compositions suggests the presence of dipole molecules.

4. Very accurate measurements on viscous, dipole materials of high purity would be of great value in checking and extending the theory.

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The results given in this paper are believed to be of value both to electrical engineers and to physicists. It is hoped that they will help to interest practical students of dielectrics in a physical theory which hitherto has been tested mostly only at frequencies and on materials remote from practical purposes.

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## Discussion

**F. M. Clark:** Mr. Kitchin is to be congratulated for presenting to a body of engineers DeBye's di-polar theory. With the proper understanding of the tenets of this theory, we have a means of explaining a large number of the difficulties which we now meet in our study of insulation.

The discussion of DeByé's theory has been voluminous in the past few years. The important contribution which Mr. Kitchin has made is the presentation of an explanation of power factor in the same terms as physicists are now explaining, or attempting to explain, the dielectric constant. Di-polar materials have been considered to possess an inherently high power factor. Mr. Kitchin shows that this is not necessarily so under all conditions of measurement. Given the right condition of test the power factor may be relatively small.

It must be borne in mind that di-polar materials possess fundamentally less resistance to direct conduction and a greater tendency to electrolytic type conduction. In going from a non-polar to a di-polar material, therefore, we are passing from a substance of high resistivity and a conduction which is chiefly electronic to a material of relatively low resistivity and a conduction which may be partly or entirely of the electrolytic type. The non-polar materials are of low power factor. The di-polar materials, according to Mr. Kitchin, may possess either high or low power factor depending upon the conditions of test.

Mr. Kitchin illustrates the effect of solidification by reference to test data on ether. Ether solidifies at −113 deg. cent. From

this fact it is rather difficult to understand the conclusions drawn by the author. However, reference to the original work of Isnardi shows that there is a slight error in the curve cited by Mr. Kitchin. The maximum reached by the original author was obtained at approximately −110 deg. cent and a marked drop in dielectric constant occurred between −110 and −115 deg. cent. With this correction in Mr. Kitchin's curve, the conclusions which he draws appear justified.

Reference is made to Bock's work with glycerine. Glycerine solidifies at a low temperature but when once solid melts on warming up at about +15 deg. cent. With dielectric-constant measurements under decreasing temperature, therefore, we are testing a viscous material. With dielectric-constant measurements with rising temperature after the glycerine has once solidified we are dealing with a solid. If the work of Bock were done on glycerine as the temperature was slowly lowered, the data obtained apply directly to the phenomenon with which Mr. Kitchin is dealing, orientation in a viscous liquid. If the work of Bock were done on a rising temperature, the phenomenon is that of orientation in solids under a slowly alternating field. Perhaps Mr. Kitchin can tell us the conditions under which Bock's work was carried out.

**G. F. Kennedy:** I might explain that there is a difference in dielectric characteristics between Hercules "FF" wood rosin and Hercules "FF" wood rosin No. 11. The rosin on which these electrical measurements were made is Hercules "FF" wood rosin No. 11, which is superior in dielectric characteristics to the Hercules "FF" wood rosin because the water-soluble material which is present in all rosins, both gum and wood, has been removed.

Rosins contain, for the most part, unoxidized and oxidized abietic acid or its isomers, the unoxidized acid having the formula $C_{20}H_{30}O_2$ and the oxidized acid the formula $C_{17}H_{26}O_5$.

Oxidized abietic acid is insoluble in 70 deg. Baumé petroleum ether at 25 deg. cent., so that the percentage of this material in rosins can readily be determined. Rosins contain, on an average, about 7 per cent of oxidized abietic acid and increase or decrease in viscosity at a given temperature in proportion to the amount of the oxidized acid present. Commercial abietic acid, which contains only ¼ per cent of oxidized abietic acid is of comparatively low viscosity.

There is a marked change in power factor as we pass from Hercules "FF" wood rosin No. 11 to commercial abietic acid. The peak is at 122 deg. fahr. and at that temperature, at 60 cycles frequency, is 4.6 per cent, diminishing to 0.2 per cent at 176 deg. fahr. This is due to the removal of the oxidized abietic acid.

I agree with Mr. Kitchin and Mr. Davis that rosins, both gum and wood, will progressively give off water on heating, for oxidized abietic acid has an oxyhydryl group which dehydrates completely by simple heating to 130 deg. cent., (266 deg. fahr). Water cannot be obtained from unoxidized abietic acid.

In regard to rosin oil, if you heat rosin or abietic acid to a relatively high temperature, you will change the characteristics of the abietic acid by breaking down the carboxyl group and you will form what is termed pyrogenated abietic acid. Rosin oil consists of this pyrogenated abietic acid entrained in the distillation dissolved in the liquid constituents of the oil which are many and of a complex nature. This pyrogenated abietic acid will in time crystallize from the oil. It will not oxidize, however, but for some reason that we have not yet determined, has poor dielectric characteristics.

**A. Nyman:** The dipole theory appears to be a very beautiful theory and apparently the tests of Mr. Kitchin are substantiated very fully. I am not quite in agreement with some of the conclusions of Mr. Kitchin. As I understand it, the reduction of the power factor at the higher temperature is explained by the reduction of viscosity and therefore a freer movement of the dipoles. However, the reduction of the power factor at the lower temperatures apparently is not explained except for Mr. Kitchin's suggestion that it may be due to the reduced conductivity; in other words, to a higher resistance and therefore a reduction of losses due to that.

It is my understanding that if you try to account for dielectric losses by conductivity alone, the results will be very inconsistent. In other words, the conductivity could account for only a very small proportion of absorption losses. On the other hand, the suggestion was made by Mr. Kitchin that it may be due to the fact that the dielectric constant is reduced; in other words, the picture I get is that when the substance cools down, the viscosity is increased enormously, but because the dipole can't orient itself as freely as before, the dielectric constant has been reduced.

Referring, for instance, to Fig. 5 of Mr. Kitchin's paper, I notice that the dielectric constant from the maximum position of about 3.4 has been reduced to about 2.7; that is about 20 per cent. At the same time, if you compare the maximum of the power factor, which is over 5 in all the three curves, it has been reduced to a figure which is probably around 0.3, that is a reduction of about 95 per cent. It seems to me there is a possible explanation of this which is not mentioned in the paper. On observing the figures, in particular Fig. 4, it seems to me that the maximum of power factor is in each case at a portion of the viscosity curve where there is the maximum change; in other words, the viscosity passes from a high value of the solid to a fairly low value of the liquid.

I have a mental picture of the change in the composition of the material with regard to dipoles during this passage from high viscosity to low viscosity. I have a picture that some of these dipoles change from sort of a solid state to a liquid state; other dipoles still remain solid, and you get a kind of a mixture of two kinds of dipoles, the solid dipoles and the liquid dipoles. As the temperature rises, more and more of these dipoles become liquid and therefore the whole substance approaches to a condition of a liquid, and the viscous substance that I have a mental picture of is a mixture of the two kinds of dipoles. If that picture is correct—and I am not at all sure that my physics of the thing is correct—then it seems to me that there is a possibility of applying again the Maxwell Law to this mixture; in other words, as long as you have a mixture of two kinds of dipoles, you have really two kinds of dielectrics, and the Maxwell theory was based on this unhomogeneous material which contained two types of dielectrics. So, although the statement of the author discards Maxwell's theory, it seems to me that it is possible to bring it back again on this supposition that there are two kinds of dipoles in a viscous substance.

**J. B. Whitehead:** Mr. Kitchin's work is the first attempt that I have seen to apply Debye's theory of dielectric constant to the phenomenon of energy losses in dielectrics. For some time past I have been not only making some study of Debye's theory but I have talked with a number of physicists, with the idea of developing whether or not in its present state it offers promise of explaining the losses that we find in dielectrics. I may say that my interest has been particularly directed to losses at relatively low frequencies, such as we have in commercial circuits. I have not been able to develop from the physicists any such suggestion; in fact, their opinion has been that a correlation could only be found at the very high frequencies which Mr. Kitchin has employed.

The suggestion has been that the amounts of energy that are involved in the oscillation or rotation of the dipole molecules at ordinary frequencies are quite small, and of quite different order of magnitude from those that we meet in the dielectrics of insulation. So I am particularly glad to see this work of Mr. Kitchin's, but I should like to point out that it is confined to this high range of frequencies, where it is to be expected that some such relationship would be found. The correlation that he has brought out, however, between power factor and viscosity, seems to be exceedingly important because it ties in a fundamental

physical theory with the ordinary physical constants of materials that we work with.

**C. F. Hill:** (communicated after adjournment) The paper by Mr. Kitchin is a very interesting one in that it is a first attempt by the practical scientist towards the use of Debye's theory of dipoles in the solution of our dielectric problems. Since the dipole theory has become pretty well established, it has been evident that it might form a rational explanation of many or most of our dielectric phenomena. The time lag of orientation of the polar molecule, for example, is similar to dielectric absorption as is also the time lag of the associated Kerr effect. Viscosity takes care of losses which we associate with absorption so that even for solids we need not disturb our ideas in accepting such an explanation. While it might be questioned if there are dipoles in solids, we do find the Kerr effect in glass for example with a very large time lag which suggests that it might be worth while to attempt to apply the theory there. The very long chains forming molecules in most of our solid dielectrics could even be imagined to have a temporary electric moment under voltage.

Mr. Kitchin's paper especially points out the effects of viscosity. We are reminded that a viscous displacement theory by Pellat forms one of our dielectric theories and gave us the equation for the absorption curve, $i = A\ t^{-n}$ which so far most accurately fits experimental results. The fact that many liquids which possess dipoles and show very large variation of the dielectric constant, apparently lose most of the dipole characteristics when they pass to the solid form may point to the fact that we are overlooking the dipole nature of molecules in many of our solid dielectrics. It seems probable that a study of the Kerr effect might throw some light on this as even in glass where it has been detected it may be masked by other effects.

An experience which was not understood at the time but which may have a bearing on the subject is worth repeating. Three or four years ago some condensers filled with transformer oil were used in a million-cycle broadcasting circuit. The oil apparently deteriorated very rapidly and became so hot it could not be used. A saturated oil such as Nujol was substituted and ran perfectly cool. Later both oils ran perfectly cool under ten million cycles. Mr. Kitchin's results show a resonance frequency for which power factor reaches a maximum. The voltage for broadcasting purposes was relatively high of course and might possibly magnify the effect but the results conform to Mr. Kitchin's results. This also raises the question if a decided increase in voltage used in power factor measurement might not be used to advantage in such a study. The difference between the saturated and unsaturated oil molecules (degree of symmetry) may have some influence on the tendency towards dipole characteristics.

Turning to Debye's theory, it would seem most logical to assume a rather stable dipole molecule considerably below the ionization voltage. Debye has assumed his dipoles to have a definite moment. The fact, if true, that substances with strong dipole characteristics have relatively high conductivity, would not necessarily come from Debye's theory, in my opinion. Debye's theory would also impose another characteristic on dielectric absorption if the two are the same, namely, that the space charge should be uniformly distributed. Some results by Joffe, for example, show a non-uniform distribution.

Some experimental work which has not been explained as yet is that of Eguchi on Elektrets. Since he used Canawba wax and resin it seems logical that his permanent polarization effects must be closely associated with the phenomena just reported by Mr. Kitchin. The conclusion that one should draw from Mr. Kitchin's paper, it seems to me, is that Debye's dipole theory may have considerable to do with our anomalous dielectric properties and can have without upsetting our ideas to any extent.

**D. W. Kitchin:** In order to avoid misconceptions regarding the scope of the orientation theory, the fact should be emphasized more strongly than was done in the paper, that it does not apply to every type of power loss. Its value lies in the explanation it gives of the anomalous changes of dielectric constant, the pronounced power-factor maxima in the corresponding temperature regions, and the influence of viscosity on both the electrical and optical properties.

An example may serve to make the above point clear. Mr. Clark, if I understand his discussion correctly, calls attention to the fact that although dipole materials have inherently high power factor, the magnitude depends on the conditions of measurement. In rosin we may have at least two types of power factor, which I have called "dipole power factor" and "leakage power factor." Curve $P_3$, Fig. 5, gives, I believe, a clear separation of the two effects. The 60-cycle power factor at 210 deg. fahr. is the sum of the dipole power factor and the leakage power factor. The leakage power factor becomes negligible below 210 deg. fahr., while above 210 deg. the 60-cycle dipole power factor is negligible. The orientation theory obviously does not apply to the rising 60-cycle power factor above 210 deg. But at 1,000,000 cycles, the dipole power factor is a maximum at 210 deg., and at 10,000,000 it is at about 250 deg. At some temperature, say at 270 deg., $P_3$ would cross $P_2$ and the power factor be equal for both 60 and 1,000,000 cycles and yet be due to entirely different mechanisms in the two cases.

I believe Mr. Clark is correct in his point that dipole materials in general have higher conductivity than non-dipole substances. Examples are water, nitrobenzene, and the alcohols, which have rather high conductivity. A dipole molecule may be considered as an intermediate stage between a neutral molecule and a dissociated molecule, and in a dipole liquid it is probable that some dissociation occurs. In such a case a high power factor at low frequency would be almost entirely due to leakage in spite of the presence of dipoles. But by increasing the testing frequency over a very wide range, we should reach a region where the dipole power factor was predominant. The curve power factor *vs.* frequency should therefore give a $V$ curve, in some cases a very wide, shallow one.

Mr. Clark's remarks regarding Fig. 1 and the results of Bock with glycerine bring up two interesting points. The dielectric constant of ether instead of rising continuously as the temperature is lowered to the freezing point (−116 deg. cent.) reaches a sharp maximum at −108, or 8 deg. above the freezing point. Similar behavior is shown by other liquids in Isnardi's paper. One explanation would be that near the freezing point, the mutual influence of the molecular fields becomes so strong that response to the external field is negligible.

Bock measured the change of dielectric constant with temperature at 1.36 meters. The maximum was 36 at about 50 deg. cent., dropping to about 4 at 0 deg. He quotes the results of Graffunder who made similar measurements at 400, 670, and 1000 meters. At these lower frequencies the maxima reached about 56. The course of the dielectric constant curves is quite similar to those of rosin and castor oil, except that the effect is greatly accentuated by the much larger change of dielectric constant. Undoubtedly, both workers were dealing with glycerine in the supercooled state so that the behavior resembled that of rosin and castor oil.

It should be pointed out that rosin, glass, fused quartz, and similar materials, although popularly termed "solids," are in reality supercooled liquids of extremely high viscosity and hence media in which dipole molecules can turn if given sufficient time. Therefore, the dipole effect may occur at the proper frequencies, in some cases, conceivably 1 cycle per hour, day, or month, although it might be masked by leakage and other effects. As shown in Section $V$-3, the relaxation time of the dipoles in rosin at rosin temperature is of the order of 1 minute and there is no reason to suppose it may not be much longer in still more viscous media.

The permanent "elektrets" of Eguchi in which polarization exists for years may be an example. An alternative explanation of these elektrets would be that the rosin-carnauba mix has sufficient plasticity, or minimum shearing stress, to resist permanently the tendency of the molecules to return to a random distribution. Adams (*Franklin Inst. Jl.*, Vol. 204, p. 469, 1927) has shown that when the temperature is raised the rate of decay of polarization becomes measurable.

The suggestion of the physicists consulted by Professor Whitehead that the energy loss due to orientation at low frequency is negligible is correct, I believe, except in the case of liquids of extreme viscosity. In other words, there exists a sort of reciprocal relation, as pointed out in Section *V*-2-*c*, such that if you increase the viscosity 1000-fold by lowering the temperature, you get the power factor maximum at roughly $\frac{1}{1000}$ of the first frequency. I believe, therefore, that rosin and similar materials offer an exception to the point that the dipole power loss effect is confined to radio frequencies. The thing I have wanted to emphasize in my paper is the fact that the dipole effect is not confined to any particular range, but depending on the molecular size, the viscosity, and the temperature, may occur anywhere in the electrical spectrum from billions of cycles per second in the case of non-viscous liquids to 60 cycles and doubtless lower in very viscous supercooled liquids. Failure to detect a dipole power factor may therefore merely mean that the tests are not made in the range where it becomes predominant. Ordinarily, however, the materials used in cables, etc., would not be expected to exhibit the effect at ordinary frequencies and I believe Professor Whitehead is perfectly justified in not applying the dipole theory in his work.

Mr. Nyman's hypothesis of a mixture of solid and liquid dipoles to which Maxwell's theory would still apply is ingenious, but not in accordance with the facts. He speaks of a change from liquid to solid state in rosin in the temperature range where the slope of the viscosity curve changes rapidly. But no solidification takes place in this region. Moreover, by changing the testing frequency the power-factor peak may be shifted to correspond to quite different portions of the viscosity curve. Fig. 5 gives a clear example of this point.

Professor Karapetoff has asked about the connection between this paper and that by Kitchin and Müller in the December *Physical Review*. The two papers deal with practically the same data but from different points of view. Anomalous dispersion in dipole liquids has been familiar to physicists since the classic experiments of Drude. But in practically every case it was confined to the extremely high frequency region of the electrical spectrum. The fact that the phenomenon could by choice of suitably viscous materials having large molecules be found at frequencies as low as 60 cycles should be of great interest.

On the other hand, as Professor Whitehead has pointed out, it has been difficult to see how the dipole theory could be applied to practical insulating materials. The present paper was given in the hope of showing the conditions under which the dipole theory could be so applied.

# A Graphical Theory of Traveling Electric Waves
## Between Parallel Conductors

BY VLADIMIR KARAPETOFF[1]
Fellow, A. I. E. E.

*Synopsis.—The analytical theory of simple traveling waves along parallel conductors is well known. Its disadvantage is that the relationships among the incoming, reflected, transmitted, and absorbed currents and voltages are expressed by a number of simultaneous equations difficult to grasp, from a physical point of view, in their entirety. For this reason, a graphical theory has been developed according to which all the quantities involved are represented in a so-called star diagram and the whole phenomenon conveyed to the eye quantitatively.*

*The star diagram is then applied to the following practical cases of a simple rectangular-front non-attenuated long traveling wave:*

*(a) Reflection from and absorption in a non-inductive terminal resistance;*

*(b) Reflection from an open-circuited and from a short-circuited end of a line;*

*(c) The case of critical resistance;*

*(d) Repeated reflections from the ends of a line;*

*(e) Discharge of a wave to the ground, directly or through a resistance;*

*(f) Passage of a wave through the junction point of two conductors having different values of surge impedance;*

*(g) Same as (f), only the junction provided with a series or shunted resistance,*

*(h) Effect of a lumped series inductance or shunted capacitance in reducing the steepness of a wave front.*

*In each case, the results agree with those deduced analytically, for example by R. Rüdenberg in his book "Elektrische Schaltvorgänge." Some reciprocal relationships are pointed out at the end and further problems suggested to which the star diagram may be applied.*

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### INTRODUCTION

THE purpose of this paper is to explain graphically the fundamental properties of electric traveling waves between parallel conductors (such as transmission lines), especially where a wave strikes an obstacle, for example, a resistance, an inductance, an open end, a junction point between a cable and an overhead line, etc. While the general analytical theory of such waves is well known (see the references below), a graphical representation of the quantities involved is believed to be new, and it may give a clearer idea of the important relationships useful in the solution of some practical problems. The so-called *star diagram,* developed for this purpose, is the foundation of the graphical treatment given below.

### 1. GENERAL PROPERTIES OF ELECTRIC WAVES TRAVELING ALONG PARALLEL CONDUCTORS

Whenever some electric or magnetic conditions in a circuit are changed suddenly, a traveling electromagnetic wave starts from the point of disturbance and is propagated into the various conductors which constitute the circuit or the network. The wave is gradually attenuated because of the resistances of the circuit, is reflected at local obstacles, experiences changes in the values of its current, voltage, and shape upon entering conductors of different characteristics, etc. Ultimately, its energy is completely converted into heat.

Traveling hydraulic waves of similar character occur, for example, in a city water supply system. Should a pipe burst and the pressure be suddenly lowered locally, a wave of depression would start from the fault and spread over the whole system. Should a valve be suddenly closed in one of the branch pipes, the water hammer so created would also spread in the form of a pressure wave throughout the system, and after numerous reflections its power converted into vibrations and heat.

In this paper, only long rectangular-front waves (Fig. 1) are considered, first, because they are simpler to be treated theoretically, secondly, because other kinds

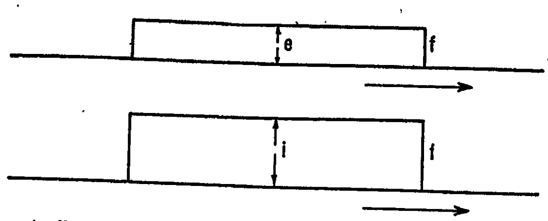


FIG. 1—A LONG RECTANGULAR-FRONT TRAVELING WAVE OF CURRENT AND VOLTAGE

of waves may be composed out of such simple waves, and thirdly because they are at least as harmful for electrical apparatus (if not more so) as any other waves known. In Fig. 1 the voltage wave, $e$, and the current wave, $i$, are shown separately, although in reality $e$ and $i$ are but two characteristics of the same traveling wave of electric energy. The direction of propagation is indicated by the arrowheads and the wave front is denoted by $f$.

The actual physical conditions corresponding to the symbolic representation in Fig. 1 are shown in Fig. 2. $B$ is a battery, $s$ is a switch, and $a$ and $b$ two parallel conductors. When the switch is closed, a wave starts to the right, charging the conductors. This means that an electrostatic field is established, shown by the vertical lines, and a magnetic field, indicated by the

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dots and the crosses. These two fields, together with a cross-section of the conductors, are also shown to the right.

Theory and experience show that such a wave, between two parallel conductors of comparatively small cross-section, is propagated nearly at the velocity of light (or of other electromagnetic disturbances) in the dielectric medium in which the conductors are immersed. Thus, for overhead lines the wave velocity is that of light in air, for a cable it is that of electromagnetic waves in impregnated paper, etc. This does

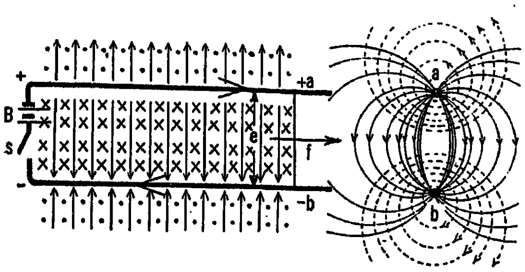


FIG. 2.—THE MAGNETIC AND ELECTROSTATIC FLUXES ACCOMPANYING A TRAVELING WAVE BETWEEN PARALLEL CONDUCTORS

not mean that the actual carriers of electricity in the conductors (ions) move at such enormous velocities, nor is it necessary to assume this velocity of motion for the electrostatic and magnetic lines of force. All we know is that the electromagnetic state advances at this velocity. This difference between the velocity of propagation of a wave (or state) and that of the material particles in it, is well illustrated for water waves in W. S. Franklin's "Electric Waves," pp. 15 to 25. If the depth of a canal (Fig. 3) is $D$, the velocity of propagation of the wave front is

$$v = \sqrt{g D} \qquad (1)$$

where $g$ is the acceleration due to gravity. On the other hand, the average velocity, $v$, of the particles of water is much lower, being represented by the expression

$$v = V\,(h/D) \qquad (2)$$

In an electric conductor, the velocity of motion of electrons which constitute the current is quite low, but as soon as the electrons in a cross-section begin to move, they disturb the equilibrium and cause the electrons in the next cross-section also to move. So the velocity of the wave itself simply characterizes the rate at which consecutive carriers of electricity are set in motion. Similarly, it is not necessary to assume the lines of force to be traveling at the velocity of light. New lines of force are built up as soon as a voltage and a current reach a point on the line, while the older lines of force remain at places where they have been created.

This difference between the two velocities may be also illustrated as follows: Think of a front of soldiers with their heads turned to the right, and let each be instructed to begin turning his head slowly to the left as soon as he sees the preceding man beginning the motion of his head. It will take but a fraction of a second for each man to begin his motion after his neighbor, so that the wave front of motion will be propagated along the row quite *rapidly*, whereas the actual motion of the heads may be quite *slow*.

There is nothing contradictory in the fact that the current in Fig. 2 seems to flow in an open circuit. Positive electricity may be thought of as flowing through the upper conductor to the right, creating a displacement current in the dielectric downward and thereby forcing a flow of positive electricity in the lower conductor to the left. Or else, the same process may be thought of in terms of negative electrons, with the polarity and direction of motion reversed. Similarly, there is no contradiction in the existence of a considerable voltage difference, $e$, between two adjacent points of the same conductor, just before and behind the plane $f$. New magnetic lines of force arising in that plane induce an e. m. f. which balances the voltage $e$. The relationship shown in Fig. 2 could not exist if $f$ were a stationary plane, but as the front of the wave moves at the proper velocity, the transient conditions at new points of the conductors make stationary values of $e$ and $i$ possible behind.

We shall limit our discussion to conductors whose ohmic resistance and leakage conductance are negligible. In other words, the attenuation of a wave will be disregarded. Not only is the theory much simplified thereby, but in a study of destructive action of traveling waves it is not safe to count on an attenuation because in a most unfavorable case a wave may originate in the immediate vicinity of a device upon which it impinges, so that there is practically no attenuation.

With this assumption, and denoting the inductance of

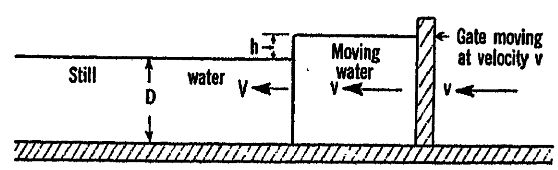


FIG. 3—A WATER WAVE IN A CANAL AS AN ANALOG TO AN ELECTRIC TRAVELING WAVE

a line or cable by $L$ and its capacitance by $C$, both per unit length, the following well known relationships hold true.[2]

(a) The velocity of propagation of a traveling wave is

$$V = 1/\sqrt{L\,C} \qquad (3)$$

With proper units, for overhead conductors, $V$ comes out equal to the velocity of light in air. In a power cable, the velocity of propagation is much lower, of the order of magnitude of one-half of that along overhead conductors.

(b) When a wave is propagated along an ideal

2. For a proof, see the references at the end of the paper.

conductor with uniformly distributed $L$ and $C$, its shape remains undistorted and identical with that at the entrance into this particular conductor. A sudden change in line constants, or a concentrated (lumped) impedance, causes changes in the values of $e$ and $i$, and in the shape of the wave front.

(c) For a given conductor, the ratio of a wave voltage to its current is constant; that is

$$e = Z\,i \qquad (4)$$

where

$$Z = \sqrt{L/C} \qquad (5)$$

$Z$ is known as the *surge impedance* of the line. The reciprocal of $Z$,

$$Y = \sqrt{C/L} \qquad (5a)$$

is called the *surge admittance* of the line. In preliminary estimates, $Z$ may be taken equal to 500 ohm for overhead lines and to 50 ohm for power cables. Whereas $L$ and $C$ usually are understood to be per unit length, $Z$ is expressed in ohms (not in ohms per unit length).

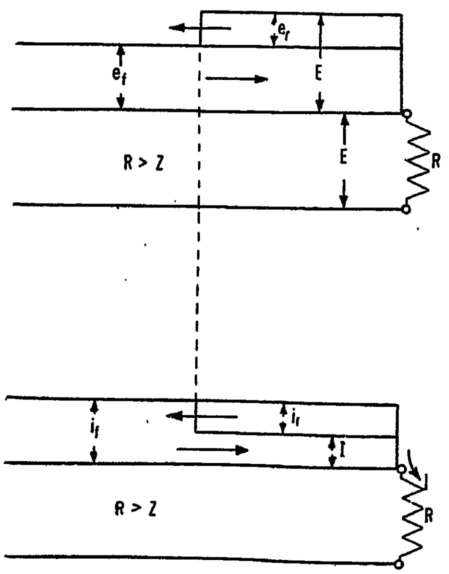


FIG. 4—REFLECTION OF CURRENT AND VOLTAGE WAVES AT THE END OF A LINE BRIDGED BY A RESISTANCE

(d) The electrostatic energy of a traveling wave, per unit length, is $0.5\,C\,e^2$; the corresponding electromagnetic energy is $0.5\,L\,i^2$. In a pure traveling wave, such as are considered here, the amounts of these two energies are equal, so that

$$C\,e^2 = L\,i^2 \qquad (6)$$

Extracting a square root of both sides of this equation, gives Equation (4). Thus, the law of surge impedance follows from that of equipartition of energy, or vice versa. At a terminal, or at a joint of two conductors with different values of $Z$, some magnetic energy is converted into electrostatic, or vice versa.

## 2. THE STAR DIAGRAM OF A RESISTANCE TERMINAL LOAD

In Fig. 4, a single-phase line of surge impedance $Z$ is shown bridged by a lumped resistance $R$ at one end. A traveling electric wave which is propagated from left to right, is partly reflected and partly enters the resistance, to be absorbed there. Knowing the voltage and the current in the original wave, it is required to determine those in the reflected and absorbed parts of the wave. The voltage notation is shown in the upper sketch, the current notation in the lower sketch. Of course, the same transmission line and the same wave of energy are meant in both sketches, the division having been made for the sake of clearness.

The quantities in the impinging or *forward-going* wave are provided with the subscript $f$ (corresponding to the subscript $v$ in Rüdenberg's book); the quantities in the reflected or *return* wave are distinguished by the subscript $r$. The arrowheads shown are in fact superfluous because the subscript itself indicates the direction of the motion of the wave with respect to $R$. The quantities $e_f$ and $i_f$ are shown above the axes of abscissas to indicate that in this particular case they are positive. The positive sign of $i_f$ means that the current is flowing to the right. If $i_f$ were negative, the current (positive ions) would be flowing to the left, whereas the wave itself (the front of the electromagnetic state) would still be moving to the right. This happens in a wave of depression.

The wave $e_r$ is shown on top of $e_f$ to indicate that its voltage is also positive and is added to $e_f$. On the other hand, $i_r$ is shown negative, that is, subtracted from $i_f$. The negative sign of $i_r$ means that the current flow is to the left. In this case, this happens also to be the direction of the motion of the wave front. In other cases, however, for example if $R$ were quite low, $i_r$ may be positive, the actual motion of positive ions being to the right, and the motion of the wave front to the left.

In this paper, the fronts of the waves marked $e_f$ and $i_f$ are always assumed to move to the right, and those of $e_r$ and $i_r$ to the left. The actual current flow, whether in the wave $i_f$ or $i_r$, is always to the right when these currents are shown positive, and vice versa.

The quantities $i_r$ and $e_r$ have no real existence. After the incoming wave, $e_f$, $i_f$, has reached the resistance $R$, the voltage changes from $e_f$ to $E$ and the current from $i_f$ to $I$. The region in which this change has taken place spreads to the left at a velocity equal to that of the incoming wave. It is more convenient, however, to consider the actual voltage $E$ as a result of superposition of the incoming voltage $e_f$ and a fictitious reflected voltage, $e_r$, so that

$$E = e_f + e_r \qquad (7)$$

Similarly, the actual current $I$, flowing into the resistance $R$, may be thought of as an algebraic resultant of the currents $i_f$ and $i_r$; in other words,

$$I = i_f + i_r \qquad (8)$$

In addition to these equations, we also have the following relationships, according to Equation (4):

$$e_f = Z\, i_f \qquad (9)$$

$$e_r = -\, Z\, i_r \qquad (10)$$

The minus sign in Equation (10) is necessary because when $e_r$ is positive and moves to the left the actual current in the reflected wave also flows to the left, so that $i_r$ is negative. For the resistance $R$ itself, we have

$$E = I\, R \qquad (11)$$

The foregoing five equations contain six quantities, $e_f$, $e_r$, $i_f$, $i_r$, $E$, and $I$. If one of these quantities, for example $e_f$, is given, the equations may be solved for the remaining five. The algebraic solution and an analysis of the results will be found in some of the references given below. Here we propose to solve the problem graphically, using the *star diagram* shown in Fig. 5.

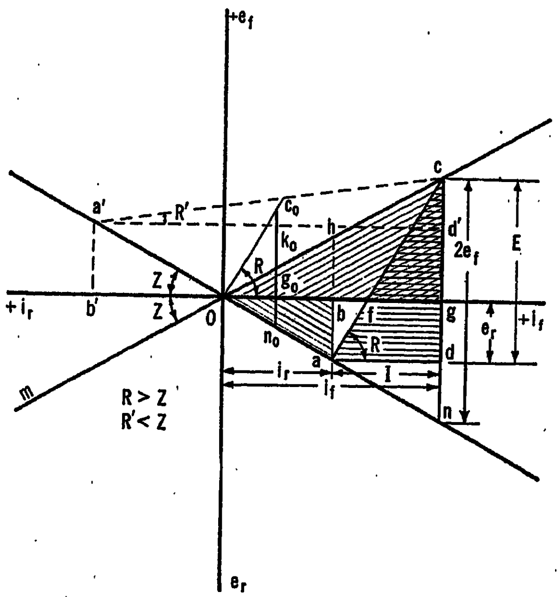


FIG. 5—A STAR DIAGRAM OF THE CURRENTS AND VOLTAGES SHOWN IN FIG. 4

In Fig. 5, positive values of $i_f$ are measured along the axis of abscissas, to the right of $O$; positive values of $i_r$ to the left of $O$. Positive values of $e_f$ are measured along the axis of ordinates upward, positive values of $e_r$ downward. The two inclined axes, $m\,c$ and $a'\,n$, are drawn at an angle to the axis of abscissas corresponding to the surge impedance $Z$. This means that if, on the chosen scale of currents, $O\,g_0$ equals one ampere, then $g_0\,k_0$ and $g_0\,n_0$ are each numerically equal to $Z$ volts on the chosen scale of voltages. Consequently, in accordance with Equation (9), any point on the line $m\,c$, such as $c$ or $h$, gives values of $e_f$ and $i_f$ which satisfy Equation (9). In other words, the line $m\,c$ takes the place of Equation (9). For the same reason, the line $a'\,n$ takes the place of Equation (10). It will be seen that for points on this line to the right of $O$, $e_r$ is positive and $i_r$ is negative. The opposite is true for points to the left of $O$.

Let now $O\,g_0$ be numerically equal to one ampere, as before, and let $g_0\,c_0$ be numerically equal to $R$ volts. Then any right-angle triangle similar to $O\,g_0\,c_0$ will give values of $E$ and $I$ which will satisfy Equation (11). It remains to choose the size and the position of a triangle in such a manner as to satisfy Equations (7) and (8) as well. It will be seen that the triangle $a\,c\,d$ satisfies these conditions. Its vertical side, $E$, is equal to the sum of the voltages $e_f$ and $e_r$, and its horizontal side, $I$, is equal to the difference between $i_f$ and $i_r$. However, in this particular case $i_r$ is negative so that in reality $I$ is equal to the algebraic sum of $i_f$ and $i_r$.

Thus, in place of solving the foregoing five simultaneous equations, we simply mark on the star diagram the point $c$ corresponding to the given value of $e_f$, draw $c\,a$ parallel to $c_0\,O$ and complete the triangle $a\,c\,d$. All the five unknown quantities can then be scaled off.

The product of the wave voltage by the corresponding current gives the power in the wave, that is, the electric energy which passes through a given point on the line, per unit time. Therefore, the area of the triangle $O\,c\,g$ is a measure for the energy flowing towards the resistance $R$, per unit time. Similarly, the area $O\,a\,b$ is a measure for the energy reflected and flowing into the line per unit time. The area $a\,c\,d$ represents the energy absorbed in the resistance per unit time. Accordingly we must have

$$\text{area } a\,c\,d = \text{area } O\,c\,g - \text{area } O\,a\,b \qquad (12)$$

This relationship may be proved geometrically as follows: The areas $a\,h\,c$ and $a\,b\,g\,d$ are equal because the altitude $a\,d$ is the same, and the base, $a\,h$, of the triangle is twice the base, $a\,b$, of the rectangle. Subtracting the area $a\,b\,f$ from both and adding $f\,c\,g$, we find that

$$\text{area } a\,c\,d = \text{area } b\,h\,c\,g \qquad (13)$$

But the area $b\,h\,c\,g$ is equal to the difference between the areas $O\,c\,g$ and $O\,h\,b$, and this proves Equation (12).

Throughout this paper, the important triangles which characterize electric power are cross-hatched in the same manner, as indicated in the following table:

TABLE I

| Kind of energy | Direction of shading |
|---|---|
| Incoming wave....... | diagonally rising to the right |
| Reflected wave....... | diagonally rising to the left |
| Locally absorbed...... | horizontal |
| Transmitted through the junction...... | vertical |

The latter shading is not used in Fig. 5, but will be found in some other star diagrams below, for example in Figs. 13 and 16.

Instead of scaling off the unknown currents, voltages, or impedances from the diagram, they may also be determined analytically from the geometry of the figure. For example, the triangles $a\,c\,n$ and $O\,c_0\,n_0$ being similar, we have

$$2\,e_f/E = (Z + R)/R \qquad (14)$$

from which any of the four quantities may be computed if the remaining three are known.

In the particular case shown in Fig. 5, $R$ is greater than $Z$, and, as a result, the reflected voltage is positive and the reflected current negative, as shown in Fig. 4. For another value, $R'$, less than $Z$, the relationships become those indicated by the dotted triangle $a' \, c \, d'$. The reflected voltage, $a' \, b'$, is negative and the reflected current, $O \, b'$, is positive.

*Critical Resistance.* An important case arises when $R$ equals $Z$. The triangle of reflected energy, $O \, b \, a$, collapses to point $O$ and the triangle $a \, c \, d$ coincides with $O \, c \, g$. This means that $e_r = i_r = O$, and the whole energy of the incoming wave is absorbed in the resistance $R$. This fact is receiving an increasing attention, both in the design of protective apparatus and in impulse testing where reflected waves are liable to vitiate the results.

When $R > Z$, the wave is reflected without a reversal in the sign of the voltage, but with a current reversal (Fig. 4). Consequently, the total "piled up" voltage $E$ is higher than that in the incoming wave. Such a combination of $Z$ and $R$ (or of $Z$ and some other device) may be conveniently called a *step-up joint*, by analogy with a step-up transformer. Also, a resistance $R$, greater than $Z$, may be called a step-up load. On the other hand, when $R < Z$, the voltage $E$ at the joint is lower than $e_f$, because $e_r$ is negative. In such a case, $I$ is greater than $i_f$ because $i_r$ is positive. Such a joint may be called a *step-down joint.* A resistance $R$, which is lower than $Z$, may be called a step-down load.

Whether a load is of step-up or step-down nature, some energy of an incoming wave is reflected and has to be dissipated elsewhere. Thus, from the point of view of energy absorption, a critical resistance, $R = Z$, deserves serious attention. It is possible that in the future lightning arresters and other over-voltage protective devices will be provided with such a series resistance (as is sometimes recommended in European literature). This resistance would absorb an incoming wave and thus prevent subsequent line oscillations due to repeated reflections, sometimes accompanied by high-voltage resonance oscillations in transformer windings. On the other hand, it must be remembered that such resistances would be large and expensive because the line current may follow the breakdown.

## 3. Wave Reflection at an Open End and at a Short-Circuited End of a Line

In the star diagram shown in Fig. 5, the terminal resistance $R$ may have any value from zero to infinity. When $R$ is infinitely great, $a \, c$ becomes a vertical line, point $a$ moves to point $n$, and the triangle $a \, c \, d$ degenerates into the straight line $n \, c$. Thus, when a wave is reflected at an open end of a line, the conditions are as shown in Fig. 6. The reflected voltage is equal to the incoming voltage, so that the actual voltage at the open end rises to the value $E = 2 \, e_f$. The reflected current is equal and opposite to the incoming current, to make the total current at the open end equal to zero. This is to be expected since there is no further conductor for the current to flow into. The cross-hatched triangles of incoming and reflected energy are of the same shape and area, the whole energy of the incoming wave being reflected back into the line. Thus, at an open end of a line a *total step-up reflection* takes place, the voltage of a wave being reflected without a reversal in sign, whereas the current is reversed in sign. The magnetic energy of the wave is converted into electrostatic. In Fig. 5, for any value of $R$ between $R = Z$ and $R$ = infinity the reflection is of the same "step-up" kind, except that it is *partial* and not total,

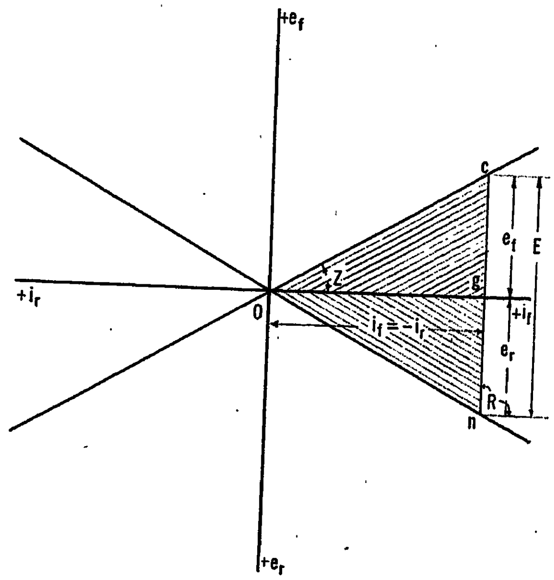


Fig. 6—A Star Diagram Showing the Reflection of a Wave at an Open End of a Line

the balance of the energy being absorbed in the resistance.

When the transmission line is short-circuited, that is, when $R = o$, the hypotenuse $a \, c$ in Fig. 5 becomes horizontal and the star diagram assumes the shape shown in Fig. 7. The energy is again completely reflected, but with a reversal in the voltage sign, so as to make $E = O$. On the other hand, the current is doubled, and the whole electrostatic energy of the wave is converted into electromagnetic. This is a case of *total step-down reflection.* A similar kind of step-down reflection takes place at any value of $R$ between zero and $Z$ (Fig. 5), except that the reflection is *partial,* and some energy is absorbed in the resistance.

Since the energy of a traveling wave is totally reflected whether the end of a line is open or short circuited, it is of interest to consider a few of the simplest cases of multiple reflection of waves at both ends of a line. This will give an idea of the most dangerous conditions which may be expected in practise.

(*a*) *Line Open at Both Ends.* Consider a fairly long line and a comparatively short thunder-cloud

over its middle portion. This cloud will induce a certain distributed electric charge in the nearest part of the line. When the cloud is discharged to the ground or to another cloud, the line charge is released and one-half of it starts in each direction in the form of a traveling wave. Let the voltage of each wave (or impulse) be $e$, the current $i$, and the length $l$. Having reached one of the ends of the line, the impulse is stopped, its current becomes zero, and its voltage rises to $2\,e$. But such a "chunk" of electricity of length $0.5\,l$ cannot remain in an equilibrium with the uncharged portion of the line, and so a return wave starts immediately. The voltage again drops to $e$ and the current becomes $-\,i$, because at these values Equations (4) and (6) are satisfied. The same process is repeated at the other end, and the impulse would persist indefinitely if it were not for the line resistance

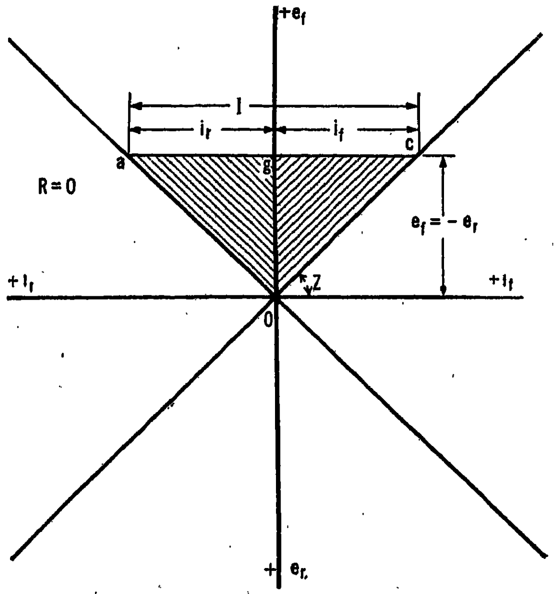


FIG. 7—A STAR DIAGRAM FOR THE REFLECTION OF A WAVE AT A SHORT-CIRCUITED END OF A LINE

and leakage, both of which exponentially reduce the impulse to zero as it travels along the line.

(*b*) *Line Grounded at Both Ends.* The phenomenon is similar to that described above, except that at each end the current is doubled and the voltage is instantly reduced to zero, to reappear in the reflected wave with a reversed sign. There is a general notion among engineers that grounding a line would automatically conduct dangerous impulses to the ground. It will be seen from the foregoing discussion that a traveling wave or impulse is simply reflected from a zero-resistance terminal and continues to surge back and forth until dissipated in the line itself. The only difference with an open end is that the current is doubled, instead of the voltage. The proper way to absorb a wave effectively is to provide a terminal resistance $R$. If $R = Z$, the whole impulse will be absorbed on the first trip; otherwise, it may take two or more reflections to reduce the energy to a negligible amount.

(*c*) *Line Open at One End and Short-Circuited at the Other End.* The reflections occur as described above, only the impulse approaches the open end of the line each time with its voltage polarity reversed, owing to a preceding reflection from the short-circuited end. It also approaches the latter end with its current polarity reversed, due to a reflection at the open end. Thus, after four trips back and forth, the impulse starts on its fifth trip with its current and voltage polarities the same as in the beginning.

(*d*) *Battery at One End, Line Open at the Other End.* We shall assume the battery to be of zero internal impedance and of voltage $e$. At the instant $t = 0$ the battery switch is closed and remains closed. A wave of voltage $e$ and current $i$, the latter determined by Equation (4), starts from the battery towards the opposite end of the line. Let $T$ be the time required for the wave front to reach the open end, this interval of time being determined by the length of the line and the velocity of propagation. The subsequent history of the wave is shown in Table II.

TABLE II

| Wave trip No. | Wave voltage | Wave current | Resultant voltage | Resultant current |
|---|---|---|---|---|
| 1 | $e_f\,(+)$ | $i_f\,(+)$ | $e$ | $+\,i$ |
| 2 | $e_r\,(+)$ | $i_r\,(-)$ | $2\,e$ | 0 |
| 3 | $e_f\,(-)$ | $i_f\,(-)$ | $e$ | $-\,i$ |
| 4 | $e_r\,(-)$ | $i_r\,(+)$ | 0 | 0 |
| 5 | $e_f\,(+)$ | $i_f\,(+)$ | $e$ | $+\,i$ |

The reflected voltage on the first return trip (trip 2) is positive, the reflected current negative. Hence, in the parts of the line over which the reflected wave has passed, the actual voltage is $2\,e$ and the current is zero, as indicated in the last two columns of the table. The battery, being of zero impedance, acts towards the returning wave as a short-circuited end of the line, the battery voltage being used up in maintaining the first outgoing wave. This explains the third line in the table, the values in the last two columns being obtained by adding $-\,e$ and $-\,i$ to the preceding values. After a second reflection from the open end, the reflected wave starts with a voltage $-\,e$ and a current $+\,i$, wiping out the previously existing resultant values $+\,e$ and $-\,i$, and clearing the line of all current and voltage. Thus, when a wave starts again on its fifth trip, the same conditions obtain as on the first trip, and were it not for the energy dissipation in the resistance of the line, the phenomenon would go on indefinitely.

Let a cathode ray oscillograph be connected at the open end of the line to measure the voltage there, and another similar oscillograph connected at the battery end to measure the line current. With an ideal line, the records would be of the shape shown in Fig. 8 by heavy lines, in accordance with Table II. The voltage at the receiver end is zero until the wave reaches it at the instant $t = T$. Then the voltage rises to $e$, and at

the same instant (because of the reflection) rises to 2 $e$. The oscillograph indicates this latter value until the reflected wave front reaches the battery and returns with the sign reversed, wiping out 2 $e$. For the next two trips the voltage at the receiver end is zero, then again rises to 2 $e$, etc. The current wave may be explained similarly.

Real oscillograms look more like the irregular curves indicated by lighter lines. Because of a steady absorption of the energy of the wave in the resistance of the conductors, the wave front does not remain abrupt,

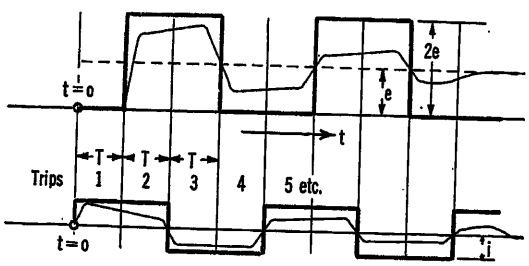


FIG. 8—GENERAL SHAPE OF THEORETICAL AND ACTUAL OSCILLOGRAMS OF A WAVE VOLTAGE REFLECTED AT THE OPEN END, AND OF THE CORRESPONDING CURRENT AT THE BATTERY END

and moreover there is a steady diminution in the amplitudes of both the voltage and the current. With the battery voltage equal to $e$, ultimately the receiver-end voltage acquires this value, and the flow of current stops.

(*e*). *Battery at One End, the Line Short-Circuited at the Other End.* This case differs from the preceding one only in the character of the reflection from the distant end, namely, the wave is reflected with a reversal in the sign of the voltage, whereas the current is doubled. This is shown symbolically in Table III.

It will be seen that in this case the resultant current increases indefinitely, as would be expected with an ideal short-circuited line. In reality the current tends to a finite limit determined by the resistance of the line.

Let an oscillograph be connected at the short-circuited end of the line. It will record zero current for an interval of time $T$ (Fig. 9) after the closing of the switch, that is, until the wave reaches the short-circuited end. Then the current will suddenly rise to the value $i$ in the wave, and immediately increase to 2 $i$, because of a step-down reflection. The latter value will persist until the reflected wave will have traveled back and forth once, at which instant the current will rise to 3 $i$ and immediately to 4 $i$; etc. There would be no object

TABLE III

| Wave trip No. | Wave voltage | Wave current | Resultant voltage | Resultant current |
|---|---|---|---|---|
| 1 | $e_f$ (+) | $i_f$ (+) | $e$ | + $i$ |
| 2 | $e_r$ (−) | $i_r$ (+) | 0 | + 2 $i$ |
| 3 | $e_f$ (+) | $i_f$ (+) | $e$ | + 3 $i$ |
| 4 | $e_r$ (−) | $i_r$ (+) | 0 | + 4 $i$ |
| 5 | $e_f$ (+) | $i_f$ (+) | $e$ | + 5 $i$ |

in connecting an oscillograph to measure the voltage at either end of the line. At the battery end the voltage is always equal to $e$ and at the other end it is always equal to zero. By connecting an oscillograph across the line, at its middle point, a record would be obtained shown in Fig. 9. The voltage is zero during a time interval equal to 0.5 $T$, then it rises to the value $e$ and remains at this value until the wave front has reached the short-circuited end and back, wiping out + $e$ with − $e$. The phenomenon is repeated with each new passage of the wave front over the mid-point of the line.

The resistance, $R$, of the line modifies these theoretical curves, as shown by the irregular lighter lines. The current finally reaches the value $i_0 = e/R$, and the voltage across the middle of the line tends to the ultimate value 0.5 $e$.

## 4. DISCHARGE TO GROUND OVER A RESISTANCE OR DIRECTLY

In Fig. 10, $Z_1$ and $Z_2$ are two conductors of different surge impedances, for example a cable and an overhead line. At their junction, $A$, an overvoltage protective device is placed consisting of a sphere gap with a resistance $R$ in series. Both conductors are initially charged to a voltage insufficient to break down the gap, and then this voltage is gradually increased to a value $E_0$

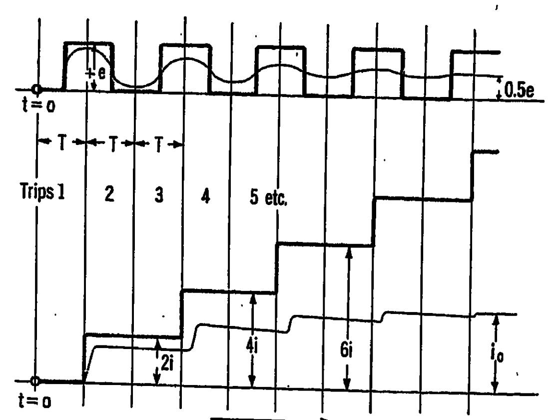


FIG. 9—GENERAL SHAPE OF THEORETICAL AND ACTUAL OSCILLOGRAMS OF A WAVE CURRENT REFLECTED AT THE SHORT-CIRCUITED END OF A LINE, AND OF THE CORRESPONDING VOLTAGE IN THE MIDDLE OF THE LINE

at which the gap breaks down. The subsequent phenomenon is as follows:

(a) A current, $I$, flows to the ground.

(b) The voltage between $A$ and the ground drops to a value $E = I R$, where $E$ is less than $E_0$. The resistance of the arc is assumed to be negligible in comparison to $R$.

(c) Waves of equal voltage depression start in both directions; in conductor 1 this wave is denoted by $e_{r1}$ ($e$ − return − one) and in conductor 2 by $e_{f2}$ ($e$ − forward − two).

(d) Currents begin to flow into the ground connection from both conductors. The current wave in conductor 1 is denoted by $i_{r1}$; the current is positive

because it flows to the right, although the wave itself is propagated to the left. The current $i_{f2}$ in conductor 2 is negative because it flows to the left, although the wave is propagated to the right. With equal values of voltages, the currents in the two waves are unequal; see Equation (4). The double arrowheads at the top of the figure indicate the directions of the currents themselves, the single arrowheads at the bottom show the direction of propagation of the waves.

The quantitative relationships are shown in the star

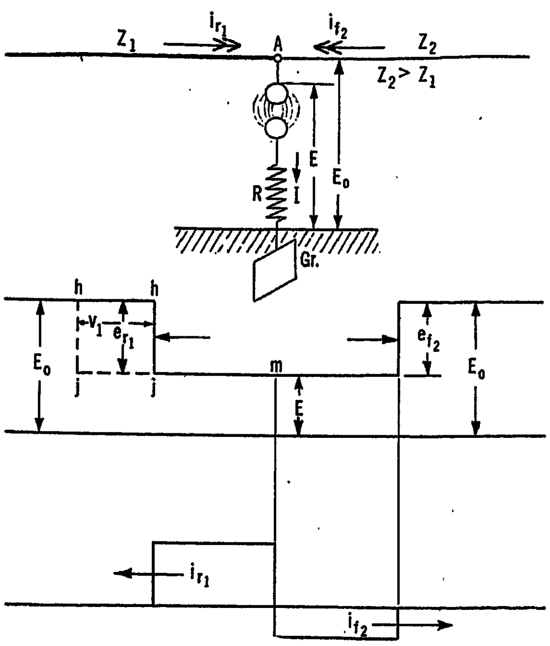


FIG. 10—TRAVELING WAVES CAUSED BY BREAKDOWN OF A PROTECTIVE GAP WITH A RESISTANCE IN SERIES

diagram in Fig. 11. It will be noted that the positive directions of $i_{f2}$ and $e_{f2}$ are the same as those of $i_{r1}$ and $e_{r1}$. The triangle $a\ c\ d$ represents the current and the voltage in the ground resistance; the triangle rests on the rays $a'\ a''$ and $c'\ c''$ drawn at angles corresponding to the surge impedances $Z_1$ and $Z_2$ respectively; see the explanation to Fig. 5. The triangle $O\ a\ b$ gives the current and the voltage in the wave in conductor 1, the triangle $O\ d\ g$ those in conductor 2. These three triangles satisfy the following conditions:

$$E = R\,I;\ e_{r1} = -\,Z_1\,i_{r1};\ e_{f2} = Z_2\,i_{f2} \qquad (15)$$

$$I = i_{r1} - i_{f2};\ E = E_0 + e_{r1} = E_0 + e_{f2} \qquad (16)$$

Since there are no other conditions to be satisfied, Fig. 11 gives a correct representation of the phenomenon shown in Fig. 10. In checking Equations (16), it must be remembered that the quantities $i_{f2}$, $e_{r1}$, and $e_{f2}$ are negative, so that an addition in Fig. 11 means a subtraction in Equation (16), and vice versa. We also have

$$\text{area } a\,d\,c\,s = \text{area } a\,p\,q\,s + \text{area } p\,q\,c\,d \qquad (17)$$

The area $a\ d\ c\ s$ is a measure of power input into the resistance $R$; the area $a\ p\ q\ s$ is a measure for the power input into $R$ from conductor 1, and the area $p\ q\ c\ d$ for that from conductor 2.

The wave energy relations are also indicated by the shaded areas as follows: in Fig. 10, consider the wave front $h\ j$ at a certain instant of time and the same front one second later, at $h'\ j'$. The distance between the two is equal to the velocity $v_1$ of wave propagation in the medium surrounding conductor 1. During the second under consideration, part of the initial stored electrostatic energy on the length $j\ j'$ of the line is converted into the electromagnetic energy of the current $i_{r1}$ on the same length of the line, and an amount of energy equal to $E\ i_{r1}$ is delivered to the resistance $R$. The amounts of electrostatic and electromagnetic energy on the part $j\ m$ of the line and in the region to the left of $j'$ need not be considered since no energy changes take place in these regions during the second under consideration. Thus, the energy equation is

$$0.5\,E_0^2\,v_1\,C_1 = 0.5\,E^2\,v_1\,C_1 + 0.5\,i_{r1}^2\,v_1\,L_1 + E\,i_{r1} \quad (18)$$

In this formula, the expression on the left-hand side represents the initially stored electrostatic energy, and the terms on the right-hand side represent the remaining electrostatic energy, the electromagnetic energy, and the input into $R$ (from the left) respectively. Eliminating $v_1$ by means of Equation (3) and using the value of $Z_1$ from Equation (5), Equation (18) becomes

$$0.5\,E_0^2/Z_1 = 0.5\,E^2/Z_1 + 0.5\,i_{r1}^2\,Z_1 + E\,i_{r1} \qquad (19)$$

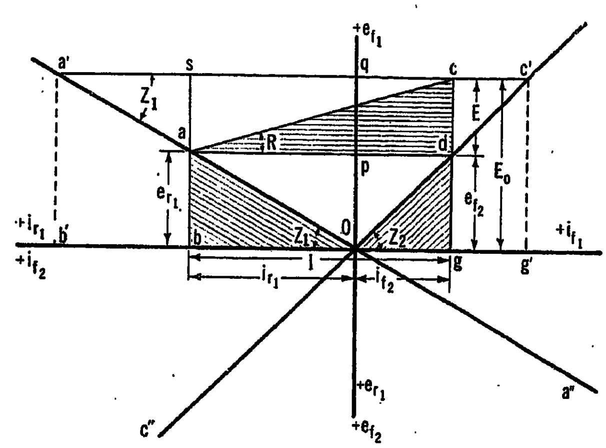


FIG. 11—A STAR DIAGRAM OF THE CURRENTS AND VOLTAGES SHOWN IN FIG. 10

In Fig. 11, this equation corresponds to the relationship

$$\text{area } a'\,q\,O = \text{area } a'\,s\,a + \text{area } a\,p\,O + \text{area } a\,s\,q\,p \quad (20)$$

In the triangle $a'\ q\ O$ the vertical side is equal to $E_0$ and the horizontal side is $E_0/Z_1$; hence its area is equal to $0.5\,E_0\,(E_0/Z_1)$. Similarly, the area $a'\ s\ a$ is equal to $0.5\,E.\,(E/Z_1)$. The horizontal side of the triangle $a\ p\ O$ is $i_{r1}$; hence, its vertical side is $i_{r1}\,Z_1$, and the area equals $0.5\,i_{r1}\,.\,(i_{r1}\,Z_1)$.

With a given initial voltage $E_0$, the triangle $a'\ O\ c'$ is fixed. The power absorbed in $R$ is represented by the rectangle $a\ s\ c\ d$ inscribed in this triangle, and the area

of the rectangle is a function of the resistance $R$. When $R$ (and consequently $E$) decreases without limit, the rectangle approaches in the limit the straight line $a'c'$ and its area tends to zero. When $R$ increases without limit, and consequently $E$ approaches $E_0$, the rectangle approaches the straight line $O\,q$ and its area also becomes infinitesimal. Thus, there is a value of $R$ for which this area, and consequently the power absorbed in the resistance, becomes a maximum.

By writing down an analytical expression for the area of the rectangle as a function of $E$, and equating its first derivative with respect to $E$ to zero, the area may be shown to be a maximum when $O\,p = p\,q$, that is, when

$$E = 0.5\,E_0 = -\,e_{f2} = -\,e_{r1} \tag{21}$$

In this case, the area of the rectangle is equal to one-half of that of the triangle $a'\,O\,c'$. This means that with the most favorable value of resistance $R$ one-half of the stored energy of the line is drained to the ground on the first trip of the waves of depression. On this trip, the initial voltage, $E_0$, is depressed to one-half of its value. If the distant ends of the two conductors are open, the waves will be reflected without change in the sign of the voltage and the remaining half of the stored energy will be drained on the return trip. With any other value of ground resistance, more reflections are necessary in order to discharge the line completely.

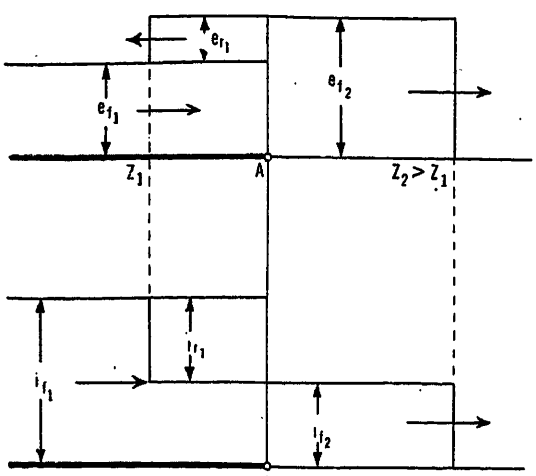


FIG. 12— A PARTIAL REFLECTION OF A TRAVELING WAVE AT THE JUNCTION POINT OF TWO CONDUCTORS WITH DIFFERENT SURGE IMPEDANCES

The most favorable value of $R$ may be calculated as follows:

$$a\,d = E\,R^{-1} \tag{22}$$

$$a\,d = b\,O + O\,g = -\,e_{r1}\,Z_1^{-1} - e_{f2}\,Z_2^{-1} \tag{23}$$

When the condition (21) is fulfilled, Equations (22) and (23) give

$$R^{-1} = Z_1^{-1} + Z_2^{-1} \tag{24}$$

In other words, the optimum resistance is equal to the combined surge impedance of the two conductors in parallel.

When $R = O$, and consequently $E = O$, the initial line voltage is depressed to zero and the largest possible current (equal to $a'\,c'$) flows to the ground. However, practically no energy is conducted to the ground, and the potential energy of the charged line is merely converted into kinetic. At the distant open end, the voltage $e_{r1} = -\,E_0$ is reflected and the wave comes back again raising the line voltage to $E_0$, only with the sign reversed. Thus, protective devices (which do not allow the line current to follow the discharge) may be assisted in their performance by a series resistance of the

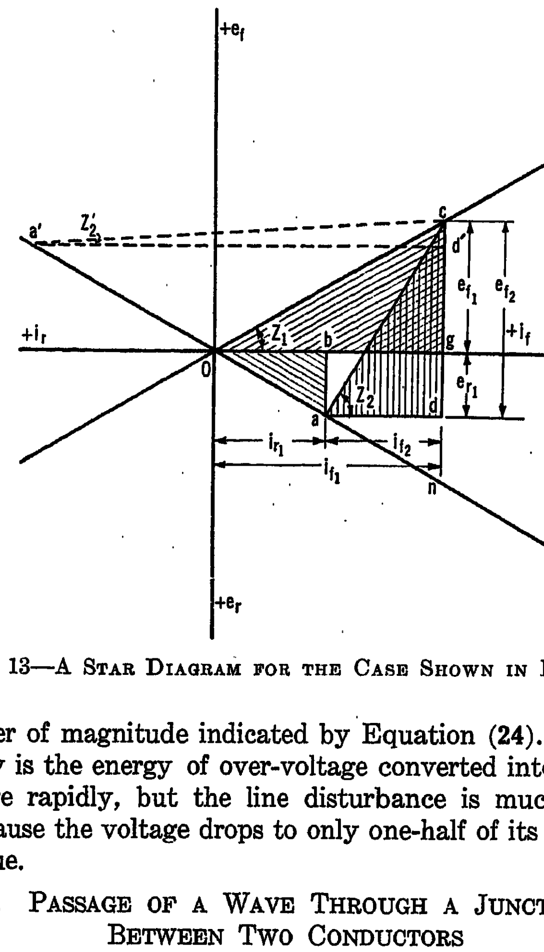


FIG. 13—A STAR DIAGRAM FOR THE CASE SHOWN IN FIG. 12

order of magnitude indicated by Equation (24). Not only is the energy of over-voltage converted into heat more rapidly, but the line disturbance is much less because the voltage drops to only one-half of its initial value.

## 5. PASSAGE OF A WAVE THROUGH A JUNCTION BETWEEN TWO CONDUCTORS

(a) *Plain Junction.* In Fig. 12, a junction of two conductors is shown at $A$. The surge impedance, $Z_1$, of the conductor 1 is assumed to be smaller than that, $Z_2$, of the conductor 2. A traveling wave, $e_{f1}$, $i_{f1}$, coming from the left, strikes the junction point $A$. Because of $Z_2$ being greater than $Z_1$, the junction point possesses step-up characteristics, so that the transmitted voltage, $e_{f2}$, is greater than $e_{f1}$, whereas $i_{f2}$ is smaller than $i_{f1}$. The reflected wave is indicated by $e_{r1}$, $i_{r1}$.

The corresponding star diagram is shown in Fig. 13, and is practically identical with that in Fig. 5. The transmitted quantities, $e_{f2}$ and $i_{f2}$, take the place of $E$ and $I$, and the surge impedance $Z_2$ is substituted for $R$. The triangle $a\,c\,d$ is cross-hatched vertically, to indicate

that the corresponding power is transmitted through and not absorbed at the junction (see Table I). Equations (7) to (11) hold true in this case, except for changes in the notation. The subscripts $f$ and $r$ become $f\,1$ and $r\,1$, $Z$ becomes $Z_1$, $R$ is changed to $Z_2$, and $E$ and $I$ to $e_{f2}$ and $i_{f2}$ respectively. For this reason, the star diagrams in Figs. 5 and 13 are identical. When $Z_2$ is smaller than $Z_1$, the triangle of the transmitted wave assumes the shape $c\,a'\,d'$, and the joint acquires step-down characteristics.

(*b*) *Closing of a Switch.* Assume an open switch at $A$ (Figs. 12 and 14), and let the conductor $Z_1$ be

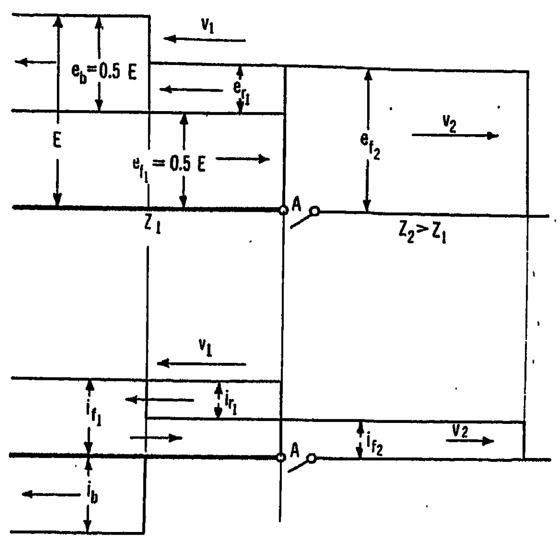


FIG. 14—TRAVELING WAVES CAUSED BY CLOSING A SWITCH

permanently connected to a source of d-c. voltage $E$, there being no traveling waves as long as the switch is open. When the switch is closed, a wave, $f\,2$, starts forward from the junction. The energy of this wave comes at first from that stored in conductor 1, until the source $E$ has been reached; hence, a wave of depression, $r\,1$, is produced in conductor 1. The problem is to determine the intensities of these two waves.

In order to be able to use the star diagram and to avoid a discontinuity in the phenomenon, due to the closing of the switch, the initial static distribution of electricity of voltage $E$ in conductor 1 may be thought of as a result of superposition of two equal waves traveling in the opposite directions. The forward going wave will have the voltage $e_{f1} = 0.5\,E$ and the corresponding current, $i_{f1}$, determined by Equation (4). The other wave, moving to the left and distinguished by the subscript $b$ (balancing), is of the voltage $e_b = 0.5\,E$, and its current is $i_b = -\,i_{f1}$. The sum of these waves will give a static distribution of electricity of voltage $E$ and current zero.

We shall assume the switch to be closed at the instant when the front of the wave $f\,1$ has just reached the switch, and the tail end of the $b$ wave has just passed it. This assumption satisfies the following three conditions:

(a) The actual static distribution $E$ extends to the switch previous to its being closed.

(b) The wave $f\,1$ meets a closed switch and proceeds into the second conductor as though through a plain junction (Figs. 12 and 13), that is, without a discontinuity in the circuit conditions.

(c) The closing of the switch has no effect upon the wave $b$ because at that instant this wave already lies entirely within conductor 1 and moves away from the switch.

Thus, in so far as the component wave $f\,1$ is concerned, the star diagram in Fig. 13 applies, provided that $e_{f1}$ is put equal to $0.5\,E$. To the left of the front of $r\,1$, the distribution is static, of voltage $E$. All the currents and voltages are indicated in Fig. 14.

(*c*) *Series Resistance between Conductors.* This case (Fig. 15) is similar to that shown in Figs. 12 and 13, except that a resistance $R$ is interposed between the two conductors, to absorb part of the transmitted wave energy. The surge impedance $Z_1$ is assumed to be

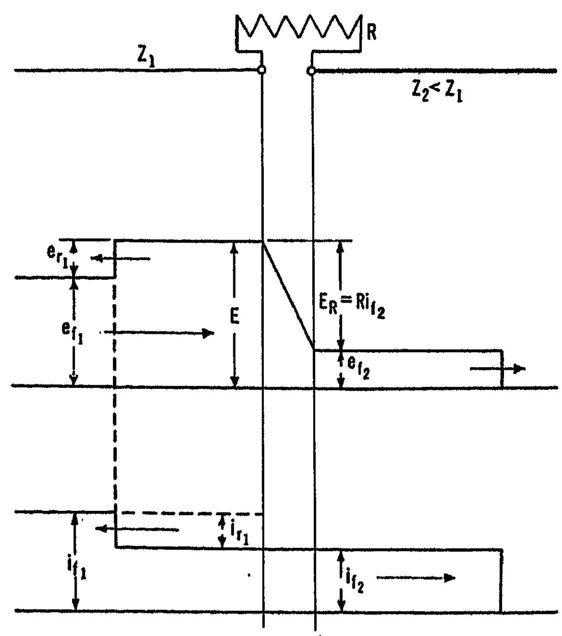


FIG. 15—REDUCTION IN THE AMPLITUDE OF A TRAVELING WAVE BY A SERIES RESISTANCE

greater than $Z_2$, and the resulting conditions are shown in the star diagram in Fig. 16. A similar diagram may be drawn for the case when $Z_1$ is smaller than $Z_2$.

The star diagram shows the following conditions fulfilled: Equation (4) for the waves $f\,1$, $r\,1$, and $f\,2$; Equation (11) for the resistor $R$; Equation (7) for both conductors; and Equation (8) for conductor 1. Since there are no other conditions to be fulfilled, the diagram gives a complete solution of the problem. The four triangles are properly cross-hatched to indicate the amounts of energy incoming, transmitted, reflected, and absorbed (see Table I). The positive directions of $e_{f2}$ and $i_{f2}$ are not indicated on the coordinate axes;

the physical sense of the problem is such that these quantities can have only positive values.

(*d*) *Shunted Resistance between Conductors.* The diagram of connections is shown in Fig. 17, where $p$ is a resistance to the ground. The total current arriving at $A$ is divided into two parts: a portion, $I_p$, flows to the ground; the remainder, $i_{f2}$, passes into the second conductor.

The corresponding star diagram is shown in Fig. 18. The familiar four triangles are shaded to indicate the amounts of energy incoming, transmitted, absorbed, and reflected. The relations obtaining between the various currents may be checked with reference to the waves themselves shown in Fig. 17; the same applies to the voltages.

It will be seen from Fig. 18 that $a\,d = E_p\,p^{-1}$, $d\,m = E_p\,Z_2^{-1}$, and $s\,m = E_p\,Z_1^{-1}$. But $a\,d$ plus $d\,m$ is greater than $s\,m$; hence

$$p^{-1} + Z_2^{-1} > Z_1^{-1} \tag{25}$$

This means that the total value of the combined impedances of $p$ and $Z_2$ in parallel is less than $Z_1$; consequently, the joint may be expected to possess step-down characteristics. In fact, $e_{f2}$ is smaller than $e_{f1}$, although $Z_2$ is greater than $Z_1$. Without $p$, the joint would have step-up characteristics (Figs. 12 and 13).

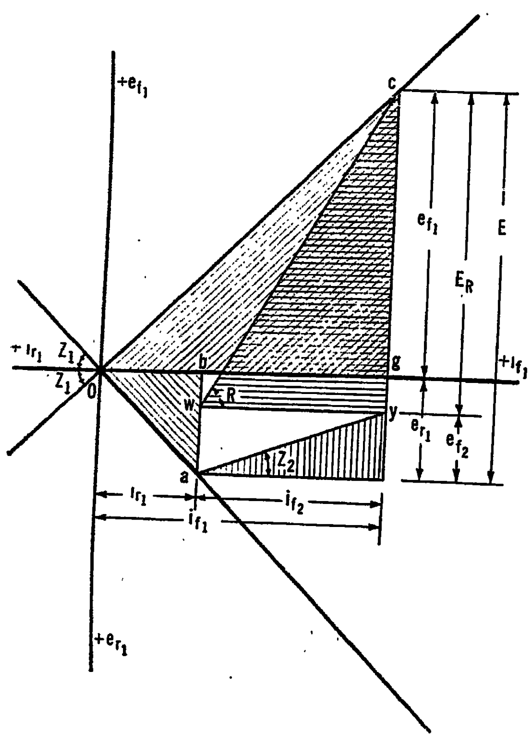


FIG. 16—A STAR DIAGRAM OF THE CURRENTS AND VOLTAGES IN FIG. 15

## 6. EFFECT OF A LUMPED SERIES INDUCTANCE IN REDUCING THE STEEPNESS OF A WAVE FRONT

In Fig. 19, a junction of two conductors is shown, with an inductance $L$ between them. This inductance will be assumed to be devoid of both resistance and capacitance, and, electrically speaking, so short as to be practically concentrated at one point. When a vertical-front traveling wave, $f\,1$, first strikes the inductance, only an infinitesimal current can pass through, because of the counter—e. m. f. $E_L = 2\,e_{r1} = L\,d\,i_{f2}/d\,t$. In other words, at the first instant the wave front is reflected as if from an open end of a line (Fig. 6). This

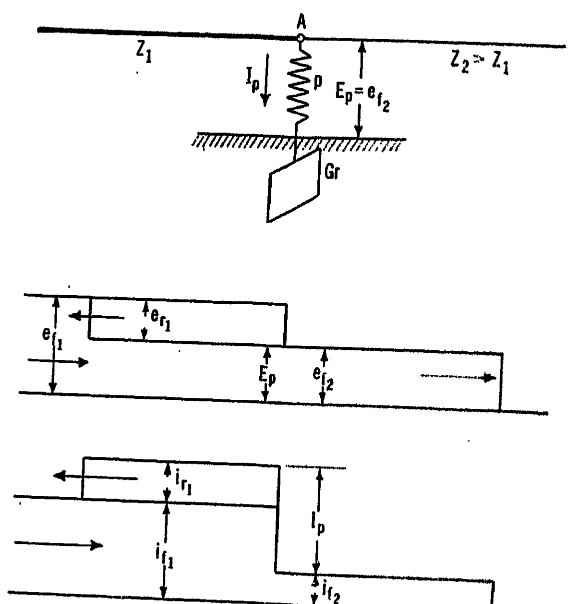


FIG. 17—REDUCTION IN THE AMPLITUDE OF A TRAVELING WAVE BY A SHUNTED RESISTANCE

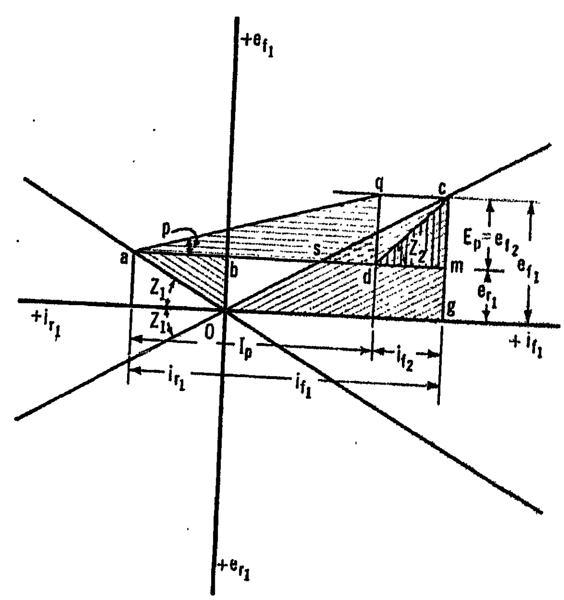


FIG. 18—A STAR DIAGRAM OF THE CURRENTS AND VOLTAGES IN FIG. 17

is indicated in the sketch of the wave by making $a\,b = b\,c$, and also by making $m\,n$ stand not only for $i_{f1}$, but also for $-\,i_{r1}$ at the first instant, so that the total current at $m$ is equal to zero.

As more and more current trickles through $L$, the value of $d\,i_{f2}/d\,t$, and consequently of $E_L$, decreases; a voltage, $e_{f2}$, is built up across the line 2, and a trans-

mitted wave, $f_2$, begins to be propagated in the second conductor. In Fig. 19, this wave is shown *in space*, but an identical curve would indicate the current and the voltage at $B$ *in time*, because the wave is propagated in the conductor 2 without distortion and any value found along the conductor at some time was produced at point $B$.

After an infinitely long time, the current flow through the inductance will become steady and the voltage $E_L$ will approach zero. In reality, these conditions are reached within a small fraction of a milli-second. The junction then behaves as though $L$ did not exist (Figs. 12 and 13). The purpose of the inductance is to make the wave front in conductor 2 more slanting and therefore the wave itself less harmful to the connected apparatus. It will be seen, however, that this advantage is gained at the cost of a steep reflected wave front in conductor 1.

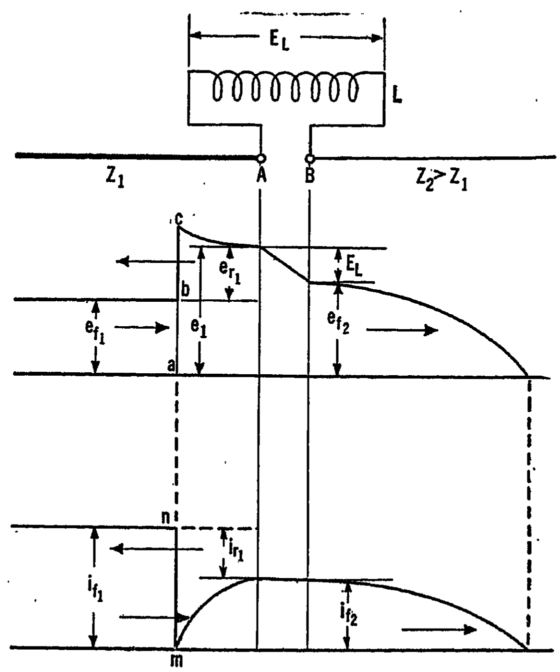


FIG. 19—ATTENUATION OF A WAVE FRONT BY A SERIES INDUCTANCE

The quantitative relationships are shown in Fig. 20, where the familiar four triangles may be distinguished by the usual cross-hatching. Since all the currents and voltages are now functions of time, the latter must be introduced into the star diagram explicitly. This is done by means of an exponential "timing" curve, $a_0\,u\,w$, whose equation is deduced below. This curve gives values of $E_L$ for various instants of time. Knowing $C\,c = E_L$ at a given instant, and the slope of $Z_2$, the triangle $c\,a\,d$ for the wave $f\,2$ may be drawn; the triangle $a\,b\,O$ for the reflected wave, $r\,1$, becomes thereby determined. The problem is therefore solved for that particular instant.

At any instant the power in the incoming wave must be equal to the sum of the amounts of power in the reflected and transmitted waves plus the rate of increase in the energy stored in $L$. Graphically this means that

$$\text{area } O\,C\,g = \text{area } O\,b\,a + \text{area } a\,c\,d + \text{area } h\,c\,C \qquad \textbf{(26)}$$

The area $h\,c\,C$ is a measure for the rate of energy storage in $L$ because the base, $c\,C$, of the triangle is equal to $E_L$ and the altitude, $d\,a$, equals $i_{f2}$. To prove Equation (26) we write

$$\text{area } a\,h\,c = \text{area } a\,b\,g\,d \qquad \textbf{(27)}$$

because the triangle and the rectangle have the same altitude, $a\,d$, but the base of the triangle is twice that of the rectangle. Subtracting the area $a\,b\,f$ from both sides of Equation (27) and adding the area $f\,c\,g$, we obtain

$$\text{area } b\,h\,c\,g = \text{area } a\,c\,d \qquad \textbf{(28)}$$

We also have

$$\text{area } O\,b\,h + \text{area } h\,c\,C = \text{area } O\,b\,a + \text{area } h\,c\,C \qquad \textbf{(29)}$$

Adding Equations (28) and (29) term by term, gives Equation (26).

At the instant $t = O$, when the front of the wave $f\,1$ has just reached the inductance $L$, the junction behaves like an open end; $E_L = 2\,e_{f1} = C\,a_0$, and the triangle $a\,c\,d$ becomes the straight line $a_0\,C$. The transmitted current, $i_{f2}$, is zero, and $i_{r1} = -\,i_{f1}$. After an infinitely long interval of time, $E_L = c\,C$ becomes equal to zero, and the triangle $a\,c\,d$ assumes the position and shape $A\,C\,D$. The diagram then becomes identical with that in Fig. 13.

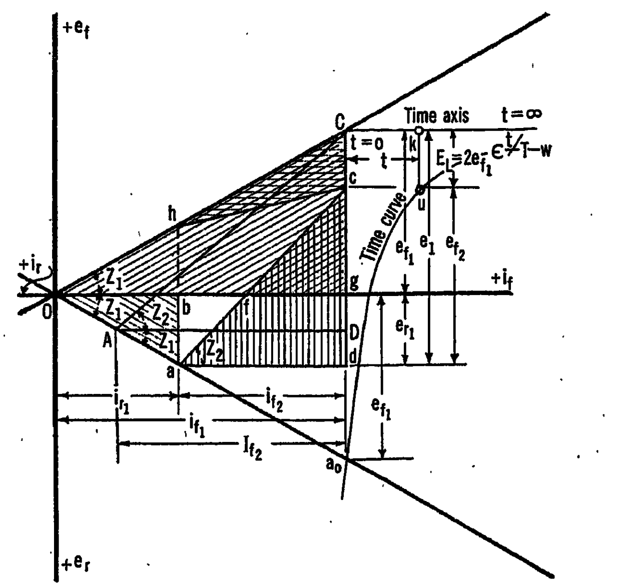


FIG. 20—A STAR DIAGRAM CORRESPONDING TO FIG. 19, WITH AN EXPONENTIAL TIMING CURVE

Thus, the whole process of smoothing down the wave front by an inductance is represented in the star diagram by a gradual transformation of the straight line $a_0\,C$ into the triangle $A\,C\,D$. For any position of point $c$, the length $c\,C = k\,u$ gives the value of $E_L$, and the corresponding abscissa, $C\,k$, gives the instant of time at which this value of $E_L$ occurs.

*Equation of the Timing Curve.* At any instant of time, $t$, the difference between the voltages $e_1$ and $e_{f2}$ (Fig. 19) is equal to the voltage $E_L$ across the inductance. Hence

$$e_1 = e_{f2} + L\, d\, i_{f2}/d\, t \tag{30}$$

To eliminate $i_{f2}$ from this equation, we write (Fig. 20)

$$C\,D = I_{f2}\, Z_2; \quad D\, a_0 = I_{f2}\, Z_1 \tag{31}$$

where $I_{f2}$ is the final value of $i_{f2}$ at the time $t$ = infinity. Adding the two Equations (31), gives

$$2\, e_{f1} = I_{f2}\,(Z_1 + Z_2) \tag{32}$$

so that

$$I_{f2} = 2\, e_{f1}/(Z_1 + Z_2) \tag{33}$$

Consequently, at any instant $t$,

$$i_{f2} = 2\, e_{f1}\,(1 - \epsilon^{-t/T})/(Z_1 + Z_2) \tag{34}$$

The reason for the added exponential factor in Equation (34) is as follows: In a straight-forward analytical solution of the problem (Rüdenberg, p. 397) one would supplement Equation (30) by a number of linear algebraic equations such as Equations (7) to (11). The result of elimination of all but one variable ($i_{f2}$) will be a linear differential equation of the first order, with constant coefficients. It is well known that such an equation is satisfied by an exponential function. This function must furthermore be such that at $t = O$, $i_{f2} = O$ and at $t$ = infinity $i_{f2} = I_{f2}$, the latter being given by Equation (33). It will be seen that the exponential term in the parentheses in Equation (34) satisfies these conditions. $T$ is the still unknown *wave time constant* of the circuit.

To determine $T$, substitute the value of $i_{f2}$ from Equation (34) in Equation (30); the result is

$$e_1 = e_{f2} + 2\, e_{f1}\,(L/T)\,\epsilon^{-(t/T)}/(Z_1 + Z_2) \tag{35}$$

At $t = O$, $e_1 = 2\, e_{f1}$ and $e_{f2} = O$. Substituting these values in Equation (35), we get

$$T = L/(Z_1 + Z_2) \tag{36}$$

Thus finally

$$e_1 = e_{f2} + 2\, e_{f1}\,\epsilon^{-t/T} \tag{37}$$

This means that the voltage across the inductance, or the equation of the exponential curve in Fig. 20, is

$$E_L = 2\, e_{f1}\,\epsilon^{-t/T} \tag{38}$$

$T$ being given by Equation (36).

The foregoing theory and the star diagram have been deduced for the general case of $Z_1$ different from $Z_2$. If $Z_1 = Z_2$, the equations and the diagram are somewhat simplified. Another interesting case is that of $Z_2$ equal to zero, that is, when the inductance $L$ is bridged across the end of the line $Z_1$ (Rüdenberg, p. 405). In Fig. 20, $a\,c$ becomes a horizontal line which moves from $a_0$ to $C$. It will be readily seen that this represents a gradual transition from an open line to a short-circuited line (Figs. 6 and 7).

## 7. Effect of a Lumped Shunted Capacitance in Reducing the Steepness of a Wave Front

In Fig. 21, the vertical front of a traveling wave, $f\,1$, is assumed to arrive at the junction of the conductors $Z_1$ and $Z_2$ at the instant $t = O$. Some of the energy is reflected, some goes to charge the capacitor $C$, and the remainder passes into the conductor $Z_2$. Assuming the capacitor to be initially uncharged, it will act at the first instant like a short-circuit to the ground, so that the total voltage at the junction drops to zero and no current passes into the second conductor. After the capacitor has been fully charged (which theoretically takes an infinite time, but in reality occurs within a small fraction of a milli-second), it exerts no further effect on the wave, and we again have the phenomenon shown in Figs. 12 and 13.

The waves shown in Fig. 21 represent the distribution of the voltage and the current in space, but similar curves can be drawn to represent variations of the same quantities in time, at the junction point, for the reason explained in application to Fig. 19. At $m$ and $n$, the

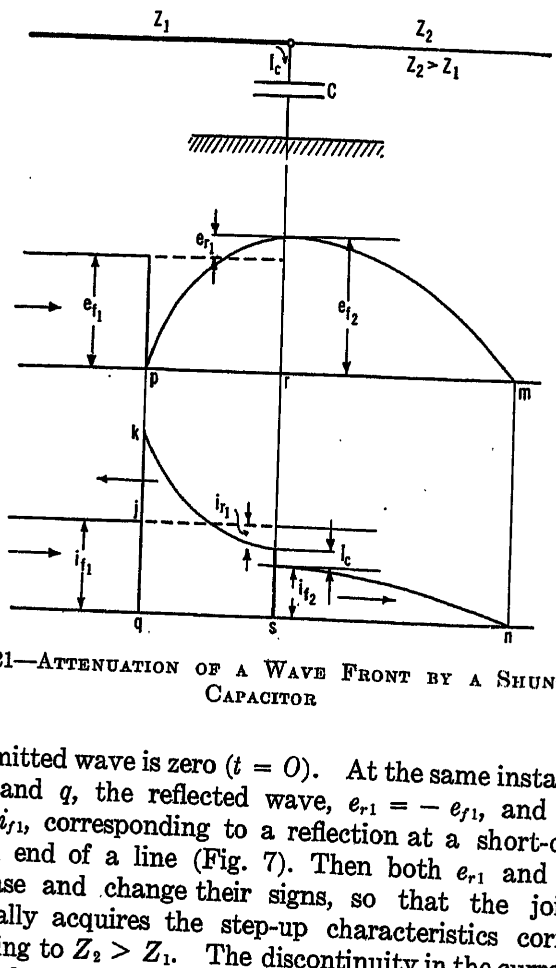


Fig. 21—Attenuation of a Wave Front by a Shunted Capacitor

transmitted wave is zero ($t = O$). At the same instant, at $p$ and $q$, the reflected wave, $e_{r1} = -\,e_{f1}$, and $i_{r1} = +\,i_{f1}$, corresponding to a reflection at a short-circuited end of a line (Fig. 7). Then both $e_{r1}$ and $i_{r1}$ decrease and change their signs, so that the joint gradually acquires the step-up characteristics corresponding to $Z_2 > Z_1$. The discontinuity in the current at $s$ is due to the charging current, $I_c$, of the capacitor, the instant $s$ being an intermediate one, before the capacitor is fully charged. At $t$ = infinity, this discontinuity is reduced to zero.

It will be seen that a capacitor smooths down the front of the transmitted wave, but causes a steep front of the reflected current wave and increases the total current in the first conductor to a double value. It is

instructive to compare the curves in Fig. 21 with those shown in Fig. 19.

The star diagram of the waves in Fig. 21 is shown in Fig. 22. The four cross-hatched triangles represent the amounts of instantaneous power, incoming, outgoing, reflected, and absorbed in the capacitance. The exponential curve $a_0\, u\, w$ gives values of the capacitor current, $I_c$, for different instants of time, and corresponds to the $E_L$ curve in Fig. 20. At the instant $t = O$, $I_c = a_0\, C$, corresponding to a short-circuited end of the conductor 1 (Fig. 7). At $t$ = infinity, $I_c = O$, and the relations are represented by the triangle $A\, C\, D$, as in Fig. 13. The whole process of the passing of the wave over the junction point is therefore represented in the star diagram as a motion and increase in size of the triangle $a\, c\, d$ from an infinitesimal magnitude at point $a_0$ to the final position $A\, C\, D$. During this motion, the sides of the triangle remain parallel to themselves.

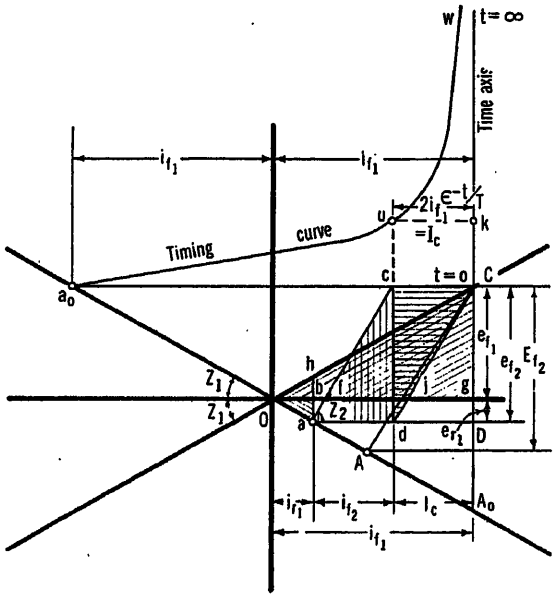


FIG. 22—A STAR DIAGRAM CORRESPONDING TO FIG. 21, WITH AN EXPONENTIAL TIMING CURVE

The law of conservation of energy requires that

area $O\, c\, g$ = area $O\, b\, a$ + area $a\, c\, d$ + area $c\, d\, C$ **(39)**

The latter area is equal to the product of the capacitor current by the voltage at its terminals, and therefore is a measure for its power input. To prove Equation (**39**), we shall first compute the area of the trapezoid $h\, b\, g\, C$. The sum of its parallel sides is equal to $d\, c$ and its altitude is $b\, g$. Hence,

$$\text{area } h\, b\, g\, C = 0.5\, d\, c \times b\, g \qquad \textbf{(40)}$$

Next we compute the area of the trapezoid $c\, a\, d\, C$. Its altitude is $d\, c$ and the sum of its parallel sides is equal to $b\, g$. Consequently

$$\text{area } c\, a\, d\, C = 0.5\, d\, c \times b\, g \qquad \textbf{(41)}$$

Thus, the areas of the two trapezoids, $h\, b\, g\, C$ and $c\, a\, d\, C$, are equal. Adding the area $O\, h\, b$ to the first, and $O\, b\, a$ to the second, Equation (**39**) is proved.

*Equation of the Timing Curve.* When $t = O$, $e_{f2} = O$; when $t$ = infinity,

$$E_{f2} = 2\, e_{f1}\, Z_2/(Z_1 + Z_2) \qquad \textbf{(42)}$$

see Equation (**14**). The voltage $e_{f2}$ varies exponentially between these two limits, for the reason explained in the derivation of Equation (34). Thus, at any instant $t$,

$$e_{f2} = 2\, e_{f1}\, Z_2\, (1 - \epsilon^{-t/T})/(Z_1 + Z_2) \qquad \textbf{(43)}$$

where $T$ is the unknown wave time constant of the circuit. For the junction point we have

$$i_{f1} + i_{r1} = i_{f2} + C\, d\, e_{f2}/d\, t \qquad \textbf{(44)}$$

Here

$$I_c = C\, d\, e_{f2}/d\, t \qquad \textbf{(45)}$$

is the charging current flowing into the capacitor. Substituting the value of $e_{f2}$ from Equation (**43**) in Equation (**44**), we get

$$i_{f1} + i_{r1} = i_{f2} + 2\, e_{f1}\, (C/T)\, Z_2\, \epsilon^{-(t/T)}/(Z_1 + Z_2) \qquad \textbf{(46)}$$

At $t = O$, $i_{f2} = O$, $i_{r1} = i_{f1} = e_{f1}/Z_1$. Substituting these values in Equation (**46**), gives

$$T = C\, Z_1\, Z_2/(Z_1 + Z_2) = C/(Y_1 + Y_2) \qquad \textbf{(47)}$$

where the surge admittances $Y_1$ and $Y_2$ are the reciprocals of $Z_1$ and $Z_2$; see Equation (**5a**). Equation (**45**) becomes

$$I_c = 2\, i_{f1}\, \epsilon^{-t/T} \qquad \textbf{(48)}$$

This is the equation of the timing curve in Fig. 22. The foregoing relationships are somewhat simplified when $Z_1 = Z_2$. The hypotenuse $a\, c$ is then parallel to $O\, C$, and point $A$ coincides with $O$. To deduce the case of a capacitor bridged across the end of a line, put $Z_2$ = infinity. The triangle $A\, C\, D$ degenerates into the line $C\, A_0$. Thus, the reflection begins at $a_0$, as though with a short-circuited end of the line, and gradually changes to $A_0$ as with an open-end (Figs. 7 and 6).

## 8. RECIPROCAL RELATIONS

There are certain formal analogies between some of the cases considered above, worth noting. Some of these similarities become particularly clear by comparing the corresponding star diagrams in pairs, and turning one of each pair by 90 deg. It then looks very much like the other diagram. The following cases may be mentioned in particular:

| Case | *vs.* Case | Figs. |
|---|---|---|
| Open end | Shorted end | 6 and 7 |
| Series resistance | Shunted resistance | 16 and 18 |
| Series inductance | Shunted capacitance | 20 and 22 |
| Current curve | Voltage curve | 19 and 21 |
| Voltage curve | Current curve | 19 and 21 |

These formal graphical analogies follow from the equations underlying the phenomena. For example, where with a parallel connection currents are added, voltages are added with a series connection, etc. See A. Russell, "Alternating Currents," Vol. I, Chap. 21; the principle of duality and reciprocal theorems.

The subject is not followed here any further, but it

is quite possible that in some more complicated cases of protective apparatus, having obtained a solution for a certain combination, a solution for another arrangement of apparatus may be "guessed at" on the basis of the foregoing analogies and reciprocal relations.

### 9. Further Applications of the Star Diagram

The purpose of this paper being to explain the general theory and the use of the star diagram, only a few of the simplest applications have been considered. The next step will be to apply this diagram to more involved practical cases. The following problems on reflection and refraction of traveling waves may be suggested in particular:

(a) A transmission line which is divided at a point into two or more branches with different values of $Z$.

(b) Combinations of capacitance and inductance for protective purposes; for example, those shown in Rüdenberg's book on pp. 430 to 442.

(c) Same, with damping resistances; *ibid.*, pp. 442 to 446.

(d) Influence of short connecting conductors between two long conductors; *ibid.*, pp. 418 to 430.

(e) Graphical theory of short rectangular-front pulses, rather than long waves; *ibid.*, pp. 400, 404, 409, 443, 444, 445. Problems on short pulses may be solved by assuming an indefinitely long rectangular-front wave followed by an identical wave of opposite polarity which wipes out all but the short front portion of the original wave. Perhaps it is only necessary to draw star diagrams for both waves, with a proper time lag between them, and add the results algebraically.

(f) Mutual induction caused by traveling waves; *ibid.*, pp. 387 to 396.

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*Archiv für Elektrotechnik:* consult the late volumes for numerous valuable articles on the subject.

# Theory of the Deion Circuit-Breaker

BY J. SLEPIAN*

*Synopsis.—Three major features incorporated in the Deion circuit-breaker are discussed. They are: deionization at solid surfaces, the function of the static balancer, and cold electrode arcs.*

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THE switching of electric power circuits calls for elements which, subject to control, shall function sometimes as good electrical conductors and at other times as good insulators. When "closed" the element must pass hundreds or thousands of amperes with only at most a few volts drop; its resistance or impedance must be of the order of a fraction of an ohm. When "open" it must withstand hundreds or thousands of volts, with the passage of at most a few milliamperes; its resistance must be in the hundreds of thousands of ohms. Also it must be able to change from the one state to the other in a fraction of a second. So far, the only materials found which can meet these requirements are the gases, and arcs in air, and arcs in the vapors and decomposition products of oil are regularly serving to control power circuits. Careful study shows that the arc instead of being merely an unpleasant accompaniment of the opening of a switch, plays a very necessary and desirable part, and that if the arc did not occur spontaneously on separating contacts, it would have been necessary for us to discover or invent it or its equivalent for the purpose of circuit interruption.

Recognizing the importance of the arc in switching equipment, the company with which the author is associated five years ago began an extensive theoretical and experimental study of the electric arc as it appears in switches, and more particularly the study of what happens when the change-over occurs from the state of conductor to the state of insulator, that is at the mement of extinction of the arc. Some of the results of this study have already been presented.[1] As was perhaps to be expected, the study revealed some new and interesting possibilities in the application of arcs to circuit interruption and the Deion a-c. circuit-breaker in which these possibilities have been developed, promises to take its place with or perhaps replace the oil circuit breaker for certain classes of high power work.

While there are many details of the Deion circuit-breaker which are of great scientific interest and which required months and even years of intensive work for their mastery, there are three general principles which stand out above the others and which will be described here.

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1. *Extinction of an A-C. Arc*, J. Slepian, A. I. E. E. Quarterly TRANS., Vol. 47, October 1928, p. 1398.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

## I. DEIONIZATION AT SOLID SURFACES

The subject of the rate of recovery of the dielectric strength of the space carrying a short a-c. arc immediately after arc extinction has been treated in some detail in a previous paper.[1] There it was shown that the ability to withstand the first few hundred volts was recovered almost instantly, but that later increments of dielectric strength were recovered at a very much slower rate. This is brought out in the curve of Fig. 1 which is derived from the data of Fig. 6 of the paper referred to.

Theory and experiment indicate that the first 250 volts are borne almost entirely by a thin layer of gas immediately adjacent to the cathode. Electrons readily leave this layer, but others to replace them cannot enter from the metal. The positive ions discharge into the cathode. Thus this cathode layer is deionized exceedingly rapidly.

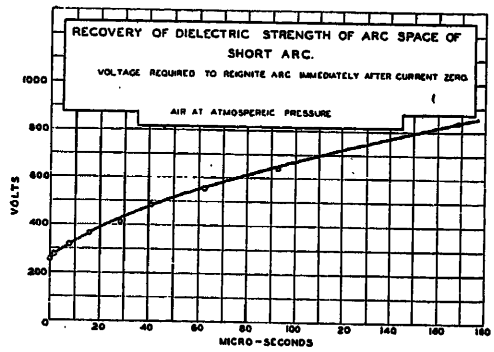


FIG. 1

The subsequent slow further growth of dielectric strength is due to the further growth of the deionized cathode layer, and the disappearance of the ions in the other parts of the gas space by recombination.

It is evident then that it is the slow rate of recombination of the ions in the arc space away from the electrodes which limits the applicability of the arc in air for interrupting high voltages. A fairly obvious suggestion would be to reduce as far as possible the arc space remote from a cathode, and so far as possible cause all the arc to play in space close to a cathode; in other words to use a large number of short arcs in series. This is what is done in the Deion circuit-breaker.

As at present developed, the Deion breaker consists of a stack of copper plates 1/16 in. thick separated by 1/16 in. spacers. The arc which is drawn on contacts below this structure is blown by a magnetic field into

the stack. The arc is thus broken into short arcs in series, each of 1/16 in. length. There are thus in each inch of structure eight cathodes with their eight immediately adjacent rapidly deionizing gas layers. Immediately after the current passes through its zero value in its normal cycle each cathode layer is almost instantly deionized, and acquires the ability to withstand 250 volts much faster than any practical power circuit of corresponding voltage can supply the 250 volts. Thus the voltage necessary to reignite the arcs after the current zero is eight times 250 or 2000 volts per inch of structure. Hence the structure will interrupt circuits whose voltage is not over $2000/\sqrt{2}$ or 1414 volts r. m. s. per inch length.

This seems like a very high voltage for arcs in air,

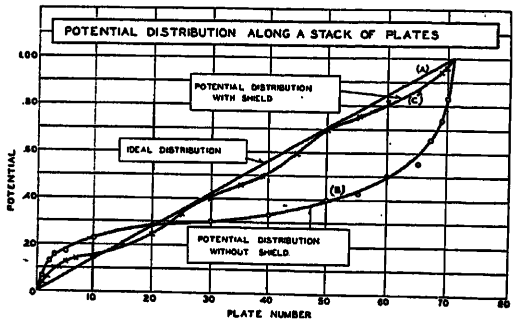


FIG. 2

but it by no means represents the limit. The plates in the Deion circuit-breaker have been made 1/16 in. thick for the sake of thermal capacity, so that it may operate many times without an excessive rise in temperature. So far as concerns extinction of the arc, the plates could be 1/32 in. thick or 1/64 in. thick. The spacing between the plates was made 1/16 in. so as to permit free motion of the individual arcs. Experiment shows that 1/32 in. spacing still leaves sufficiently free motion of the arc. Using these last figures there would be 21.3 plates per inch, and therefore the structure would interrupt 3760 volts r. m. s. per inch.

In the present Deion breaker the circuit volts per plate is kept very much less than the theoretical limit of 175 volts r. m. s.; in fact it is less than 130 volts. This is partly for the sake of having a factor of safety, and partly because when a voltage is impressed upon a long stack of plates insulated from one another, the potential does not divide among the plates in a uniform manner. This lack of uniform voltage distribution is compensated for in the Deion breaker by a static shield as described in the next section but the compensation cannot be made exact so that a sufficient margin between the theoretical limit and the working volts per plate is necessary.

## II. The Function of the Static Shield

When voltage is applied to the ends of a long uniform stack of plates insulated from one another, the potential does not divide uniformly among the plates, but the potential differences between successive plates at the end of the stack may be many times the potential differences between successive plates in the middle of the stack. This is a consequence of the elementary principles of electrostatics, and need not be gone into in more detail here. The example of this phenomenon best known to the electrical engineer is probably the non-uniform distribution of potential among the units of a long string of suspension insulators.

The distribution of potential among the members of a stack of plates in a Deion breaker has been studied experimentally by Mr. B. P. Baker. A resistance-bridge method with telephone receiver as null instrument proved satisfactory. The stack studied consisted of 72 plates of 1/16-in. copper, spaced 1/16 in. apart, each plate having the shape of the letter P with round part 17 cm. diameter, and straight part 17 cm. long.

When unshielded, the potential distribution was found to be that shown by curve *b*, Fig. 2. The departure from uniform distribution given by curve *a* is very great. The point of inflection of curve *b* marks the potential of space remote from the stack. That it did

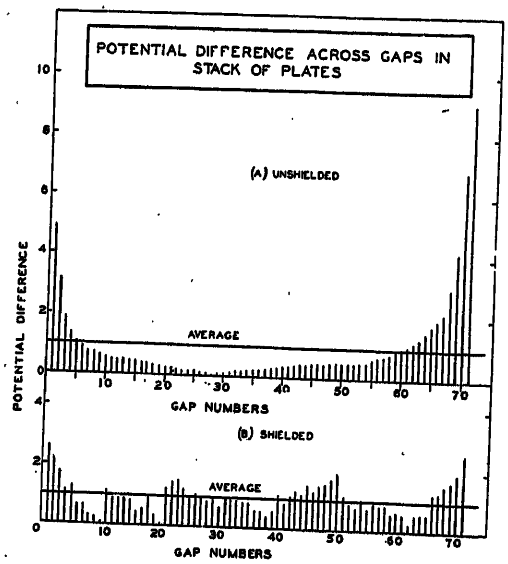


FIG. 3

not occur at the center of curve *b*, and that curve *b* was not symmetrical, are due to the influence of the frame supporting the stack.

Fig. 3A gives the relative potential differences across the gaps formed by the plates. It shows that the end gap received an impressed potential 9.1 times as great as the average voltage per gap. So large a ratio might be expected to diminish seriously the average volts per gap upon which the breaker could operate, and this had been found to be the case.

A static shield was then made up consisting of a micarta cylinder formed to fit over the stack, and with pieces of tin foil imbedded in it of such shape and size as to make uniform the distribution of potential among

the plates. The resulting curve as determined experimentally is given by *c*, Fig. 2. This curve follows the ideal distribution more closely, and as shown in Fig. 3B, the maximum gap potential is 2.6 times the average as compared with 9.1 for the unshielded stack. The curve of Fig. 2C is for the first static shield made. Later designs of static shield give still better results.

At first thought it would seem that the average volts per gap at which the unshielded stack would operate in a Deion breaker would be reduced in the ratio 9.1 so that if 176 volts r. m. s. is the theoretical limit for a single gap, 176/9.1 or 19.4 volts per gap would be the limit for the unshielded stack, and 1375 volts r. m. s. would be the limit for the whole stack. Similarly for the shielded stack, 176/2.6 or 67.8 volts r. m. s. would be the limiting average voltage per gap, and 4810 volts r. m. s. would be the limit for the whole stack.

Actually, however, the unshielded stack was found by test to be good for nearly 6000 volts r. m. s. and the shielded stack for more than 10,000 volts; how much more was not determined. The limit for the case of perfect uniformity of voltage distribution would be about 12,500 volts.

To explain these results it is necessary to examine in more detail the mechanism of reignition of an arc immediately following a current zero. In the previously quoted paper[1] it was pointed out that the almost instantly acquired ability to withstand 250 volts had a close relation to the voltage required to maintain a glow on copper electrodes. A glow is a discharge requiring several hundred volts at its cathode as compared with 10 or 20 volts required by an arc. It seems probable that when an arc space is broken down by the application of more than 250 volts immediately following a current zero, the current is first passed in some form of discharge like a glow, and only after a little time does the discharge break down into an arc. The time to change from the glow form to the arc will be very short if considerable current (that is several amperes) flows in the glow, and will probably be only a few hundredths of a microsecond. If, however, the current flowing in the glow is limited (that is a fraction of an ampere) the time for change from glow to arc may be many microseconds).

In the Deion breaker, immediately after the current zero, the voltage tending to reignite the arc in the stack of plates rises at a rate depending on the nature of the circuit in which the breaker is working. In a fast practical power circuit this reignition voltage may reach its maximum value in about one microsecond.[1] During this period of rising reignition voltage the overstressed gaps in the deion stack may be subjected to more than 250 volts, even though the average volts per gap impressed is much less than 250. These gaps then pass current, but the magnitude of this current is limited to that of the charging current of the remainder of the structure where the gaps are not broken down. If this charging current is small the discharge in the overstressed gaps will remain in the glow form, and will require in each gap a voltage of not less than 250 volts. The overstressed gaps then continue to hold more than the average voltage per gaps so that the remaining gaps are not overstressed and the arcs in the structure do not reignite.

If the charging current is too high, however, the discharge in an overstressed gap may break down from the glow form to an arc. It then takes only 20 volts which is much less than the average volts per gap, and thus causes more voltage to be thrown upon the remaining gaps. This may make the next overstressed gap break down into an arc with further raising of the voltage on the remaining gaps. This process may continue until all the remaining gaps are overstressed, and then reignition of all the arcs in the stack takes place.

As an example, consider the unshielded stack of 72 plates described above when operating in a circuit of 5000 volts r. m. s. or 7070 volts crest. This corresponds to 99.6 volts per gap average, and at this voltage the breaker was found to work. Assume that this voltage tending to reignite the arcs is applied in one microsecond at a uniformly increasing rate so that the average rate of application of voltage per gap is $99.6 \times 10^6$ volts/sec. The electrostatic capacity of a single gap is approximately 150 micro-microfarads. The average charging current per gap is therefore $99.6 \times 10^6 \times 150 \times 10^{-12} = 0.015$ ampere.

The charging current of the end gap by Fig. 3A is 9.1 times as great as this, or 0.136 ampere. After the critical voltage 250 volts is reached the current is carried principally as a glow in the end gap. The time taken to reach 250 volts in the end gap is $250/(9.1 \times 99.6 \times 10^6)$ or $0.28 \times 10^{-6}$ sec. The glow therefore carries 0.136 amperes for $0.72 \times 10^{-6}$ seconds. After this time the transient is over and on 60 cycles the glow current is correspondingly reduced. A copper cathode can carry 0.08 to 0.10 amperes as a glow indefinitely without changing into an arc.[2] Hence it would seem that 0.136 amperes would remain as a glow for $0.28 \times 10^{-6}$ seconds.

At 7000 volts r. m. s. for which the unshielded stack fails, the charging current, and therefore also the current in the glow when the end gap discharges is 0.190 ampere, and the time that the glow must hold is increased to $0.61 \times 10^{-6}$ seconds.

For the shielded stack at 10,000 volts r. m. s. at which it works in a breaker, the rate of rise of voltage in the end gap is by Fig. 3B $2.6 \times 10{,}000 \times \sqrt{2} \times 10^6/71$ or $518 \times 10^6$ volts/second. The charging current is then 0.078 amperes, and the glow should carry this current indefinitely.[2] Actually it needs to

2. W. Burstyn, *Elektrotech. Zeitsch.*, Vol. 41, p. 505, 1920. The current 0.32 amperes given by Burstyn is the current taken by the closed contacts in a 440-volt circuit. On allowing for the drop in the glow, 310 volts, the current on opening the contacts would drop to 0.095 ampere.

carry it only for $(1 - 250/518) \times 10^{-6}$ or $0.48 \times 10^{-6}$ seconds. It seems then that with this size of plates, and with the shielding as obtained in Fig. 3B there is a large factor of safety in operating the 71 gaps at 10,000 volts or at 140 volts r. m. s. per gap average.

## III. Cold Electrode Arcs

As explained in Section I, the efficacy of the Deion breaker resides in having all parts of the arc in close vicinity to deionizing surfaces. This is accomplished in the heavy current switch by blowing the arc into a stack of closely spaced plates so that no part of an arc is farther than 1/16 inch from a deionizing cathode. If the short arcs in this structure stood still for the duration of one-half cycle, 1/120 second, the structure would quickly be destroyed by the welding together of some plates and the burning of holes through others.

At the time this work was begun, it was believed that the cathode of an arc was necessarily at a very high temperature. In fact, the theory generally accepted then required thermionic emission of electrons from the cathode for the maintenance of the arc, and for most metals the temperature for so intense a thermionic emission is far above the boiling point. How well entrenched this theory was may be seen by referring to a paper by K. T. Compton, (*Phys. Rev.*, Vol. 21, 1923, p. 269), and Seeliger, "Physik der Gasentladungen" (Leipzig 1927, p. 360 *et. seq.*). On this account much work was done on speeding up the deionization of an arc by causing it to play through the openings of gauze sheets, and thus avoiding the development of arc terminals on the deionizing structure.

However, in the course of this work, it was forced upon the author's attention that sometimes arcs were obtained which did not have a hot cathode. Therefore, although it required considerable courage to take a stand opposite to that espoused by so many eminent authorities, the thermionic emission theory of the cathode of an arc was abandoned, and arcs with cold cathodes were accepted as possible. Experiment soon showed that by moving the terminals of the arc sufficiently rapidly over the electrode surfaces melting could be avoided even for very heavy currents. Arcs of more than 20,000 amperes have thus been carried on copper electrodes for more than 0.01 seconds with only slight oxidation of the electrodes.

A theory of the cathode of an arc was also developed based on the hypothesis that the metal itself is not necsarily at a temperature sufficient for thermionic emission but that a layer of gas or vapor immediately adjacent to the cathode is so intensely ionized, perhaps in virtue of very high temperature, that the arc current can be carried to the cathode by positive ions only.[3]

Shortly after this, papers by Stolt[4] described experiments with rapidly moving arcs of moderate current (up to 12 amperes) which seemed incompatible with the thermionic theory of the cathode of an arc, and this, with some theoretical work on heat balance at the cathode, is causing general abandonment of the thermionic theory.[5] Another theory of the cold cathode arc has been proposed by Langmuir,[6] who states that electrons are drawn from the cathode by intense electrostatic forces arising from space charges developed close to the cathode. However, a theory of the cold cathode arc is not essential for the understanding of the Deion breaker. Merely the possible existence of arcs with cold cathodes must be accepted.

In the Deion breaker the melting of the electrodes is prevented by causing the arc terminals to move very rapidly over the electrode surface by means of a magnetic field. The first experiments were carried out with a stack of long straight plates as the deionizing structure. It was found, however, that the velocity of the arc terminals necessary to prevent melting of the electrodes was so great that for 10,000 amperes, plates more than ten feet long would be necessary. These would be prohibitive from the standpoint of size for most applications, and would require too expensive a magnetic field structure.

This difficulty was surmounted by causing the arcs moving with high velocity to retrace over and over again an annular path. On adopting this expedient a very important new advantage was obtained. The deionizing structure became an almost completely closed structure. The arc, when once driven in, could not get out again, and had to stay in until its extinction at the end of the half cycle. Thus the danger of the arc getting across live parts outside the switch and causing short circuits was practically eliminated.

To calculate the speed necessary to prevent melting of the electrodes it is necessary to know the current density, and the voltage of the arc. Observation of the oxidized trail of arcs with currents about 10,000 amperes led to the estimate of 30,000 amperes/cm.$^2$ as the current density in the cold cathode arc. The voltage of the 1/16-inch long cold cathode arcs was found to increase with current, (rising characteristic) and for more than 2000 amperes is given by

$$E = 30 + 0.79 \times 10^{-3}\, I \qquad (1)$$

where $I$ is the current in amperes. The influence of the current then is a moderate one, and it is not until currents approaching 40,000 amperes are reached that the arc voltage is doubled.

The watts input per cm.$^2$ of arc section is then

$$W = (30 + 0.79 \times 10^{-3}\, I) \times 30{,}000 = 9 \times 10^5 + 23.9\, I \qquad (2)$$

The diameter of the arc section, assuming it to be circular, is given by

$$1/4\, \pi\, d^2\, 30{,}000 = I \qquad (3)$$

3. J. Slepian, *Phys. Rev.* 27, p. 407, 1926.
J. Slepian, *Jour. Franklin Inst.* 201, p. 79, 1926.
4. H. Stolt, *Ann. d. Physik* 74, pp. 80-104, 1924.
H. Stolt, *Zeits. f. Physik* 262, pp. 95-101, 1924.
5. K. T. Compton, A. I. E. E. Trans., Vol. XLVI, 1927, p. 868.
6. I. Langmuir, *Zeits. f. Physik* 46, p. 282, 1927.

A point of the electrode surface will be exposed to the action of the arc for the time taken by the arc section to pass over the point. For a point over which the center of the arc section passes this time will be

$$t = \frac{d}{\nu} \tag{4}$$

where $\nu$ is the velocity with which the arc moves.

Lastly, the temperature rise $T$ in degrees $C$ at the surface of a semi-infinite solid into whose surface $W_1$ watts per cm.[2] is flowing is given by

$$T = \frac{2\,W_1}{4.18\,\sqrt{\pi\,k\,c\,\delta}}\sqrt{t} \tag{5}$$

where $k$ is the heat conductivity of the solid in cal/cm.[3] deg. cent. $C$ is the heat capacity in cal/cm.[3] deg. cent., and $\delta$ is the density in grams/cm.[3] Substituting (3) and (4) in (5)

$$T = 2.19 \times 10^{-2}\,\frac{W_1\,I^{1/4}}{\sqrt{k\,c\,\delta}\;\nu^{1/2}} \tag{6}$$

$$\nu = 4.8 \times 10^{-4}\,\frac{W_1^2}{k\,c\,\delta}\,\frac{I^{1/2}}{T^2} \tag{7}$$

For copper we may take $k = 0.72$, $c = 0.096$, and $\delta = 8.0$. Let $T$ be the temperature rise for melting, *i. e.*, 1000 deg. cent. Then (7) becomes

$$\nu = 8.69 \times 10^{-10}\,W_1^2\,I^{1/2} \tag{8}$$

Of the energy developed in the arc given by (2), it is uncertain how much goes into the cathode, and how much into the anode. A pessimistic estimate of the velocity needed to avoid melting will be obtained if all the heat $W$ is assumed to go into one electrode, that is $W_1 = W$; an optimistic estimate will be obtained if the heat is supposed to divide equally between the electrodes, *i. e.*, $W_1 = 1/2\,W$. Substituting from (2) then, these two estimates become

$$\nu = 8.69 \times 10^{-10}\,(9 \times 10^5 + 23.9\,I)^2\,I^{1/2} \quad \text{(pessimistic)} \tag{9}$$

$$\nu = 2.17 \times 10^{-10}\,(9 \times 10^5 + 23.9)^2\,I^{1/2} \quad \text{(optimistic)} \tag{10}$$

The velocities given by (9) and (10) are quite large numerically for large currents as is seen in the following table. Experimentally the necessary velocities seem to be nearer to the optimistic figures than the pessimistic.

TABLE I

| $I$ Amperes | Velocity to prevent melting meters per second | |
|---|---|---|
| | Pessimistic | Optimistic |
| 4,000 | 543 | 136 |
| 8,000 | 925 | 231 |
| 12,000 | 1350 | 338 |
| 16,000 | 1805 | 451 |

It will be observed that the speeds in the table are all greater than the velocity of sound. This occasioned no particular difficulty. Magnetic fields of about 0.3 $I$ gauses were sufficient to give these velocities.

In the above calculation it was assumed that the arc section was coherent and circular. This was probably the case for small currents. Above 15,000 amperes there was some evidence from the character of the trails, and by indications of probe electrodes, that the arc was broken up into several separate arcs in parallel. It may be that at this high current in the correspondingly high field, the air in the 1/16-inch space was set into such violently turbulent motion that the arc was broken up. If the arc is divided into several parallel arcs, the velocity needed to prevent melting is lessened.

The above calculations give the temperature rise at a point in the electrode surface for a single passage of the arc over it. After the arc passes, the temperature of the point drops to nearly its original temperature before the arc reaches it again. With each rotation of the arc, the plate as a whole is raised in temperature, so that this rise in plate temperature must be added to the temperature of the point which the arc is passing over. Calculation readily shows, however, that this rise in temperature of the plates as a whole is too small to affect the above calculation seriously.

## Discussion

For discussion of this paper see page 545.

# The Structural Development of the Deion Circuit-Breaker up to 15,000 Volts

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Associate, A. I. E. E.

and

B. P. BAKER[1]
Associate, A. I. E. E.

## INTRODUCTION

Up to the present time, the interruption of an alternating-current circuit has been accomplished generally in one of two ways. The arc may be drawn between contacts located in some insulating liquid such as oil. In this case, the insulating value of the oil depreciates with each current interruption until it reaches a point where it must be renewed. On the other hand, the arc may be drawn in air with no means of extinguishing it other than lengthening it to such an extent that the generated voltage is no longer able to maintain it. For modern generating voltages, this requires arcs of great length and, for the upper range of transmission voltages, results in arc lengths which are impractical. As a result of this limitation, the oil circuit breaker has assumed a position of paramount importance on modern operating systems.

There has been a growing demand for some time, on the part of operators, for a circuit-interrupting medium which does not involve the use of oil. The chief reasons for this demand are removal of possible fire hazards and simplification of maintenance problems. The demand has been recognized by manufacturers and although research work toward this goal has been carried on for a number of years, no satisfactory, general-purpose apparatus of this kind has been placed on the market up to the present time.

At the Westinghouse Electric and Manufacturing Company, fundamental research in this field has been carried on for a long period and has served to give a deeper insight into the nature of arc conduction. It has suggested the use of means for deionizing the path of an arc drawn in air other than by merely extending it to a great length. Experimental circuit interrupters have been made utilizing such deionizing means in a variety of forms and the name "Deion circuit-breaker" has been applied to these devices.

The work on all of the methods of deionizing an arc stream has contributed much to the fundamental knowledge of arc phenomena and some of them may be developed further for practical application in the future. One of the most promising of these various methods was developed and applied to the Deion circuit breaker described in this paper.

In this circuit-breaker, an arc is drawn in air and forced into a deionizing chamber where it is broken up into a multiplicity of short arcs which are moved over metal plates at a velocity sufficient to prevent burning. This movement of the arc is maintained over an annular path until the current wave reaches zero, after which the arc stream between the metal plates is deionized, quickly changing from a good conductor to a good insulator. A further discussion of this theory of deionization is found in other papers presented before this and previous Institute meetings.[2]

Development of the "Deion" principles has been carried to the point of building and testing circuit-breaker structures up to 15-kv. ratings with rupturing capacities comparable to some present-day heavy-duty oil circuit-breakers in the power-house class. These breakers have been subjected to extensive laboratory and field tests, successfully interrupting three-phase grounded and ungrounded short circuits above 15,000 amperes at 12,000 volts consistently. The results of a recent series of field tests with one of these breakers is the subject of another paper presented before the Institute.[3]

## GENERAL CONSTRUCTION

The 15-kv. Deion circuit-breaker shown in Fig. 1 is made up of three single-pole units, each consisting of a deionizing chamber, an arc-drawing mechanism with main contacts for carrying load current, and a controlling mechanism. The deionizing chamber consists essentially of a stack of thin copper plates spaced a short distance apart to form a series of gaps, the general arrangement being shown in Fig. 2. In these gaps are placed insulating spacers which enclose arc runways, each having a straight entering portion and a circular portion. One of these plates with its insulating spacers in position is shown in Fig. 3. Approximately 130 volts r. m. s. has been found to be a safe working voltage for each gap. On this basis, a large number of gaps is required for a 15-kv. circuit-breaker. These gaps are divided into groups and separated by coils connected in such a way that the magnetic fields of adjacent coils are in opposition, which causes flux to be diverted radially through the gaps, as shown in Fig. 4. The terminals of these radial field coils are so connected that they are automatically inserted in the circuit by the arc itself as it moves into the deionizing chamber. This is accomplished by short plates placed directly under the radial field coils for the purpose of developing

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*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

2. *Extinction of an A-C. Arc*, J. Slepian, A. I. E. E. Quarterly Trans., Vol. 47, Oct. 1928, p. 1398.

3. "Field Tests on the Deion Circuit Breaker," B. G. Jamieson, see p. 535.

a high voltage on the section of arc subtended by the coils, as discussed later in this paper.

A blow-out magnet of conventional design, while adequate for small deionizing chambers working at low voltages, did not develop a suitable blow-in field for the larger chambers necessary for 15-kv. service. A blow-in magnet of entirely new design was, accordingly, constructed for this breaker. In this magnet, the coils are wound so as to cover the entire space in which the arc is drawn and extended. The cores on which these coils are wound are fabricated of iron and insulating material in such manner that the field in the air gap is concentrated at the necessary points for rapid movement of the arc into the deionizing chamber. The cores and coils are supported by a laminated return circuit passing over the top of the deionizing chamber. This return circuit acts also as a partial return path for the flux of the radial field coils previously referred to, and as a shield against stray magnetic fields. The general appearance of this magnet may be seen by referring to Fig. 1.

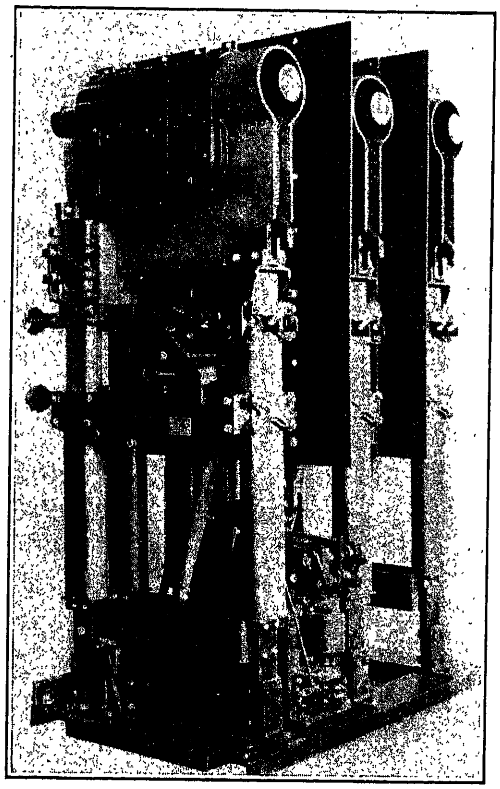

FIG. 1—THREE-POLE DEION CIRCUIT-BREAKER WITH CONTINUOUS RATING OF 2000—AMPERES—15,000 VOLTS

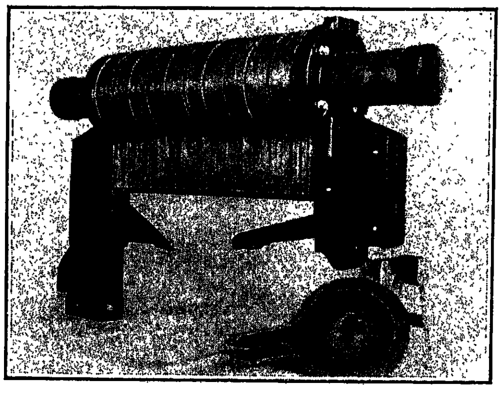

FIG. 2—DEIONIZING CHAMBER WITH BLOW-IN MAGNET AND ELECTROSTATIC SHIELD REMOVED TO SHOW GENERAL ARRANGEMENT OF THE PLATES, RADIAL FIELD COILS, AND VENTS

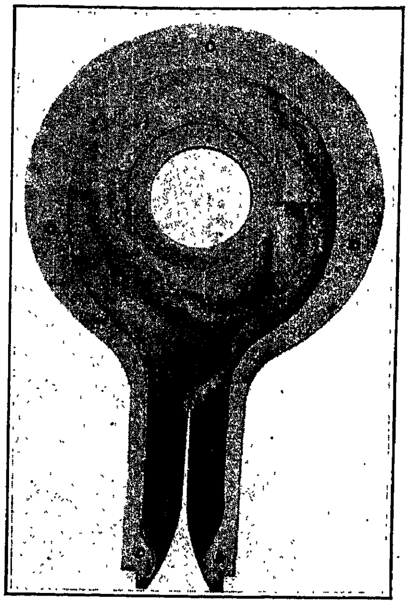

FIG. 3—PLATE FROM DEIONIZING CHAMBER WITH INSULATING SPACERS IN POSITION, SHOWING THE TRAILS LEFT BY THE ARC TERMINALS IN MOVING OVER THE RUNWAY

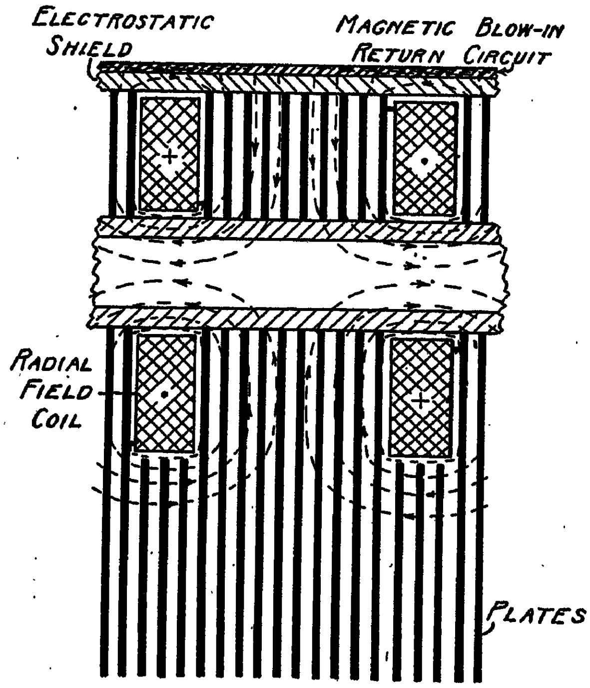


FIG. 4—SCHEMATIC RELATIONSHIP OF PLATES AND RADIAL FIELD COILS FOR THE DEIONIZING CHAMBER, SHOWING THE MANNER IN WHICH THE RADIAL FIELD IS PRODUCED

Due to the high inductance of these blow-in field coils, it is necessary to use a special form of intermediate auxiliary contact when the continuous current-carrying capacity of the breaker is such as to make it inadvisable to connect the coils permanently in series with the main circuit. These contacts are of heavy section and are operated at high pressure. In order to obtain positive,

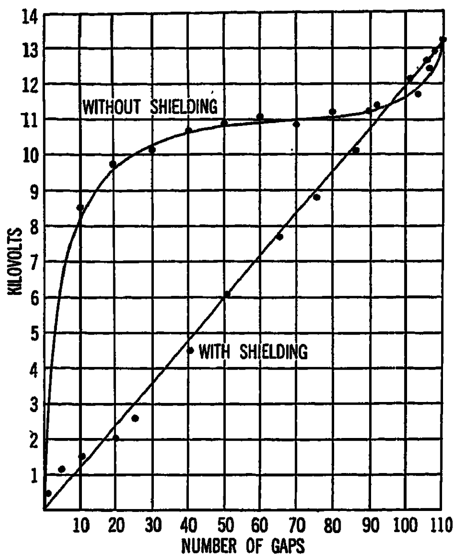


Fig. 5—Distribution of Potential Across the Gaps of the Deionizing Chamber with and without the Electrostatic Shield

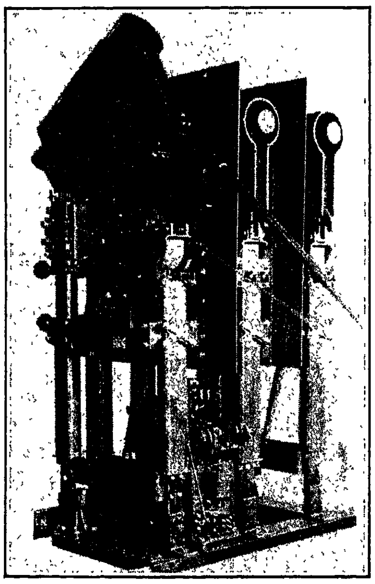

Fig. 6—Deion Circuit-Breaker with One Deionizing Chamber Raised for Inspection

quick transfer of the current to the blow-in coils, without undue flashing, this contact is equipped with an arc chute employing the Deion principle which was previously developed for use on standard industrial contactors.

A practically uniform distribution of recovery voltage over the gaps between metal plates in the deionizing chamber is an essential condition for satisfactory operation of the Deion circuit-breaker. In the longer deionizing chambers required for the higher voltages, it is necessary to use a shielding device to prevent a concentration of voltage across the end gaps, due to the electrostatic capacity of the metal plates to surrounding space. The electrostatic shield, which also serves as insulation between the deionizing chamber and the blow-in magnet, has layers of metal foil imbedded in it. These layers of metal foil are so shaped and located that each plate is forced to assume its proper electrostatic potential. Fig. 5 shows potential distribution across the deionizing chamber. The deionizing chamber, the blow-in magnet, and the elecstatic shield form a complete structural unit which is hinged at the rear support so that it may be rotated upward for inspection as shown in Fig. 6.

This unit is mounted at the top of insulating uprights which also support the main current-carrying parts and

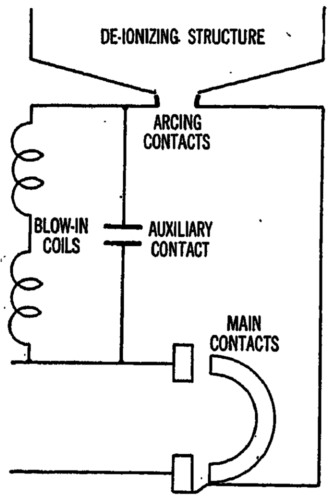


Fig. 7—Electrical Circuit of a Deion Circuit-Breaker in Which the Blow-in Field Coils are Normally in Parallel with the Main Contacts

the arc drawing mechanism. These uprights are securely anchored to a structural steel base as shown in Fig. 1. A movable parallel-path brush of conventional design, engaging stationary contact blocks, completes the main power circuit in the breaker. The moving arcing contact is carried on an individual arm, so linked to the main current-carrying contact as to secure proper sequence of opening and closing and arranged to provide a high speed of separation in drawing the arc. The arcing contacts are of familiar design, the moving element rolling on the stationary member and drawing the arc at the tips. Movement of the arcing contact operating arm also operates the intermediate transfer contacts which are isolated at the rear of structure. Operation of this transfer contact is such as to insure proper sequence of opening and closing with the main arcing contacts.

The electrical circuit of a Deion breaker is shown in Fig. 7. On opening the circuit the main contacts part

first, followed by the transfer contacts and then by the arcing contacts. As the transfer contacts part, sufficient voltage is developed across them to deflect the current to the blow-in coils. This insures the presence of a blow-in field when the arc is drawn.

The insulating uprights carrying the deionizing chamber and the arc drawing mechanism comprise a complete pole unit which may be operated by its own individual closing mechanism as a single pole breaker. Three of these pole units may be assembled on a structural steel base and operated by a single closing mechanism, for three-phase service. Barriers between pole units permit a spacing the equivalent of that in a modern oil-insulated breaker of comparable rating. This circuit-breaker is well adapted to use in isolated-phase service either with individual closing mechanisms or through remote control from a common mechanism.

## Operation

The theory of the Deion circuit-breaker deals only with an arc after it has been drawn and is in no way connected with the method of securing a tripping impulse. Inasmuch as standard closing mechanisms are used to operate this breaker, tripping may be obtained in any conventional manner applicable to modern breakers of other types. The operation of the breaker up to the time of drawing the arc is similar to that of conventional circuit interrupters drawing an arc in air.

When the arc is drawn on the arcing contacts, the action of the blow-in field moves its terminals onto stationary arc horns very quickly, permitting the movable arcing contact to continue its opening stroke independent of the motion of the arc. The speed of arcing contact separation is sufficient to insure adequate break distance of the contacts at the time of interruption. To prevent possible retardation due to movement of the arc into a closed chamber, vents are provided at each end. As shown in Fig. 8, parallel copper plates spaced quite close together are placed in the vents to confine the arc and at the same time permit passage of air from the chamber. As the arc travels up the horns past the vents, it impinges on the metal plates of the deionizing chamber. As illustrated in Fig. 3, the lower end of these plates have a tapered slot. When a number of these plates are stacked together, these slots form a groove, roughly V-shaped, into which the arc is forced. The contour of the groove is such that as the arc moves upward, its cross section is decreased and the current density increased, with a corresponding increase in arc voltage. When a sufficiently high arc voltage is reached, the arc strikes to the plates, forming a series of short arcs which move into the circular portion of the plates under the influence of the blow-in field.

The portion of the groove below each radial field coil is made up of short plates introducing several gaps across the terminals of the coil. The increase in arc voltage resulting from the presence of several short series arcs in parallel with the coil insures sufficient voltage across its terminals to deflect the current into the more inductive path of the coil winding. Under the influence of the radial fields, the short arcs trace an annular path around the circular portion of the plates. This motion continues until a zero of current is reached, at which time deionization prevents further flow of current. Under these conditions, arcs have been found to move around this path more than 15 times in ½ cycle, with a velocity several times that of sound. This high speed results in what has been termed the "Cold Cathode Arc."[4]

Fig. 3 shows a plate taken from a Deion circuit-breaker on which approximately 300 rupturing tests were made, where arc terminals of currents up to 14,000 amperes have passed over the metal surface thousands of times without deterioration.

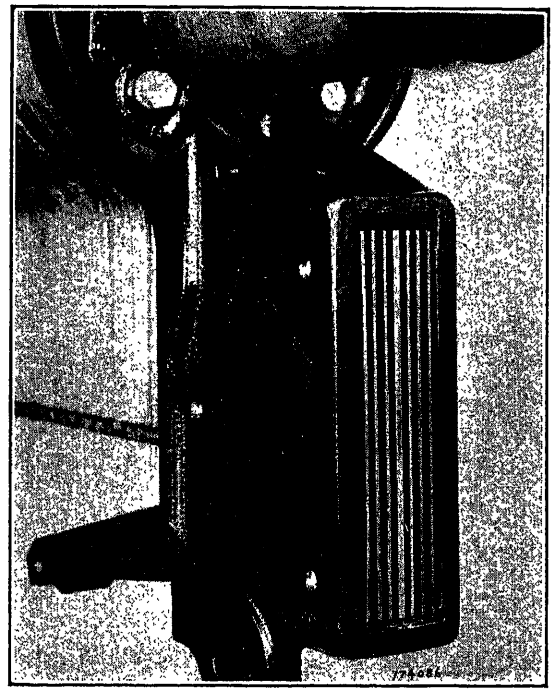

Fig. 8—Vent of Deionizing Chamber Showing Parallel Copper Plates

The mottled area extending from the end of the slot upward and around the circular portion of the plate is the retraced trail left by the arc. This mottled marking is only a very thin film of copper oxide and does not affect the operation of the breaker.

## Tests

The first experimental Deion circuit-breakers built and tested were single-pole units. A large number of tests were made at 13,200 volts, 60 cycles, with two 20,000-kv-a. generators supplying the short-circuit current. The magnitude of short-circuit current was controlled by series air-core reactors. Currents interrupted varied from 1 ampere to 17,000 amperes r. m. s.

3. J. Slepian, *Jour. Franklin Inst.*, 201, p. 79, 1926; J. Slepian, *Phys. Review*, 27, p. 407, 1926.

at 13,200 volts and to 28,000 amperes r. m. s. at 6600 volts.

Figs. 9, 10, and 11 show typical oscillograms taken during these tests. The oscillogram shown in Fig. 9 was made on one of the earlier designs, while Figs. 10 and 11 were made on a later model. It may be noted that all of these oscillograms have the same general features, the most noticeable of which are the shape of the arc voltage wave and the short period of arcing.

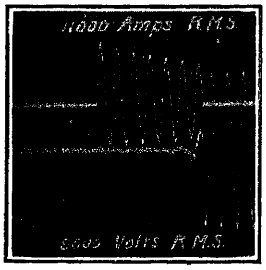

FIG. 9—OSCILLOGRAM OF SINGLE-PHASE SHORT CIRCUIT INTERRUPTED BY A SINGLE-POLE DEION CIRCUIT-BREAKER

This is the characteristic form of arc voltage produced by a Deion circuit-breaker. The voltage is at first very low, while the arc is on the contacts and horns, and it may be regarded as simply an arc in open air. After it has been broken up into a series of short arcs and moved into the deionizing chamber, the arc voltage varies slightly with the current.

At currents above a few hundred amperes, the arc

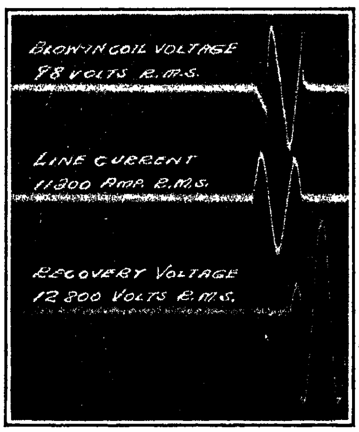


FIG. 10—OSCILLOGRAM OF SINGLE-PHASE INTERRUPTION ON A SINGLE-POLE DEION CIRCUIT-BREAKER. (NOTE THAT VOLTAGE ACROSS THE BLOW-IN COIL WAS MEASURED IN THIS CASE)

voltage has a rising characteristic with increasing current, resulting in a convex arc voltage wave as shown in the oscillograms. At very low currents a concave arc voltage wave is obtained which is characteristic of the usual inverse arc voltage-current curve. The increasing arc voltage with decreasing current does not result in voltage surges as the current approaches zero. As the current wave passes through zero and is unable to increase in the opposite direction, the voltage reverses and rises almost instantaneously to a value depending on circuit conditions.

From the foregoing, it is apparent that a given deionizing structure will have a constant arc-voltage characteristic. That is, the arc voltage will depend only upon the current and the deionizing structure and will be independent of the open-circuit voltage. For large values of current, this arc voltage is high enough to reduce the current to less than its normal value. This is especially apparent when a given deionizing structure is tested at considerably less than its normal voltage.

In no case have tests shown that the Deion circuit-breaker has any undesirable effects on the system to which it is connected. In view of the fact that the arcing time is very short as compared to most conventional circuit-breakers, it is possible for the Deion cir-

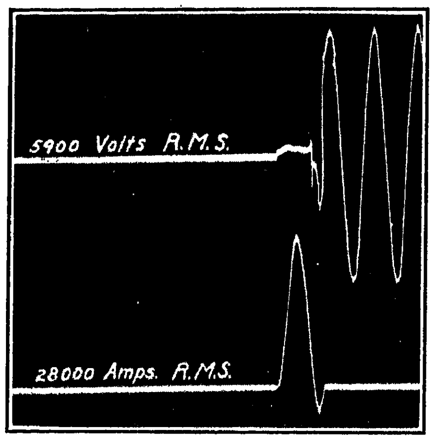


FIG. 11—OSCILLOGRAM OF TEST ON SINGLE-POLE, 15-KV. DEION CIRCUIT-BREAKER, SHOWING ITS OPERATION AT LESS THAN ½ NORMAL VOLTAGE

cuit breaker to remove short circuits from a system in a minimum of time with a minimum of system disturbance.

## A THREE-POLE 15,000-VOLT DEION CIRCUIT-BREAKER

The three-pole, 15,000-volt Deion circuit-breaker described in this paper was given several series of interrupting tests, a part of which were laboratory tests and the remainder field tests. The most comprehensive laboratory series consisted of 250 rupturing tests at currents varying from 13,100 amperes to 586 amperes at 13,200, 7600, and 6600 volts, grounded and ungrounded, with both star and delta generator connections. These tests were made at an average of 12 tests per day, the highest number on a single day being 32. The total of 250 tests was made without a failure to clear the circuit and with very little maintenance. One hundred and fifty of these tests were made with no maintenance whatever. During the remainder, only minor adjustments were made and several of the arcing contacts were renewed. At the end of these tests, the breaker was in satisfactory operating condition.

Fig. 12 shows current and voltage characteristics of an ungrounded short circuit interrupted by this Deion circuit-breaker. The generator voltage in this test was

13,200 line-to-line and the average current in the three phases was 8170 amperes r. m. s. The oscillogram shown in Fig. 13 was made on the same test by means of instantaneous watt oscillograph elements and from it the total arc energy can be obtained. Fig. 14 is a side view of the breaker made simultaneously with these oscillograms, while the breaker was interrupting the circuit. Fig. 15 is similar to Fig. 12 except that the short-circuit is grounded, the average of the three currents in this case being 9630 amperes r. m. s. The photograph made with this oscillogram is shown in Fig. 16, which is a rear view of the breaker. Light from the arc as it moves up the horns may be seen through the vent of the right-hand pole. The light from the arcs in the other two poles is not visible because the camera was not directly in line with the vent plates. It is also possible to see the arc chutes of the transfer contacts, directly below the vents.

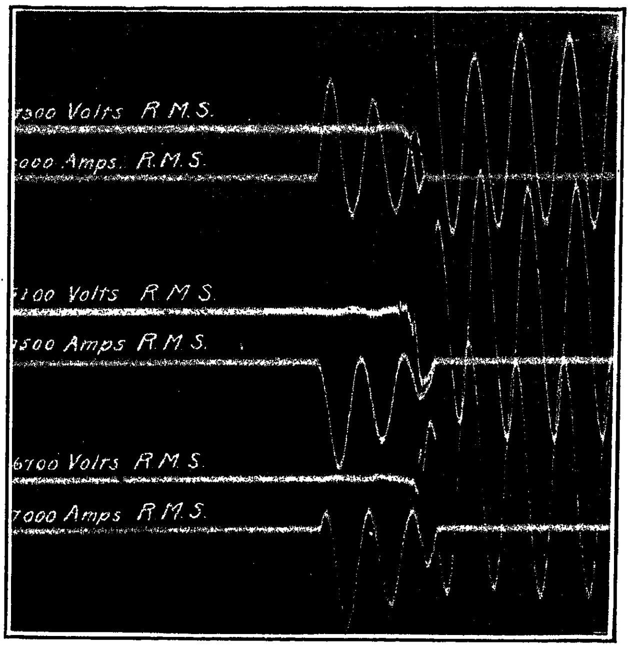


Fig. 12—Oscillogram Showing Line Current and Line to Neutral Voltage in a Three-Phase Ungrounded Short Circuit, Interrupted by a Three-Pole Deion Circuit-Breaker

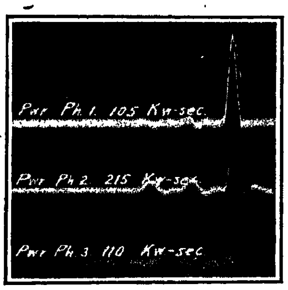


Fig. 13—Instantaneous Power Oscillogram Made Simultaneously with Oscillogram Shown in Fig. 12

It is apparent that a large part of the arc energy liberated in interrupting a short-circuit will be absorbed by the metal plates of the deionizing structure. This arc energy has been measured in a large number of tests by means of the oscillograph, as mentioned before. Fig. 17 shows the results of some of these tests, the total arc energy per three-phase interruption being plotted against the average of the r. m. s. currents interrupted in the three phases. The average arc energy per interruption per pole is small in comparison to the large thermal capacity of the deionizing structure, so that the breaker is capable of withstanding much more severe operating duty than is encountered in modern applications. To investigate this, several series of laboratory tests were made on the three-pole breaker described in this paper. These series consisted of 12-CO interruptions at two minute intervals in a total time of 24

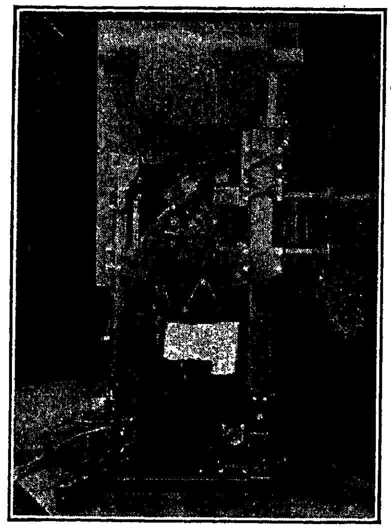

Fig. 14—View of Deion Circuit-Breaker Taken Simultaneously with Oscillogram Shown in Figs. 12 and 13

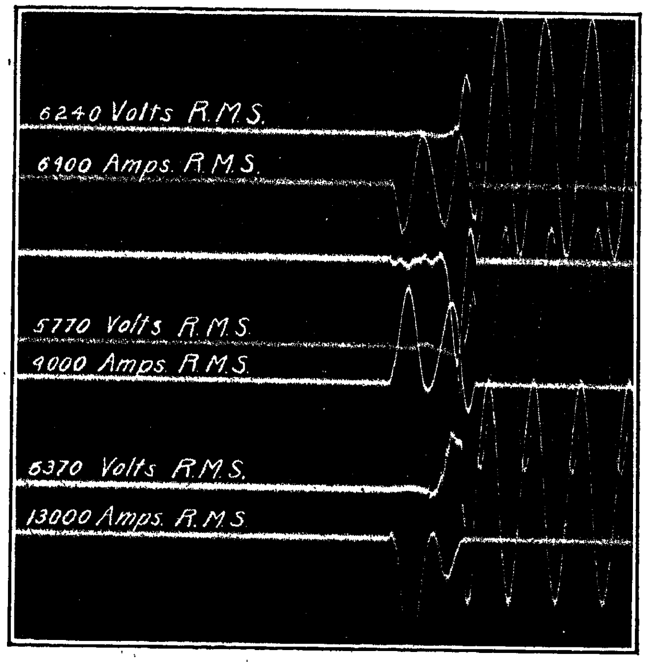


Fig. 15—Oscillogram of a Three-Phase Grounded Short Circuit Interrupted by a Three-Pole Deion Circuit-Breaker

minutes, with the average value of the r. m. s. currents interrupted exceeding 8000 amperes. Currents interrupted in individual phases varied from approximately 5000 amperes to 12,000 amperes r. m. s. At the end of

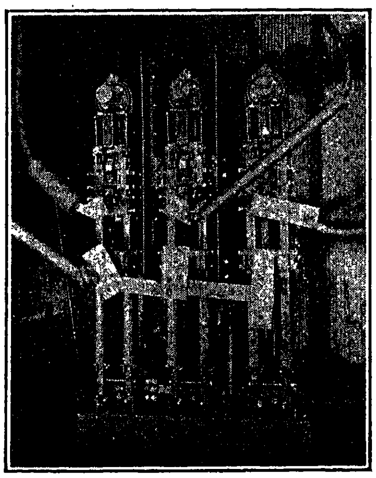

FIG. 16—VIEW TAKEN WITH OSCILLOGRAM SHOWN IN FIG. 15

each one of these series of tests, the breaker was in satisfactory condition to perform further interrupting duty. In all series of tests in which the deionizing structure was heated to its limiting temperature, it was only necessary to allow the breaker to cool before further interruptions could be made.

It is obvious that a given deionizing structure will have a thermal limitation in regard to the number of interruptions it can make in rapid succession. It is also obvious that the thermal capacity of the deionizing structure can be controlled, and that by proper design a given structure can be made to have a factor of safety over any practical operating condition.

## APPLICATION

From the manner in which the Deion circuit-breaker functions, it is apparent that zero points in the current wave play a most important part in its operation, which makes it most effective as an a-c. device. Strictly speaking, the type of deionizing chambers referred to in this paper is applicable to d-c. circuit interrupters, but there are some indications at present that the voltage at which a given structure will function on d-c. may be in the order of 0.2 of the limiting a-c. voltage, so that the advantages to be gained by the use of this form of structure on d-c. circuits are not at the present time outstanding. Due to extensive development over a long period, sufficient data and experience have been obtained to make possible the extension of the Deion principles to a large number of different classes of commercial a-c. switching apparatus. Industrial contactors operating on the Deion principles have been in service in considerable numbers for periods up to one year operating at as high as 440 volts. A limited number of three-phase Deion circuit-breakers has been operating under service conditions on 2300-volt circuits for more than one year. The performance of both the industrial contactors and the 2500-volt circuit breakers has been entirely satisfactory and represents a considerable advance over results obtained with conventional magnetic blowout devices applied to this class of service. Extensive tests have also been made on Deion circuit-breakers at 4500 and 7500 volts, the results of which warrant the belief that they can be placed in heavy duty service without encountering serious difficulties. This general development has led up to the building and testing of heavy-duty three-pole Deion circuit-breakers for operation in the 15,000-volt class. The results obtained in laboratory and field tests, as presented in this and other papers before the Institute, warrant the belief that Deion circuit-breakers as at present developed are applicable through the power-house class of breakers at modern generating voltages.

With reference to higher voltages, there appears to

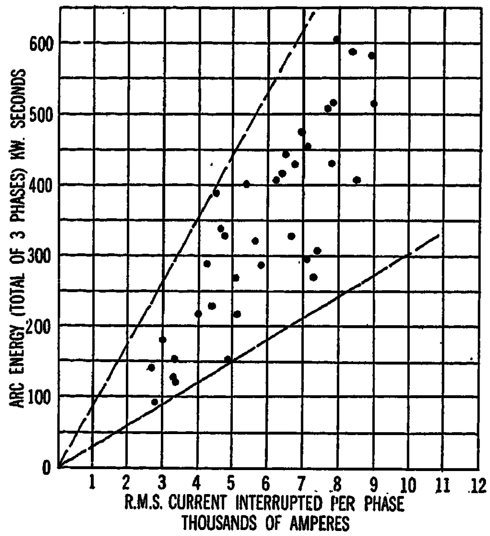


FIG. 17—CURVE SHOWING RELATIONSHIP BETWEEN TOTAL THREE-PHASE ARC ENERGY AND R. M. S. CURRENT INTERRUPTED ON A THREE-POLE 15,000-VOLT DEION CIRCUIT-BREAKER

be no great difficulty in extending the Deion principles beyond 25 kv. However, there are certain detail problems involved which are not yet worked out and the discussion of developments along this line will be left for future papers.

## ACKNOWLEDGMENT

This paper would be incomplete without acknowledgment to the Commonwealth Edison Company of Chicago for its courtesy in extending the use of its operating equipment for test purposes and to the various members of its engineering and testing organization for their efficient arrangements made for conducting the tests.

Acknowledgment is made also to Mr. H. M. Wilcox for his guidance in carrying on some phases of work on the Deion circuit-breaker, and for his very material assistance in the preparation of this paper.

## Discussion

For discussion of this paper see page 545.

# Field Tests of the Deion Circuit Breaker

BY B. G. JAMIESON[1]

Fellow, A. I. E. E.

*Synopsis.—The operating principles of a new type of air circuit breaker for alternating currents are herein outlined together with results of field test of a 2000-ampere, 15,000-volt, three-phase unit. The special testing facilities of the Commonwealth Edison Company used for the test, the performances of the air circuit breakers under test, and the effects of the test currents on the power system are described.*

* * * * *

THE power industry has been confronted by switching problems from its infancy. Development of the a-c. generator opened up vast possibilities for the deconcentration of power units in localities most favorable for generating purposes with transmission of power over long distances to consumers. With the advent of cheap power, consumption in the most favored industrial and mercantile districts increased, calling for larger and larger concentrations of generating equipment with more extended and complicated distribution systems. At each step in this advance, the question of adequate and dependable apparatus for control of the flow of energy has arisen for solution.

With the growth of the power industry, the oil circuit-breaker has assumed a commanding position as the best-adapted apparatus available for switching of power circuits in normal service and, through various combinations of control relays, for clearing these circuits under fault or abnormal operating conditions. This has come about if not with full agreement of the operating companies, at least with their acquiescence, since there was admittedly no known interrupting medium better adapted to the service.

Acquiescence on the part of the operators, however, was not entirely without misgivings and the hazard involved in the use of large quantities of oil in connection with the interruption of heavy short circuits has been ever present in the operators' minds and has been emphasized by their representative associations. Co-operation between operating companies and manufacturers has resulted in the development of types of construction which make this hazard somewhat more remote in the modern oil circuit-breaker, and as a further safeguard, some circuit-breakers have been installed out of doors although their logical location is inside the power station with the exception of breakers operating at the higher transmission voltages which admittedly must be located out of doors. This latter move has, however, increased the work and cost of maintenance, due to difficulties encountered from the presence of moisture in oil together with other troubles arising from varying weather conditions, so that the possibility of service interruption is still present in some measure.

In view of these facts, the attitude of the power industry has been one of open-mindedness toward the advent of any means of circuit interruption applicable to heavy-current circuits in the generating and distribution voltage classes, which should not involve the use of oil. If this interrupting means be applicable in the transmission voltage class as well, so much the better, but air insulation for electrical clearances runs into considerable distance for the upper range of transmission voltages and the conventional oil breaker in this class may carry advantages in the form of compactness, due to its oil insulation, that offset other disadvantages. By far the greater proportion of present-day switching is carried out on the low side of the transformers, and any practical interrupting medium which will remove the oil hazard from this class of switching service has a permanent field of usefulness.

It was with considerable interest, therefore, that the author of this paper was informed by a responsible manufacturer of switching apparatus that a new type of circuit-breaker, operating without oil, has been developed to the point of interrupting consistently the maximum short-circuit current available from a 40,000 kv-a. test circuit operating at 13,200 volts. Further discussion disclosed that the interrupting performance of this device, called the Deion circuit-breaker, is based on a means for deionizing the arc stream at the zero point of the current wave to such extent that the impressed voltage is not sufficient to re-establish the circuit and permit current to pass on the succeeding alternation. By this means, an arc is extinguished in air without resorting to the expedient of extending it to great length as is characteristic of the present-day conception of the air-break circuit-breaker. A discussion of the theory of this device and its operation is contained in other papers[2] to be presented before this meeting of the Institute and need not be presented in further detail here.

Upon the statement of the manufacturers that the development of this circuit-breaker had reached a point where they were desirous of obtaining data as to its

1. Vice-President, Engineer of Inside Plant, Commonwealth Edison Company, Chicago, Ill.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

2. *Theory of the Deion Circuit-Breaker*, by Dr. J. Slepian.

*Structural Development of the Deion Circuit-Breaker up to 15,000 Volts*, by R. C. Dickinson and B. P. Baker.

performance under interrupting duty on an operating system, arrangements were made for a series of tests at the Crawford Avenue Station of the Commonwealth Edison Company, and it is the purpose of this paper to discuss these tests together with the results obtained and the impressions formed from performance of the breaker on test.

### The Deion Circuit-Breaker

The breaker supplied by the manufacturer for these tests was a three-pole, 2000-ampere, 15,000-volt,

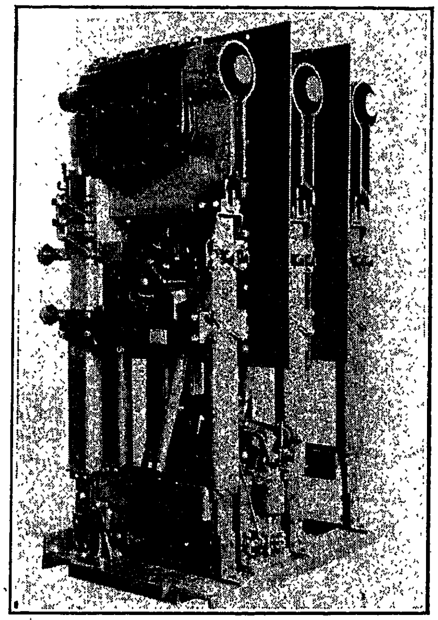

Fig. 1—Three-Pole, 2000-Ampere, 15-Kv., Deion Circuit-Breaker

Used at Crawford Avenue test circuit for the group-phase tests

electrically-operated Deion circuit-breaker of the multiple-single-pole form of construction as shown in Fig. 1. The three-pole units were mounted upright on a common structural steel base at 16-inch centers, and were operated through a single shaft by means of a conventional solenoid mechanism. The base or common mounting frame is at ground potential but the contact operating linkage for each pole unit is alive at line potential and is mounted on upright insulating posts supported by the grounded base. Operation of this linkage is obtained through insulating pull rods. The deionizing chamber which is the real interrupting medium, is mounted at the top of the structure in such relationship to the contacts that an arc drawn from the arcing members is moved into the chamber by a magnetic blow-in field of new design especially developed for this device. This breaker being of 2000-ampere normal current-carrying capacity, the blow-in coils are of necessity in shunt relationship to the main power circuit and are switched into series relationship with this circuit by a pair of auxiliary transfer contacts operating in proper sequence with the main current-carrying and the arcing contacts.

The manufacturer stated that this breaker was the first model of a commercial form of the Deion circuit-breaker which had gradually been evolved from development work covering an extensive laboratory testing experience with a number of experimental forms of construction. It was pointed out, however, that as a broader experience was obtained with the operation of this breaker under service conditions, and its application was extended to installations of varying requirements, conditions would undoubtedly be encountered pointing to the desirability of modification in some details. The general form of construction he believes well adapted to the application of this principle in service.

At the end of the initial series of tests with the breaker in the form supplied by the manufacturer, the

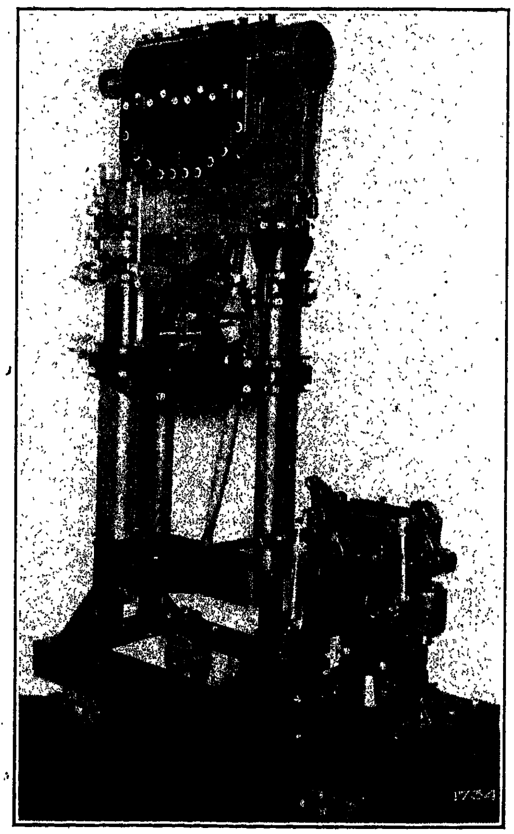

Fig. 2—One of the Deion Circuit-Breaker Pole Units Used for the Isolated-Phase Tests

author of this paper suggested that a further series of tests be made on the basis of an isolated phase application. The three-pole units were accordingly mounted on individual structural steel bases, each with its individual-solenoid closing mechanism, and connected to the testing circuit as individual units with no mechanical tie between poles other than the short-circuiting bar which was connected across the terminals of the three units. Fig. 2 shows the general arrangement of one of

the pole units used for this test. It was at first proposed to use the identical units for this test which had been tested on the previous series but the time allotted for completion was short and it developed that the necessary alterations could be completed more quickly by diverting three additional pole units then nearing completion in the factory and this course was finally adopted.

## Test Equipment and Methods

A testing circuit with permanently installed recording equipment had been set up at Crawford Avenue Station over two years ago in connection with the general testing program of the Commonwealth Edison Company. Considerable pains were taken to make this installation conform in all respects to the rules for standard test procedure adopted by the Institute. The purpose of this installation was not solely to conduct interrupting tests on circuit-breakers but to make a study as well of the effects of a fault and its interruption on the remainder of the system linked to the fault. It has been apparent for some time that inability of large operating systems to withstand shocks is a serious limitation to their usefulness as sources of power for commercial and industrial purposes. The question assumes increasing importance as the trend continues toward larger and larger concentrations of generating equipment on the one hand and the linking of heavy systems through interconnection on the other. A study of these limitations is essential to the successful operation of large power systems and this phase of the work was, accordingly, given a prominent place in the program of tests. Such a study does include, however, consideration of various means of removing a fault and the time required for the clearing agent to perform this function.

The test circuit proper is connected to the 12,000-volt distribution system through suitable switches in Crawford Avenue Station. It consists essentially of approximately 500 ft. of 22-kv. underground cable and roughly the same length of three-phase overhead line mounted on wooden poles and insulated for 66-kv. Sheet metal houses of portable construction are located at intervals directly under the overhead line with tapped connections bringing the three phases into a rack in each house. These houses are used as testing cells and as enclosures for the permanent circuit equipment, some of which is indoor apparatus. Hand-operated disconnects are located in the overhead line at the pothead connection to the cables, permitting complete isolation of all test apparatus without recourse to switches in the station. Two backing-up oil circuit-breakers are permanently connected on the test circuit side of the disconnects. One of these breakers is used for closing purposes in the event of tests being made on the "CO" basis and its controls are arranged to retain the contacts in the closed position until tripped manually in order that its opening operation may not cloud the results to be obtained from the test breaker. The second breaker is arranged to open through relay control at a predetermined time interval after the short circuit has been applied, to clear the circuit in event of failure of the test breaker. Fig. 3 shows a schematic diagram of the test circuit connections and its relation to the distribution system.

A larger house of the same general construction as the test cells is used as a control room with all circuit controls and recording instruments permanently installed. A 125-volt battery is installed for solenoid operation, and 110- and 220-volt a-c. circuits are also available for control purposes when desired. The control room is also used as an observation post when tests are in progress. Telephone connections to the load dispatcher's desk and to other stations are available for the coordination of test operations with general operation of the system.

The main control table has been especially developed

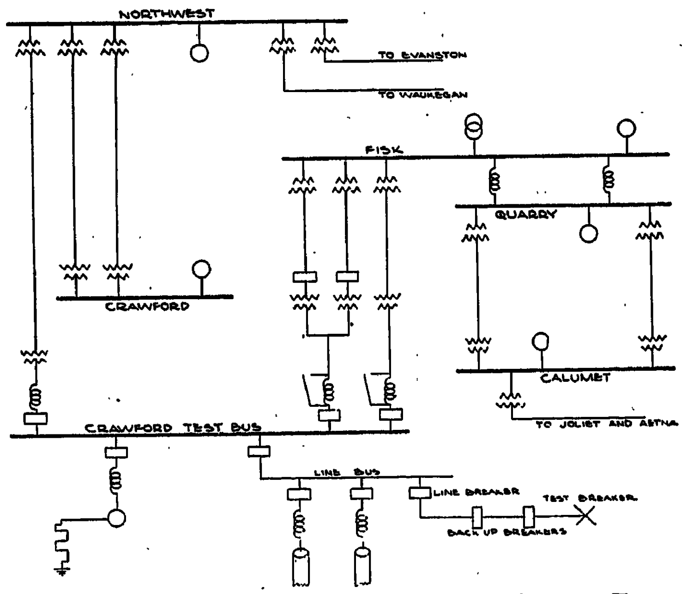


Fig. 3—Elementary Diagram of Crawford Avenue Test Circuit and System Connections

for use with this circuit. All test operations are controlled through a motor-driven controller, consisting essentially of a number of adjustable cams rotating on a common shaft and so arranged as to close or open the desired control circuits in proper sequence and at predetermined time intervals. Two years experience with this controller indicates that it is far preferable to control through relay operation since all operations are related to a common source, the revolving shaft, and variables are thus reduced to a minimum. Strictly speaking, test operations conducted with this device are electrically non-automatic but it is felt that the ability to increase or decrease the duration of short circuit accurately in very short steps is a valuable feature in conducting tests of this nature.

Two six-element Westinghouse oscillographs are mounted on the control table and permanently con-

nected to the motor-driven controller. The use of permanent connections throughout permits standardization of calibration for oscillograph circuits and test operators soon become sufficiently familiar with the circuit to make proper adjustment of resistances for any given oscillograph element and for any system set-up to obtain satisfactory films without preliminary trial tests. Adequate test records are thus obtainable with a minimum of test operations. Dark-room facilities are provided in the control room, permitting the development of films as each test is made in cases where it is desirable to study the results of one test before proceeding with the next.

As a part of the study of the system under fault conditions, Hall recorders are installed at points in the system where experience indicates the greatest voltage disturbance will occur. In this manner, valuable information is obtained as to the transient effects at different points due to various system set-ups under fault conditions. The operation of this device and the results obtained through its use have been discussed in a paper[3] previously presented before the Institute.

In the two years' experience with this test circuit, covering several hundred interrupting tests, the question of safety has been given first consideration. A full testing crew has been trained to the point of complete familiarity with the test circuit and their individual duties in connection with its operation. A test chief, at least two oscillograph men, and six men stationed at telephones are on duty at every test as well as several mechanics for the performance of necessary alterations, renewing of oil, etc., between tests and for patrol of the grounds while actual tests are in progress. Representatives of the Engineering Department and other interested parties are always present as observers. Arrangement for tests are always made with the load dispatcher one or more days in advance, and the man responsible for the system set-up is in direct telephone communication with the load dispatcher, the operating gallery of Crawford Avenue Station, and with the operators of any neighboring stations at which disturbance is anticipated, during the progress of all tests. To obtain the benefit of high generating capacity on the system, all tests are made during regular working hours of the business day, rather than at night or on Sundays.

In order to secure protection for the generators, the usual practise is followed of so arranging the system set-up that no power bus is left without the protection of a reactor. In addition to the backing-up breaker in the test circuit, two more protective breakers are involved in the test bus at Crawford Avenue Station as protection for the generators, although control relays are so set that these circuit-breakers are called upon for service only in event of failure for any reason of the first backing-up breakers to function, or in the event of a fault between this breaker and the test bus.

3. *The Hall High-Speed Recorder*, E. M. Tingley, A. I. E. E. Quarterly Trans., January 1928, p. 252.

All of this testing experience was brought to bear on the two series of tests made with the Deion circuit-breaker. As shown in Fig. 3, one 66-kv. cable line to Northwest Station and three 22-kv. cables to Fisk Street Station were connected to the 12,000-volt test bus at Crawford Avenue Station, and thus to the test circuit for these tests. Various transformers and reactors were shorted out of these circuits as the tests proceeded, in order to secure the desired steps in the range of short circuit current values. In keeping with the usual test practise on this circuit, no temporary reactors were used in the test circuit nor were any temporary cross connections made in station circuits except in the case of one test for which it was desired to secure a very low value of short circuit current (approximately 1100 amperes) and which it was not feasible to obtain using the available system connections.

With the reactance of these circuits reduced to the minimum value consistent with the safety provisions outlined in a previous paragraph, the maximum short circuit current at the test breaker was approximately 10,000 amperes, average current interrupted for the three phases. Under these conditions, and for all short circuit currents up to this value, the currents in the three phases were approximately equal, the current decrement is comparatively small, the degree of assymmetry is not very pronounced, and the delta recovery voltage is approximately equal to the initial voltage.

When tests with the Deion circuit-breaker had reached this point of maximum current obtainable with the portion of the system connected, the breaker's performance in interrupting the circuit with a very short duration of arcing had been sufficiently consistent to warrant the belief that it would function satisfactorily over the next succeeding steps in the current range and a generator was, accordingly, connected direct to the test bus in Crawford Avenue Station in order to increase the current value at the point of short circuit. This means of securing additional power had never been attempted for in previous circuit-breaker tests, due to the fear of severe system disturbance following a prolonged duration of fault with the current values involved. The results obtained were, therefore, regarded with considerable interest aside from performance of the test breaker.

Three different generating units, varying in capacity from 60,000 to 75,000 kv-a., were used in this manner during the series of tests with the Deion circuit-breaker. These generators are driven by steam turbines and it was considered advisable to have them carrying some load rather than to run light when the short circuit was applied, in order to minimize the possibility of tripping out the steam end. They were, accordingly, operated to feed power into the system during these tests in amounts varying from 10,000 to 38,000 kw. The circuits were so arranged that closing in on the short in the test circuit had the effect of short circuiting

the generator, and clearing the test circuit automatically returned the generator to the system again. Permanent generator reactors to the value of one-eighth of an ohm were connected between the generator and the test bus. The generator neutral was grounded through approximately four ohms resistance with no neutral ground other than this on the various buses involved in the circuit. Tests made with the system alone were entirely ungrounded. No extensive cable system was directly connected to the test bus except that two idle three-phase cables were allowed to remain connected at times to note the effect on arc rupture.

The addition of steam driven generating capacity proved very satisfactory on the Deion circuit-breaker tests, and added quite materially to the current values obtainable on the test circuit. For tests made on the "OCO" basis, the load on the generator was varied from 10,000 to 28,000 kv. With this maximum load, the first-cycle short-circuit current was approximately 19,000 amperes r. m. s., and the current interrupted approximately 14,000 amperes, average for the three phases. For "CO" tests, the generator load was increased to a maximum of 38,000 kw. for tests where the motor-driven controller was set to give a duration of short circuit of from two to three cycles. This maximum load gave first-cycle short-circuit currents of approximately 30,000 amperes r. m. s., and the current interrupted was approximately 22,000 amperes, average for the three phases.

A total of 38 tests was made with a generator feeding varying amounts of load into the system and in all cases, the generator returned automatically to feeding the system as soon as the short was cleared, without system disturbance other than four volt dips in lamp voltage in a few cases and without injury to the generator. The maximum duration of short circuit was 16 cycles, varying from this time down to four cycles for maximum currents, except for two tests in which difficulty was encountered in operation of the test breaker and in which the circuit was cleared by the backing-up breaker after a period of 49 cycles (as determined by the motor-driven controller). On none of these tests did the generator drop out of step nor was any effect noted on low voltage releases on the system and but very little effect on lamps in the station. No so-called transients appeared during the course of these tests.

Connection of the generator to the test bus produced a noticeable effect on short circuit characteristics as compared with tests which involved use of the system alone, and this effect became more marked as the loading of the generator was increased. Considerable variation appears between the values of short circuit current for the individual phases, the degree of assymmetry is more pronounced, the current decrement becomes larger, and at the maximum currents tested, several cycles were required for the delta recovery voltage to approximate the initial voltage.

In general, the test procedure as outlined here has proved very satisfactory. It is believed that the short circuit current value of 30,000 amperes r. m. s. on the first cycle, obtained on the Deion circuit-breaker tests, represents about the maximum to which the test circuit as at present equipped should be subjected (if safety of operation is to be maintained). Future developments in connection with the testing program of the Commonwealth Edison Company contemplate further tieing the 66-kv. transmission system to the 12,000-volt bus through three 20,000 kv-a., 66,000/12,000-volt transformers. It is estimated that when backed by the entire system, short circuit currents of 50,000 amperes may be obtained at 12,000 volts with almost instant 100 per cent recovery voltage. With suitable modifications in the test circuit, interrupting tests at these values are contemplated without fear of the results.

## Description of Group-Phase Tests

The first series of Deion circuit-breaker tests was made with the structure shown in Fig. 1. The three-pole units of this breaker were mounted on a common frame and operated by a single-solenoid mechanism through a common shaft. For purposes of simplicity and identity, tests made with this form of the Deion breaker will be referred to as "group-phase" tests in the remainder of this paper. For the same reasons, tests made with the single pole units each with an individual closing mechanism, as shown in Fig. 2, will be referred to as "isolated-phase" tests since these tests were intended to simulate isolated phase conditions.

The test breaker for the series of group-phase tests had been subjected to previous interrupting duty under laboratory conditions, in connection with the manufacturer's development work. According to his statement, this duty covered the range of current values from 500 to 9000 amperes with voltages from 6600 to 13,200 at 60 cycles, together with some 25-cycle tests. He states further that the deionizing chambers on this breaker had never been dis-assembled since the start of these laboratory tests, and that no other maintenance had been given the breaker during this time, except that it had been equipped with a new set of arcing contacts and auxiliary transfer contacts just previous to its delivery at Crawford Avenue, the purpose being to note the deterioration of these contacts on interrupting duty under system conditions.

In laying down a program for these tests, the question of duty cycle arose. It was desired that these tests should conform in all respects to the rules for standard test procedure adopted by the Institute and these rules stipulate that the test duty shall consist of two "OCO" operations with a two-minute interval. This rule, quite obviously, was adopted with the characteristics of the oil circuit-breaker in mind. It was believed that this duty cycle should be regarded as a minimum requirement but that the characteristics of the Deion circuit-breaker would permit more latitude in the

selection of a duty cycle than was practicable with the oil breaker, and it was felt that data should be obtained from these tests to determine the effect of a more severe duty cycle. The manufacturer was of an open mind on the subject and it was finally decided to retain the time interval of two minutes between operating cycles but to increase the number of operations to three. This number was selected rather as a matter of convenience than with any reference to breaker characteristics. The oscillograph equipment in use with the test circuit allows three exposures at short intervals after which a sufficient interval of time must elapse for insertion of a new film. The 3-OCO duty cycle, therefore, became a convenient one and was used throughout these tests. After some tests had been made, the time interval was reduced to one minute to observe the effect on the performance of the breaker, and as there was no apparent difference in performance, the remainder of the tests were made on the basis of three "OCO" operations at one-minute intervals.

In accordance with the usual test procedure, a 25-kv. potential test to ground was applied across the three phases of the complete testing circuit, including the test breaker, for one minute. This test was satisfactory and the interrupting tests proceeded as soon afterward as the system set-up could be made.

The current value for the first interrupting test was approximately 1100 amperes and required the special system connection previously referred to. This current value was the lowest of the series and the test is of interest as showing the interrupting performance of the Deion breaker at currents comparable to those involved in normal switching operations. Of the nine phase interruptions on this test, seven phases were interrupted with two half-cycles of arcing while two phases showed arcing in parts of three half-cycles. Analysis of these results indicates that the arc is not necessarily extinguished at the first zero occurring after it originates. Apparently an appreciable time required for the blow-in field to move the arc into the deionizing chamber and in the event of a zero occurring during this transmission period, the arc is not extinguished but persists until it has entered the chamber. The results of subsequent tests indicate that arcing may appear in parts of more than one half-cycle on any interruption, with the possible exception of very heavy currents having relatively large inherent blow-in effect.

The six succeeding tests were all made within a period of slightly less than two hours and used the system only as a source of power. Short-circuit current values ranged from 2000 to 10,000 amperes in the six steps. For the 18 three-phase interruptions in this group, in no case did any phase show arcing for more than two half-cycles and in many cases, the arcing was confined to a single half-cycle.

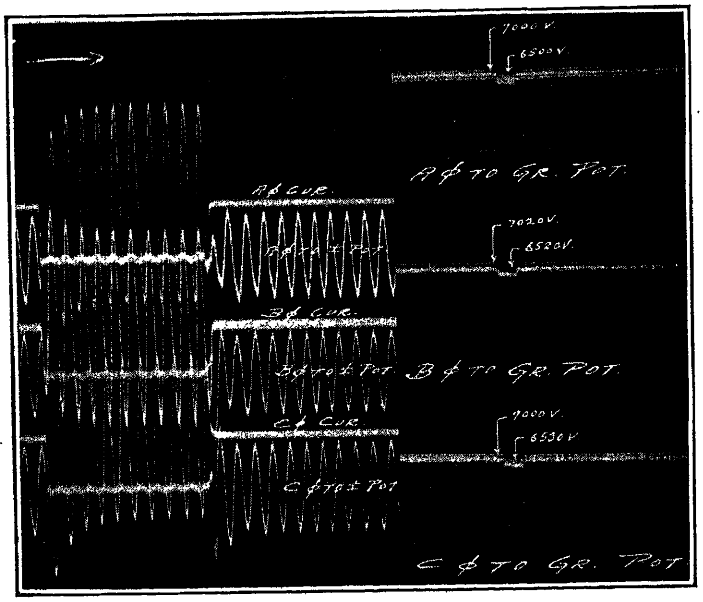


Fig. 4—Oscillogram of Short-Circuit Test at 4200 Amperes

Together with Hall record, taken during group-phase tests
(For test data, see Table I, Test No. 11-7)

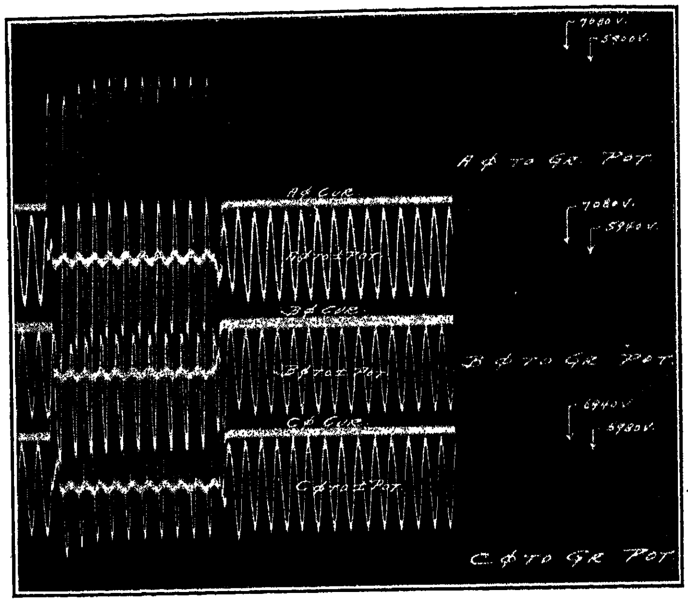


Fig. 5—Oscillogram of Short-Circuit Test at 10,000 Amperes

Together with Hall record, taken during group-phase tests
(For test data, see Table I, Test No. 11-19)

Fig. 4 shows an oscillogram of a three-phase interruption on one of these tests in which the current interrupted was approximately 4200 amperes. Together with this oscillogram is shown the record obtained during this test from the Hall recorder at Fisk Street Station, five miles away, which was the nearest generating station supplying the energy. In Table I, Test No. 11-7, are given the data for this test which is representative of the breaker performance in this portion of the current range. Fig. 5 shows another

TABLE I
TABULATED DATA TAKEN FROM OSCILLOGRAMS SHOWN IN FIGS. 4, 5, 6, AND 7

| Test No. | R. m. s. current | | | | | | R. m. s. volts | | Arcing period | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Initial | | | Interrupted | | | Before test | Re-established | (Cycles) | | |
| | A | B | C | A | B | C | | | A | B | C |
| 11- 7 | 5,820 | 5,100 | 7,300 | 4,440 | 4,120 | 4,230 | 11,600 | 10,800 | 0.500 | 0.500 | 0.445 |
| 11-19 | 13,000 | 11,000 | 11,700 | 10,150 | 10,000 | 9,740 | 12,000 | 11,600 | 0.500 | 0.334 | 0.445 |
| 11-27 | 15,700 | 19,700 | 20,300 | 11,500 | 12,400 | 11,700 | 12,400 | 10,200 | 0.445 | 0.445 | 0.400 |
| 11-31 | 21,700 | 25,100 | 29,600 | 15,400 | 14,100 | 16,200 | 12,700 | 11,300 | 0.450 | 0.330 | 0.330 |

representative oscillogram with the corresponding record from the Hall Recorder, made during one of these tests at 10,000 amperes. Data for this test are given in Table I, Test No. 11-19. This was the highest current value obtained, using the system as a sole source of power.

The performance of the test breaker having been consistent throughout the tests up to this point, it was decided to connect No. 2 Generator at Crawford Avenue to the test bus in order to increase the value of short circuit current. Four tests were made under this condition with the generator load varying from 10,000 to 28,000 kw. and the short circuit current value from 10,000 to 15,000 amperes. One test was made approximately one-half hour after the series of six tests just described and the remaining three the next morning in a period of slightly over one hour.

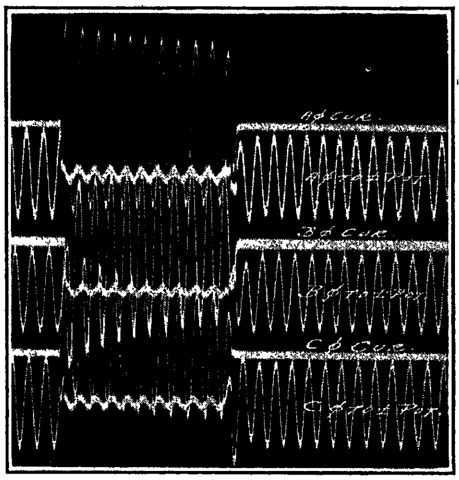

FIG. 6—OSCILLOGRAM OF GROUP-PHASE SHORT-CIRCUIT TEST AT 12,000 AMPERES

(For test data, see Table I, Test No. 11-27)

Of the twelve three-phase interruptions included in these four tests, eleven were satisfactory as to performance of the test breaker. Fig. 6 shows a representative oscillogram of a test at approximately 12,000 amperes, and Fig. 7, one at 15,000 amperes. No records were made with the Hall recorder during this series. The data for these two tests are given in Table I, Tests No. 11-27 and 11-31.

On the twelfth and last three-phase interruption, heavy flashing was observed on the breaker which persisted until the circuit was cleared by the backing-up breaker. The reason for this flashing was not entirely clear but upon examination of the breaker, it was found that two pieces of arc-resisting material had become loosened and dropped out of the breaker at some time during the test. It was concluded that this material had fallen between breaker parts of opposite potential, thus establishing an arc outside the control of the magnetic blow-in field and this arc had persisted until the circuit was cleared by other apparatus. Developments on subsequent tests, to be discussed later in this paper, throw additional light on the performance of the breaker during this test.

As nearly as could be determined, flashing on this test had started in the contact operating levers of Phase $C$ pole unit and moved to a position across the terminal studs of that phase. A short-circuiting bar had been placed across the lower studs of the three pole units for the group-phase tests and through this bar, an arc finally became established across the terminal studs of each pole unit. The breaker was still in good condition, aside from some scorching of insulating surfaces, and a light deposit of carbon.

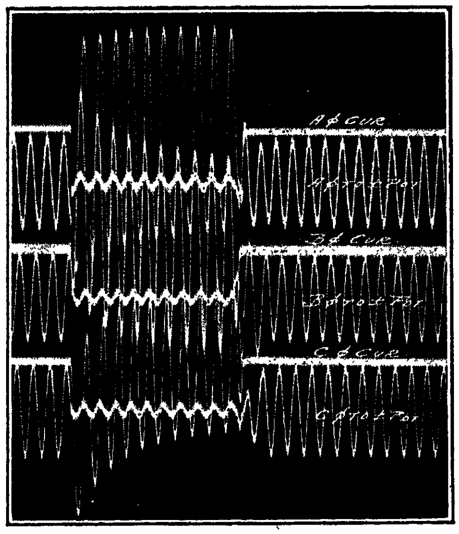

FIG. 7—OSCILLOGRAM OF GROUP-PHASE SHORT-CIRCUIT TEST AT 15,000 AMPERES

(For test data, see Table I, Test No. 11-31)

A review of the test results obtained up to this point showed that eleven interrupting tests had been made

with the breaker at short circuit currents of from 1100 to 15,000 amperes r. m. s., average for the three phases. Of the 33 three-phase interruptions involved, breaker performance on 32 had been satisfactory and consistent while one was unsatisfactory and this at a current value slightly lower than had previously been interrupted by the breaker. This demonstration of breaker performance was believed to be sufficiently satisfactory to warrant further testing work and the author of this paper suggested a series of tests, using the three pole units operating independently without a mechanical tie, which should simulate isolated-phase conditions.

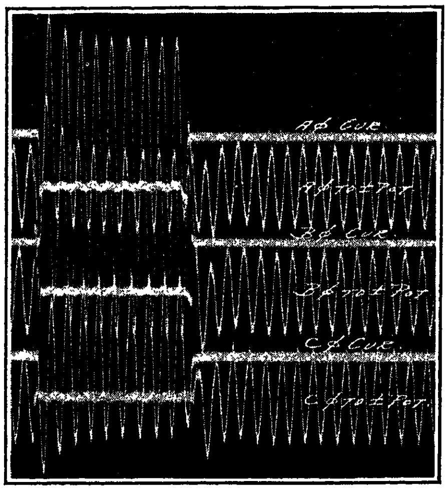

FIG. 8—OSCILLOGRAM OF SHORT-CIRCUIT TEST AT 4000 AMPERES TAKEN DURING ISOLATED-PHASE TESTS

(For test data, see Table II, Test No. 14-5)

## DESCRIPTION OF ISOLATED-PHASE TESTS

Three single-pole Deion circuit-breaker units, as shown in Fig. 2 and previously described in this paper, were supplied by the manufacturer for the series of isolated-phase tests. These units were new, having had no previous interrupting duty, and were assembled in this form especially for these tests so that a period of about 30 days elapsed between the group-phase and the isolated-phase tests.

The three-pole units were mounted in the test cell on approximately 30-inch centers which was about the maximum spacing the cell would allow leaving the desired clearances and working space. It was believed that this spacing would permit interruption of short circuits without appreciable magnetic influence due to inter-phase action. Barriers of ½-inch thick asbestos were erected between the poles in such manner as to preclude the possibility of flashing between phases. The short-circuiting connection was placed across the lower terminal studs of the three units as in the group-phase tests, but there was no other mechanical tie between the units.

The first five tests were made, using the system only as a source of power and with short circuit currents of from 2500 to 10,000 amperes. All of these tests were made on the basis of a 3-OCO duty cycle at one-minute intervals as in the group-phase tests. Of the 15 three-phase interruptions involved, in no case did arcing appear on any phase for more than two half-cycles and in a great many cases, it appeared in one half-cycle only. Fig. 8 shows a representative oscillogram of the breaker interrupting approximately 4000 amperes and Fig. 9, a test at approximately 7500 amperes. The data for these tests are given in Table II, Test No. 14-5 and 14-11.

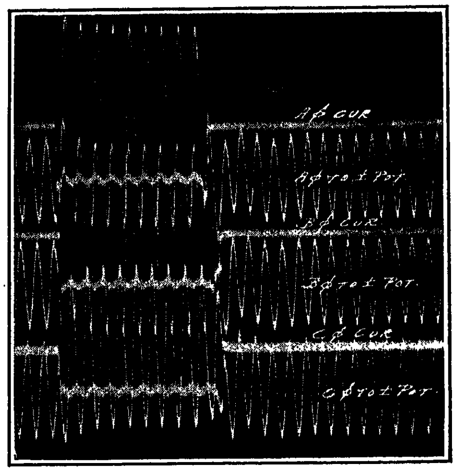

FIG. 9—OSCILLOGRAM OF SHORT-CIRCUIT TEST AT 7500 AMPERES TAKEN DURING ISOLATED-PHASE RESTS

(For test data, see Table II, Test No. 14-11)

One single-phase short circuit test was made with the maximum current available on this set-up. For this test, the pull rod was disconnected on Phase $A$ with the contacts in the open position, and the remaining two poles operated as usual on the 3-OCO duty cycle. No unusual features were observed in connection with this test. It may be noted that an effect on the test breaker similar to that of a single-phase short circuit

TABLE II

TABULATED DATA TAKEN FROM OSCILLOGRAMS SHOWN IN FIGS. 8, 9, 10, AND 11

| Test No. | R. m. s. current | | | | | | R. m. s. volts | | Arcing period | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Initial | | | Interrupted | | | Before test | Re-established | (Cycles) | | |
| | *A* | *B* | *C* | *A* | *B* | *C* | | | *A* | *B* | *C* |
| 14-5 | 5,120 | 6,520 | 6,240 | 4,030 | 4,030 | 4,350 | 11,500 | 11,000 | 0.445 | 0.445 | None |
| 14-11 | 8,740 | 9,340 | 8,620 | 7,760 | 7,300 | 7,780 | 11,700 | 12,200 | 0.334 | 0.665 | 0.334 |
| 14-31 | 22,000 | 14,400 | 20,600 | 11,600 | 11,000 | 11,400 | 12,800 | 11,300 | 0.443 | 0.521 | 0.626 |
| 14-43 | 22,000 | 25,400 | 25,600 | 14,200 | 15,900 | 15,600 | 12,200 | 11,800 | 0.500 | 0.500 | 0.700 |

was observed in several cases of the preceding three phase interruptions, in which one phase cleared slightly before the other two and one of the remaining pole units apparently did all the work of interruption since little or no arcing appeared in the third phase. In all of these cases, arcing in the second phase was confined to two half-cycles or less.

Performance of the test breaker having been satisfactory up to the point of maximum current available with the system set-up, No. 1 generator at Crawford Avenue was connected to the test bus and a 3-OCO test was made at a current value of approximately 13,000 amperes. The test breaker interrupted the circuit each time but some flashing was observed on Phase *A* at the third interruption. Inspection of the test breaker indicated that a small arc had been established across the terminals of the blow-in coil, which in this case were quite close together. The oscillogram showed that Phase *B* had functioned normally in interrupting its circuit but that Phase *A* had started to open about three cycles before the other two and that the arc had persisted in this phase until Phase *C* opened and cleared the circuit.

It was concluded that breakdown across the coil terminals had deprived Phase *A* of its blow-in field, resulting in a very slow movement of the arc toward the deionizing chamber and this arc apparently had not completely entered the chamber when Phase *C* cleared the circuit. It was believed advisable to inspect the deionizing chamber on Phase *A* before proceeding with further tests and it was completely dismantled. No signs of injury were found other than slight burning at the entrance of the chamber, evidently caused by slow movement of the arc. After a thorough cleaning the chamber was re-assembled and mounted on the pole unit.

The next test consisted of three "OCO" operations at 3500 amperes, using the system only, in order to check the performance of the breaker before proceeding with heavier currents. A single "OCO" test was then made, using the generator, with a short circuit current of approximately 14,000 amperes. The test breaker interrupted this circuit with arcing in parts of two half cycles on Phase *A* and with a single half cycle of arcing on the two remaining phases. The oscillogram showed, however, that Phase *A* was cleared two cycles before Phases *B* and *C*. The reason for this difference in operation was not entirely clear and the test was repeated with approximately the same result. A cycle-counter test on the three-pole units with no load showed that Phase *A* was opening consistently from 2 to 3 cycles ahead of the other two. Closing power for the isolated phase tests was applied to the three closing coils in parallel. Further investigation of the pole unit for Phase *A* showed that its contacts were not reaching the full-closed position before the closing current was cut off by the auxiliary switches on the other two pole units, which resulted in this unit beginning to open earlier than the rest. In view of these facts, it was decided that certain alterations should be made in this breaker before undertaking further "OCO" tests at heavy current values.

This difference in operation, however, did not affect the performance of the breaker when tested on the "CO" operating cycle and it was decided to proceed with testing on this basis in order to obtain data as to the breaker's interrupting performance at still heavier currents. All succeeding tests with this breaker were, accordingly, made with operating cycle of three "CO" interruptions at one minute intervals except the first which was a single "CO" interruption at approximately 7000 amperes, using the system only, as a check on the timing of the control circuit. The next test consisted of three "CO" interruptions with a current value of approximately 7500 amperes, using the system only, as a further check. This test was satisfactory and No. 3 generator at Crawford Avenue was connected to the test bus for all succeeding tests.

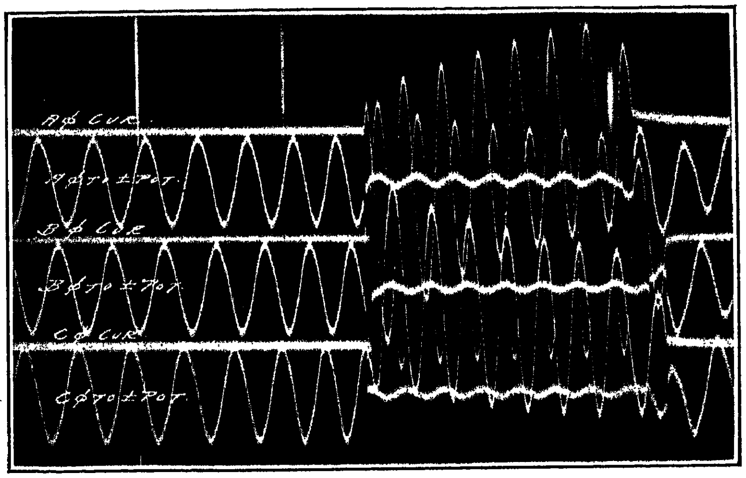

FIG. 10—OSCILLOGRAM OF ISOLATED-PHASE SHORT-CIRCUIT TEST AT 11,000 AMPERES

(For test data, see Table II, Test No. 14-31)

A series of tests was then made with 5 steps in current value, the first of which was 11,000 amperes. The generator load was then increased for the next two tests, in two steps, to a maximum of 38,000 kw., giving a short circuit current value of approximately 13,000 amperes. The duration of short circuit for the first three tests has been from seven to eight cycles and, for the next two tests, this time was reduced in two steps to a minimum of 2½ cycles in order to secure the benefit of increase in current value due to displacement. The maximum value of current interrupted on these tests was 22,400 amperes r. m. s., average for the three phases, and the maximum current interrupted in any single phase was 25,800 amperes r. m. s. Figs. 10 and 11 show oscillograms made during these tests, the corresponding test data being shown in Table II, Tests No. 14-31 and 14-43.

Performance of the test breaker had been satisfactory throughout these five steps in current value of from 11,000 to 22,400 amperes. The 15 three-phase interruptions covered by the tests had all been made within a period of 4 hr. 45 min., and the last twelve interruptions were made within a period of 1 hr. 40 min. The deionizing chambers had become very warm due to the rapid interruption of heavy currents, and it was decided to make an additional test with the same set-up in order to observe the breaker's performance under extreme temperature conditions within the deionizing chamber.

Of the three interruptions on this test, the first was normal and the second not greatly abnormal but, on the third interruption, very heavy flashing was observed on all phases and this flashing presisted until the circuit was cleared by other apparatus. Inspection of the test breaker showed that an arc had been established between opposite potentials in the contact operating levers of all pole units and had apparently persisted there until the circuit was cleared. Insulating surfaces were badly scorched and there was a considerable deposit of carbon, but otherwise the breaker was not seriously damaged.

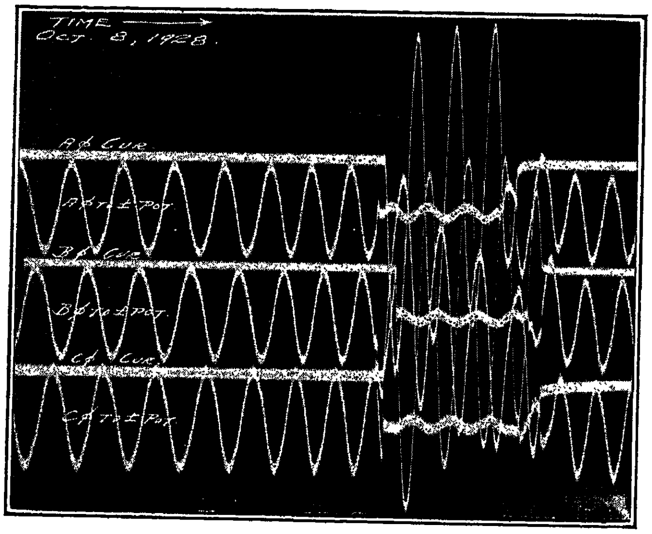


Fig. 11—Oscillogram of Isolated-Phase Short-Circuit Test at 15,200 Amperes

(For test data, see Table II, Test No. 14-43)

The oscillogram for the third interruption showed that the short-circuit current value at the time the breaker started to open was slightly less than 15,000 amperes r. m. s., average for the three phases, or about 67 per cent of the current value successfully interrupted on a test shortly before this. In view of this fact, and of the results of the preceding tests, it was concluded that this last test had demonstrated a limitation of the Deion circuit-breaker, not in maximum current rupturing ability, but in the severity of the operating duty to which it may be subjected.

## Summary of Observations and Deductions

*Severity of Test Duty Imposed.* It was anticipated in arranging for these tests that the Deion circuit-breaker would yield a performance on the same test duty superior to that of an oil breaker because of the absence of oil, higher speed, and definiteness of arc suppression. Accordingly, the number of cycles of operating duty was increased from two to three and the time interval between cycles reduced from two minutes to one minute. Also, while no special effort was made to increase the number of tests to a maximum in a repetitive sense as in all system tests certain time allowances must be made for necessary supervisory functioning and for ascertainment of effects on the system, it was realized that the factors which were assumed would enable the more severe operating duty would also allow a considerable increase in the number of tests possible within a given period. Also it was appreciated that the service requirements for this type of breaker would not necessitate the extremes required of breakers in industrial service from the standpoint of frequent operation.

It was expected that the choice of this more severe operating duty would expedite the whole schedule of tests toward the end that the limitations of the breaker would become apparent more quickly and in a sense procedure on this basis might be regarded as an approach towards a destruction test rather than the establishing of safe operating interrupting capacity rating.

*Speed of Test Program.* On the first day of the group phase tests a total of 21 three-phase interruptions with currents varying from 2000 to 10,000 amperes was made in a period of 2 hr. 23 min. without evidence of limitation on the part of the breaker. On the succeeding day nine three-phase operations with currents varying from 11,000 to 15,000 amperes were made in a period of 1 hr. 9 min. This series of tests was ended by a disability of the breaker which, upon investigation, appeared to be partially due to heating of the deionizing chamber.

On the first day of the isolated phase tests, 12 three-phase interruptions with currents varying from 7000 to 13,000 amperes were made in the period of 1 hr. 42 min. The test breaker interrupted all of these short circuits when difficulty of a mechanical nature ended this series.

On the final day of the isolated phase unit tests, a total of 15 three-phase operations with currents varying from 12,600 to 22,400 amperes was made in the period of 2 hr. 40 min. These tests were made in the afternoon with breaker starting at a temperature somewhat above the ambient due to seven three-phase interruptions at varying currents made during the morning. This series of tests was ended by the temperature of the deionizing chamber becoming excessively high at the beginning of the final group of tests.

*Performance of Test Breaker.* In the foregoing text mention was made of the fact that the group-phase

breaker had undergone a protracted series of tests in the factory, and it should be stated that the isolated-phase breaker was assembled on short notice especially for the field tests and perhaps, therefore, not a commercial standard to the same extent as the group-phase breaker Considerable difficulty was experienced in the mechanical adjustments on the isolated-phase breaker, but all of the original contacts with the exception of one remained intact throughout the tests.

Very few renewals were required for the group-phase breaker. Throughout its test there was no renewal of insulation except that following the final interruption which was a failure due to the excessive temperature, the renewal of insulated parts in the vicinity of the arc contacts would have been required for further testing. During the tests there were no renewals of arcing of auxiliary contacts and despite the previous factory tests the main current carrying contacts were apparently in a condition to carry normal rated current without excessive heating at the end of the Chicago tests.

Summarizing the performance, the two breakers jointly interrupted 82 short circuits. Of the 82 operations, two were unsuccessful in clearing the circuit. These two failures are especially interesting from the point of view comparable with oil breaker failures. In neither of these cases was there any damage to the test house or barriers or other equipment in the test circuit. There was no resultant fire and due to the absence of oil-throwing the work of preparing the test cell for succeeding tests was very much expedited. The fact that practically instantaneous determination of the extent of the damage was possible should be a feature of paramount interest to all operating companies.

Another feature of interest in these tests was the extent to which it was possible to visualize the performance. In contrast to the oil breaker, witnesses at a distance of one hundred yards could judge from the flashes any irregularities in the closure or displacement of time with regard to the sequential phase operations or inequalities in amount of gas or smoke emitted. Moving pictures which were taken disclosed the presence of gas and sparks to an extent not possible with the naked eye, and it might be well to re-emphasize the importance of such records. At the same time it is realized that only experience in judging the performance by the movie camera films or actual presence at the same tests justifies conclusions from either of these visual indications of performance.

*Limiting Factors in Interrupting Capacity.* In this series of tests certain factors recognized as determinants of interrupting capacity were carefully measured or observed. They are: maximum current values, resultant temperatures, mechanical strength, arcing time, and also restoration voltage. Of these the current values reached 25,800 amperes without definite signs of distress on the part of the breaker. On account of the rapidity of successions of the test cycles the temperatures reached a value which appeared to limit the safe interrupting capacity of the breaker. Need of reinforcement of the mechanical strength in the construction of blow-out coils was indicated. One factor, however, which is regarded as a limiting influence in the operation of oil breakers, namely, the production of restoration voltage of relatively high value, as far as this factor may be attributed to the action of the breaker, was absent, and since according to the theory of the Deion breaker the extinguishing of the current at the zero value is a fundamental characteristic of its operation, the Deion breaker should be given full credit for the absence of this limiting feature of performance.

*Indeterminate Factors of Interrupting Capacity.* While these tests indicated a superiority of performance of the Deion breaker from relatively low ranges of current values, its performance at extremely low values such as that due to magnetizing current or charging current of cables was not disclosed, and this test must be made before the performance of the breaker can be compared to that of an oil breaker. Neither did the tests permit observation of the action of the breaker on closure under the maximum current duty since these values were employed only in the "CO" tests.

Attention is again invited to the fact that while the performance of the breaker must be ultimately judged from service on an operating system, the expected advantages of the Deion breaker were established by the test performance. Also it appears that the limitations of the breaker are more capable of definite determination than those of oil breakers, and finally it should be remembered that the tests as conducted were but tests of a unit the first of a type and that as such they may be fairly construed to indicate ultimate results from the deiónizing type of circuit breaker not possible with types having a less scientific principle of arc disposal.

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## Discussion

### THEORY OF THE DEION CIRCUIT BREAKER

(Slepian)

### STRUCTURAL DEVELOPMENT OF THE DEION CIRCUIT BREAKER

(Dickinson and Baker)

### FIELD TESTS ON THE DEION CIRCUIT BREAKER

(Jamieson)

New York, N. Y., January 30, 1929

**P. H. Chase:** These papers very forcibly illustrate the effectiveness of a determined attack upon a major problem in the electrical field; the solution undoubtedly resulted from research of a broad basic character and also, which is very important, from the willingness to consider and accept new theories.

Undoubtedly the oil circuit breaker of the present familiar type is the most unsatisfactory piece of apparatus on the power system. This is probably not due primarily to defects in workmanship, design, or application, but is due largely to the fact that the problem of arc extinction is an extremely difficult one, and the recognized methods of accomplishing it leave much to be desired, particularly when the requirements are severe. While the problem is an old one, it is acute at this time when

the tremendous size of power systems and the tendency to link them together through heavy ties has resulted in the existence of short-circuit conditions that are very difficult to handle.

The successful development of the deion breaker is of particular interest to the central-station engineer. It opens a vista of many possible revolutionary changes in design, construction, and central-station practise and promises many long-desired operating advantages. I want to refer to a few of them.

Of major importance is the promise of elimination of oil from all circuit breakers, with several desirable consequences: a pronounced decrease in life hazard; a decrease in the liability of property damage, with resultant lower fire insurance rates; elimination of oil-handling equipment; less complicated fire-fighting equipment; possibly less space required for isolation of equipment, and decreased inspection and maintenance costs.

Then, there is another series of possibilities that grow out of the apparent increased reliability and speed of operation. Among the foremost are the facilitation of relaying and the improvement of the selective clearing of faulty equipment in lines, which has an immediate effect in better service. Again, the higher speed of operation should have a decided improvement on the system operating stability.

In connection with high-voltage transmission lines, we should have more hope of clearing insulator flashovers on one phase, before there is even time for the arc to travel in a high wind and involve adjacent phases or adjacent circuits. There should be decreased damage at the point of fault. All of these point toward better operating reliability and better service afforded by the central stations.

**S. M. Dean:** There is one thought that occurs to me which is aside somewhat from the engineering aspect of the question. Those of us who have to deal with budget approval as well as system plans usually find the least definite part of our presentation to our executives to be the question of oil circuit-breaker performance. Inevitably they ask the question, "When we spend all this money are the infernal things going to blow up in ten years or in five years?" That question is very difficult to answer. We are going to be in a much better position to show them that in case the problem is attacked basically.

*Operating Speed.* On cable systems, the C–O operation of this breaker is of decided importance, and its high speed on such a cycle is very gratifying. Requirements as to stability are very uncertain. One may desire to connect a cable system in one manner today only to find it must be connected in another tomorrow, and to be able to have a margin in the speed of the circuit breaker is very useful.

*Interrupting Capacity.* While the tests to date on this breaker have been a repeated series of C–O or O–C–O tests, on cable systems particularly the single C–O shot is the thing of importance. The large interrogation point in our minds is, What will it do on the single shot? Will it open a short circuit of 1,500,000 kv-a. or is there some inherent limitation to a lower value? What immediately interests us on our particular system is higher interrupting tests on a single C–O basis.

*Maintenance of the Breaker.* In spite of our best efforts to deal with condensation and dampness we have had trouble. We don't want to be pessimists, but if insulations less subject to moisture troubles can be employed it will better satisfy us.

Safety revolves immediately around the question of no oil. To companies who look forward to large concentration in built up city areas with indoor switching, the elimination of oil, and with it the fire and explosion hazard, is a very real advantage.

**S. M. Viele:** From the point of view of a railroad man very much interested in heavy electric traction, the development of a circuit breaker which does not embody the use of oil, is a subject of great interest to railroads contemplating any major electrification.

Our past experience with oil, which is probably not materially different from that of the central station people, as well as the expense of its use, warrants the statement that oil should be reduced to a minimum. Oil elimination from circuit breakers offers the greatest opportunity in reduction of attendant hazard. This is particularly true in unattended substations. The development of air circuit breakers of the deion or equivalent type, for potentials to 25 kv., is one of major importance.

There is one thing, however, that must not be overlooked in this development, that is, the time required for operation must be reduced from that now utilized in the usual breaker equipment. It is essential that development should embody a time of operation for the selection of the circuit, relay operation, and functioning of the breaker not exceeding the remainder of the half cycle of current wave during which the short circuit occurs.

In other words, interruption of the circuit should take place approximately at the first zero of the current after the inception of the short circuit.

Naturally, faster performance than the above should not be carried to a point where such advantage is offset by the resultant increased potential of the circuit interrupted.

Such time performance as previously recommended would increase the dependability of the breaker equipment, increase the stability of systems, decrease the territory effected by short circuits, and eliminate many of the dilatorious effects which now result from short circuit.

We hope for the complete success of the development of an oil-less circuit breaker.

**R. C. Mason:** The interest of the Deion circuit breaker to electrical engineers centers chiefly in the high voltage which may be satisfactorily opened with only moderate dimensions. Besides this feature, there are involved other theoretical considerations, concerning the nature of the arc, which are of interest to physicists.

Dr. Slepian and his associates have brought to the attention of physicists most forcibly the fact that cold-cathode arcs can exist. It has long been known that discharges in gases could be obtained with cold electrodes, but always the discharges were glows, with very small current densities, of the order of milliamperes, and with large cathode falls, of the order of hundreds of volts. Here, however, has been described a heavy-current arc, with a current density of 30,000 amperes per sq. cm., and with a cathode fall certainly not over 30 volts, maintained between electrodes which were relatively cold. For some three or four years, the old idea that thermionic emission of electrons from the cathode was the essential and complete mechanism of the arc has been failing to explain completely some types of arcs. In some arcs, only part of the current could be given in that way. Now, we are offered evidence of an arc in which it is hard to imagine that any appreciable thermionic emission takes place.

To account for that part of the current not carried by electrons, two theories have been proposed: one, by Slepian, suggests that positive ions, arising from intense thermal ionization close to the cathode, carry the current to the cathode; the other, by Langmuir, proposes that a very high field pulls electrons out of a relatively cold electrode. The latter theory is possible, even though the total cathode fall is only a few volts, if it is concentrated in such a small space that the gradient is of the order of millions of volts per cm.

The volt-ampere characteristic of an ordinary arc is negative; that is, the voltage decreases with increasing current, and may be represented by an equation of the type,

$$V = a + b/I^n$$

From Equation (1) we see that the arc voltage of the cold-cathode arc in the Deion circuit breaker increases with increasing current, as given by,

$$V = a + b\,I^n$$

Arcs of this type, then, should operate stably in parallel, because of their positive volt-ampere characteristic. It is interesting to note that evidence is found of the existence of parallel arcs.

Whatever may be the final theory of the cold-cathode arc, the work of Dr. Slepian should offer a further stimulus to the study of a new type of arc phenomenon.

**J. W. McNairy:** Until recently the oil circuit breaker was practically unchallenged for applications above 2300 volts; however, successful commercial operation on 12-kv. single-phase circuits of air breakers of somewhat different design from that described in the papers presented is now an accomplished fact, the rupturing capacity being over 35,000 amperes.

We have followed with particular interest the series of papers presented by Dr. Slepian covering laboratory investigations of the phenomena associated with the electric arc in air as related to the performance of air circuit breakers. These papers bring up many interesting points, particularly with reference to the effect of metallic plates and screens introduced into the arc stream at right angles.

In following Dr. Slepian's theory as outlined in connection with the Deion breaker, it appears that the basic principle involved is the utilization of the rapid increase of dielectric strength of the ionized gas because of the quick accumulation of space charges immediately adjacent to numerous cathodes in series at the *instant the current wave* passes through its normal zero.

However, on examining the oscillograms contained in the papers by Mr. Jamieson, and Messrs. Dickinson and Baker, it was noted that in the majority of cases interruption occurred in advance of the time of normal zero current as indicated by the preceding half cycle and that the current just prior to the interruption was, in many cases, in excess of 50 per cent of the preceding current peak. Figs. 8 and 10 of Mr. Jamieson's paper and Fig. 12 of the paper by Messrs. Dickinson and Baker are cited as examples.

It appears that the normal current rise has been stopped and the current decreased to zero considerably in advance of the time of normal zero by some action in the circuit breaker other than the deionization of the gas adjacent to the cathodes which is assumed to take place only at the time the current would normally be zero or very near zero. In view of the above, it appears that there must have been some phenomena which resulted in a very considerable increase in the resistance of the arc while carrying relatively high currents, the resistance being sufficient to reduce the short-circuit current at a rate much faster than the normal decrement. The currents involved are too great to have been carried as a glow discharge at the cathode and following the preceding current zero.

Therefore the characteristics of this breaker under discussion, from the standpoint of performance in the circuit as indicated by the oscillograms, appear not to differ greatly from existing types of air and oil breakers for similar service designed to reduce ionization of the gas when carrying instantaneous short-circuit currents of considerable magnitude.

This theory is further substantiated by Figs. 13 and 14 of Messrs. Dickinson and Baker's paper. The average power dissipation in one unit of the breaker over a period of one-half cycle is of the order of 20,000 kw., the maximum being very much greater in an operation involving 8500 amperes and 6000 volts for the unit, the product of which is 51,000 kw. Since the existence of voltage across the breaker simultaneously with current through it results in power liberated in the arc, it would appear that the phenomena associated with the arc in the breaker while carrying heavy short-circuit currents are of as great importance in studying its performance as the phenomena at the normal current zero. If the recovery of dielectric strength occurred principally at zero no appreciable energy would be dissipated in the arc during the interruption. The standpoint is further borne out by the marked increase in temperature of the metallic disks noted during the short-circuit tests.

I cite this characteristic in particular as I shall presently show a slide of an oscillogram taken during tests on an air circuit breaker designed to bring about deionization of the main body of the arc while the short-circuit current is flowing.

In the opening paragraph of the papers by Messrs. Dickinson and Baker a statement is made to the effect that, with the exception of the oil breaker, the only other method generally used for extinguishing an arc in air has been by lengthening to such an extent that the circuit voltage would no longer maintain it. In view of this statement it seems desirable to call attention to the type of air circuit breaker previously referred to and now in commercial operation, the successful operation of which is dependent largely upon deionization of an arc in air. A photograph was taken of such a circuit breaker when interrupting a single-phase short circuit on a single element at 13,000 volts, 25,000 amperes r. m. s., the entire voltage after interruption appearing across this single element. This photograph demonstrates visually that the length of arc obtained was only a fraction of that which would be necessary in open air to insure interruption. The successful operation is, therefore, attributed to rapid deionization of the main body of the arc.

In this connection it has been repeatedly demonstrated that it is entirely possible to open circuits of from 1 to 5 amperes at very high voltages by unconfined arcs in open air. Disconnecting switches capable of opening exciting currents for large transformers at 135 kv. with only a few feet of arc have been in successful operation for some time. Since there is only one cathode involved, the successful interruption of the circuit appears to be due to the relatively rapid deionization of the main body of the arc by recombination resulting from rapid cooling, and not by a space charge accumulating at the cathode. As the current is increased, however, the energy dissipation is not sufficient to prevent continuous thermal ionization unless auxiliary means for energy dissipation or extremely long arcs are provided.

The method involved in the breaker consists primarily in taking advantage of the possibilities of energy dissipation in greatly accelerating the deionization of the main body of the arc so that the resistance can be made to increase during the time when the current through the arc is comparatively heavy, as well as at the normal zero of the current wave.

In a breaker of this type the arc is subdivided into a number of very narrow parallel streams, the columns of ionized gas being flattened in such a way as to bring all portions into intimate contact with a cooling medium, in this case made of insulating materials. The ionized gas is moved under the action of blowout coils so that fresh cooling surfaces are continually encountered by the arc stream.

It is usually assumed that ionization in the main body of the arc is maintained thermally since the voltage gradient is not sufficient for ionization by collision. The temperature of the main body of the arc is greatly lowered and re-combination of ions accelerated by the above arrangement, particularly during periods near zero of the current wave where the energy supplied is relatively small.

By virtue of the cooling agents introduced the energy required for maintaining sufficient ionization to insure a stable arc is greatly increased and the resultant voltage which can be developed even with relatively heavy currents through the arc is correspondingly increased. Oscillograms taken during the test of this circuit breaker show that deionization takes place both with current through the breaker and at the normal zero point of the current wave.

This arrangement is particularly effective for high-voltage direct current where no zero point is available.

Photographs and comparative records taken in open air have shown conclusively that the arc length necessary in a device of this type is only a fraction of that required in open air.

Another method utilized on breakers of this type is the introduction of arcing horns of high-resistance material so that mechanical resistance is introduced as the arc travels along the

horns absorbing considerable energy. This is a further important factor in the successful performance.

One of the problems usually encountered in the design of circuit breakers, particularly the air type, which depend for their successful operation upon the action of magnetic coils in series with the circuit, is the loss of effectiveness at relatively low currents.

For a-c. applications the most difficult practical case, as cited by Mr. Jamieson, is usually that involving the interruption of low-power-factor exciting currents for one or any number of transformers at normal voltage. Inasmuch as the breaker, subject of the papers presented, depends for its success upon the rapid circular movement of the numerous arcs around the deionizing plates, it seems reasonable to assume that at relatively small currents the action of the blowout coils is weakened and if the current becomes sufficiently low, this action might cease altogether. While currents of 1 or 2 amperes, as pointed out previously, may be broken on the main contacts of an air break without the necessity of deionization by auxiliary means, currents of the order 25 to 100 amperes particularly at low power factor usually require the operation of the arc-extinguishing agents. A statement of the observed effect of the reduced speed of rotation of the arcs at currents of this order on the temperature of the cathode spots and the effect on burning and the amount of metallic vapor liberated would be of considerable interest.

Dr. Slepian's theory of the cold cathode arouses considerable interest. The effectiveness of rapidly moving the arc over arc horns in limiting burning is unquestioned. This reduction in burning, however, is usually attributed to the wide distribution over a considerable area of the total burning which might otherwise be concentrated on a single cathode spot, a slight amount of burning always being present. The importance of preventing oxidation or burning of the metallic plates cannot be over-estimated because of the danger of strike-back through this metallic vapor. I believe this phenomenon has been observed by Dr. Slepian and commented on in one of his previous papers on this subject.

The development of several arcs in parallel, noted by Dr. Slepian, is interesting under conditions of heavy currents. The question naturally arises as to whether or not with very heavy currents the number of arcs might not increase to such an extent that the burning would be practically continuous on the surfaces of the deionizing plates. It would be interesting to know whether or not Dr. Slepian considers that the maximum current which the breaker is capable of rupturing at normal voltage is limited by the increase in the multiplicity of the arcs. It seems reasonable to assume that as the current is increased the volume of ionized gas necessary to carry this current until interruption occurs must necessarily increase.

The effect of temperature on the interrupting ability of the Deion breaker, as noted in Mr. Jamieson's paper, (p. 544), prompts the question as to what the temperature of the ionizing plates was at the time failure to interrupt occurred. It would appear that in order to impair seriously deionization of the region adjacent to the cathode, the temperature would have to be sufficiently high to maintain a considerable thermal ionization either of the air in contact with the plates or the thermal emission from the plates themselves.

**R. M. Spurck:** Mr. Jamieson in his paper brings out that the performance of the breaker in interrupting charging currents and exciting currents of transformers had not been disclosed. It seems necessary to complete the test record of the breaker to include such tests. They are particularly important because in the interruption of such currents, the voltage tending to re-establish the arc as the current passes through zero is a maximum under those conditions. Also, at the low currents, the blow-in effect is probably very small.

Another feature on which I should like to hear discussion is that of the insulation of the Deion chamber. When the breaker is open, this chamber is connected across the open contacts of the breaker. What insulation test across the contacts at 60 cycles will the breaker in the open position withstand? Also, how is this insulation affected by arcing in the Deion chamber?

As I understand Dr. Slepian's theory, the Deion chamber described has as a limit the ability to extinguish an arc in a circuit having a re-establishing voltage of 12,500 volts. Will the breaker operate satisfactorily in interrupting a circuit between two systems that may fall out of synchronism? In this case, on a 15,000-volt system, the *normal* re-establishing voltage would be about 17,000 volts.

**J. D. Hilliard:** Mr. Slepian's clear statement of his conception of the theory of operation, his determination of the limiting value of factors affecting satisfactory operation, and his mathematical treatment of the subject will greatly aid in the development of this interrupting device, which is one of the earliest methods of interrupting an electric circuit.

The claim is made that the recovery voltage of the Deion breaker is less than with an oil circuit breaker doing the same duty, but no statement is made as to the magnitude of the recovery voltage in the two cases. Personally, I believe the difference, if there is any, will be found to be small. In testing for recovery voltage on various breakers it is necessary that all breakers be tested on the same circuit, under the same conditions, because the circuit itself largely determines this voltage. They should also be given a considerable number of shots as the record of a single shot is of only small value. Tests as above indicated in which alternate shots would be made with the various breakers so that the characteristics of all could be definitely determined would be of the greatest interest.

The claim is also made that the Deion breaker interrupts at the zero value of the current wave; the same statement is true for the ordinary oil circuit breaker and also for a properly constructed magnetic-blowout oil circuit breaker.

Mr. Jamieson stresses the "absence of oil or oil fire." It is not necessary that either oil throw or an oil fire at the breaker accompany the interruption of an electric circuit by an oil circuit breaker. As a matter of fact, the modern oil circuit breaker does not throw oil or emit a flame during the interruption. The breaker may have an internal short circuit but no oil or fire will appear at the breaker as it is all conducted away by the waste piping connected to the breaker and without ignition or spilling of oil. Personally I believe the oil circuit breaker would stand a good chance of winning in a competitive test as its thick steel jacket prevents any ionized gas escaping to cause a short circuit on adjacent conductors. I am certain that the oil circuit breaker would stand an internal short circuit of longer duration and then be placed in operation with less repairs than would be required by any high-voltage air breaker, because usually a small quantity of oil to replace that blown out through the safety blow-off pipe and perhaps the renewal of burned arcing contacts is allt hat is required. Breakers with pressure-limiting devices have been given many severe tests and an oil breaker so equipped should be no more liable to explosion than the ordinary steam boiler with a proper safety valve attached and provision is now made for attaching the pressure limiting devices to all General Electric Company's modern breakers.

I anticipate that for low-voltage currents, air breakers of various kinds will prove satisfactory but I am skeptical concerning their success at higher voltages, as the ionized gas liable to be ejected at times will be a great hazard. The starting of an arc by the ejection of ionized gas has been observed by means of the motion-picture camera and this demonstration has proved the absolute necessity of preventing its escape if one is to safeguard against short circuits.

**F. C. Hanker:** Those of us who have known of Dr. Slepian's research have been greatly interested in the very early studies which he has made of the extinction of the arc. He has described those in some of his previous papers. The simplicity

with which he described those years of research makes it difficult to evaluate the importance of the results, but the effect is recognized by those who have followed circuit-breaker development.

Mr. Chase has called attention to the possible effects of this development on the design of switchhouses. That is one of the important features that will come out of a construction differing from existing devices and which eliminates the oil tank around the contacts.

In the early days, the first way known of interrupting a circuit was by separating the contacts in air. These varied from the crude plug-type switches to the more fully developed stick-type circuit breakers and were sufficient for the limited generating capacity.

The next development was the oil breaker. Some twenty-five years ago, we considered the results of the development as sufficient. We soon found, however, that with concentration of power there were limitations. While I am hopeful that Mr. Hilliard's prophecy will come true, unfortunately we have had failures on oil circuit breakers.

Mr. Jamieson called attention to the acquiescence of the operators to the development of the oil circuit breaker. All operators as well as manufacturers will agree there has been a very considerable advance in the design of oil circuit breakers to meet the conditions of today. Those who have been in contact with operating men have heard of their objection to oil breakers quite frequently. They haven't been at all hesitant in bringing the necessity of the elimination of the oil hazard to our attention. There have been important improvements in the mechanical structure of breakers but nevertheless all of those developments are in the direction of greater mechanical strength to resist higher pressures and interrupt the circuit by what may be termed "brute force." It is refreshing to have a development such as Dr. Slepian's which gives us what we might call a scientific method of interrupting a circuit.

It is hoped that the development of the Deion breaker which has been tried out in service on lower voltages, and is now being tried out on higher voltages, will follow through on the advantages that have been shown in the past. All the operators will welcome the elimination or minimizing of the present hazard of very high-power circuit interruption.

**J. B. MacNeill:** The present status of this development is about as follows. First, 2500-volt service: we have a small number of these breakers in actual service on our shop circuits. Second, 15,000-volt service: We are now building 6 single-pole units for 12,000-volt isolated-phase service on Mr. Jamieson's system in Chicago. These breakers are to be installed, I believe, in Washington Park Substation sometime in the near future to obtain actual field experience.

The extension of the development to other fields seems quite easy. For instance, the problem that Mr. Viele raised of handling 11,000-volt contact-line circuits on railway electrifications is being worked out and shows great promise. The extension to 25,000-volt service is actually being worked on and bears considerable promise. It is probable that within a year we shall have field data for 25,000 volts comparable to those which have been secured in Chicago at 12,000 volts.

With the deionizing chamber developed to the point of positive knowledge that it will handle 130 volts per gap up to say 25,000 or 30,000 amperes, there is a definite unit to work with which can be built up in series as necessary, or possibly, as has been suggested, in parallel. The use of these chambers in series makes the application of the device to 25,000-volt service a problem somewhat secondary to its development for 12,000 volts.

We expect to be able to present something on these other developments before the Institue in due time. At present the device is not offered for general sale. The art of switching has grown up empirically one step at a time, and it is not well to launch a radically new type of device on the market without the most complete operating experience.

Mr. McNairy and Mr. Spurck raised certain questions regarding the action of this device on small current values. You may be quite sure that the Chicago people did not overlook that point; although their system was not arranged so that the device could be demonstrated at low currents, they requested quite a comprehensive set of tests, which have since been made.

**C. A. Corney:** It is certainly gratifying to central-station engineers to learn of a development such as has been described in this paper. Any development which points the way to the possible elimination of oil as a medium in which to extinguish arcs will be carefully scrutinized and no doubt rapidly adopted as soon as it may have proved itself adequate and dependable in service.

There are several inherent features of the circuit breaker here described which appeal to me as particularly advantageous.

1. Elimination of oil hazard—as already mentioned.
2. Extremely rapid breaking of the arc once established resulting in:
   a. Reduced system disturbance at time of fault
   b. Lessened tendency to shake off synchronous apparatus
   c. Better selectivity in relaying, as a consistently quick break should permit closer timing among associated relays in any group, as it eliminates one of the variables now encountered in the overall timing of any system.
3. The apparent ability of the breaker to handle repeated short circuits without having to be derated should be particularly advantageous where the breaker is to be applied to automatic reclosing duty.

There would appear to be no reason why this breaker should not be built in the three-pole class using the truck type of construction with its attendant advantages. It should also be lighter in weight and more accessible for repairs than the equivalent oil circuit breaker.

**K. B. McEachron:** The Deion circuit breaker is of very great interest to me because it employs principles which have been used for quite a good many years in lightning arresters.

Early in the arrester art it was found that if an arc was subdivided into a large number of small arcs, it was possible to extinguish quite large currents. The theory at that time was not so well known as Dr. Slepian has shown us today, but this principle was employed in actual lightning arresters of that time.

It is quite well known that a single gap connected to a circuit whose voltage does not exceed 220 volts will suppress quite large follow currents. It has been common practise in designing lightning arresters for such service to use only single gaps when the voltage did not exceed 220 volts. This voltage limit is not far from the 250 volts mentioned by Dr. Slepian in his paper.

A single gap on high voltage, however, is quite another proposition. There a very small current will maintain an arc. It seems to be necessary to divide that arc into a large number of small arcs in order to stop the flow of current by the use of gaps.

The use of the static shield is also a principle which has been used for a long time in connection with the control of voltages across gaps. It was called an antenna in connection with a lightning arrester and its purpose was to distribute the impulse voltage across the gaps so that they would break down in cascade.

I should like to raise one or two questions which occur to me because of lightning-arrester practise. It is a race, as Dr. Slepian says, between the rate of deionization and the rate at which the voltage across the gaps builds up. I am wondering if we do have circuits in practise where the rate at which the voltage builds up will exceed the rate of deionization.

In connection with that same question, it would be interesting to know whether or not in the reactive circuit without the capacity in parallel the voltage is still 250 volts for recovery.

I should like to know whether or not there has been any attempt to use any other metals than copper. There has been a use in the past of metals containing zinc and other low-melting materials in an effort to produce the so-called non-arcing metal.

What does Dr. Slepian feel is the limitation in voltage of such a circuit breaker? Is it limited in ability to hold the disks in line by the static shield or is it limited in some other manner in voltage?

**L. B. Chubbuck:** (communicated after adjournment) To any engineer who has had experience with oil circuit breakers for many years, the Deion oil-less circuit breaker is of great interest. While oil circuit breakers have recently been greatly improved there is always present the hazard of oil fire due to poor maintenance, the use of too small a breaker, etc. Short-circuit tests on the Deion circuit breakers are remarkable in their absence of noise or blaze as experienced with conventional air breakers. The Deion breaker is fast, and evidently requires little maintenance as evidenced by the large number of consecutive interruptions at the high test currents. The breaker is very accessible for inspection, and I assume in the final design provision will be made for readily replacing the deionizing chamber or plugging in a complete pole unit.

**R. H. Park:** (communicated after adjournment) I understand that the abscissas of the curve of Fig. 1 were calculated, not tested, and that the calculation did not consider the effect of the capacity between turns of the coil used. I believe that this fact has resulted in an error in the abscissa scale. The effect of this capacitance will be to increase the values of time shown on the present scale. Thus 0 might be replaced by 40, 40 by 55, etc. If this is the case, as appears probable, the theory propounded by the author of almost instantaneous recovery to a strength of 250 volts, would have to be discarded.

**A. H. Kehoe:** (communicated after adjournment) In a symposium on an outstanding development such as the Deion circuit breaker, it is easy to overlook the valuable contribution made by the Commonwealth Edison Company in permitting its system to be used for high-capacity tests required to establish ratings of the new switch. In fact, Mr. Jamieson's paper records, by inference only, what appears to me to be one of his principal accomplishments; this was to obtain consent for such tests of those in responsible charge of his company's operations.

The effects of the tests on this system should act as a stimulus to other operating companies in supplying the necessary capacity for similar tests when proper results can be obtained in no other way than to utilize the operating system as the laboratory.

Being one of the operators mentioned by Mr. Jamieson who has frequently stated his misgivings concerning existing oil circuit breakers, it seems but just to the manufacturer to state that to me the development of the Deion circuit breaker appears to mark the turning of the ways, and the time may come when the circuit breaker can be treated like other electrical equipment rather than as an explosive hazard of large dimensions. It is hoped that this development will act as an incentive to all manufacturers to make material improvements in their present lines of switches.

**P. H. Thomas:** (communicated after adjournment) The physical or mathematical explanation as to the exact manner in which the electrons constituting the current in an electric arc escape from the body of the cathode into the arc space is given differently by different observers. I should like to suggest a conception or picture of this operation which I have held for over 20 years and which has seemed to me to fit the observed results very well. This conception was worked out well to explain the action of the cathode in mercury vapor rectifier and allied apparatus.

As stated by Dr. Slepian, there seems to be no serious resistance to electrode flow of current from a cold cathode; this is true not only of liquids such as mercury but of metals. The most extreme example of which I have knowledge is the use of a cold mercury vapor vacuum device with mercury electrodes as a breakdown gap for operating high-frequency Tesla coils. In such operation there must be enormous amperages passing with the current flow starting and stopping, corresponding to exceedingly high alternating frequencies. The operation of this device as far as I know it suggests no limitations as to the suddenness with which current may be produced nor its amount and the low temperature of electrodes appears to have no retarding effect; not only this, but these very high abrupt currents are at very low voltages once the flow is started. Thus, not only is a cold cathode no restraint on very heavy flow of current but apparently, at least in vacuum devices, the greater the current the easier the flow, as far as the cathode is concerned. I should perhaps say that this statement is not based on direct scientific measurements but on the results and observation of a large amount of laboratory work with various types of mercury vapor apparatus.

Aside from the *energy* required for passing an electron from a cathode to the vacuum space, a certain strong electric field or electric *force* is required to separate the electron from the solid matter of the electrode. This may be looked upon as the supplying of the necessary force to overcome the attraction of the electron lying outside of a solid surface for this solid body; such an electron has no difficulty moving within the body of the electrode as ordinary current, since attraction of the solid particles on one side of an imaginary plane through the electron is balanced by the attraction of the solid particles on the other side. Suppose now at the point of exit of the electron from a cathode there is a very violent vaporization of the solid material of the electrode concentrated at a small area on this surface and that the concentration of evaporation is so great that the density of the material changes by gradual stages from the solid condition behind the surface outward until at some relatively large distance from the position of the surface the density becomes reduced until the normal gaseous pressure appears. That is to say, the concentration of evaporation at the critical location on the surface is so great and diminishes so gradually that there is no line of demarcation between the solid material and the vacuum space. In such a case the electron could presumably pass from the cathode into the vacuum space by way of this path through a gradually reducing density without the necessary existence of any overwhelming electrical force to produce the motion. No such condition could reduce the amount of *energy* required for separating the electron from the electrode.

It would appear that this condition of high vaporization is exactly the condition that would be expected to exist at the cathode spot of an arc. It could exist only as long as there is sufficient current to supply enough energy for the necessary amount of evaporation. The necessary densities may easily exist at small currents on account of the possibility of the phenomena involving very restricted areas.

This theory is very strongly supported by calculations made, as I understand it, in connection with mercury vapor rectifiers of commercial proportions showing that the gas pressure at the immediate surface of the cathode spot exceeds one atmosphere during operation.

If this view can be co-ordinated with the fundamental theory of electronic phenomena it very beautifully explains the behavior of the cathode carrying current in a vacuum in many different applications. In this view the force resisting starting of current flow in an unexcited cathode is somewhat analogous to surface tension and when the cathode is excited the surface tension may be said to be punctured with a blast of vaporized material and electrons passing out through the hole.

Cannot this analogy be extended so that we may imagine the inside of a solid or liquid metallic conductor to be the equivalent of a vacuum free space for the passage of electrons and carrying space charges during current flow, with collisions and losses of energy as in the vacuum space and operating at a low voltage and that the electrode starting resistances and energy losses are related to the passing of the electrons by the boundary surfaces between the two spaces.

**Joseph Slepian:** The curve of Fig. 1 of my paper, which I

consider of rather fundamental importance, was derived by computation from tests which consisted of determining the limiting value of shunting resistance which would just cause an arc to go out in a particular circuit. It was found that when the circuit voltage had a peak value of less than 250 volts, the arc would go out without any shunt. This surely means that the arc space recovered the ability to withstand 250 volts faster than the circuit could develop 250 volts, immediately after the current zero. I described this fact by saying that the arc space recovers the ability to withstand 250 volts, "almost instantly" as compared with its ability to withstand larger voltages. Of course, strictly, the recovery cannot be instantaneous.

The curve of Fig. 1 was calculated by an approximate method and I have since calculated it more carefully and obtained a considerable departure from that given in the paper. The "almost instantaneous" recovery of 250 volts is, of course, still retained, but the rate of recovery of higher reignition voltages is much slower. This is shown in the accompanying figure. The curve is the envelope of the exponential curves which show the recovery-voltage transient following arc extinction. I have left in the figure portions of these exponential curves to show their relation to the resultant curve.

Mr. Park has raised the question as to the influence of distributed capacity in the circuit upon these calculations. I have already discussed this point in my former paper (see reference 1 of the present paper.) In the particular series of tests from which the curve was derived the current was limited by a small reactor whose natural period was of the order of 100,000 cycles. A quarter period of this frequency would be 2½ microseconds and I conclude, therefore, that the recovery of the first 250 volts of dielectric strength of the arc space is "almost instantaneous" at least to this extent. There are, however, good reasons drawn from the physical theories of the arc to believe that this recovery of dielectric strength is much faster. It is generally believed that a cathode of an arc requires either a very large gradient at the cathode surface or a high temperature in the gas space next to the electrode. The high gradient is maintained by space charges and to maintain these space charges requires a current density of several thousand amperes per sq. cm. This puts a limit to the density of ionization which is required for the striking of the arc at low voltage. It may readily be shown that the density of ionization will fall far below this figure due to recombination in a fraction of a microsecond. On the basis of high temperature in the gas next to the electrode also, we are lead to a similar result. The layer of gas involved is only 0.0001 cm. thick and this will cool in a very small part of a microsecond. Hence, on either basis after a small fraction of a microsecond, the arc must be restruck by break-down of the gas in a manner similar to that in which ordinary spark-gaps break down and such a break-down requires a minimum of several hundred volts.

Mr. McNairy has pointed out that in the Deion circuit breaker the zero point of the current wave is advanced to a considerable extent at the moment of arc extinction and that the current is considerably reduced while the arc is in the deionizing chamber, and asks whether some aid in arc extinction does not arise thereby. To a certain extent, there is some advantage resulting from the advancing of the zero of the current wave and the reduction in the maximum value of the arc current. As a result of the former, the voltage tending to reignite the arc does not rise to the peak of the generated voltage but to a smaller value, and the latter causes the rate of application of this voltage to the arc terminals immediately after the current zero to be slower than it otherwise would be. However, no advantage of this is taken in the Deion circuit breaker. Enough plates are used so that the arc will be extinguished even if the voltage tending to reignite rises to the full peak value of that generated in the circuit, and the plates recover the ability to withstand this voltage faster than it can be supplied by any practical circuit. Hence, they do not need the slowing down of this rate of rise resulting from the reduction in current.

This advance of the current zero and reduction in the maximum current in the arc is a consequence of the voltage required to maintain the arc in the deionizing chamber. This voltage is not negligible in comparison with the circuit voltage but in the case of the 15,000-volt Deion breaker amounts to 3000 volts for smaller currents and rises to 6000 volts for very large currents. This arc voltage is something that we would have much preferred to do without. If it were possible, we would bring the deionizing means into play only at the current zero, and leave the arc absolutely unimpeded until the current zero. Then there would be no energy liberated as heat in the switch and the ideal circuit breaker would have been obtained. However, the deionizing means while waiting for the current zero does affect the arc and abstract energy from it, although this energy is small compared to that abstracted by most other forms of circuit breakers. It is conceivable that we might take advantage of the effect of the arc voltage upon the current by reducing the number of plates for a given voltage but in that case, we would require a certain minimum time of residence in the chamber by the arc prior to extinction. We prefer, however, to have the chamber ready for extinction at the first current zero and to act whether the arc enters the chamber just shortly before the zero or near the beginning of a half cycle. When the arc enters the chamber near the end of the half cycle, tests show that the breaker operates entirely satisfactorily and with correspondingly very small heating.

The only type of switch I know in which the arc energy is quite negligible, is the vacuum switch. In this switch the arc voltage is only some 20 or 30 volts and there is practically no influence upon the course of the current in high-voltage circuits. The arc is extinguished at the current zero, however, by the "almost instantaneous" disappearance of the arc-conducting vapors.

Mr. McNairy makes the point that moderate currents can be interrupted on high-voltage circuits by a single break and believes this to be at variance with the theory given in this paper which states that an unshunted single break will handle only 250 volts. I have tried always to qualify this statement by saying that it holds only for a short arc. By a short arc, I mean one which is short compared to the linear dimensions of its cross-section. For long arcs this limit of 250 volts does not hold. I brought this out in my former paper and have curves showing how circuits of considerable voltage may be interrupted in moderate space by confining the arcs so that their section will be small in comparison to their length. The curves of that paper bring out very definitely the point that Mr. McNairy makes, that arcs confined to slots will be extinguished in a circuit of given voltage at much shorter length than arcs in the open air.

Concerning the phenomenon of the existence of arcs in parallel in the deionizing chamber for the larger values of current, Mr. McNairy asks whether this produces any limitation in the current-interrupting capacity of the circuit breaker. So far as I can see it does not. As a matter of fact it would seem to me that it makes the instantaneous temperature rise of the points of the metal plate surfaces less than it would be if the arc remained a single continuous arc. Of course, the heating up of the metal plates as a whole will be larger in proportion with the larger currents. Since we have found the current density in these arcs to be 30,000 amperes per sq. cm., it is quite evident that the arc cannot have sufficient extent to cover the whole of any one plate at one time with less than millions of amperes.

The failure of the circuit breaker when the deionizing chamber was permitted to reach too high a temperature has been noted. This limitation is tied up entirely with the properties of the fibrous insulation used for separating the copper plates. Without this limitation the deionizing chamber might be expected to be effective up to the melting point of copper. By using special

high-temperature insulation for the insulating spacers between the copper plates we have operated Deion circuit breakers successfully and repeatedly when the deionizing chamber temperature was over 300 deg. cent.

Mr. Spurck has raised the question as to how a Deion circuit breaker would act in separating two 15,000-volt systems which had fallen out of step. Regarding this problem, the Deion circuit breaker is in no way different from any other circuit breaker. When two systems are 180 deg. out of phase, they are really in series with each other so far as the circuit breaker is concerned and the circuit breaker is called upon to open a 30,000-volt circuit. In general, a 30,000-volt circuit breaker would be necessary for this service. However, systems fall out of step under conditions in which the generating voltage is usually very greatly reduced and so the usual 15,000-volt circuit breakers are able to meet this exigency. There is no reason why the Deion circuit breaker should be different from others in this respect.

Mr. McEachron's points concerning the relation of the Deion development to old lightning-arrester art are very good. As a matter of fact, in my former paper I mentioned this and took some of my data from investigations made on lightning arresters some eight or ten years ago. However, although the principles seem clear and obvious now, so that it would seem almost that I should apologize for myself and my colleagues for having taken so many years to apply such well known principles, nevertheless, there was much hard work, both mental and physical, before it was realized that the reignition of an arc is a phenomenon quite similar to the break-down of spark gaps and that it can be affected in the same way by static fields.

We have tried metals other than copper for the plates in the Deion circuit breaker and with success. Iron has advantages in certain types of switches. We have used brass and even aluminum. Because of its high electrical and thermal conductivity, copper is advantageous and does not require so high a speed of the arc to prevent the melting at the surface of the electrode.

I find the discussion of Mr. Thomas very interesting and thought-provoking. There is no doubt that the obstacle to the free egress of electrons from the surface of the metal is tied up with the discontinuity in material which exists at the surface. If it were possible to have a continuous transition from the dense metal to the attenuated gas or vapor, then the work function for electrons would disappear and arcs could be struck between separated electrodes with very low voltages. I find it difficult to see how this continuous transition from metal to vapor can take place without a tremendous amount of vaporization, but I shall be open-minded on this point for we may yet discover definite proof of the loss-of-work function under certain circumstances.

**B. P. Baker:** The question of the ability of the Deion circuit breaker to interrupt low currents has been raised by several gentlemen and I believe has been suggested in the paper by Mr. Jamieson.

The Deion circuit breaker has not been tested as extensively at low currents (less than 1000 amperes) as it has been at high currents. However, several hundred tests have been made which show that it is capable of successfully interrupting currents from a fraction of an ampere to many thousands of amperes.

Operating characteristics are essentially the same at low currents as they are at high currents. The arc is usually extinguished at the end of the first half-cycle after it has passed into the deionizing plates. The time required for the arc to get off the contacts and move up the arcing horns to the deionizing structure varies inversely with the current. At 3000 amperes and above, the movement of the arc is very fast and it usually gets off the contacts and into the deionizing chamber in time to be extinguished at the end of the first half-cycle. At 2000 amperes, one or two half-cycles are required. At 1000 amperes three half-cycles may be necessary. The time in general increases as the current decreases, until at about one ampere under the most unfavorable conditions one second may be consumed. This time may be shortened by using a stronger magnetic field to move the arc; however, as long as there is no injury resulting from this slow movement there seems to be no necessity for making any changes. Below 20 amperes the operating characteristics and time required for operation varies considerably with the current conditions and applied voltage. The most highly inductive circuit where the current is limited by air-core reactors, gives the longest arcing time.

When a bank of condensers is used as the load, to simulate the charging current of a cable system, the operation is quite different from that obtained on an inductive load. At 7.8 amperes, 13,800 volts, single-phase on a single-pole unit, the arc is always extinguished upon separation of the contacts. At 17.4 amperes, 13,800 volts, single-phase, the arc is extinguished as soon as the contacts separate; however, re-ignition frequently occurs one half-cycle later, when the generated voltage reverses and conspires with the condenser voltage to give double voltage across the breaker. This re-ignition may continue until the arc moves into the deionizing structure. In this case the time duration of the arc is very nearly the same as with an inductive circuit for a similar current.

Mr. Dean raised another question with reference to the ability of the Deion circuit breakers to withstand high voltages immediately after interrupting short-circuit current and suggested the advisability of using some sort of an air disconnect switch in series with the Deion circuit breakers to take voltage off the breaker when the circuit has been opened and the breaker is standing idle in the line. In response to this question I will say that in one of our earlier designs, the 2300-volt breaker, we did make use of an air disconnect switch which was mechanically operated by the operating mechanism of the circuit breaker itself. This switch was designed so that several half-cycles after the arc had been extinguished in the deionizing chamber, the switch was automatically opened, thus removing potential from the circuit breaker. After examining this type of construction for its ability to withstand voltage we found that the gaps between the grids which were required to operate at 140 volts r. m. s., when interrupting current, were capable of withstanding a 1000-volt potential test after deionization is complete. This immediately removed the necessity for using the disconnect switch.

Insulating the remainder of the breaker seems to be only a question of following conventional practise in that there is no inherent limitation involved. We, therefore, believe that it would be cheaper and easier to insulate the low-voltage breakers a little better and eliminate the complication arising from using the disconnect switches. There are, however, some values of voltage at which it might be easier to provide a disconnect switch mechanically operated by the circuit breaker mechanism than it would be to provide sufficient insulation to give the same factor of safety and with that in mind, each breaker design is gone over thoroughly from this point of view and a decision between the disconnect switches and straight insulation is made on an economic basis.

Mr. Spurck also raised a similar question concerning the ability of the circuit breaker to withstand voltage immediately after short circuits. I believe he had in mind conditions where there might be some residual ionization left from the arc, rather than the ability of the circuit breaker to withstand voltage at some considerable time after the circuit had been opened. We have tried to make tests to answer this question, but I am not certain that the tests which have been made will quite furnish the desired information. It is not feasible to apply a test voltage of several times operating voltage immediately after a short circuit has been cleared. However, in making tests in

accordance with the A. I. E. E. standards, where voltage is left on the line for several minutes after the short circuit is interrupted, we have found no signs of distress in the Deion circuit breaker. In addition to that we have, immediately after a series of short-circuit tests, disconnected the Deion circuit breaker from the power terminals and applied test voltage without finding the least indication of trouble. From theoretical considerations we have every reason to believe that the Deion circuit breaker is just as capable of withstanding voltage a few cycles after the short circuit has been interrupted, as it is several hours after the interruption. In other words, the deionizing process is certainly complete in much less than one second.

In recognition of the fact that insulation in this circuit breaker is dependent on creepage surfaces rather than puncture through oil as in the case of the oil circuit breaker, we did not feel content with making this breaker stand 2¼ times the rated volts plus 2000 volts, as specified by the A. I. E. E. standards, but have designed the 15-kv. Deion breaker for a potential test of 50,000 volts. We feel that with this additional insulation the Deion circuit breaker will be just as capable of withstanding voltage surges as any other piece of apparatus on the line.

From Figs. 13 and 14, Mr. McNairy has calculated that the average power dissipated in one unit of the breaker over a period of one half cycle, to be of the order of 20,000 kw. He has used this figure to substantiate the theory that there must have been some phenomena which resulted in a very considerable increase in the resistance of the arc while carrying relatively high current. This phenomenon we have chosen to call "arc voltage," which for the 15,000-volt Deion circuit breaker under discussion may be represented roughly as $E = 36{,}000 + 0.0951$ for currents above 2000 amperes, where $I$ is the instantaneous current in the arc. Fig. 17 might have been used as a more striking proof of the existence of an arc voltage. In this case the highest point, 600 kw-sec. for the 3 phases, is the equivalent of an average of 24,000 kw. per phase during the half-cycle of arcing. In contrasting the theory of operation of the Deion circuit breaker with that of other circuit breakers we have been concerned but little with what happens in the arc other than at the current zero, so long as the natural or forced zero, as the case may be, does not produce any surges on the system, or impair the circuit for future use.

A further exception has been taken to a statement to the effect that, with the exception of oil breakers, the only other method generally used for extinguishing arc in air has been by lengthening. We are still of the opinion that even though it would be difficult to prevent deionization (recombination) in the main body of the arc, it is necessary to use much more lateral restriction than has hitherto been used to obtain the same amount of deionization per inch of arc as has been obtained in using short series of arcs.

# Automatic Reclosing High-Speed Circuit Breaker Feeder Equipment for D-C. Railway Service

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Associate, A. I. E. E.

*Synopsis.—High-speed circuit breakers have been used in both feeder and machine circuits of railway substations. It is the purpose of this paper to point out some of the advantages gained by placing this type of breaker in the feeder circuit and also to describe some of the more common types of feeder equipment using a well known form of high-speed circuit breaker. A brief summary of the breaker characteristics is given, together with the results of short circuit tests made on 600-volt reclosing feeders employing this type of breaker.*

*The reclosing action is ordinarily obtained by means of a relay which, in conjunction with a load indicating resistor, measures the resistance of the external circuit. The action of this relay is purposely delayed, after the opening of the breaker, in order to permit feeder conditions to reach a stable value and particularly to enable all transient and counter e. m. f. effects to disappear before the reclosing devices are given control. The latter effects usually disappear within a few seconds of opening the breaker on the usual type of railway circuits. In a majority of cases a time delay of 10 to 30 seconds (after the opening of the breaker) elapses before the reclosing devices are given the control of the breaker. After this time delay has expired and load conditions have reached the desired value the circuit breaker is reclosed.*

*In some cases, especially for sectionalizing purposes, it has been found satisfactory to take the reclosing indication and control from the load side of the breaker. This feature requires that some other breaker undertake the re-establishment of voltage on the load circuit. By means of supervisory control the range of such equipment can be extended so as to pick up a dead section during abnormal operating conditions.*

*A majority of 3000-volt installations has been made in connection with main line electrification where the desired reclosing operation is a combination of load indicating and load limiting functions. Voltage increments on the load of too large a value may increase the tractive effort to such values as to cause wheel slippage of the locomotive or snapping of draw-bars. Consequently the load voltage is raised in graduated steps by progressively short circuiting portions of load limiting resistors placed in the feeder circuit.*

* * * * *

## Characteristics of the High-Speed Air Circuit Breaker

The type of automatic reclosing equipment described in this paper makes use of a well known magnetically held high-speed air-circuit breaker. A brief explanation of this type of circuit breaker, as illustrated in Fig. 1, may aid.

The breaker is a self-contained device. It is closed by means of a closing or reset coil which seats an armature against the pole faces of the holding magnet. During this operation the auxiliary contacts move between the positions corresponding to the open and closed positions of the breaker, but the main contacts do not close until the closing or resetting mechanism is returning to the open position. Due to this function, which is obtained by a suitable system of levers, etc., the breaker is free to trip during the closing operation.

A set of springs is attached to the moving contact. These springs exert a certain amount of opening force when the breaker is held in the closed position. The above mentioned armature is also attached to the moving contacts and holds the breaker in the closed position (once the armature is seated against the holding magnet pole faces) so long as sufficient holding flux passes through the armature.

The breaker may be opened in two ways; *viz.*, by decreasing the holding coil current or by shunting flux from the holding armature. By decreasing the holding coil current, a point is reached where the opening springs over-power the holding effect and the contacts consequently open. The breaker is tripped on overcurrent by means of a trip coil or bucking bar which is so placed in relation to the magnetic circuit as to shunt or transfer enough flux from the armature, thereby decreasing the holding effect and allowing the contacts to be opened by the opening springs. When the feeder current is gradually increased a point will be reached where the breaker is tripped in the above manner. This particular trip point is sometimes designated as the "steady" current trip point, as contrasted with a trip point where the line current is rapidly increasing.

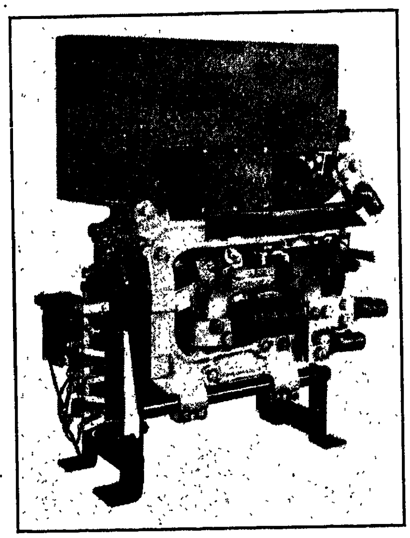

Fig. 1—600-Volt D-c. 2000-Ampere Magnetically-Held High-Speed Air Circuit Breaker

1. Switchgear Engg. Dept., General Electric Co., Philadelphia, Pa.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

By using a circuit in parallel with the trip coil or bucking bar, and proportioning the resistances and inductances of these circuits, it is possible to divide the currents by means of the resistances; when the line current is gradually increasing; and by having a relatively larger amount of inductance in the paralleling circuit or inductive shunt, it is possible to force a greater portion of the line current into the tripping circuit during rapid rates of current rise such as obtained on short circuits. This characteristic of lowering the trip point (in terms of line current) on rapid rise of current is sometimes referred to as "discrimination," since by virtue of certain load and circuit characteristics the breaker is able to trip on one value of line current when the load is gradually increasing and at another (lower) current value when a fault occurs. This feature results in decreasing the trip point of the breaker when it is most needed.

The "steady" trip point of the breaker is usually varied by changing the reluctance of the holding magnetic circuit. The amount of "discrimination" is varied by means of laminations placed on the external shunt.

The holding coil is connected in shunt with the source, which fixes the polarity of the holding flux. In order to shunt the flux, set up in the armature by the holding coil, the current in the tripping circuit must flow in a certain direction. If it flows in an opposite direction it merely tends to increase the flux in the armature as already set up by the holding magnet. Due to this relation of the magnetic circuits the tripping characteristic of the breaker may be said to be polarized and

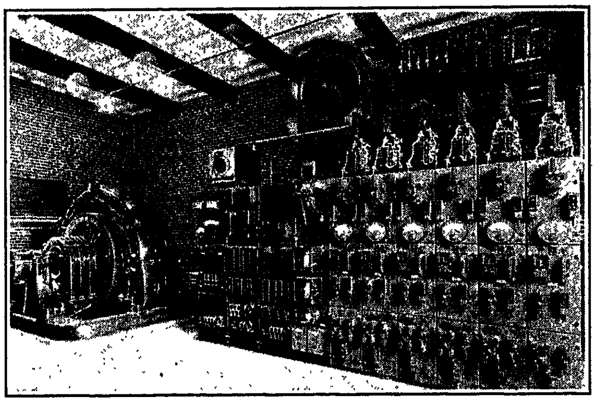

FIG. 2—AUTOMATIC RECLOSING HIGH-SPEED CIRCUIT BREAKER FEEDER EQUIPMENTS AS INSTALLED IN A 600-VOLT AUTOMATIC RAILWAY SUBSTATION

current must flow in a given direction in the line, in order to trip the breaker. Such a characteristic assists in obtaining selective tripping action when using certain system interconnections. A particular application using this characteristic is pointed out under a following heading.

The arc obtained when opening the breaker under load or faulty line conditions is extinguished by the well known combination of magnetic blowout and arc chute. The quick acting qualities of this type of breaker are obtained mechanically by using parts of small inertia combined with considerable force in the opening springs and electrically, by using a laminated structure around the tripping circuit, together with a suitable design of magnetic blow-out and arc chute.

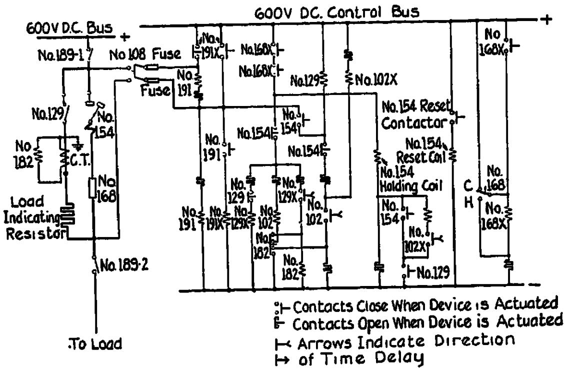


FIG. 3—ELEMENTARY CIRCUIT DIAGRAM OF AUTOMATIC RECLOSING FEEDER

For 600-volt service, using a periodic load indicating scheme, and suitable for stub-multiple feed in either direction

The mechanism which operates the auxiliary switches is arranged in such a way as to allow the breaker to open quickly without interference from the auxiliary switches.

## ADVANTAGES OF PLACING THE HIGH-SPEED CIRCUIT BREAKER IN THE FEEDER

Fig. 2 shows a typical installation of high-speed circuit breakers as part of reclosing feeder equipments in an automatic substation. By placing these breakers in the feeder circuit it is possible to remove any suddenly applied overloads or faults without any serious effect on the remaining feeders supplied from the station. The feeder handles its own circuit conditions without reflecting them back to the machine, which might cause the insertion of steps of machine load limiting resistors, reduce the voltage on the remaining feeders, and consequently decrease the car speed.

If conditions warrant the use of a high-speed circuit breaker in the machine circuit, together with corresponding breakers in the feeder circuits, it is possible to select the trip points and discrimination so as to permit the feeder breaker to trip first. This particular combination is not used frequently since it is felt that the high-speed breakers in the feeder circuits provide suitable protection to the machine under short circuit conditions, while the customary load limiting resistors afford sufficient protection on gradually increasing overloads.

There are conditions when the machines in an automatic substation are shut down, for example during light load periods, but the feeder breakers are closed so that the station bus becomes a tie point between the feeders, some of which may be feeding power into

the bus and the balance feeding outwardly. All feeder breakers in this case are connected so as to trip only on outgoing current. This inherent polarized tripping characteristic of the breaker has been described above. When a fault occurs on one of the feeders, only that particular feeder is tripped, since the remaining feeders are either carrying current below their trip points or else the current is flowing through their trip circuits in a reversed direction. This characteristic is of great

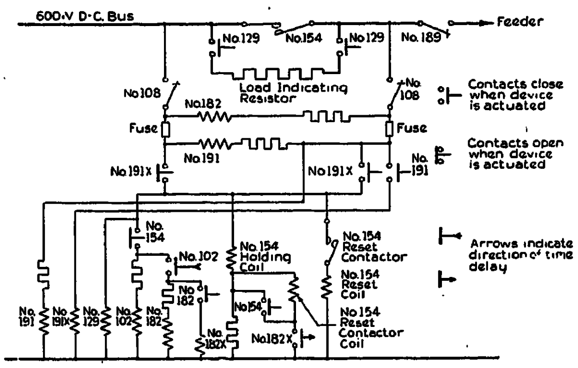


**FIG. 3—ELEMENTARY CIRCUIT DIAGRAM OF AUTOMATIC RECLOSING FEEDER**

For 600-volt service using a continuous load indicating scheme and suitable for stub-multiple feed in either direction

importance during conditions of through or back feed, when the machines in the station are shut down, or when these breakers are applied to cross-tie and sectionalizing points. When breakers having non-polarized tripping characteristics are applied to such points it will be found that it is usually necessary to rely on an excess current flowing through the breaker on the faulty circuit in order to obtain the desired tripping operation. If this condition is not fulfilled more than one breaker is apt to trip.

Other operating conditions may sometimes find two or more breakers in series (at different substations) with the fault current flowing through them in a direction to produce tripping. Due to the decreased voltage across the holding coil of the breaker nearest the fault, it will be found that its trip point is reduced below that of the breaker nearer the source. This results in tripping the breaker nearest the fault and isolates only that section of the system having the fault on it.

The use of a high-speed circuit breaker as contrasted with slower forms of circuit interrupters, results in a material reduction of the peak current of the faulty feeder. A comparison of the interrupting characteristics of common forms of air circuit breakers is given more fully in the comments pertaining to Fig. 8.

The trip-free characteristic of the high-speed breaker permits it to be closed on heavy overloads or faults where the reclosing relays are in the open position. The breaker may be closed in such cases (contrary to the indication of the reclosing devices) by means of supervisory or manual control. Where the breaker is not trip free it is desirable to have two breakers in series, one being closed before the other, thereby giving a trip-free combination.

The characteristics of this type of high-speed circuit breaker make it desirable for heavy city service and in addition for 1500- and 3000-volt applications. In some applications it is necessary to energize certain devices from a storage battery. When a storage battery is used the polarized tripping characteristic is still maintained, but the advantage of decreased holding coil voltage of the breaker nearest to the fault is lost. The number of cases where the storage battery has to be used, with a resulting sacrifice of the latter characteristic, is small and the advantages gained by a steady holding current and simpler devices, as in the case of 3000-volt d-c. circuits, appear to justify the use of a battery.

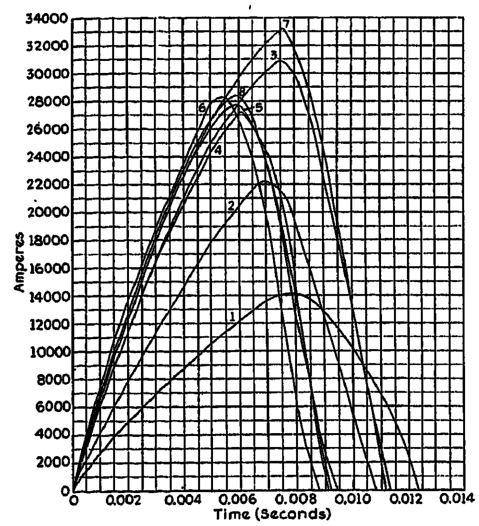


**FIG. 4—HIGH-SPEED AIR CIRCUIT BREAKER**

Opening a short circuit at 600-volts with different combinations of machines supplying power. Resistance of external circuit 0 002 ohm

| Curve No. | Kw. connected to station bus | Maximum rate of rise, amperes per second |
|---|---|---|
| 1 | 2,000 | 3,050,000 |
| 2 | 4,000 | 4,250,000 |
| 3 | 6,000 | 6,650,000 |
| 4 | 7,000 | 6,050,000 |
| 5 | 8,000 | 7,400,000 |
| 6 | 10,000 | 7,600,000 |
| 7 | 11,000 | 7,000,000 |
| 8 | 12,000 | 7,000,000 |

When long feeders of small cross-section are used, which appears to be confined principally to interurban systems as a class, the regulation of the feeder system may be so great as to cause unnecessary tripping of breakers nearest the load. Such regulation usually occurs when the machine in the substation is shut down, and an excessive drop is obtained over the positive and negative circuits under normal load conditions. However, such a system affects more devices than the mag-

netically held high-speed circuit breaker. In general, it will be found that whenever the regulation of the d-c. system is not excessive the above mentioned characteristics of the high-speed circuit breaker can be used to good advantage.

The value of circuit resistance between stations together with the size and type of load is useful in determining the application of this type of equipment.

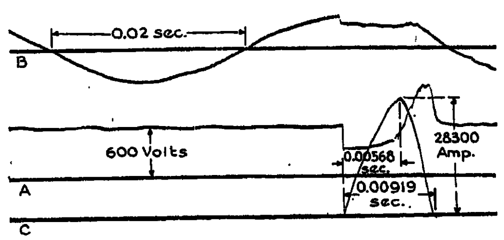


FIG. 6—OSCILLOGRAM OF SHORT-CIRCUIT CURRENT ON 600-VOLT D-C. RECLOSING FEEDER

Using a high-speed circuit breaker as shown in Fig. 1, 12,000 kw. of conversion apparatus connected to bus. External circuit resistance 0.002 ohm. Curve *A*, line volts; Curve *B*, incoming station volts; Curve *C*, d-c. feeder amperes

## DESCRIPTION OF 600-VOLT FEEDER EQUIPMENT

High-speed air-circuit breakers were first applied to d-c. reclosing service over five years ago. The construction of this type of equipment is illustrated in Fig. 2. The elementary circuit connections agree substantially with those shown in Fig. 3, the design in the latter illustration being a slight simplification of the original.

This type of feeder can operate on either stub or multiple feed and obtain its control current from either the bus or the feeder. The breaker, No. 154, is tripped on steady current, and has the discriminating feature as pointed out above. In addition, the feeder is provided with a thermostat, No. 168, which disconnects the outgoing circuit when the feeder cable has overheated. The thermostat resets after the circuit has been disconnected and sufficient time has elapsed for the feeder cable to cool.

The reclosing devices operate in accordance with the external circuit characteristics in somewhat the same manner as the circuit breaker, No. 154. A load indicating resistor of comparatively low ohmic value (usually 1 ohm) is periodically connected in shunt with the open breaker, No. 154. This connection is made by the isolating contactor, No. 129, and suitable timing relays so that a limited amount of current is supplied to the feeder circuit for load indicating purposes. A current transformer is connected in series with the load indicating resistor, and in the secondary circuit of the transformer there is connected the coil of the reclosing relay No. 182. If a short circuit exists on the feeder the rate of current rise in the load indicating circuit will be sufficient to operate the reclosing relay, No. 182, so that the circuit breaker, No. 154, will not close. After a certain time delay has elapsed, the above cycle is repeated, and kept up in this manner until suspended by manual or supervisory control, or the fault has disappeared. In case the fault disappears and the load is not excessive, the resulting rise of current is not sufficient to operate the reclosing relay, No. 182, and consequently the feeder breaker recloses.

Control power from either side is obtained by the voltage directional relay, No. 191, which is usually connected so as to close its contacts when the voltage of the feeder is slightly higher than that of the bus. This operation energizes the coil of No. 191-X, which in turn connects the control bus to the feeder side of the breaker. Under the normal operating condition the bus voltage is usually equal to or higher than that of the feeder, in which case the contacts of No. 191 are open and the coil of No. 191-X is deenergized. In this case, control power is taken from the bus side of the circuit breaker, No. 154.

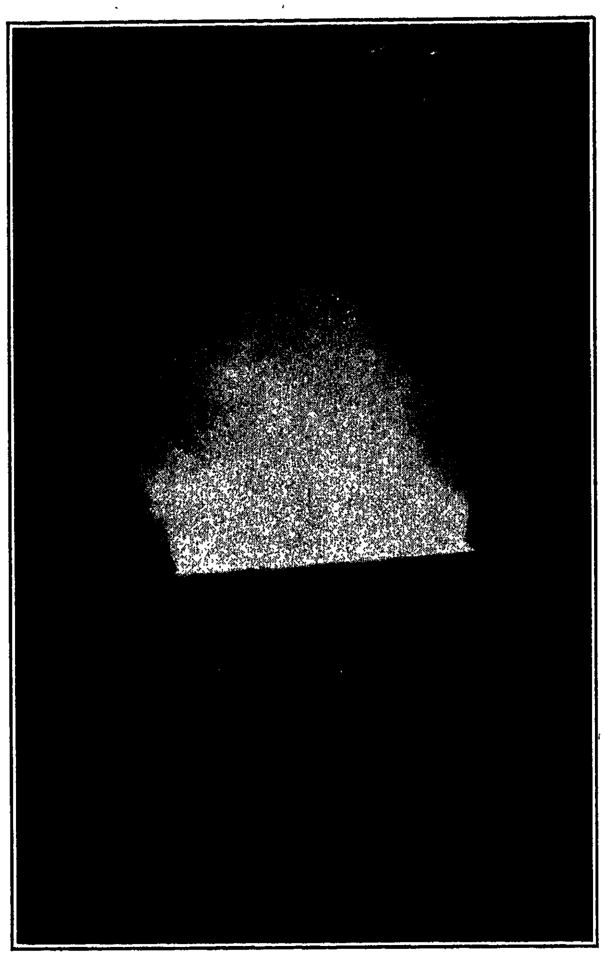

FIG. 7—HIGH-SPEED CIRCUIT BREAKER OPENING A SHORT CIRCUIT AT 600-VOLTS

Power being supplied by four 1000-kw. and four 2000-kw. 25-cycle converters. Photograph taken simultaneously with oscillogram as given in Fig. 6, (Maximum current 28,300 amperes)

Another design of reclosing feeder is illustrated in elementary form by Fig. 4. This feeder uses a continuous load-indicating scheme. In other words, the

load-indicating resistor is capable of having full operating voltage impressed continuously across its terminals. This condition is obtained during short circuit conditions on the feeder.

The reclosing relay No. 182, used in this class of feeder is of the so-called "balanced" type. The restraining (or opening) coil is connected across the load-indicating resistor. The actuating (or closing) coil is connected across the source of voltage. With a short circuit on the feeder, the restraining coil has full operating voltage impressed on it. This action causes the contacts of No. 182 to open and remain open. Sometime later, as determined by the setting of the time delay relay No. 102, the actuating coil is connected across the operating source. Both the actuating and restraining coils now have full operating voltage impressed across their respective circuits. The relay

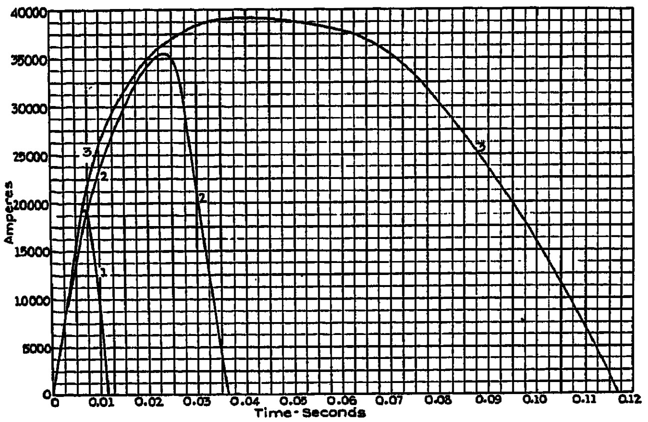


FIG. 8—COMPARATIVE TIME-CURRENT TRIPPING CHARACTERISTICS

Of correspondingly rated magnetically-held high-speed circuit breaker (Curve 1), mechanically latched circuit breaker with fast moving parts and magnetic blowout on arcing contacts (Curve 2), and mechanically-latched circuit breaker with carbon arcing contacts (Curve 3). Tests were made with the same external circuit conditions and with the same amount of connected power

(No. 182) is so adjusted that under this condition the restraining coil overpowers the actuating coil and the relay contacts remain open. As the load resistance increases, the voltage drop across the load indicating resistor decreases. This decrease also takes place across the restraining coil. Consequently, a predetermined load value is reached where the actuating coil overpowers the restraining coil and the relay contacts close. The latter operation energizes an auxiliary relay, No. 182-X, which in turn closes the circuit breaker, No. 154. The reclosing relay, No. 182, is so adjusted that it operates at a fixed voltage ratio over a wide range of operating voltage. By this characteristic the feeder recloses on a fixed value of load resistance, regardless of the value of the source voltage. In case another station is supplying the load (multiple feed) the feeder recloses when the same voltage ratio is obtained.

Control power is obtained from either side by means of the voltage directional relay, No. 191, and its auxiliary, No. 191-X. An isolating contactor No. 129, is used to disconnect the load indicating resistor when the feeder is taken out of service, by opening the control power switch No. 108. The maximum amount of current taken by the resistor is usually greater than that which can safely be interrupted by No. 108.

### RESULTS OF CURRENT INTERRUPTING TESTS ON 600-VOLT RECLOSING FEEDER

The number of applications of reclosing feeders to 600-volt railway systems is obviously in excess of those at the higher voltages. Consequently, the number of tests made at 600-volts exceeds those at 1500 and 3000 volts. A few of the results of current interrupting tests might be briefly pointed out, especially those on the 600-volt systems. Tests have been made at the other voltages in order to establish the correctness and suitability of the design, but these tests were not carried to the extent of those made at 600 volts.

An extensive set of tests was made on a street railway property using the type of feeder as illustrated in Fig. 2. One feeder was installed in a station where the connected synchronous converter capacity (25 cycles) could be varied, in certain steps, from 2000 to 12,000 kw. One set of tests was made in which the short circuit was applied immediately outside the station, giving an external circuit resistance of 0.002 ohm. The results of these tests as given in Fig. 5 show that the peak current increased as the connected machine capacity increased from 2000 to 6000 kw. From 6000 to 12,000 kw. there was no corresponding increase, in fact most of the peak values were slightly under the peak value corresponding to the 6000 kw. condition. These tests show that as the generating or conversion capacity of the station increased from 2000 to 6000 kw., the peak current was largely determined by certain characteristics of the machines. Beyond the 6000 kw. condition, the characteristics of the external circuit predominated, so that the amount of connected machine capacity, beyond this point, no longer served as an indication of the maximum current to be expected when interrupting the short circuit with a high speed circuit breaker. With a short circuit more remote from the station it was found that the peak value was obtained at 4000 kw.

Fig. 6 is reproduced from an oscillogram taken with 12,000 kw. connected to the bus. The feeder current curve, $C$, corresponds to curve No. 8 in Fig. 5. The maximum rate of rise of current in these tests was approximately 7,000,000 amperes per second. Fig. 7 gives an indication of the amount of arc obtained when interrupting the current as given in Fig. 5, curve No. 8, and Fig. 6. This photograph was taken during the same time that the current was being recorded by the oscillograph. The above series of tests was made by having the breaker in the closed position and then applying the short circuit by another device. In other

words, the breaker operated on a *C O* cycle, which represents the normal interrupting cycle for reclosing service. This same type of breaker, as illustrated in Fig. 1, has successfully interrupted currents as high as 61,500 amperes at 600 volts, on a *C O* cycle. In this particular set of tests the total external circuit resistance was 0.001 ohm and the connected machine capacity consisted of four 4000-kw. 25-cycle synchronous converters. The maximum rate of rise in the latter set of tests was approximately 14,000,000 amperes per second.

The comparative time-current tripping characteristics of the high speed and other more common forms of circuit breakers may be obtained from Fig. 8. These tests were made at 600 volts, with the same external circuit conditions and with the same amount of connected conversion apparatus. Curve 1 shows the time-current tripping characteristic of the high-speed circuit breaker as illustrated in Fig. 1. Curve 2 is the corresponding characteristic of a relatively fast panel-mounted mechanically latched circuit breaker having a magnetic blowout associated with the arcing contacts. Curve 3 illustrates the corresponding characteristic of the usual panel-mounted air circuit breaker (mechanically latched) having carbon arcing tips.

The ratio of the overall times of these breakers (from 0 to 0 current) is approximately in the order of 1:4:12. Curve 2 shows how the overall time of the usual panel-mounted device can be decreased by making it faster mechanically, so that its tripping mechanism is quickly released and the moving parts start sooner, together with using a magnetic blowout to extinguish the arc in a shorter time. The type of breaker, whose characteristic is covered by curve 2, is fast enough so that its tripping mechanism is released during the opening operation of the high-speed circuit breaker. The latter time, however, is not sufficient for the slower device to operate and extinguish the arc, but does permit it to unlatch and drop open shortly after the faster device has interrupted the circuit.

## Description of 1500-Volt Feeder Equipment

High-speed circuit breakers have been applied to 1500-volt reclosing service. Such an equipment, as illustrated in Fig. 9, makes use of the continuous load indicating scheme as outlined in connection with Fig. 4. As installed, the high-speed circuit breaker is mounted in back of the panel on suitable supports, so that ample clearance is obtained in the vicinity of the arc chute. The design and operation of the 1500-volt high-speed breaker agrees substantially with that of the 600-volt breaker, as may be seen by comparing Figs. 1 and 9.

The control of the equipment illustrated in Fig. 9 is so arranged that if a disconnecting switch is accidently opened while the feeder is carrying current, the corresponding high-speed breaker will open. This operating sequence is obtained by means of suitable auxiliary switches on the disconnects, which are opened when the disconnecting switch is being opened, thereby de-energizing the holding coil of the high-speed circuit breaker. This particular feeder is arranged so that it can be opened by supervisory control and have the reclosing function restored by the same means.

The load indicating resistor is shown to the left of the panel, while the resistors for the various 1500-volt coil circuits are shown on a frame swung out to the right of the panel.

A number of these reclosing feeders are in service and are giving successful results.

Another application of this type of equipment has been made to cross-tie and sectionalizing points. Fig. 10 illustrates a typical system consisting of three generating stations, designated as Nos. 1, 2, and 3,

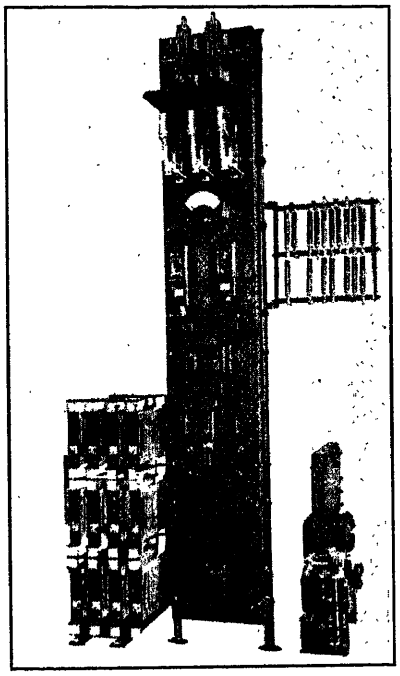

Fig. 9—1500-Volt, 2000-Ampere D-C. Reclosing Feeder

**Using a high-speed circuit breaker and a continuous load-indicating scheme. Circuit breaker and load-indicating resistor shown respectively, to the right and left of the panel**

feeding four track circuits. At convenient points along the system there are installed certain cross-tie and sectionalizing busses. This arrangement results in confining the fault to a particular section without a loss of service on other sections and, in addition, permits the feeder and trolley copper to be used to advantage during normal operating conditions.

If a short circuit should occur on Track No. 1 in the section fed from Station No. 2, not only will the corresponding feeder breaker at Station No. 2 be tripped, but also the two breakers which feed each end of the faulty section from the adjacent cross-tie and sectionalizing busses will be tripped. The selective operation of the breakers at these points is obtained by having them trip on outgoing current only, which is readily available due to the polarized tripping characteristics of the type of high-speed breaker covered by this

paper. Another characteristic used advantageously in such an installation is the reduced holding coil voltage of the breaker nearest the fault. This characteristic together with current flowing in the proper direction gives the desired selectivity. Breakers on a remote bus section distribute the current between themselves and even though the current flowing through them is in the proper direction for tripping, it is usually found that this current is considerably less than that through the breaker feeding the fault. In addition, the voltage at the remote bus is much higher than that of the busses adjacent to the fault so that the trip point is not

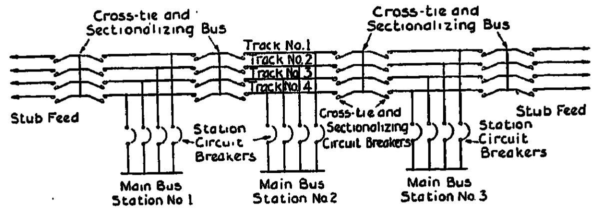


FIG. 7—TYPICAL ONE-LINE DIAGRAM SHOWING APPLICATION OF CROSS-TIE AND SECTIONALIZING CIRCUIT BREAKERS

To a four-track system. Current must flow away from cross-tie and sectionalizing bus in order to trip circuit breaker on faulty section

lowered by the same amount as that of the breakers at each end of the faulty section.

The breakers usually reclose when voltage has been re-established for a definite time on the faulty section, either by the station breaker or by the sectionalizing breaker at the other end of the section. By means of supervisory control it is also possible to reclose the sectionalizing breaker from its corresponding bus. Due to the fact that no load indicating equipment is used under this condition and that the breaker may consequently close on a fault, the control is so arranged that the breaker can close only once when supervisory control calls for closing from the bus. If this provision were not made, it would be possible for the breaker to "pump" due to a fault, while the supervisory devices were holding their "closing" indication. If the breaker does trip, it is necessary for the load dispatcher to turn the supervisory control to the "trip" position and then call for closing again. This method of control has been used for a number of years in connection with oil circuit breakers operated in conjunction with supervisory systems.

The stub feeders at the end of the system obviously obtain their control from the bus at all times. Those feeding in multiple with other feeders can reclose from the bus (as mentioned above) or in response to voltage restoration, on the track circuit. The normal reclosing operation is on voltage restoration. Provisions are made in the control so that if a breaker is reclosed from the bus, thereby energizing its line, the connection for reclosing on voltage restoration can be made without dropping out the breaker. This permits a transfer in the reclosing indication without an interruption to the load.

Fig. 11 illustrates a truck-mounted unit for 1500-volt cross-tie and sectionalizing service. The breaker is located in the upper portion of the truck where suitable clearances and barriers are provided. The covers of the 1500-volt relays, which are mounted on the panel at the front of the truck, are arranged so that they cannot be removed until the truck is withdrawn from the housing. Accidental contact with the 1500-volt circuit is averted by means of the latter arrangement and, in addition, by placing the balance of the 1500-volt devices in back of the steel panel.

DESCRIPTION OF 3000-VOLT FEEDER EQUIPMENT

The application of reclosing feeders to 3000-volt d-c. railway circuits includes a number of problems not encountered at the lower voltages. Principal among these is the fact that this voltage has been applied to main line electrification with long trains and also with longer distances between stations. Tests have been

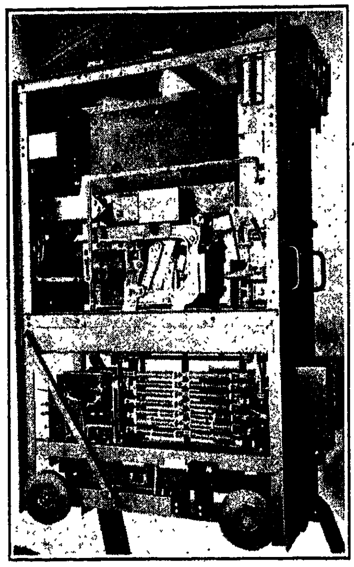

FIG. 11—SIDE VIEW OF TRUCK-MOUNTED 1500-VOLT 2000-AMPERE EQUIPMENT

With high-speed circuit breaker, for cross-tie and sectionalizing purposes (Barriers removed)

made in the field, showing that if too large a voltage increment is placed on the locomotive there is danger of suddenly increased tractive effort which will cause the wheels to slip or the draw-bars to snap.

The investigation of this problem has led to the development of a combined load-indicating and load-limiting feeder. The load-indicating resistor determines whether a short circuit exists on the feeder and if load conditions warrant reclosure. If so, the load-indicating resistor is short circuited, leaving the lower ohmic-value load-limiting resistor in circuit. The purpose of the latter resistor is gradually to raise the feeder voltage by suitable increments.

This sequence is accomplished by using the connections and equipment given in Fig. 12. Two high-speed breakers, Nos. 172 and 173, are used in the feeder circuit. At this voltage a high-speed breaker has about the same cost as that of any other interrupter that may be used as a resistor shunting device, No. 173. Upon the occurrence of overload either No. 172 or No. 173 opens, followed by the opening of the other breaker. After a time delay the load indicating devices come into

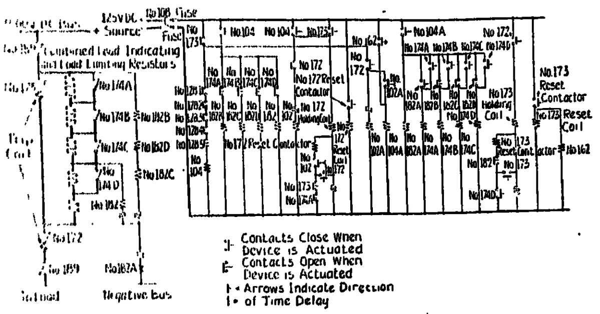


FIG. 12—ELEMENTARY CIRCUIT DIAGRAM OF 3000-VOLT COMBINED LOAD-INDICATING AND LOAD-LIMITING RECLOSING FEEDER

With resistors arranged to apply voltage in graduated steps

action and, if conditions warrant, the load indicating resistor is short circuited by device No. 174-A, which in turn is controlled by the reclosing relay No. 182-A. Four steps of load limiting resistance now remain in service. Three of these steps are progressively short circuited by suitable contactors controlled by the reclosing relays Nos. 182-B, 182-C, and 182-D. These particular relays are so interlocked that one cannot operate until the previous one has functioned at its particular value of drop out voltage. In other words, the relays close the contacts progressively when the difference of voltage across the open breaker terminals is less than a certain amount. These amounts become smaller as the last steps are being short circuited. The last relay in the sequence, No. 182, recloses the high-speed circuit breaker, No. 173.

In this way the load limiting portion of this type of feeder serves as an accelerating resistor. The most severe condition is one where the locomotive is in full parallel outside the station and is being fed from the remote station. The difference in voltage across the open terminals of the reclosing feeder breaker, is equal to the drop in the positive and return from the remote station. Such differences in voltage may be as high as 1200 volts. Under these conditions the reclosing feeder has to decrease this difference in voltage, by suitable steps, to zero or, in other words, raise the locomotive voltage from 1800 to 3000 volts.

In order to proportion suitably the resistors used in the load limiting portion of the feeder it is necessary to know not only the size and type of the load, together with other operating conditions, but also the resistance between stations, in the positive and return circuits.

Extensive tests have been made on this type of feeder. A typical train load of suitable tonnage was first supplied through a feeder having sufficient resistance to give the effect of the feeder and return of the remote station. The reclosing feeder was permitted to operate when the train was immediately outside the station, thereby giving the greatest difference in voltage across the reclosing feeder terminals. Fig. 13 is typical of the results obtained from these tests. Curve *A* is the current supplied from the remote station. Initially no current is being supplied by the nearby station. As soon as the combined load-indicating and load-limiting resistors are connected in the line, a slight current is obtained from station B, and a corresponding decrease at A. The next step in the curves is due to short circuiting the load indicating resistor. This action is followed by the progressive short circuiting of the steps

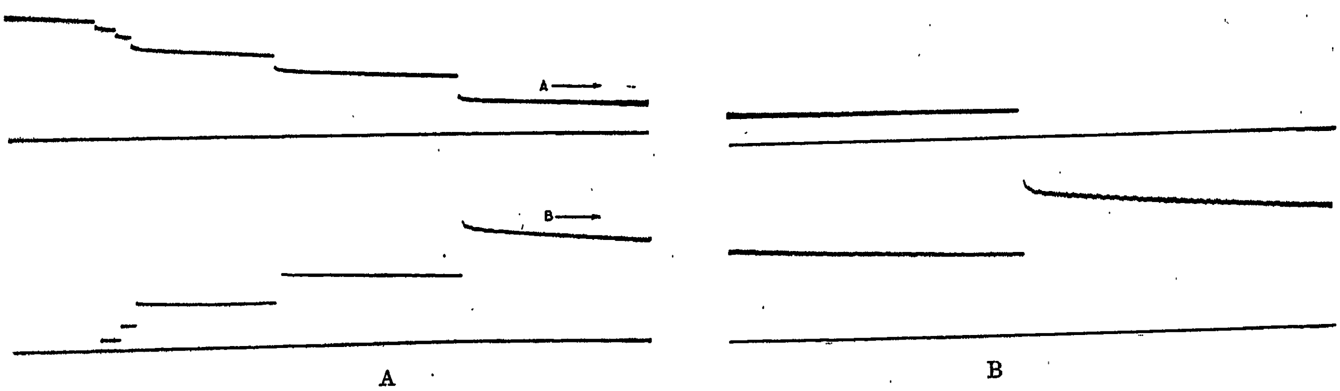


FIG. 13—OSCILLOGRAM SHOWING THE MANNER IN WHICH A 3000-VOLT LOAD-INDICATING AND LOAD-LIMITING RECLOSING FEEDER

Goes into service and takes over the load initially supplied from another source. Curve *A*, current supplied to load from remote source. Curve *B*, current supplied to load from nearby source. Time progress from left to right

of load limiting resistor, attended by increases in current from station B and corresponding decreases from station A. When the last step of resistance is short circuited, curve A decreases to zero and station B takes the entire load, since the load is practically outside the station in which the reclosing feeder is located. In this set of tests the controller was in full parallel. No jolts took place in the acceleration of the locomotive although it was apparent on the locomotive voltmeter that the voltage was being raised.

The resistors are protected by thermostats which disconnect the feeder and suspend further reclosing operation until the resistors have cooled and the thermostats have reset.

In the foregoing text the reclosing characteristic of the feeder is given more consideration than that of the high-speed circuit breaker. It is felt, however, that the characteristics and advantages of the high-speed circuit breaker as pointed out under other headings do not require repetition at this point. The protection afforded to the 3000-volt machines makes it a valuable part of the reclosing equipment.

## Discussion

**Chester Lichtenberg:** This paper indicates that the high-speed breaker is particularly suited for railway service on d-c. circuits of 1500 volts and above and for a-c. service up to 15,000 volts.

The high-speed circuit breakers described have been in use for about fifteen years. They consist essentially of a pair of contacts which in opening project an electric arc into a chute. Under the influence of a magnetic field that electric arc is distended and the circuit is interrupted.

The high-speed circuit breaker is quite different from other breakers in that it has certain unusual time characteristics. The most prominent, of course, is the short time between the initiating of the opening impulse and the time the circuit is interrupted. This period is interesting to examine in detail. First, there is a fixed time depending upon the operation of certain relaying equipment. This can be made a relatively short time. Then, there is the time for the mechanical parts of the breaker to go through the performance of unlatching, whether the retention be mechanical or electrical. This is followed by the time for the contacts to part, the time for the arc to be sprung, and the time for the arc to be distended until the circuit is interrupted. All of these times in the high-speed type of breaker have been reduced. It responds and opens in about one cycle of a 25-cycle current.

Mr. Anderson has given in great detail some very interesting control circuits for these high-speed circuit breakers. They may appear to be rather simple, but as he points out in connection with the tripping and reclosing of a circuit which feeds a resistor load like lamps or heaters, sometimes a little change in the circuit makes a big change in the operation.

The earlier high-speed circuit breakers were used for tripping loads irrespective of the reclosing features. In the latter designs, it has been made apparent that not only must the breaker trip rapidly but it also must reclose as soon as the circuit condition makes reclosure safe.

In some of the earlier reclosing designs, the circuit breaker was tripped and reclosed repeatedly through a resistance to try the circuit. In the latest designs the circuit is opened and left open until the feeder circuit has been found to be in a sufficiently good condition to permit reclosure and the breaker to stay reclosed.

**A. E. Anderson:** The description of the high-speed circuit breaker at the beginning of the paper is rather brief, but those who would like to obtain more detailed information concerning the design of the circuit breaker may find certain papers, by Messrs. J. W. McNairy[1] and J. F. Tritle,[2] which have been presented at meetings of the Institute in the past, of interest.

A question has been raised on the widespread use of high-speed breakers in the feeder circuits. The principal advantages of the high-speed circuit breaker in feeder circuits may be covered by 1500- and 3000-volt applications. On such systems, it is practically necessary to have the high-speed circuit interrupter.

The amount of maintenance required for this type of high-speed circuit breaker is not necessarily excessive. Some of the earlier forms have been in service for ten years without any considerable amount of maintenance, and the present design, as covered by this paper, has been in service for approximately eight years without any unnecessary amount of maintenance being required.

Fig. 8 shows the comparative time-current tripping characteristic of three common types of circuit breakers on d-c. short circuits. The a-c. type of high-speed circuit breaker will open the short-circuit current in one cycle or less on a 25-cycle system.

1. A. I. E. E. Trans., Vol. XLV, 1926, p. 962; Vol. 47, October 1928, p. 1276.
2. A. I. E. E. Trans., Vol. XLI, 1922, p. 262.

# Uses of Radio as an Aid to Air Navigation

BY J. H. DELLINGER[1]

Non-member

*Synopsis.—Use of radio for guiding airplanes along fixed airings during fog or other conditions of low visibility is the principal topic of this paper. A directive radio beacon system is described together with a receiving system which gives simple and direct visual indications of the location of the airplane. Methods of air navigation on other than established courses are also discussed as well as simple radio communication between plane and ground.*

* * * * *

THE possibilities of radio as an aid to flight are being actively developed. This development includes the following lines:

A. Communication
B. Course navigation
C. Field localizing
D. General

Under D are included miscellaneous developments, for example, the use of radio methods in connection with altimeter devices. This paper is largely devoted to item B, presenting a successful system of guiding airplanes along fixed airways during fog or low visibility. I shall speak particularly of the work of the Bureau of Standards because I am most familiar with that. It is, however, only one of many organizations pursuing active work and making contributions in this field. These organizations include transport companies, communication corporations, and research organizations, as well as government departments.

At the present time the transportation of passengers by air is far from the ideal service expected in the future. Genuine service of interest to the public can hardly be said to be available until the air traveler can count on a scheduled service as regular as the railway trains, independent of weather or other contingencies. The present nullification of the most essential feature of the air passenger travel as a serious service arises entirely from the hazards of weather. As we shall doubtless learn from other accounts of progress in the various phases of aviation, all other limitations are in a fair way to be overcome. Airways and airports are being provided in abundance, aircraft of adequate strength and stability are more and more available, every provision of comfort and convenience is offered the air traveler, and yet air traffic can still be halted when low visibility prevents the pilot from seeing his landmarks or lights on the ground.

It is impossible to exaggerate the solitude and helplessness of an airplane flying in dense fog. Deprived of all landmarks, under incessant strain at the controls to maintain equilibrium and direction, the aviator must frankly abandon dependence upon his senses and navigate according to the information conveyed by his instruments. It is contrary to all human instinct to throw overboard the testimony of the senses and stake life itself on a mute instrument dial. Not every pilot can do it, and unquestionably the oceans hide the sad remains of more than one hero whose only mistake was failure to learn "instrument flying" before he essayed the great adventure.

By means of the familiar instruments such as the altimeter, turn indicator, and compass, a pilot can continue flying in fog, but it is only by radio means that he can be certain to keep on a given course and find his landing field when the ground is invisible. Accurate as a compass may be, it cannot tell the pilot how much he is drifting sidewise due to cross winds, nor what actual progress he is making forward because of the unknown effects of head or tail winds. Unless radio aids are used, fog always brings the hazard of getting off the desired course into unfamiliar or dangerous areas, and also makes even the possibility of a landing unknown.

By radio means, however, particularly by the use of the radio beacon system which is being established on the airways of the United States, the pilot can, regardless of fog, keep accurately on his course, know the points he is flying over, and proceed unerringly to the landing field. This, I believe, largely destroys the menace of fog. When this system is fully established there is every reason to believe that the last great obstacle to safe flying will have been conquered, scheduled flights will be dependable, and passenger flying can be considered established as a serious service.

*Directional Radio Off the Airways.* Before describing the radio beacon system I should like to indicate briefly the possibilities of navigation by radio on other than the established airways. The beacon system will mark out the airway routes but will give no aid to the flyer on an independent course. There are several ways in which radio can be adapted to this navigational need.

One is the system used in Europe: radio direction-finding stations are maintained by the governments at

1. Chief of Radio Section, U. S. Bureau of Standards, Washington, D. C.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*



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various airports, and each airplane carries both a transmitting and a receiving set. Upon request by radio from an airplane, two or more of the direction-finding stations determine the direction of travel of radio waves from the airplane; combining their determinations they calculate the airplane's position and send this information by radio to the airplane.

A second means of radio navigation for the independent flyer is the use of a radio direction finder on the airplane. By steering a course in the indicated direction of a radio station on the ground, the airplane can be certain of reaching that point, the accuracy of the indicated direction increasing as the objective is approached. Direction finders are used extensively as a navigational instrument on marine vessels and on lighter-than-air craft. On airplanes their use is difficult. It is considerably more difficult to protect them from error and disturbances caused by the engine ignition and other sources aboard an airplane. They have been used successfully to some extent, and their use will doubtless increase; they do, however, require expert handling. This method of navigation has the inherent limitation that it does not prevent wind-drift from shifting the air-plane off its course; the method does eventually bring the airplane to its destination, although by a circuitous route if there is a side wind.

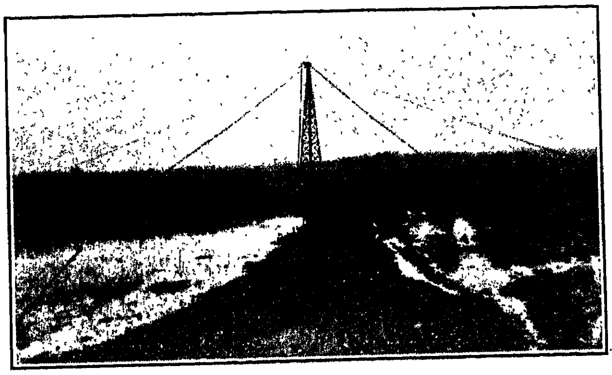

Fig. 1—The First Radio Beacon Tower on the Civil Airways

Experimental station of the Bureau of Standards at College Park, Md.

A third method of furnishing navigational aid to the independent flyer is the rotating radio beacon. This is a radio transmitting station, located at an airport, which has a rotating directive antenna. This causes a sort of beam of radio waves to sweep constantly around. A special signal indicates when the beam sweeps through the north. A pilot listening for this beacon's signal with his receiving set can determine his direction by the time elapsing between the north signal and the instant when the beam is heard with maximum (or minimum) intensity. The elapsed time is determined by means of a stop-watch, which can be calibrated to read direction.

*The Airway Radio Beacon.* The radio beacon system for the United States airways has been designed to operate with the minimum of apparatus and attention on the airplane. The objective in its development was to place a simple visual indicator on the airplane instrument board to tell the pilot whether he is on the course or how far off, which should operate without any effort or attention by the pilot. This has been successfully accomplished, and navigation over the official air routes thus has the advantage of a superior means of radio navigation not available to the independent flyer off those routes. The three methods of radio navigation for the independent flyer described in the foregoing require the pilot to listen with headphones through the roar of noise on the airplane. Also, each of them requires other apparatus besides a radio receiving set; thus in the first method the airplane must carry a radio transmitting set, in the second a direction finder, and in the third a special type of stop-watch.

While the radio beacon system for the airways attained practical development only this year, its origin goes back to 1920. At the request of the War Department, the Bureau of Standards undertook to develop a directive radio system for airplane navigation. A method was devised in which radio waves were transmitted alternately from two directive antennas placed at an angle with each other. Equality of signal intensity from the two antennas along a certain line or zone determined a course which an airplane could follow. The system was tried out successfully in Washington and in Dayton, Ohio. In succeeding years the Army engineers at Dayton developed the system further.

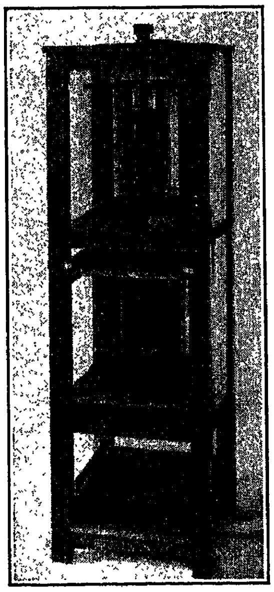

Fig. 2—Goniometer used in Radio Beacon Station

To orient the course marked out by the beacon in any desired direction.

When the Aeronautics Branch was formed in the Department of Commerce in 1926, it determined that radio aids would be necessary on the civil airways, and

assigned their development to the Bureau of Standards. As part of this work, the Bureau undertook to perfect the radio beacon, particularly by developing a visual indicator so that a pilot would have a direct indication, on his instrument board, of his location.

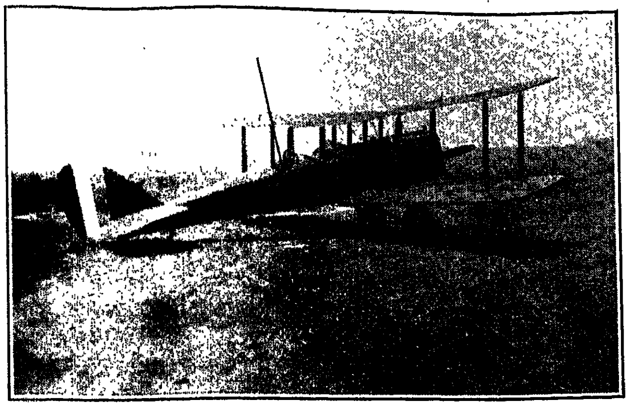

FIG. 3—AIRPLANE SHOWING 10-FOOT VERTICAL ROD ANTENNA

The required radio equipment on the airplanes is reduced to a short pole antenna and a simple receiving set weighing a few pounds, plus the indicator on the instrument board which tells the pilot whether he is on the course or how far off. All of the expensive and powerful apparatus necessary for the system is on the ground, maintained by the Government.

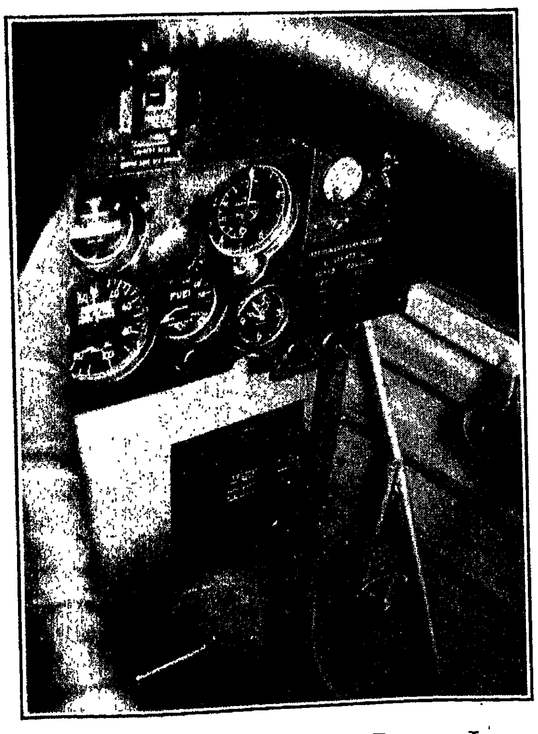

FIG. 4—INSTRUMENT BOARD WITH BEACON INDICATOR MOUNTED ABOVE THE OTHER INSTRUMENTS

The radio beacons operate in the frequency band 285 to 315 kilocycles. Airway radio telephone stations are to communicate with airplanes in flight, in the band 315 to 350 kilocycles. These are allocated to air service by the 1927 International Radio Convention. For the present the beacons are adjusted to the frequency of 290 kilocycles, and the telephone stations to 333 kilocycles.

The directive radio beacon is a special kind of radio station, usually located at an airport, just off the landing field. Instead of having a single antenna like an ordinary radio station, it has two loop antennas at an angle with each other. Each of these emits a set of waves which is directive, *i. e.*, it is stronger in one direction than others. When an airplane flies along the line exactly equidistant from the two beams of radio waves, it receives signals of equal intensity from the two. If the airplane gets off this line it receives a stronger signal from one than the other.

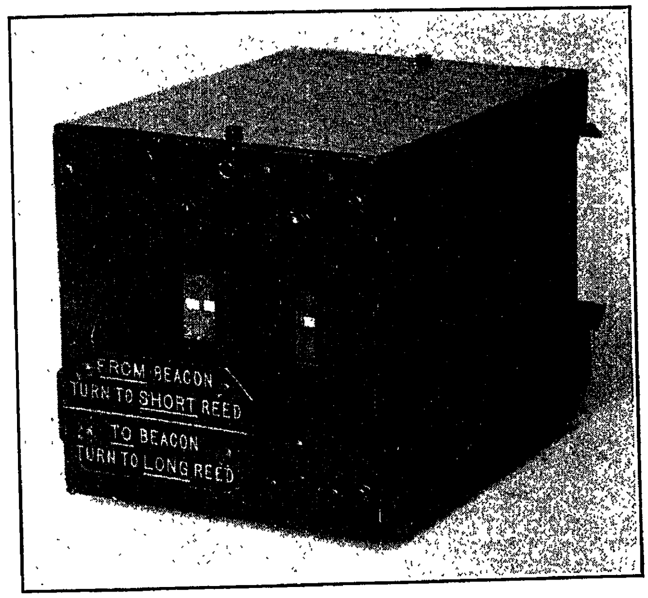


FIG. 5—VISUAL BEACON INDICATOR

**Comprising the pair of reeds to show deviations to either side of course and single reed (at right) which indicates when airplane is passing over a marker beacon**

The current in the two antennas is of exactly the same frequency, but is modulated at a different low frequency in each, *i. e.*, the current in one antenna has a tone of 65 cycles impressed on it, and the current in the other antenna has a tone of 85 cycles impressed on it.

The indicator on the instrument board of the airplane shows when the signals from the two beams are received with equal intensity, by means of two small vibrating reeds. When the beacon signal is received the two reeds vibrate. The tips of these reeds are white in a dark background so that when vibrating they appear as a vertical white line. The reed on the pilot's right is tuned to a frequency of 65 cycles and the one on the left to 85 cycles. It is only necessary for the pilot to watch the two white lines produced by the vibrating reeds. If they are equal in length, he is on his correct

course. If the one on his right becomes longer than the other, the airplane has drifted off the course to the right. If he drifts off the course to the left, the white line on the left becomes longer. Thus if the pilot leaves the regular course either accidentally or to avoid a stormy area, the radio beacon will show him the way back.

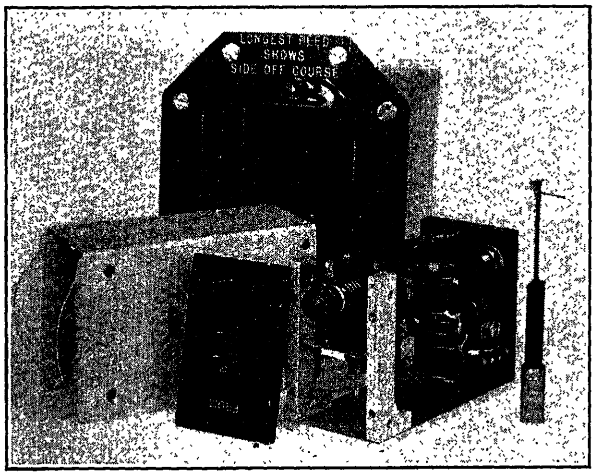


FIG. 6—INTERIOR OF BEACON INDICATOR

**Showing electromagnets, pair of reeds in place, lamp, and detail of detached reed**

The whole receiving system comprises a small indicator unit on the instrument board weighing one pound, a receiving set weighing less than 10 pounds, and a 10-pound battery. The same receiving set can be used to receive radiotelephone messages, by plugging in a pair of headphones. The receiving system is very little affected by interference, including static, other radio stations, and airplane ignition interference, which has hitherto been the bar to satisfactory use of radio on airplanes.

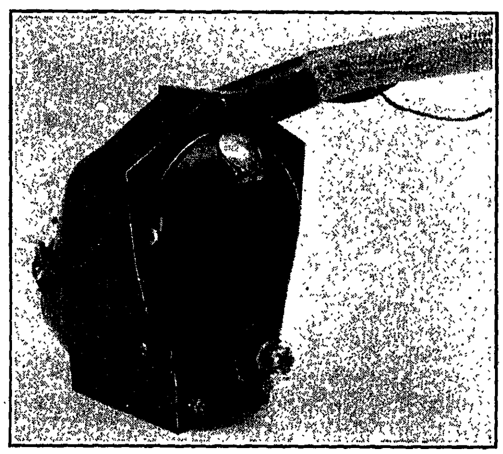

FIG. 7—COMPLETELY SHIELDED MAGNETO

**Illustrating how all parts of the electrical circuits on the airplane have to be enclosed in a a grounded metal sheath**

The beacon stations will probably be placed at airports in general averaging about 200 miles apart. The Airways Division of the Department of Commerce Aeronautics Branch has begun a program of installing them on the various airways. The directive beacons, with a straight airway between them, will be supplemented by small marker beacons at intervals (perhaps 20 miles) along the route. These are simply very low-power radio transmitting stations serving as mile-posts. A characteristic signal from a marker beacon will show on the visual indicator aboard the airplanes what point is being flown over.

Thus the radio beacon system guides the airplane along the airway regardless of fog, informs the pilot of the distance passed over, and brings him to the landing field. There are two other services which directional radio can eventually perform to complete the conquest over fog, the providing of a field localizer and a landing altimeter, *i. e.*, to mark out clearly the landing area and to indicate distances above ground in the act of landing. While it is not yet certain whether radio or other methods will be the best means to provide these two services, the need of a field localizer is already partially met. When the pilot arrives at the radio beacon station and flies over it, there is a sudden deflection of his indicator which enables him to ascertain the location of the radio beacon station within 100 feet. This is accomplished by virtue of the peculiar properties of the vertical pole antenna on the airplane, and is of material assistance when landing during poor visibility.

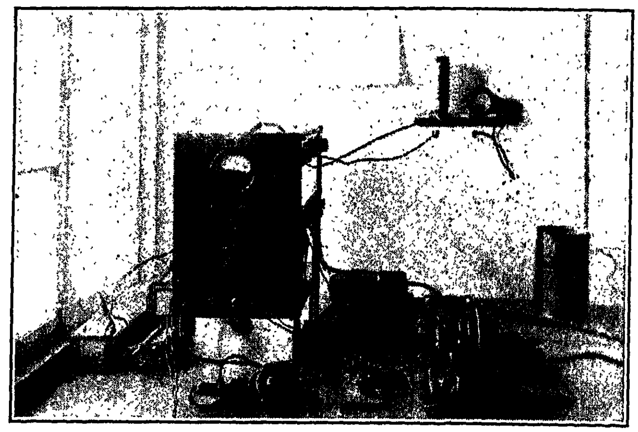

FIG. 8—TRANSMITTER FOR MARKER BEACON

**A 10-watt transmitter located at intermediate points between airports to show distance flown along the airway**

The practicability of this system, both for course navigation and field localizing, may be illustrated by a recent trial flight. On a day of low visibility, a pilot unfamiliar with the route took the air in Philadelphia for Washington with no maps or instructions as to landmarks; he was told to proceed to Washington (a distance of 120 miles) and land at College Park field solely in accordance with the guidance given by the beacon indicator on his instrument board. He not only flew in a straight line to Washington, but when over College Park field, which he had never seen before,

the special deflection of the indicator told him he was at his journey's end, whereupon he landed.

Valuable as directional radio is, it is perhaps not as fundamental a service to the aviator as simple radio communication between airplane and ground. On the United States civil airways, radio telephone stations are being installed to inform the pilots of weather and landing conditions. This instantaneous service is a powerful addition to flight safety. The radio telephone messages may be received by the same simple receiving set used for the radio beacon signals.

As time goes on there will be more and more demand for two-way telephony between airplane and ground.

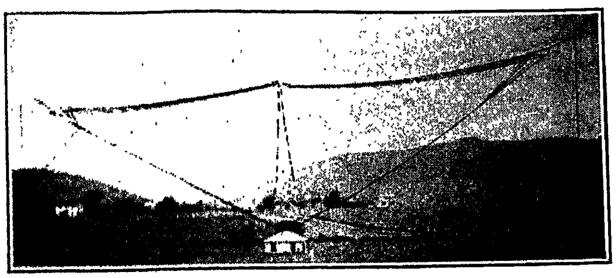

FIG. 9—RADIO TELEPHONE TRANSMITTING STATION AT BELLEFONTE, PA.

The first of the airways stations established to give weather and other information to airplanes in flight

A number of demonstrations has shown that such communication can readily be provided with quality sufficiently good to justify connection to the regular telephone exchange. In some of the demonstrations, officials sitting at their desks in Washington had two-way conversations on their regular desk telephones with other persons in an airplane. On one of these occasions Assistant Secretary MacCracken at his desk in Washington demonstrated the applicability of this service by warning the occupants of the airplane of the rising of a severe storm near the landing field.

The possibilities of radio in flying have been illustrated in some of the spectacular transoceanic flights. The Southern Cross, on its remarkable trip from California to Australia in the summer of 1928, was in touch with the world throughout the trip, by means of high-frequency (short wave) radio communication. The successful flight of Goebel and Davis from California to Hawaii in 1927 was made by the aid of the radio beacon of the aural type. A long flight over sea, terminating in a relatively small objective like the Hawaiian Islands, is extremely hazardous if undertaken without radio aid. Navigation by compass is subject to the indeterminate effects of wind drift, and the airplane's path may easily be shifted entirely away from the objective.

Any practical scheme for transoceanic air service would seem to require directional radio aid. It would be imperative for a system such as that involving a number of seadromes anchored at intervals across the ocean. Navigation on such a system without directional radio could not be considered; there is no other known means of being sure to arrive at the next airdrome.

Exploration by air is another instance where radio must be used. An exploring party takes unnecessary risks if it neglects directional radio aids to reach its objective or to find the way back to its base. This is recognized by Commander Byrd who is taking direction finding equipment along on the airplanes which he will use in exploring the Antarctic Continent.

The principal use of radio, however, will doubtless come on the regular commercial airways. The radio beacon system developed for airways is now being subjected to the test of routine operation.

As the radio aids have been slow in coming, compared with the advances in airplane design, engine reliability, and airway development, there has been a constantly increasing percentage of aviation accidents due to the hazards of weather. Radio seems the answer to those hazards, and there is ground for hope that not only this percentage of accidents but the whole number of accidents will become vanishingly small when the present possibilities of radio are realized in practise. Commercial reliability of air travel seems to depend directly upon the use of radio.

## Discussion

**H. W. Drake:** I should like to ask what is the reaction of the aviators themselves to devices of this character? I am led to ask that question because of some familiarity with the feelings of locomotive enginemen in regard to the numerous improvements that are brought about in railroad practise from time to time.

**Harry Diamond:** The psychology is apparently the same in aviation as in railroading. The average pilot is not sufficiently trained in flying by instruments. The instruments, being mechanical, are subject to failure and therefore, even though the percentage of failure is very, very small, the pilot prefers to use his instincts.

The owners of aircraft are strongly in favor of the beacon system since they think it will incline their pilots to rely more upon instruments and thereby to fly by instruments. Such pilots as have used it on the lighted airways, on the transcontinental route, are very strongly in favor of it.

# The Predominating Influence of Moisture and Electrolytic Material Upon Textiles as Insulators

BY R. R. WILLIAMS[1]
Non-member

and

E. J. MURPHY[1]
Non-member

***Synopsis.***—*Attention is called to the practical importance of water in all insulators and especially to the extreme electrical sensitivity to moisture of textiles as a class.*

*Significant amounts of electrolytic impurities occur in many insulators.*

*In textiles in the presence of moisture such impurities are responsible for very conspicuous features of electrical behavior. Data are given showing their effect on the insulation resistance of cotton and silk.*

*The insulation resistance of textile fibers in moist air rises greatly with duration of d-c. voltage, accompanied by many evidences of electrolysis of aqueous solutions of impurities in the textile.*

*The instantaneous insulation resistance of fibers decreases with increase of the measuring voltage as previously shown by Evershed. However, this fact does not necessarily support his idea of kinetic redistribution of water in textiles, as this behavior is also compatible with the nature of electrolytic conduction.*

*Electrolytic impurities may be washed out of textiles and substantial practical improvements effected thereby. The increase of resistance is of the order of 50 times.*

*Fibers are classified according to their electrical behavior in a manner which is also in harmony with their chemistry as follows;*

*(1) Animal fibers.*

*These are of protein nature and are characterized by high moisture content at ordinary humidities and by great electrical sensitivity to further increments of moisture, yet possess excellent insulating properties under usual atmospheric conditions.*

*(2) Vegetable fibers.*

*These are of cellulosic nature and are characterized by lesser moisture absorption and lesser electrical sensitivity to further increments of moisture, yet possess relatively poor insulating properties over the range of prevalent atmospheric conditions.*

*(2a) Cellulose acetate.*

*This absorbs little water and accordingly has excellent insulating properties under like conditions.*

*The differences in electrical behavior of the two main classes of textiles are believed to be due to differences in the space patterns according to which water is distributed within the individual fibers. The patterns are probably determined directly or indirectly by the chemical composition of the fibers and associated with the colloidal structure.*

* * * * *

A GREAT diversity of materials is used for insulating purposes. No simple descriptive term includes them all as the term "metals" includes commercial conducting materials. Yet in spite of this diversity it is to a great extent the quantity and mode of distribution of water in all insulators that determines their relative excellence. Were it not for the accumulation of moisture in it or on it the cheapest and mechanically most convenient material could, with rare exceptions, be used for the most exacting service.

At first glance it might seem possible to select insulating materials very simply according to moisture content, but a few illustrations will serve to show that wide contrasts exist in the response of insulations to a given amount of moisture. At one extreme is gutta percha, the classical insulation of submarine cables. If dry at the outset, it very gradually absorbs one or two per cent of moisture from the sea, but undergoes only a slight change in electrical characteristics in the process. Thereafter its water content and electrical properties are extremely stable in use. Rubber insulations used in air partake of these properties to some degree. Fluctuations in their electrical behavior never are large or sudden so long as they are mechanically intact. At the opposite extreme are the textile insulations which, especially if unimpregnated, are subject to every whim of the weather. Their water contents rise suddenly with corresponding changes in the relative humidity of the atmosphere, and the dielectric qualities faithfully reflect the moisture supplied from the air. A one or two per cent increment of moisture affects gutta percha scarcely at all but an equal amount has a most profound effect on the textiles.

The phenolized fibers, the impregnated papers, the cellulose esters, insulating varnishes and enamels, as well as glass and porcelain are intermediate between the "waterproof" insulations and the textiles in their sensitivity to atmospheric moisture. We refer to these insulations, of course, in the forms in which they are ordinarily used, for brevity neglecting distinctions which might properly be made as to relative importance of surface and volume characteristics in the several cases.

Diverse as are the insulators in use, they have another common property of importance. They often contain, or have deposited on their surfaces, electrolytic material which dissolves in the absorbed water to form conducting solutions which are injurious to the insulating qualities of the material. This fact seems to be second in importance only to the prevalence of water in insulating materials. These electrolytic substances may be present as part of the natural constituents of the insulating material or as accidental contaminants; they may consist of the saline or organic constituents of the vegetable tissues which furnished the raw material, of by-products of the processes of manufacture, of degradation products of the insulating substances resulting from atmospheric oxidation or hydrolysis, or

1. Both of the Bell Telephone Laboratories, Inc., New York, N. Y.

*Presented at the Winter Convention of the A. I. E. E., Jan. 28-Feb. 1, 1929.*

of atmospheric dust. Illustrative of the diversity of electrolytic material in commercial insulating materials are the natural ash constituents of textiles, pulp woods and other materials of vegetable origin; saline diluents of dyes used in fibrous materials; the quebrachitol of the latex of the rubber tree; acid resins produced by the atmospheric oxidation of rubber and gutta percha; and the free phenol present in phenol condensation products.

While the importance of moisture and of electrolytic contaminants in practical insulations has long had some recognition by electrical engineering opinion, especially in the telephone field, the foregoing general philosophy has been emphasized in the minds of the authors and their associates by the results of extended experimental studies of submarine insulation[2] and of textiles.[3]

Important contributions to the knowledge of the quantitative relations between the electrical properties of insulating materials and the moisture which they take up from the air have been made by Evershed,[4] Curtis,[5] Kujirai and Akahari,[6] Setoh and collaborators,[7] and other investigators. But in no published work, so far as we are aware, have data been given showing the quantitative relationships between the electrical properties of textile insulations and the electrolytic material which they contain. Data of this kind were obtained in the investigation of textiles mentioned above. Part of these data have been reported elsewhere,[3] but the investigation is being continued and a further report will be made when it is completed. It will require much further work to establish in detail the importance of contamination with aqueous solutions of electrolytes for every commercial insulating material. However, the presentation of the main thesis as a general one is abundantly justified by our constantly growing experience with cases in which such contamination of a variety of insulating materials has actually been found responsible for poor insulating qualities and for corrosion of metallic conducting or supporting elements in contact with them in electrical systems. This paper is intended to emphasize the importance of moisture and electrolytic material on the behavior of textiles as insulators and to discuss briefly the relation of electrical characteristics to physical structure and chemical constitution, so far as possible with the available facts.

## General Characteristics of Textiles

It is obvious that the rapidity of response of textiles to atmospheric moisture is due first of all to their fibrousness which permits ready access to the interior of the mass through the large surfaces exposed. By contrast the relative stability of rubber insulations for example is clearly due in part to the smaller ratio of surface to volume.

Since textiles are composed of fibers, it might seem that the resistance of a thread or the serving on a wire should depend largely on the resistances of the contacts between fibers. Further, the fibers themselves have superficial irregularities which would suggest that their resistance might vary widely from fiber to fiber of the same material. Table I-A shows that single fibers of cotton and silk have a resistance[8] which, considering the nature of the material, is surprisingly uniform for different fibers taken from the same material. Similarly, Table I-B shows that threads[9] of cotton and silk also have a uniform resistance; this suggests that the interfiber contacts do not have a large effect on the resistance of the thread as a whole. Table I-C shows the resistance of the servings on wires; the resistances are for short twisted pairs (2 in. long). This shows also that even where the voltage is applied transversely to the long axis of the fibers,—which would tend to make contact resistances more important than when the voltage is applied parallel to the long axis—the resistance of different samples of the same material is fairly uniform. These facts suggest that interfiber contact resistances are only secondary or negligible in determining the resistance of a thread or other mass of fibers. Further evidence of this is given by the data in Table II, which show that, even when the length of a thread considerably exceeds the length of a single cotton fiber, the resistance is approximately proportional to the length; if interfiber resistances were large, the resistance per unit length would increase considerably with the length of the thread measured.

The above results also suggest that electrical conduction takes place primarily through moisture in the interior of the fibers rather than through moisture condensed on their surfaces. Other evidences that this is the case may be found in the relationships of conductivity to humidity, moisture content, and electrolyte content, as well as the absence of any obvious relationship between the physical dimensions of dif-

2. Williams, R. R. and Kemp, A. R., *Jour. Frank. Inst.*, 35 (1927). Lowry, H. H. and Kohman, G. T., *Jour. Phys. Chem.* 31, 23 (1927).

3. a. Murphy, E. J. and Walker, A. C., "Electrical Conduction in Textiles. I. Dependence of the Resistivity of Cotton, Silk, and Wool upon Relative Humidity and Moisture Content," *Jour. Phys. Chem.* 32, 1761, (1928).

b. Murphy, E. J., "Electrical Conduction in Textiles. II. Alternating Current Conduction in Cotton and Silk," *Jour. Phys. Chem.* 33, (1929), p. 197.

c. Murphy, E. J., "Electrical Conduction in Textiles. III. Anomalous Properties of Conduction in Textiles," *Jour. Phys. Chem.* 33, (1929).

4. Evershed, *Inst. of Elec. Eng. Jl.* (London) 52, pp. 51-83, 1914.

5. Curtis, Bur. of Standards, *Sci. Paper No.* 234 (1915).

6. Kujirai and Akahari, *Sci. Papers, Inst. Phys. & Chem. Res.* (Tokyo), 1, pp. 94-124, 1923.

7. Setoh and Toriyama, *Sci. Papers Inst. Phys. & Chem. Res.* (Tokyo), 3, pp. 285-323, 1926.

8. The experimental procedure is described elsewhere.[3a]

9. Because of their uniformity, small samples of thread (½ in. lengths) have been used in this laboratory as a convenient means of comparing the insulating quality of cottons and other textiles.

ferent classes of fibers and their electrical behavior.

While the form of the sample is not of predominating importance with reference to the insulation resistance of either cotton or silk, the marked contrast, except at very high humidity, between cotton and silk, in all forms of samples should be noted. Both these facts and other available data justify the inference that the dielectric properties of textiles are determined primarily by the composition or internal structure of the fibers, not by the twist of threads or the lay of servings.

TABLE I

A. RESISTANCE OF COTTON AND SILK FIBERS

*R* is the resistance in megohms of a single fiber ½ in. long. Time allowed for equilibrium, 20 hr. or more. Room temperature.

| Humidity, 99 per cent (about) | | | Humidity, 77 per cent (about) | | |
|---|---|---|---|---|---|
| | *R* | | | *R** | |
| Fiber No. | Cotton | Silk (Tussah) | GroupNo.* | Cotton | Silk† |
| 1 | 2600 | 3300 | 1 | 563,000 | |
| 2 | 4700 | 4500 | 2 | 736,000 | |
| 3 | 3800 | 5140 | 3 | 680,000 | |
| 4 | 5600 | 4740 | 4 | 822,000 | |
| 5 | 3000 | 6650 | 5 | 625,000 | |
| 6 | 4600 | | 6 | 577,000 | |
| 7 | 6000 | | 7 | 822,000 | |
| 8 | 4500 | | 8 | 733,000 | |
| 9 | 5500 | | 9 | 736,000 | |
| 10 | 6000 | | 10 | 830,000 | |
| 11 | 4000 | | 11 | 653,000 | |
| 12 | 4000 | | 12 | 824,000 | |
| | | | 13 | 760,000 | |
| | | | 14 | 867,000 | |
| | | | 15 | 725,000 | |
| | | | 16 | 938,000 | |
| | | | 17 | 820,000 | |
| | | | 18 | 682,000 | |

*Each group consisted of 60 single fibers attached to the electrodes so that they were in parallel. *R** is the average resistance per single fiber calculated from that of 60 in parallel. The experimental technique is described elsewhere (3a).

†The values of *R** for silk at this humidity were found to be of the order of several million megohms, *i. e.*, beyond the limit of accurate measurement with equipment available at the time the study of single fibers was under way.

B. RESISTANCE OF COTTON AND SILK THREADS

Length ½ in. Temperature 25 deg. cent.

| | Resistance Megohms | |
|---|---|---|
| Sample | Cotton Humidity 77% | Silk (Spun) Humidity 90% |
| 1 | 4160 | 21,100 |
| 2 | 4220 | 22,700 |
| 3 | 4100 | 28,000 |
| 4 | 3730 | 14,500 |
| 5 | 4020 | 30,600 |
| 6 | 3820 | |
| 7 | 3900 | |
| 8 | 3715 | |
| 9 | 4050 | |
| 10 | 4100 | |
| | Aver. 3982 | 23,380 |

C. RESISTANCE BETWEEN TWISTED PAIRS OF COTTON AND SILK INSULATED WIRES

Humidity 77 per cent Temperature 25 deg. cent.

| | Resistance Megohms | |
|---|---|---|
| Sample | Cotton | Silk (Tussah) |
| 1 | 5.45 | 2200 |
| 2 | 3.96 | 1890 |
| 3 | 5.20 | 1685 |
| 4 | 3.96 | 1685 |
| 5 | 3.50 | 2250 |
| 6 | 4.05 | 1970 |
| 7 | 5.60 | |
| 8 | 5.15 | |
| 9 | 4.37 | |
| 10 | 4.76 | |
| 11 | 4.00 | |
| 12 | 4.37 | |
| | Aver. 4.53 | 1942 |

TABLE II

RESISTANCE OF DIFFERENT LENGTHS OF COTTON THREAD

Humidity about 77 per cent room temperature

| | Resistance Megohms | |
|---|---|---|
| Length Inches | Total | Per Inch |
| 0.5 | 21,700 | 43,400 |
| 1. | 41,750 | 41,750 |
| 3. | 153,000 | 51,000 |

The moisture content of each sort of textile depends directly on the humidity of the atmosphere. Fig. 1 shows the best data available for the moisture content of silk, wool, cotton, and cellulose acetate in equilibrium with air over considerable ranges of relative humidity. The data for silk and wool were taken from a paper by Schloessing;[10] those for cotton are due to Urquhart and Williams;[11] while those for cellulose acetate represent the figures of Wilson and Fuwa,[12] who also give corresponding data for several textiles and many other substances. It is sufficient for our present purpose to emphasize the orderly dependence of moisture content upon the relative humidity of the atmosphere without

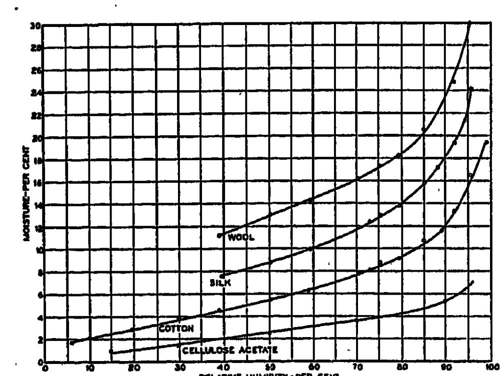


FIG. 1—DEPENDENCE OF MOISTURE CONTENT OF TEXTILES UPON RELATIVE HUMIDITY OF ATMOSPHERE WITH WHICH THEY ARE EQUILIBRATED

discussing secondary phenomena or the full significance of the curves.

The relation of electrical behavior of each textile to relative humidity is also very close. Fig. 2 shows the

10. Schloessing, Th., *Bul. Soc. Encour. Indust. Nat.* 8, 717 (1893); C. R. 116, 808, 1893. Text. World Record, Boston, Nov., 1908, p. 219.

11. Urquhart and Williams, *J. Textile Inst.* 15, 143, (1924).

12. Wilson, R. E. and Fuwa, Tyler, *Ind. & Eng. Chem.* 14, 913 (1922).

insulation resistance of each of the above fibres plotted against relative humidity over the upper part of the range of atmospheric humidities. It is not practicable to plot the resistance over the entire range of humidity directly in this way, on account of the wide range of insulation resistance values which are obtained. In order to depict the fact that there is a consistent relationship throughout the range, we have plotted in Fig. 3 Log Insulation Resistance *vs.* Relative Humidity as far as values are at present available. When considered together, these three charts show that the insulation resistance of a textile depends on its moisture

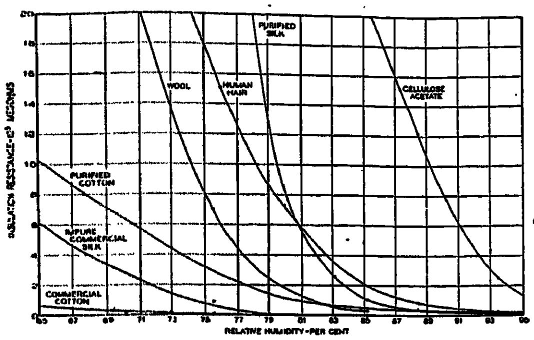


FIG. 2—INSULATION RESISTANCE OF ½-IN. LENGTHS OF TEXTILE THREADS AS AFFECTED BY RELATIVE HUMIDITY OF ATMOSPHERE

The purified cotton and purified silk had been submitted to a washing procedure to remove electrolytes. The impure silk was a commercial specimen representing somewhat more than usual contamination with electrolytes, while the commercial cotton is representative of its class

content, which in turn is a function of relative humidity. In the series of papers previously mentioned,[3] the electrical behavior of textiles in relation to relative humidity and moisture content is discussed more fully.

Aside from the common property of dependence of electrical characteristics of textiles upon their moisture contents, several other phenomena of electrical behavior have been encountered which are common to all. If any textile within our experience, including cellulose acetate (or even glass), be brought in contact with two electrodes of opposite polarity in the presence of atmospheric moisture, the electrical properties of the material undergo a change with a rapidity dependent on the current and in turn upon the voltage, the length of path, and the humidity. Such a change in the properties of insulating materials with continued application of voltage has been discussed recently by Granier who advanced the explanation[13] that it is due

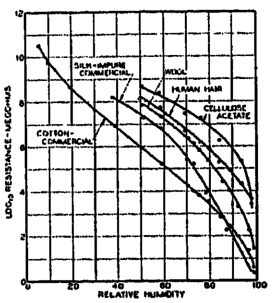


FIG. 3—LOG INSULATION RESISTANCE OF ½-IN. LENGTHS OF TEXTILE THREADS *vs.* RELATIVE HUMIDITY OF THE ATMOSPHERE

For description of samples see Fig. 2. For tabulated data see reference 3a

to the presence of electrolytic impurities in the materials. However, the great magnitude of this change which may occur in textiles when freely exposed to ordinary atmospheres seems to have been very little appreciated.

In our experiments the insulation resistance rises to a value perhaps 10 to 100 times the original value, depending on the nature and condition of the fiber. A few typical cases are given in Table III. This phenomenon will be referred to as polarization. This rise in resistance appears to be largely due to substantial denudation of some intermediate portion of the fiber of electrolytic impurities which in general tend to accumulate in the vicinity of the electrodes. The phenomenon involves the possibility of chemical reactions between the products of the electrolysis and the material of the electrodes, as well as the evolution of gases and the conversion of soluble salts into insoluble products. As regards the mineral constituents of cotton, it can be observed by ashing the polarized fiber that the ash

TABLE III

RATE OF CHANGE OF RESISTANCE WITH TIME OF APPLICATION OF VOLTAGE FOR SOME FIBROUS MATERIALS

Separation of Electrodes ⅛ in. Humidity 97 per cent (approx.) Voltage 275

| Cotton | | Silk | | Cellulose Acetate Silk | | Paper | |
|---|---|---|---|---|---|---|---|
| Time Secs. | Resist. Megohms | Time Secs. | Resist. Megohms | Time Secs. | Resist. Megohms | Time Secs. | Resist. Megohms |
| 70 | 19.8 | 30 | 8.1 | 80 | $9.3 \times 10^3$ | 55 | 4.54 |
| 150 | 33.6 | 90 | 12.8 | 1250 | $1.01 \times 10^4$ | 100 | 8.30 |
| 200 | 40.5 | 120 | 19.8 | 2300 | $1.06 \times 10^4$ | 200 | 11.2 |
| 300 | 45.1 | 153 | 26.7 | 3200 | $1.11 \times 10^4$ | 260 | 19.8 |
| 400 | 50.0 | 220 | 40.5 | | | 520 | 32.2 |
| 500 | 57.5 | 300 | 47.8 | | | 615 | 38.5 |
| 900 | 68.2 | 640 | 54.8 | | | 1000 | 42.7 |
| 1900 | 82.0 | 1250 | 58.4 | | | | |
| 3000 | 91.0 | 2100 | 79.5 | | | | |
| 4000 | 99.0 | 2320 | 107.0 | | | | |
| | | 3000 | 153.0 | | | | |

13. Granier, J., *Soc. Fran. & Elec. Bull.*, 3, 480 (1923).

lies largely in those portions which were adjacent to the electrodes and especially to the cathode. The electrical resistance is very unequally distributed along such a polarized fiber or thread, the positions of maximum resistance depending upon the conditions of polarization.[3c] Interruption of the current after polarization leads to a gradual restoration of the original electrical properties and a redistribution of the ash constituents with a speed depending largely on the humidity. Reversal of polarity is accompanied by a rapid drop in insulation resistance, followed upon continued application of voltage in the reverse direction by polarization in the opposite sense. Interruption of the circuit after polarization leaves large potential differences on the opposite ends of the fiber, which persist for several minutes. The cathode region is found to be alkaline in reaction, the anode region acidic. The polarized fiber is therefore a concentration cell.

The electrolysis of cotton may be carried out experimentally in another way. If cotton yarn is immersed in water in each of a series of cells separated from one another by a parchment paper membrane and a direct current is passed through the cells for some hours, the impurities tend to accumulate near the electrodes, with the development of acidity at the anode and alkalinity at the cathode, as is usual in the electrolysis of a saline solution. If the samples of cotton yarn be now removed and brought into equilibrium with an atmosphere of standard humidity, the insulation resistance is found to vary fairly regularly with the original position of the sample in the series of cells, being greatest in some intermediate cell and diminishing toward either electrode. The precise position of the maximum varies with the nature of impurities present in the system. The highest insulation resistance may be many times that of the original cotton.

Perhaps the most significant evidence of the importance of electrolytic impurities in silk, wool, cotton, and to some extent other textiles, is the fact that their electrical characteristics can be greatly improved by thorough washing with water though without altering qualitatively the general nature of the electro-conducting phenomena which characterize them. Chart II illustrates the result of washing upon the insulation resistance of cotton and silk threads. The improvement in insulation resistance of cotton and silk upon washing ranges commonly from fifty to one hundred fold, under any of the commonly prevailing conditions of atmospheric temperature and humidity. This improvement is accompanied by diminution of the ash content, in the case of cotton from about 1.0 per cent to 0.15 or 0.25 per cent. It produces only a slight reduction in the equilibrium moisture content of the cotton over the ordinary ranges of atmospheric humidity. The sensitivity of the washed cotton to continued application of voltage is much less than that of the original, but polarization still occurs. Commercial silks are similarly affected by washing.

If the mineral contents of cottons which have undergone washing are compared quantitatively with the original contents a decrease is observable, particularly as to potash, but the calcium and magnesium contents are much less altered. Fairly complete removal of potash is apparently essential to good electrical characteristics, but improvement electrically has been attended in some cases by an actual increase in content of alkaline earths. This suggests that interchange of electrolytic impurities between the textile and the water is involved as well as actual removal of electrolytes by the water. Thus in general hard natural waters, *i. e.*, those containing calcium and magnesium salts, have proved as good or better than soft waters when used in economically small amounts. Very exhaustive extraction with distilled water gives excellent results, though not vastly superior to washing with very dilute solutions of alkaline earth salts. Sufficiently complete and accurate analyses of samples of textiles brought into equilibrium with washing liquids and of the kind and quantity of electrolytes in the corresponding liquids have yet to be made to determine the precise importance of the composition of the saline residues. Non-saline electrolytes have also to be considered. This matter requires extended study and the experimental data are reserved for future publication.

The commercial value of such treatments of insulating yarns have proved to be very substantial. The utilization of the products forms the subject matter of another paper from the Bell Laboratories.

Another common property of textiles is known as the Evershed effect. Evershed[14] found in various insulating materials, including textiles, that insulation resistance does not obey Ohm's law but is less if a larger measuring voltage is used. Evershed's finding as to cotton has been verified by us. This result is easily obtainable if conditions are maintained so that little polarization occurs. But if extensive polarization is allowed to take place the reverse effect is observed and the ultimate resistance is higher in proportion to the voltage used. The conditions which favor polarization are, of course, considerable voltages, prolonged application, high relative humidities, and short paths through the insulation. Evershed's work apparently did not involve any special attention to the time of application of voltage.

Evershed's explanation of decreased insulation resistance with increased voltage involves the assumption that much of the water contained in insulations is originally in the form of isolated pools and therefore of no conductive effect at the instant of application of voltage. In support of his theory of "dormant" water he lays great stress upon his observation that the volume of water present in the insulating materials is far in excess of that which would be required to furnish

14. Evershed, S., *Inst. Elec. Eng. Jl.* **52**, 51 (1914).

the observed conductivity if the water were in the form of continuous filaments of uniform cross section. This argument seems impressive and conforms to our own ideas of the distribution of water in textiles. However, according to Evershed, this pool water is electrokinetically spread out into conducting films under electric stress, thus accounting for decreased resistance with increased voltage. A tendency to such movement of water cannot be denied. But it is difficult to harmonize Evershed's conception of electro-endosmotic movement of water as the predominating phenomenon with all the facts regarding textiles with which we have had experience; for example, with the fact that the Evershed effect is greatest when the electrolyte content of the textile is high. Electroendosmose in systems designed for its ready detection usually diminishes with increasing electrolyte content except when the concentrations are very small.[15] Further a decrease of resistance with increase of voltage has been noted in other systems in which electrolysis is unquestionably involved and in which electro-kinetic redistribution of water seems improbable.

Though sufficient support for an alternative cannot be furnished at present, electrokinesis does not constitute the sole possible explanation of the Evershed effect. The analogy between the moist fiber and an electrolytic cell conforms to a number of other corollary facts about the Evershed effect which are discussed in a more specialized paper by one of the authors.[3c] The various properties which have been discussed are in agreement with the view that the cardinal principle of conduction in textiles is the electrolysis of aqueous solutions.

## Distinctive Characteristics of Each Fiber Species

The several kinds of fibers exhibit a number of curious contrasts in the relation of electrical behavior to hygroscopic properties, some of which at first glance appear contradictory. For convenience in discussion let us classify the commercial fibers into two main groups: (1) the animal fibers, and (2) the vegetable fibers, and a sub-group (2a), the cellulose ester fibers of which the so-called cellulose acetate silk is the sole representative of commercial importance at present. It will be seen by reference to Fig. 1 that over the entire range of relative humidity the animal fibers, silk and wool, absorb more water than the natural vegetable fibers. This is true whether we deal with fibers in their natural impure state or after a washing process which has been shown to improve greatly the electrical characteristics of both types of natural fibers. Cellulose acetate absorbs less water at any given humidity than either class of natural materials.

We have seen that for any given kind of fiber there is an orderly dependence of electrical properties upon the moisture content of the fiber and in turn upon the relative humidity of the atmosphere. The more water present in any given fiber the poorer are the electrical properties. If the amount of water in fibers were the sole determinant of electrical characteristics we would expect the animal fibers to be, at a given humidity, the poorer electrically of the classes enumerated above. But this is emphatically not the case. With respect to electrical properties, we find that the vegetable fibers are inferior to the more hygroscopic animal fibers and are also inferior to the least hygroscopic variety of commercial fiber, *viz.*, cellulose acetate. To make the existing contrast clear we have plotted in Fig. 4 for the several textiles, Log Insulation Resistance *vs.* Log Moisture Content. When so plotted the values for each kind of fiber fall approximately on straight lines throughout the range of actual measurement. The relative position of the curves for animal fibers to the right and above that for cotton[16] means that the animal fibers have the better insulating qualities in spite of higher hygroscopicity.

The slopes of these lines have an even greater significance for they indicate the relative sensitivity of the fibers to an increment of moisture. Since the slope for the animal fibers is greater, it is evident that the animal fibers are more sensitive electrically to moisture than cotton. Under a given set of conditions they are not only wetter than cotton, but are more sensitive to the effects of further increments of moisture and yet they have a higher insulation resistance.

In one respect alone can we say that cotton is preferable as an insulating material. It has the merit of being more nearly uniform in behavior under a variety of weather conditions. When the amount of moisture taken up from the surrounding atmosphere by silk or wool is doubled, the electrical leakage increases by a factor of 50,000 to 100,000, while that for cotton rises by a factor of only 600.

The position and slope of the values for cellulose acetate are of great interest as the curve coincides with that for cotton, indicating that moisture affects these two fibers in a very similar manner. The essential electrical difference between these two appears to be satisfactorily accounted for by the fact that cellulose acetate absorbs less water than cotton at any given relative humidity of the atmosphere. The conversion of cotton which is essentially cellulose into cellulose acetate by the process of acetylation, has, as could be predicted on chemical grounds, reduced its hygroscopicity but apparently has not modified its structure greatly. Cellulose acetate is therefore put in a subgroup rather than in an independent classification.

The reader will have been led to ask several questions. Why do the several kinds of fibers differ so much in absorption of water and particularly why does not a

15. Powis, Frank, *Zeit. Physik. Chem.* 89, 91, 1914 and Burton, E. F., *Coll. Symp.*, Monograph IV, 132, 1926.

16. The threads referred to are approximately, but not precisely, of the same size. This variable is by no means sufficient to account for the higher position of the animal fibers as compared with cotton.

given amount of water affect them all alike electrically? He will want to know if the mere fact of animal, vegetable, or artificial origin is associated with a particular type of electrical or hygroscopic behavior. He will wonder whether there is any fundamental resemblance between the silk which is rapidly extruded by a worm and the hair which grows so slowly on a sheep's back. He will inquire whether there is sufficient justification for classifying other vegetable fibers with cotton. To these questions only partial and to some extent speculative answers can be given.

We are justified on chemical grounds in classifying the fibers in the same way which we have found to be convenient for discussion of their hygroscopic and electrical properties. How much importance should be attached to this correspondence between the chemical and electrical classifications cannot be determined at present. However, the correspondence seems suggestive and deserving of a brief discussion. The first class, that of animal fibers, has a common chemical nature in that they consist largely of proteins. Proteins are molecular aggregates of colloidal size composed in turn of simpler substances known as amino acids. Each of the constituent amino acids contains both an acidic and a basic group, so that in acid solutions they behave as bases and in alkaline solution they behave as acids. This so-called amphoteric property is due to the presence of an acidic oxygen nucleus and a basic nitrogen atom, which are almost invariably adjacent to one another. Amphoteric properties persist in the proteins which are formed by union of many amino acids in a single molecule. This is illustrated by the fact that either amino acids or proteins in a solution which is subjected to a d-c. voltage tend to migrate to positions intermediate between the electrodes where the acidity is such that they are equally ionized as bases and as acids. The combination of adjacent acidic and basic groups within a single molecule gives them a salt-like property which may be significant. It is reasonable to associate the hygroscopic quality of the proteins with these groups as the molecules are usually without groups of polar character other than the paired groups mentioned.

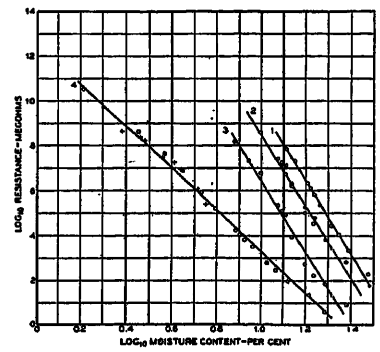

FIG. 4—INSULATION RESISTANCE AS A FUNCTION OF MOISTURE CONTENT OF TEXTILES

1. Wool yarn
2. Silk threads purified (○ sample 1 ● sample 2)
3. Silk threads impure
4. ○ Cotton threads, + cellulose acetate threads

That their common protein character is responsible in some way for the properties of principal interest from the insulating standpoint is rendered the more probable by the close resemblance of silk and wool, as shown by the approximate parallelism of their curves in Fig. 4. This resemblance is shared in considerable measure by other hairs than wool.

The second class of fibers, coming from the vegetable world, are alike in being composed of cellulose, a substance like the protein in having a high molecular weight but unlike it in that its polar groups are hydroxyls which have a faintly acidic rather than amphoteric nature. These are the groups in cellulose with which water is likely to associate itself. Such data as are available concerning vegetable fibers other than cotton, notably linen, ramie, manila hemp, and wood pulp, indicate a strong resemblance not only chemically but hygroscopically and electrically.

The sub-class embracing only cellulose acetate as a commercial fiber is chemically more neutral and non-polar in type than other cellulose fibers, with which fact it is reasonable to associate its lower hygroscopicity and consequent better electrical characteristics. It is probable that cellulose nitrate and cellulose ethers will be found to fall in this class but artificial silks other than cellulose acetate absorb more water and appear on chemical grounds to be better classified with the cellulose fibers of natural vegetable origin.

It cannot be decided from available information whether the similarities and differences in electrical properties among the textiles are traceable directly to chemical similarities and differences or indirectly to physical (colloidal) structures which in turn are determined by chemical composition. In either case it is possible to account for the high sensitivity of all the fibers to moisture and the variation in sensitivity from species to species by assumptions as to the distribution of water in them. Water which collects in any isolated form in the material will have little electrical effect compared with that which forms continuous filaments. The distinction we are making is essentially the same as that of Evershed when he referred to part of the water as "dormant," though we do not attach the same importance as he does to electrokinetic redistribution of water under electric stress. Each increment of water may be considered as undergoing partition into two portions, one causing a large increase of conductivity and one having a negligible effect, in a ratio determined in some fashion by the structure or nature of the material and the humidity of the atmosphere. The ratio of the two portions which are in equilibrium via the surrounding atmosphere will be subject to constant readjustment under changing conditions.

The fact that the electrical characteristics of the two classes of fibers as affected by moisture appear to be specific properties of the substances involved suggests

some highly regular distribution pattern of conducting water paths determined by the chemical or physical (colloidal) structure of the material. Such a regular pattern may involve only water condensed upon the surfaces of the elements of structure in such a way that the thickness of the film varies regularly from point to point through the material. Accumulation of water at thick points would have little electrical effect, while that in thin portions would be very significant. An alternative regular mode of distribution would involve water in part dispersed in solution or chemical combination within the units of structure of the material and in part in fairly uniform thin films on their surfaces, in which case the latter would have the major electrical consequence. While such a regular form of distribution seems preferable, it is perhaps not the only way of accounting for the electrical properties observed.

The curves as shown in Chart IV for both cotton and silk are straight lines within the experimental error but the assumption is not justified that they can be projected as straight lines to zero humidity. At the lower ranges of humidity the resistance of silk is so high as to exceed the limits of our present technique of measurement. Over some range below 40 per cent relative humidity it may well be that the sensitivity of the animal fibers to increments of moisture is less than that of the cellulose fibers. If so, the break in the curve would have great interest in connection with determining the mode of water distribution and in turn the colloidal structure of the materials in question. It is hoped that an extension of the study in this direction will be possible in the future.

The authors wish to acknowledge their indebtedness to their colleagues, whose names appear as authors of kindred papers, for their advice and assistance. Also we wish especially to thank Dr. Homer H. Lowry, whose discernment has stimulated the development of evidence necessary to several of the more important deductions.

## Discussion

For discussion of this paper see page 581.

# Purified Textile Insulation for Telephone Central Office Wiring

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and

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*Synopsis.—This paper outlines methods by which silk and cotton insulation can be purified and improved. It gives the results of tests on the insulation characteristics of these materials before and after purification and explains the testing procedures. One of the findings is that the purified cotton may be substituted for ordinary commercial silk.*

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IN another paper, *The Predominating Influence of Moisture and Electrolytes upon Textiles as Insulators,* (see p. 568) Messrs. Williams and Murphy have shown that the electrical properties of textiles are closely associated with their moisture content and impurities in the textiles. In particular, water-soluble salts become ionically conducting in the presence of moisture and the ions migrate along the paths of initially low resistance to the electrodes with which they react chemically, causing serious corrosion. The resulting corrosion products, themselves electrolytes, accelerate the process of current transfer and may easily lead to a complete failure of the insulating textile at the point of greatest concentration. Conversely, if the impurities are removed, the insulating properties of the textile are improved initially and, furthermore, are not subject to cumulative deterioriation due to concentration of conducting salts and electrolytic corrosion products at the weaker points. It is the purpose of this paper to show how these principles are borne out by field observations and laboratory tests, and to show in a general way the extent to which the insulating properties of silk and cotton can be improved commercially with particular application to telephone central office wiring.

Since the early days of telephone development work, silk and cotton have been the standard insulating materials for wire insulation in telephone central office apparatus, supplemented in later years by enamel insulation. Relatively low voltages have always been used in the telephone plant, 24 to 48 volts being the usual voltages which are carried continuously in cables, while intermittent a-c. and d-c. potentials generally do not exceed 100 to 150 volts. Therefore it has been generally accepted that telephone cables, once installed and properly protected from accidental high voltages, could be depended upon to have a substantially indefinite life. In general the insulation of these cables has been satisfactory, but breakdowns have occurred which could not be attributed to faulty operating conditions or to manufacturing defects. A study of this subject showed that it was possible under certain conditions to get discolored or faded spots in the insulation and corresponding corroded or pitted spots in the tinned copper conductors. It was also observed that the textile insulation at such spots showed a strong concentration of water soluble salts. Also, cables in which such conditions occurred measured relatively low in insulation resistance with the current leakage concentrated at these points. These observations led to the conclusion that silk and cotton would be decidedly improved as insulating materials if they were made less susceptible to deterioration under telephone service conditions.

Aside from the consideration of improving silk and cotton to assure greater insulation stability, considerable thought has been given to the possibility of improving the insulating characteristics of cotton to such a degree that it could be substituted for the more expensive silk. The importance of this work with respect to its bearing on the cost of telephone service can be better appreciated from the fact that about 2000 pounds of silk are required daily to provide for the growth of the country's telephone requirements, which if replaced with cotton would reduce raw material costs by a very substantial sum.

The desirability of reducing the quantity of silk required in the telephone plant does not arise entirely from this phase of the economic question. The problem of supply and demand has at times entered into the matter. For example, shortly after the close of the world war the supply of insulating silk was limited and the price prohibitively high. Substantially the same condition arose a few years later, which leads to the conclusion that silk is inherently much more subject to violent fluctuations in available supply and cost than cotton. Therefore, with demands for telephone equipment rapidly increasing, we have decidedly greater assurance of an adequate supply of insulating material at reasonable cost if cotton instead of silk is used.

## Purification Process

With the foregoing as an introduction to indicate the economic advantages to be gained by improving the electrical characteristics of cotton and silk, the following is intended to show what has been accomplished by the commercial application to silk and cotton thread of

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the processes referred to by Messrs. Williams and Murphy for removal of objectionable impurities.

Since such impurities are soluble in water, it will be inferred that the purifying process consists in a thorough washing with water. In effect, this is the case. The process, however, for both silk and cotton, being based on substantially complete removal of the ionically conducting salts, especially those of sodium and potassium, prescribes the use of water of low saline content. It also means that the washing is best accomplished by a continual flow which after passing through the textile

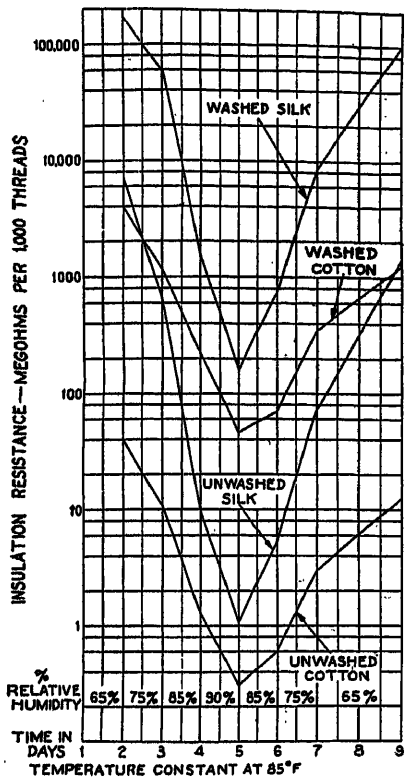


FIG. 1—TYPICAL D-C. RESISTANCE CHARACTERISTICS OF WASHED AND UNWASHED SILK AND COTTON THREADS OF EQUAL SIZE

is considered to be contaminated and is not used again.

Where cotton is to be dyed and washed, the washing consists in an additional operation applied to the cotton immediately following the dyeing operation without the necessity of drying between processes.

## CHARACTERISTICS OF PURIFIED INSULATIONS

Obviously, the first consideration in the insulation of electrical conductors is to provide an insulating medium of sufficient dielectric strength to withstand the working potentials to which they are subjected. Also, the d-c. insulation resistance must be high enough to prevent undue d-c. energy loss. A comparison of the electrical resistance of the cotton and silk at relative humidities ranging upward from 65 per cent to 90 per cent and down again to 65 per cent, before and after washing, is shown by the graphs in Fig. 1 as determined by samples prepared and tested by the method and testing apparatus shown in Figs. 6, 7, and 8 and described later. The same comparison is shown in Fig. 2 except that these graphs show the insulation resistance of wire insulated with the washed and unwashed textiles. In addition to the insulation resistance requirement, it is required that the energy losses at talking and carrier current frequencies must be maintained at the minimum point consistent with the space limitations permitted for the conductors. The effect of purification of the textiles on this characteristic expressed in capacitance and conductance, measured at 1000 cycles per sec. between the wires of twisted pairs is shown in Fig. 3 and Fig. 4. The data represented by these graphs converted into transmission loss units are illustrated in Fig. 5. As the same thickness of insulation was used in all cases, the graphs are on a comparative basis. It should be noted that the graphs are illustrative of the effects of purification on the electrical properties of cotton and silk as insulation and should not be considered as applying quantitatively to telephone circuits.

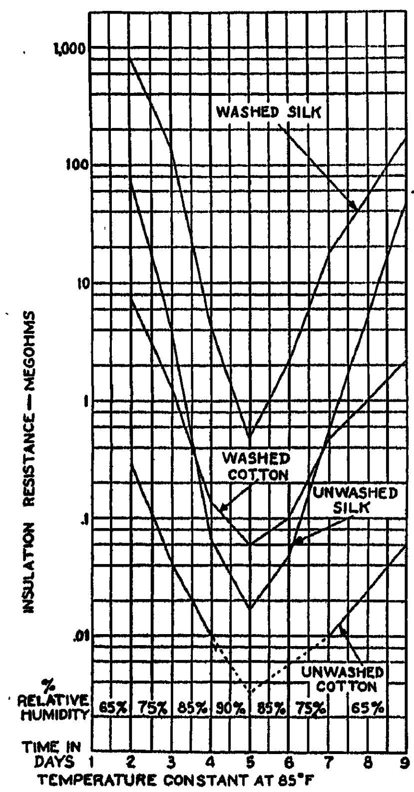


FIG. 2—D-C. INSULATION RESISTANCE OF 50 FT. OF TWISTED PAIR WIRE INSULATED WITH DOUBLE SERVINGS OF EQUAL THICKNESS

From a telephone transmission point of view, perhaps the most significant fact to be observed is the large reduction in capacitance and conductance at relative humidities of 75 per cent and higher. These characteristics which largely determine transmission efficiency are relatively low for both silk and cotton at 65 per cent and below, but in commercial textiles in general use for insulating purposes they increase very rapidly as the relative humidity increases. The characteristics of

purified textiles are not as markedly different from those of unpurified textiles at 65 per cent relative humidity as at higher humidities, but their rate of increase as the humidity increases is greatly reduced. This fact is of particular importance in the maintenance of a standard level of voice transmission through toll offices where suitable repeater gains and balance must be maintained. Losses, if fixed in value and not excessively large, can be compensated for, but if they change with every change in atmospheric moisture content the compensation problem becomes serious.

## Method of Testing

Two fundamental characteristics of silk and cotton made it necessary to do a large amount of experimental work before a practicable shop test method could be established to determine whether or not the textiles were washed to the point of meeting the requirements

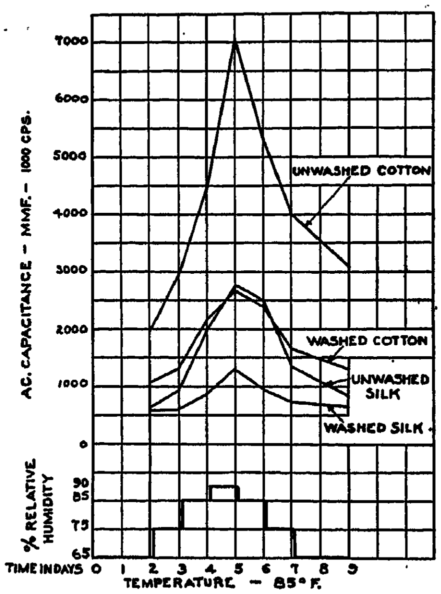


Fig. 3—A-c. Capacitance of 50 Ft. of Twisted Pair Wire Insulated with Double Servings of Equal Thickness

established. One of these characteristics is the high electrical resistance of both washed and unwashed textiles at the lower relative humidities and the other the extreme sensitivity to change, with minor change in relative humidity especially at the higher humidities. The first mentioned characteristic precludes the use of any but measuring instruments of the highest degree of sensitivity and makes desirable the use of comparatively high humidities, and the second characteristic means that the specimen must be tested under exceedingly well controlled relative humidity conditions. Furthermore, the problem is complicated by the polarization effect discussed in the paper by Williams and Murphy and the fact that this effect varies in magnitude with humidity and with the degree of purity of the textiles. The problem was finally solved by the development of the test equipment shown in Figs. 6, 7, and 8.

Figs. 6 and 7 show a heat insulated glass tank of about one cubic foot capacity fitted with an insulating cover in which holes normally closed with stoppers are used to introduce the test samples. The humidity is

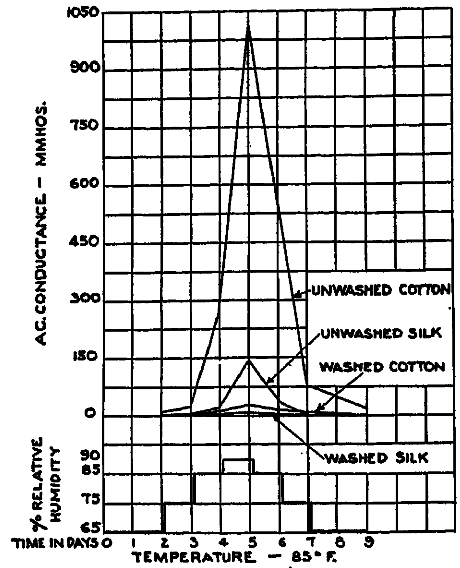


Fig. 4—A-c. Conductance of 50 Ft. of Twisted Pair Wire Insulated with Double Servings of Equal Thickness

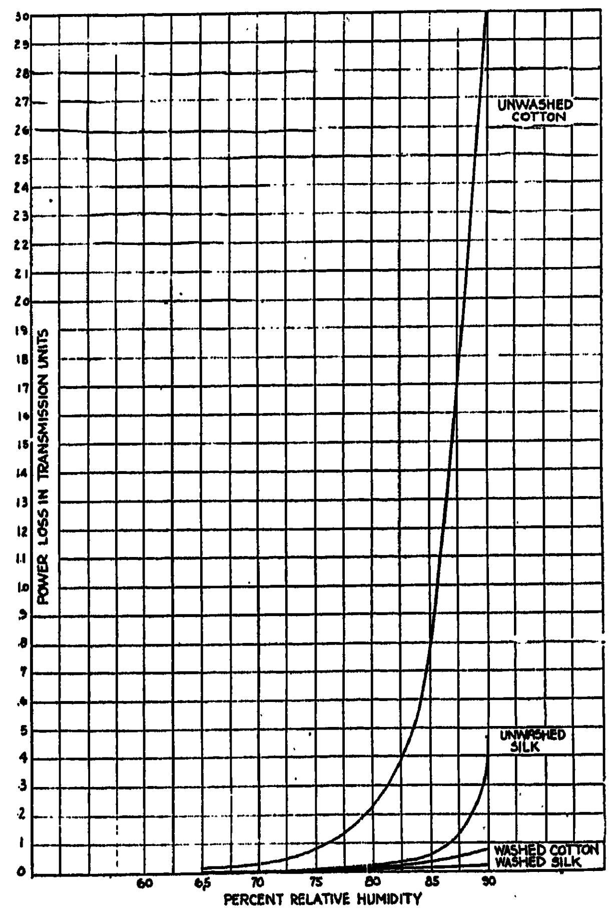


Fig. 5—Transmission Loss in 50 Ft. of Twisted Pair Wire Insulated with Double Servings of Equal Thickness

maintained by means of sulphuric acid or a saturated salt solution in the bottom of the tank and constant temperature within very narrow limits is maintained in

the tank by placing the entire assembly inside a cabinet or oven automatically controlled to ± 0.5 deg. fahr. Due to the heat insulation it has been found that temperature variations within the tank are reduced to the vanishing point for all practical purposes.

This is very important as it has been found that fluctuations in the temperature of the test chamber

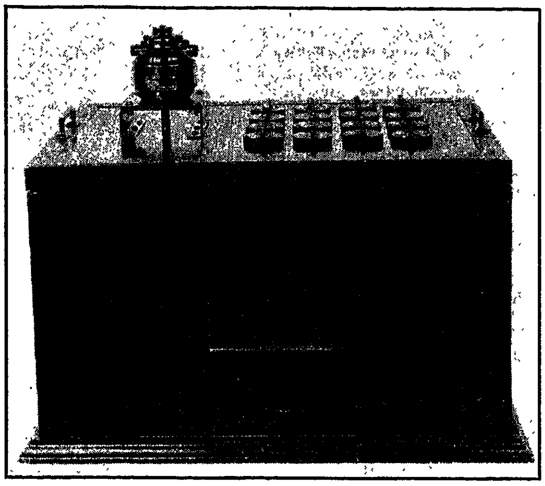

FIG. 6—HUMIDITY CABINET FOR CONDITIONING SAMPLES

introduce large errors in the insulation resistance of the samples. The errors, however, are attributed not to temperature effects on the samples but to variations in relative humidity produced by the temperature changes and the considerable time required for equilibrium to be restored after such changes occur.

Another source of error in textile testing is found in

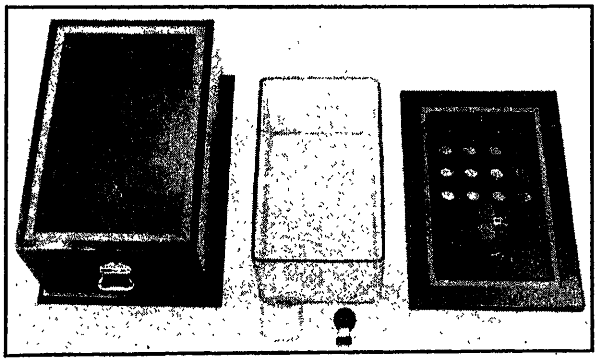

FIG. 7—HUMIDITY CABINET DISASSEMBLED

the fact that the values of insulation resistance are affected by the humidity condition to which the sample has been exposed prior to the test. To avoid error from this source, all test samples are conditioned by drying in a desiccator at the approximate temperature of the test tank before being placed in the tank.

The samples are prepared by winding a number of turns of the textile around the electrodes inserted in the stoppers as shown in Fig. 8. Care is taken not to handle the textile itself during the winding process as perspiration from the hands is likely to contaminate the thread. Samples are left in the tank over night as there is considerable evidence to show that several hours are required for them to come to complete equilibrium. A temperature of 100 deg. fahr. and relative humidity of 75 per cent has been found suitable for cotton testing and 100 deg. and 87 per cent relative humidity for silk.

The number of turns of yarn or thread wound around the electrodes will vary with the size of the thread since the winding space is fixed and a single layer of thread is applied. For No. 30/2 cotton approximately 90 turns, 180 parallel threads, have been found to give satisfactory readings. This same space accommodates about 256 turns, 512 parallel threads, of No. 62/1 spun silk.

The distance between the electrodes is not particularly critical. That is, it is not important, for ex-

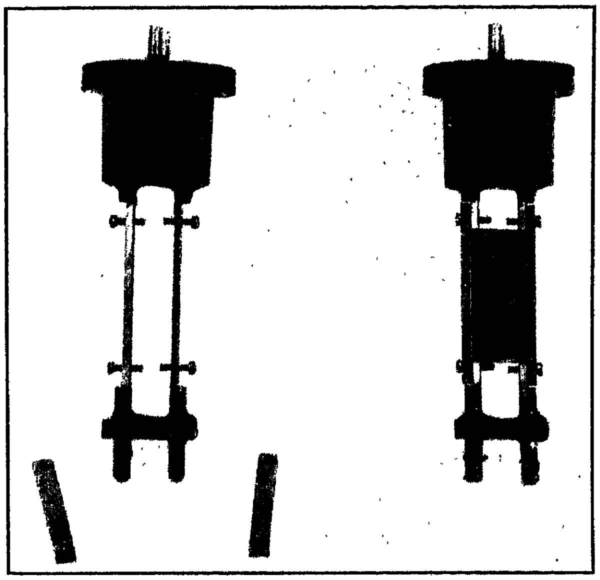

FIG. 8—ELECTRODES ON WHICH SAMPLES ARE WOUND FOR TEST

ample, whether the distance is ⅝ in. or ¾ in. It is important, however, that having decided upon a certain separation, say ¾ in., this separation be accurately maintained for all electrodes if the readings are to be comparative. Of course if the separation is too great, an unreasonable number of turns of textile is required to bring the resistance of the sample within the range of the galvanometer. On the other hand, if the separation is too small the error due to variation in separation for different sets of electrodes increases in magnitude.

In actual practise, ¾ in. separation with a winding space of 2 in. accommodating, as mentioned above, about 90 turns of No. 30/2 cotton has been found to be fairly satisfactory. This arrangement gives galvanometer readings of the order of 2000 megohms for washed silk and 1000 megohms for washed cotton as compared with 12 meghoms and 5 megohms for unwashed silk and cotton respectively. It is obviously necessary to maintain a high degree of insulation resistance between the electrodes. This is accomplished by using hard rubber for the stoppers in which they are mounted and preventing surface leakage by coating the end of the stoppers with ozokerite wax. The electrodes them-

selves are gold or platinum plated to prevent oxidation or corrosion. Observing these precautions, it is possible to obtain readings sufficiently consistent to distinguish not only between washed and unwashed textiles but to determine differences in degree of purification in various lots of washed textiles.

The question may be asked as to why 75 per cent relative humidity and 100 deg. fahr. was selected for cotton and 87 per cent relative humidity and 100 deg. for silk. These values were, within reasonable limitations, more or less arbitrarily selected and further experience may show that some other values are preferable. However, for the following reasons, 75 per cent and 87 per cent at 100 deg. fahr. were selected as offering definite promise of giving consistent results.

The main considerations in the choice of humidity conditions were, first, that the humidity should be high enough so that insulation resistance measurements of sufficient accuracy could be made using a band of threads as described above and a commercial galvanometer of reasonably high sensitivity; second, that the humidity be lower than that at which polarization effects would introduce serious error. The humidities chosen are within the range found suitable for cotton and silk under these limitations. Furthermore, these conditions are readily obtainable by the use either of saturated salt solutions or sulphuric acid solutions, thereby increasing the flexibility of the test. The temperature 100 deg. fahr. was chosen arbitrarily as one which could be maintained in the shop at any time of the year without artificial cooling.

## Application to Apparatus

From an economic standpoint the most important conclusion to be drawn from the graphs is that cotton can be improved by washing to such an extent that it becomes a better insulator than the ordinary commercial insulating silk in general use. Since the cost of washing silk and cotton is nominal, usually less than 5 per cent of the cost of the material, the engineer given purified textiles may either take advantage of marked improvement in quality of electrical characteristics by using washed silk, or may substitute washed cotton for silk and realize substantial economies without degrading the product. As an example of how this applies to Bell System apparatus, central office distributing frame wire with annual requirements of more than 400 million conductor feet is now insulated with two coverings of silk where three were formerly required. The resultant wire is superior electrically to the old wire and the annual saving in silk amounts to about 70,000 pounds.

As another example, telephone cords of various types have been reduced substantially in cost with no impairment in quality by substituting two washed cotton braids for the cotton and silk braids formerly used. Altogether, various types of textile insulated wire aggregating annual requirements in excess of two billion conductor feet have either been changed to employ washed textile insulation or are scheduled for change as soon as possible because of corresponding economies in manufacturing cost or improvement in electrical properties.

The foregoing is intended to show what has been accomplished on a commercial scale at reasonable cost in the way of improving the insulating properties of silk and cotton. There still exists a rather wide margin in insulating properties between washed silk and washed cotton at high humidities which further study may show can be reduced. The graphs do not show the magnitude of improvement in cotton which has been obtained occasionally in laboratory experiments which leads us to hope that presently it may be possible to process cotton in a way that will result in its having electrical properties equal to those of washed silk for many practical purposes.

The question naturally arises as to the permanence of the improvement effected by the purification process. We have attempted to answer this question by periodic tests of washed silk and cotton insulated wire over an extended time, the test samples being exposed to ordinary room conditions where they could accumulate the normal quantity of dust. The results show no tendency for the insulation to revert to the constants of unwashed insulation. This appears logical since there is no particular reason to expect contamination by accumulation of such impurities as sodium or potassium salts from ordinary exposure to the air. Furthermore, in service, telephone office wiring is protected from the effects of dust by braided textile coverings or by the application of waxes or varnishes where the individual wires are exposed.

## Conclusion

The discussion has been confined primarily to telephone central office cabling where silk and cotton are used in the cable core without impregnation. However, it is believed that the whole subject of purification of textiles becomes of general interest when it is stated that the improvements obtained by washing are not nullified by the supplementary use of impregnating waxes or varnishes. That is, the improvement in dielectric properties and reduced electrolysis obtained by washing and by impregnating are apparently substantially additive. While the studies have not proceeded far enough to cover comprehensively all of the better known impregnating waxes, asphalts, varnishes, etc., they have proceeded to the point where we can say that this is the case for the beeswax-paraffin waxes and certain asphaltic compounds. These findings are in line with the generally known fact that impregnation of textiles with wax compounds does not prevent, though it does retard, the absorption of moisture which in the presence of soluble salts causes conducting paths to be established, probably through the embedded textile fibers. Consequently, such materials as fabric base insulating tapes, varnished linens and cambrics, electro-magnet coil winding insulation, all being sensitive electrically to moisture, should be benefited to a

substantial degree by purification of the fibrous components.

Therefore, while there is still much to be learned about the behavior of silk and cotton with respect to their electrical characteristics under various treatments and conditions, the study has progressed to the point where the following statements can be made.

1. The removal of water soluble salts which are present in both silk and cotton not only results in a very decided improvement in their insulating properties, but reduces the sensitivity to change of the a-c. characteristics with changes in atmospheric moisture conditions.

2. The improvement which can be realized is great enough to permit the substitution of washed cotton for silk where ordinary commercial silk has been found to give satisfactory results.

3. The use of purified textiles in cables carrying continuous d-c. potential will reduce electrolysis and consequently prolong the useful life of such cables about in proportion to the extent to which the purification process is carried.

In presenting the foregoing discussion, the authors wish to acknowledge their indebtedness to engineers of the Western Electric Company whose work in cooperation with silk suppliers has been largely responsible for the development of commercial methods of purifying insulating silk. Acknowledgment must also be made of the importance of the fundamental and research work which underlies the engineering result briefly described by this paper.

## Discussion

### INFLUENCE OF MOISTURE AND ELECTROLYTES UPON TEXTILES AS INSULATORS

(Williams and Murphy)

### PURIFIED TEXTILE INSULATION FOR TELEPHONE CENTRAL-OFFICE WIRING

(Glenn and Wood)

New York, N. Y., January 31, 1929

**William Fondiller:** The problem of improving textile insulation has been attacked by various methods in the past, usually by treating the textile with some moisture-resisting compound such as wax or mineral soap in an effort to make the thread moisture-repellent. These methods have effected very slight improvement. It wasn't until the fundamental work done by Mr. Williams and his associates on the nature of the conduction in textiles that real improvement was effected.

The value of these 600,000 pounds of silk, a large part of which the authors tell us may eventually be superseded by cotton, at around three dollars a pound for silk, evidently means an economy in the neighborhood of a million and a half dollars a year.

**Bela Gati:** If the Bell concern or other manufacturers have built common-battery central offices in tropical countries, I should like to hear something about their experience in this matter.

**S. J. Rosch:** I should like to ask Mr. Williams just how the moisture content was measured. In the case of a wood-pulp paper which I know is being used in the telephone cable industry, I find that samples of the same paper given to different investigators and under the same conditions of humidity will yield different percentages of moisture.

I am also interested to know, after the cotton has been washed and used in service, just how the entrance of moisture into the fibers once more is retarded? A paper which is thoroughly dry and is brought into atmospheric conditions immediately absorbs moisture again. Even fibers which have been impregnated in waxes are not immune from absorbing moisture.

I should also like to ask Mr. Williams whether he has investigated jute in the same way as he has the cotton and silk fibers.

**E. J. Murphy:** In reply to Mr. Rosch, we didn't actually measure the moisture content on our own samples. We took the data from the literature on the moisture content of textiles and applied it to our own case. The error caused by not measuring the moisture content of the samples actually used is not large enough to have any essential effect on the conclusions to be drawn from the results we have obtained.

There is what might be called a hysteresis effect in the absorption of moisture by textiles. When the humidity is decreasing, the amount of moisture in the textiles is greater than when it is increasing. The difference is quite considerable. It amounts to as large an amount as the difference between the moisture content for humidities that are different from each other by about ten per cent, under some circumstances. Because of this "hysteresis" effect it is necessary to take into account the previous history of the samples as regards their exposure to humid air if consistent results are to be obtained by different investigators making moisture absorption measurements on the same samples under the same humidity conditions.

Washing the water-soluble material out of a textile does not prevent the entrance of moisture nor cause an appreciable reduction in the amount of moisture which it absorbs. The essential effect of washing is to increase the general level of resistance; that is, for any given moisture content, the resistance of a sample of washed cotton is from 50 to 100 times greater than that of the original cotton. This is shown by the fact that the logarithm of resistance *vs.* logarithm of moisture content curves for washed cotton samples are parallel to those for the original cotton.

In further reply to Mr. Rosch, we have done no experiments on jute so I don't know whether the resistance-moisture content relationship is similar to that of cotton or not, but it seems probable that it is since jute is a cellulosic material.

**E. B. Wood:** In view of the point which was brought up regarding trouble in tropical countries, we may say that the curves we have shown apply mainly to the problems of the telephone engineer in temperate climates. It is only those parts of the curves above perhaps 90 or 85 per cent relative humidity that might apply to the more severe conditions of the tropics and you will note that there particularly we get a great benefit from the washing of the textiles. At the lower humidities, particularly under the dry conditions which we have in the winter time, there is very little difference between the washed and unwashed materials. In other words, under dry conditions all textiles are good insulators. It is only when they get moisture in them that we get the low insulation resistance shown on the curves. Therefore, the purification of textiles, it would seem, would be of special interest to those who are interested in the question of insulation for the tropics.

Another point is the corrosion effects. The remark is made in one of the papers that we have had corrosion effects even in our temperate climates and we know from experience that the corrosion troubles are far more serious in the tropics. Since such troubles are caused largely by the presence of impurities in the textile insulation, apparatus for the tropics seems a rather important application for the use of washed textiles.

There is one more point I wish to bring out, namely, that it is not our intention to convey the impression that we are likely in the near future to abandon the use of silk. It is evident from the curves that washed silk is still appreciably superior to washed cotton, and there are many circuits in which the advantage to be gained by the use of washed silk will make it desirable to continue its use. Furthermore, to offset the tendency toward reduction of silk due to the use of washed materials, we have the ever increasing demand for textile insulation to take care of the natural growth of the telephone system.

# Vector Presentation of Broad-Band Wave Filters

BY R. F. MALLINA*
Member, A. I. E. E.

and

O. KNACKMUSS*
Non-member

*Synopsis.—The function of a broad-band wave filter of the iterative ladder type in the attenuation band, and outside the attenuation band can be explained very simply when expressed in terms of two characteristic vectors $Z_a$ and $Z_b$. Drawing the diagram of these vectors, it becomes obvious that the angle between them is the phase shift of the filter and that the natural logarithm of the ratio of their magnitudes is the attenuation.*

*The diagram also shows very plainly the relationship between a mid-series and a mid-shunt structure, and the equations for such filters can be derived in a very simple manner from the geometry of one vector triangle.*

*It is hoped that this simplified presentation of types of filters which are so extensively used in radio, acoustical, and in mechanical engineering will be helpful in understanding their physical meaning.*

## 1.0 Broad Band Filters of the Ladder Type in General

SUPPOSE we measure the impedance of a network as illustrated in Fig. 1 at the point 1 and obtain at the frequency $f$, the value $\mathcal{Z}_i = \mathcal{Z}_k$. The impedance $\mathcal{Z}_i$ we call the input impedance, $\mathcal{Z}_k$ the iterative impedance.[1]

Then we cut off section $a$, measure the impedance at point 2, and obtain again $\mathcal{Z}_i = \mathcal{Z}_k$. Cutting off sections $b$ and $c$ and always obtaining the measurement $\mathcal{Z}_i = \mathcal{Z}_k$ shows that Fig. 1 is a network whose input impedance is equal to the terminating or iterative impedance. A structure of this type is called an iterative structure. The broad-band wave filter of the ladder type is a special case of an iterative structure, and it is this type of filter with which this paper deals.

As will be seen later, it is necessary that with a change of frequency the terminating impedance $\mathcal{Z}_k$ must be varied in a certain manner so that at all times the input impedance $\mathcal{Z}_i$ is equal to $\mathcal{Z}_k$.[2]

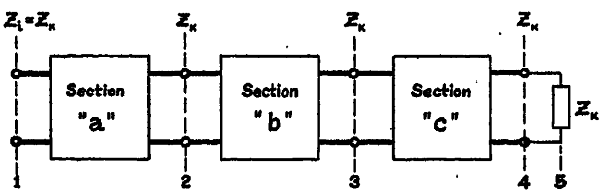


Fig. 1—General Arrangement of an Iterative Structure

A broad band wave filter structure will allow current to pass in a certain frequency band without attenuation, whereas outside this band the current is attenuated considerably.

This fact accounts for the name filter.

*1.1 Two Characteristic Types of Filter Sections.* The following two types of filter sections will make $\mathcal{Z}_i$ equal to $\mathcal{Z}_k$. The one is called a "mid-series section" (Fig. 2), and the other a "mid-shunt section" (Fig. 3). The mid-series section Fig. 2 is also called a $T$ section and the mid-shunt section Fig. 3 a $\pi$ section. There are other special types of filter sections which, however, will not be considered in this paper.

## 2.0 The Mid-Series Filter Section

*2.1 Input Impedance.* That the input impedance

MID-SERIES SECTION

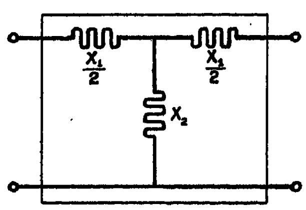


Fig. 2

MID-SHUNT SECTION

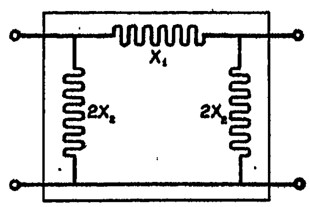


Fig. 3

Figs. 2 & 3—Two Characteristic Types of Filter Sections

$\mathcal{Z}_i$ of a mid-series filter section (Fig. 4) can be made equal to the terminating impedance $\mathcal{Z}_k$ is shown in the impedance diagram, Fig. 5.

For the purpose of our first illustration, let us choose conditions so that $\mathcal{Z}_k$ is a pure resistance.

Adding to the terminating impedance $\mathcal{Z}_k$ the series impedance $X_1/2$ the vector $Z_a$ (Figs. 4 and 5) is obtained. The sum of the reciprocal of $Z_a$ and the vector $1/X_2$ gives us the vector $1/Z_b$ whose reciprocal is then $Z_b$. Adding $X_1/2$ to $Z_b$ we find that the resulting vector $\mathcal{Z}_i$ is equal to $\mathcal{Z}_k$. In other words, having given $\mathcal{Z}_k$ and $X_1$ we choose $X_2$ to have a value such that $\mathcal{Z}_i = \mathcal{Z}_k$. The sequence of these operations is indicated in Fig. 5A.

Expressed in vector mathematics we have:[3]

$$\mathcal{Z}_k + \frac{X_1}{2} = Z_a$$

$$\frac{1}{Z_a} = \frac{|1|\,\epsilon^{j0}}{|Z_a|\,\epsilon^{j\beta/2}} = \frac{|1|}{|Z_a|}\,\epsilon^{j(-\beta/2)} = \frac{|1|}{|Z_a|}\,\overline{\backslash \beta/2}$$

*Victor Talking Machine Co., Camden, N. J.

1. As far as possible the same symbols will be employed in this paper as are used in K. S. Johnson's "Transmission Circuits for Telephonic Communication." (A complete list of symbols appears in Appendix A.)

2. In practise, of course, there is no such terminating impedance having the correct value at every frequency. However, it is possible to change certain elements of the network and obtain a close approximation to filter conditions.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

3. The symbol $|Z|$ indicates magnitude of $Z$. The symbol $Z$ without the bars represents a vector having magnitude and direction.



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In other words, if $Z_a$ is a vector with angle $\left(+\dfrac{\beta}{2}\right)$

the reciprocal $1/Z_a$ is a vector with angle $\left(-\dfrac{\beta}{2}\right)$

$$\frac{1}{Z_a} + 2\frac{1}{2X_2} = \frac{1}{Z_b}$$

$$Z_b + \frac{X_1}{2} = Z_i$$

$$\therefore \boxed{Z_i = Z_k} \tag{1}$$

These vector operations may be repeated for every filter section (Fig. 1) and the diagram (Fig. 5) will

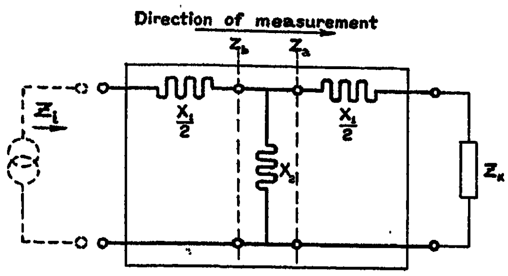



Fig. 4—Circuit of Mid-Series Filter Section

always remain the same. Therefore, we may say in general terms, that the input impedance $Z_i$ is equal to the iterative impedance $Z_k$ and this in turn is equal to the terminating impedance $Z_k$ (Fig. 1 at points 1, 2, 3, and 4).

*2.2 Iterative Impedance* $Z_k$. It is clear from the

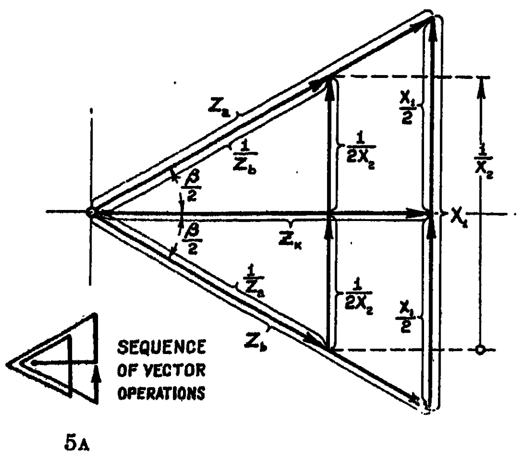



Fig. 5—Vector Diagram of a Mid-Series Filter Section

diagram in Fig. 5 that the vector $1/2\,X_2$ must be of such a value that the vector $Z_a$ is in line with the vector $1/Z_b$. If this is not the case, the input impedance is not equal to the terminating impedance and the structure is not the iterative structure described in paragraph 1.1.

From the vector geometry of Fig. 5 we obtain:

$$\frac{X_1}{2} \div \frac{1}{2X_2} = Z_a \div \frac{1}{Z_b}$$

$$\therefore X_1 X_2 = Z_a Z_b \tag{2}$$

$$\boxed{Z_a = Z_k + \frac{X_1}{2}} \tag{3}$$

$$\boxed{Z_b = Z_k - \frac{X_1}{2}} \tag{4}$$

Substituting (3) and (4) in (2)

$$X_1 X_2 = \left[Z_k + \frac{X_1}{2}\right]\left[Z_k - \frac{X_1}{2}\right] = (Z_k)^2 - \left(\frac{X_1}{2}\right)^2$$

$$Z_k = \sqrt{X_1 X_2 + \left(\frac{X_1}{2}\right)^2}$$

$$\boxed{Z_k = \sqrt{X_1 X_2}\sqrt{1 + \frac{1}{4}\frac{X_1}{X_2}}} \tag{5}$$

In Fig. 5, vector $X_1$ and vector $1/X_2$ have the $+j$ direction; such a structure is called a low pass filter. However, Equation (5) may also be obtained when $X_1$ and $1/X_2$ have the $-j$ direction. This structure is a high pass filter.

In Equation (5) the reactances $X_1$ and $X_2$ are functions of frequency.

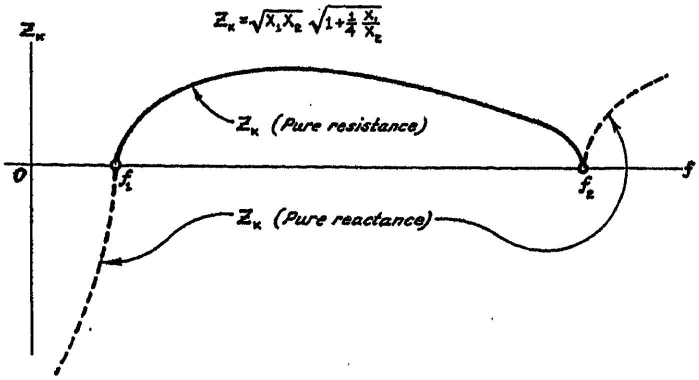



Fig. 6—Iterative Mid-Series Impedance of One Type of Band-Pass Filter

Taking the special case of a band-pass filter (Fig. 38a) we may plot Equation (5) and obtain a curve as illustrated in Fig. 6.

It appears from the figure that $Z_k$ takes the value zero at two points on the frequency scale, one at $f_1$ and one at $f_2$. They are called the cut off frequencies. As will be seen later, between these cut-off points the current passes through the structure without attenuation, whereas it is considerably attenuated outside the frequency cut-off points.

*2.3 Iterative Impedance $\mathfrak{Z}_k$ in Vector Diagram.* How the impedance $\mathfrak{Z}_k$ is changed from a resistance into a reactance at the points $f_1$ and $f_2$ may be illustrated by using the vector diagram of Fig. 5 and varying the value of the impedance vectors $X_1$ and $1/X_2$ from zero to infinity.

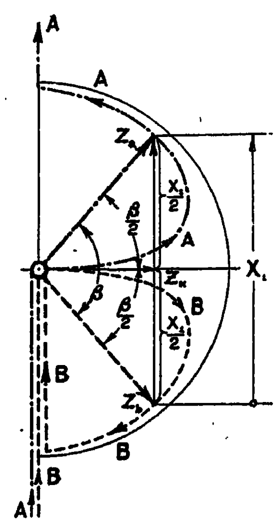



FIG. 7—POSITION OF THE CHARACTERISTIC VECTORS $Z_a$ AND $Z_b$ IN THE FREQUENCY BAND

Let us start with a frequency at which the angle $\beta$ is the angle enclosed by $Z_a$ and $Z_b$ in the band-pass filter diagram of Fig. 7.

By varying the frequency up and down (Equations (3) and (4)), the vector $Z_a$ describes the dot-and-dash line, the vector $Z_b$ the dash-line.

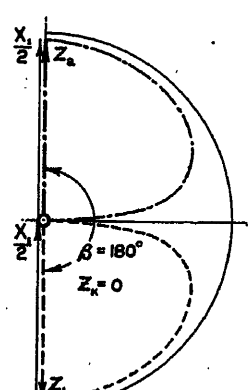



FIG. 8—POSITION OF THE CHARACTERISTIC VECTORS $Z_a$ AND $Z_b$ AT THE UPPER CUT-OFF FREQUENCY $f_2$

The arrows $A$ and $B$ indicate the directions in which the vectors $Z_a$ and $Z_b$ move when the frequency is increased.

If we increase the frequency, we get one limit for $\mathfrak{Z}_k$ as pure resistance when $\beta$ = 180 deg. (Fig. 8).

Then

$$Z_a = - Z_b = \frac{X_1}{2} \tag{6}$$

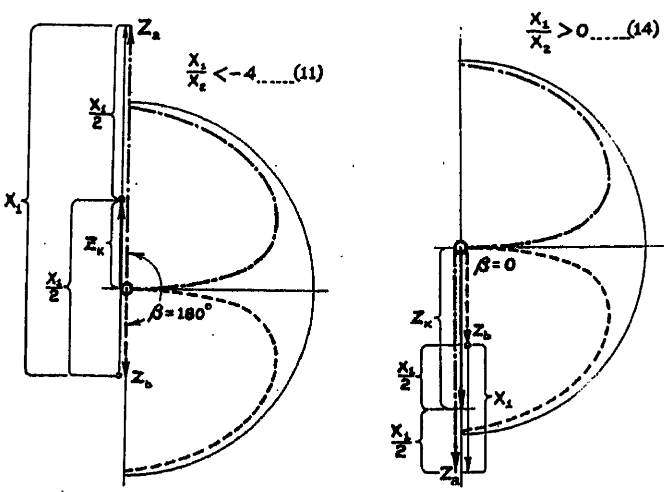
or in other words, $Z_a$ and $Z_b$ are equal in magnitude but have opposite directions. $\mathfrak{Z}_k = 0$ in this case, as is obvious from Fig. 8.

If we decrease the frequency we get the other limit for $\mathfrak{Z}_k$ as pure resistance.

Then

$$Z_a = Z_b = \frac{X_1}{2} = 0 \tag{7}$$

*2.4 Resistance and Reactance Limits of $\mathfrak{Z}_k$.* The question is now, what happens to $\mathfrak{Z}_k$ when we have increased or decreased the frequency beyond these limits.




FIGS. 9 & 10—POSITION OF THE CHARACTERSITIC VECTORS $Z_a$ AND $Z_b$ OUTSIDE THE FREQUENCY BAND

From Fig. 7 we can see that $X_1$ is the difference between the vectors $Z_a$ and $Z_b$ or

$$Z_a - Z_b = X_1 \tag{8}$$

also

$$Z_a = \mathfrak{Z}_k + \frac{X_1}{2} \tag{3}$$

$$Z_b = \mathfrak{Z}_k - \frac{X_1}{2} \tag{4}$$

These equations are also true for $\beta/2$ = 90 deg. Then $\mathfrak{Z}_k$ is zero or a pure reactance and we obtain a condition as shown in Fig. 9.

Fig. 10 represents Equations (8), (3), and (4) when $\beta = 0$.

If we let $X_1/2$ and $1/2\,X_2$ in Fig. 5 increase until $\beta$ = 180 deg. and $\mathfrak{Z}_k = 0$, we obtain a vector diagram as shown in Fig. 11. From Fig. 5 it is obvious that assuming the angle $\beta$ to be 180 deg.:

$$\frac{X_1}{2} = Z_a \tag{9}$$

$$\frac{1}{2\,X_2} = - \frac{1}{Z_a} \tag{10}$$

$$\therefore \frac{X_1}{X_2} = -4 \tag{11}$$

If we let $X_1$ in Figs. 5 and 7 decrease until $\beta = 0$, it is clear that

$$\frac{X_1}{2} = 0 \tag{12}$$

$$\frac{1}{2X_2} = 0 \tag{13}$$

$$\therefore \frac{X_1}{X_2} = 0 \tag{14}$$

In this way, we may say that assuming the vectors

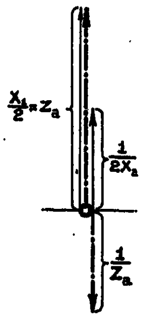


Fig. 11—Position of the Vectors $Z_a$ and $1/Z_a$ at the Upper Cut-Off Frequency $f_2$

$X_1$ and $X_2$ the vector $Z_k$ must be a pure resistance when $X_1/X_2$ is smaller than zero and greater than $-4$ (Fig. 7)

$$\boxed{0 > \frac{X_1}{X_2} > -4} \tag{15}$$

and a pure reactance when $X_1/X_2$ is greater than zero and smaller than $-4$ (Figs. 9 and 10)

$$\boxed{0 < \frac{X_1}{X_2} < -4} \tag{16}$$

Equations (11) and (14) determine the cut off points $f_1$ and $f_2$.

*2.5 Current Relations.* So far, we have considered impedance relations of the network only. It will be interesting now to see how the current passes through a mid-series filter and what the phase shift and attenuation are at various frequencies.

From Fig. 12 we see that the voltage across $Z_b$ is equal to the voltage across $Z_a$

$$I_1 Z_b = I_2 Z_a$$

$$\frac{I_1}{I_2} = \frac{Z_a}{Z_b} \tag{17}$$

$$\therefore \frac{|I_1|}{|I_2|} = \frac{|Z_a|}{|Z_b|} \tag{18}$$

This equation may be expressed in the exponential form and we obtain

$$\frac{|I_1|}{|I_2|} \equiv \epsilon^A = \frac{|Z_a|}{|Z_b|} \tag{19}$$

$$\therefore \boxed{A = ln \frac{|Z_a|}{|Z_b|}} \tag{20}$$

The index $A$ is called the attenuation constant and is the natural logarithm of the ratio of the current magnitude entering the section to the current magnitude leaving it.

*2.6 Phase Shift $\beta$.* Since in the circuit Fig. 12, the current phase shift is equal to the angle between the current $I_1$ entering the section and the current $I_2$ leaving it, and since by (17)

$$\frac{I_1}{I_2} = \frac{Z_a}{Z_b} \tag{17}$$

it is obvious that the angle $\beta$ in Fig. 7 is the phase angle of the filter (Appendix B).

In Fig. 10, $Z_a$ and $Z_b$ have the same direction, $\beta = 0$, and $X_1/X_2 > 0$.

In Fig. 9, $Z_a$ and $Z_b$ have opposite directions $\beta = 180$ deg. and $X_1/X_2 < -4$.

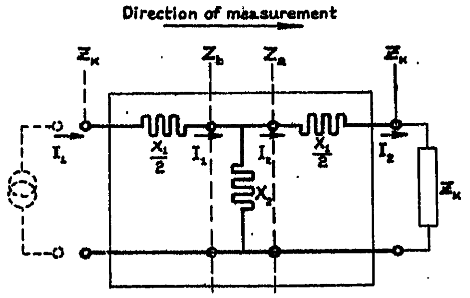



Fig. 12—Circuit of a Mid-Series Filter Section Indicating Current Entering and Leaving the Section

In Fig. 7 the phase angle changes with $Z_k$.

In general we may say that $\beta$ is the angle included by $Z_a$ and $Z_b$.

$$\boxed{\beta = \underline{/Z_a} + \underline{\diagdown Z_b}} \tag{21}$$

Fig. 13 shows that the phase shift changes very abruptly near the cut off points.

*2.7 Attenuation A.* The attenuation in circuit Fig. 12 is zero when the magnitude of the current vector $I_1$ is equal to the magnitude of the current vector $I_2$. This is the case in Figs. 7 and 8.

By Equation (19)

$$\frac{|I_1|}{|I_2|} \equiv \epsilon^A = \frac{|Z_a|}{|Z_b|} \tag{19}$$

Letting $|Z_a| = |Z_b|$ (Figs. 8 and 7).

then $A = 0$

In Figs. 9 and 10, $Z_a$ and $Z_b$ are different in magnitude and there will be attenuation of the current.

$$\frac{|I_1|}{|I_2|} = \frac{|Z_a|}{|Z_b|} = \epsilon^A \tag{19}$$

$$\therefore A = ln \frac{|Z_a|}{|Z_b|} \tag{20}$$

Assuming the special case of a band-pass filter (Fig. 38a) and plotting Equation (20) in terms of frequency, we obtain a curve which changes very abruptly near the cut off point and which indicates that there is no attenuation in the frequency band between the cut off points $f_1$ and $f_2$ or between $X_1/X_2 = 0$, and $X_1/X_2 = -4$ (Fig. 13). Here is presented the basis of the statement made in paragraph 1.0 for definition of a broad-band iterative filter.

The attenuation $A = ln\,|Z_a/Z_b|$ and the phase

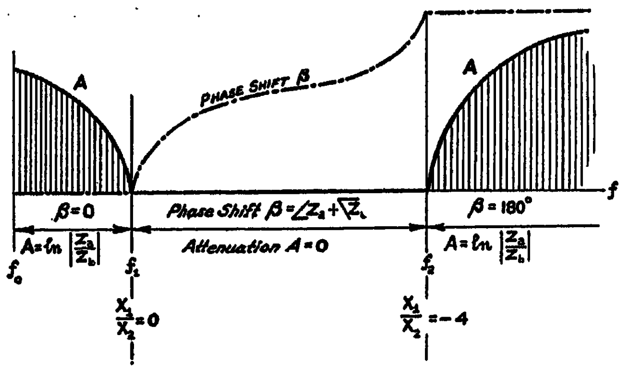



FIG. 13—ATTENUATION AND PHASE SHIFT OF ONE TYPE OF BAND-PASS FILTER

shift $\beta = \underline{/Z_a} + \sqrt{Z_b}$ can be readily expressed in functions of $X_1$ and $X_2$. (See Appendix C.)

## 3.0 THE MID-SHUNT FILTER SECTION

*3.1 Input Impedance.* On the mid-series filter, Figs. 4 and 5, we have shown that the input impedance $\mathcal{Z}_i$ must be equal to the terminating impedance $\mathcal{Z}_k$. The same must be true with a mid-shunt filter. As will be seen from Fig. 15, the vector diagram is identical for both types and the input impedance $\mathcal{Z}_i'$ is also equal to the terminating impedance $\mathcal{Z}_k'$.

Starting with $\mathcal{Z}_k'$ and $2X_2$ (Figs. 14 and 15), we obtain:

$$\frac{1}{\mathcal{Z}_k'} + \frac{1}{2X_2} = \frac{1}{Z_b}$$

$$Z_b + X_1 = Z_a$$

$$\frac{1}{Z_a} + \frac{1}{2X_2} = \frac{1}{\mathcal{Z}_i'} = \frac{1}{\mathcal{Z}_k'}$$

$$\therefore \boxed{\mathcal{Z}_i' = \mathcal{Z}_k'}$$

or the input impedance $\mathcal{Z}_i'$ is equal to the terminating or iterative impedance $\mathcal{Z}_k'$.

*3.2 Iterative Impedance $\mathcal{Z}_k'$.* The terminating impedance $\mathcal{Z}_k'$ may be obtained in a similar manner as for the mid-series filter.

$$X_1 X_2 = Z_a Z_b \tag{2}$$

$$\boxed{\frac{1}{Z_b} = \frac{1}{\mathcal{Z}_k'} + \frac{1}{2X_2}} \tag{22}$$

$$\boxed{\frac{1}{Z_a} = \frac{1}{\mathcal{Z}_k'} - \frac{1}{2X_2}} \tag{23}$$

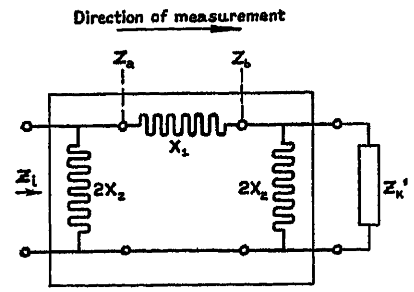



FIG. 14—CIRCUIT OF A MID-SHUNT FILTER SECTION

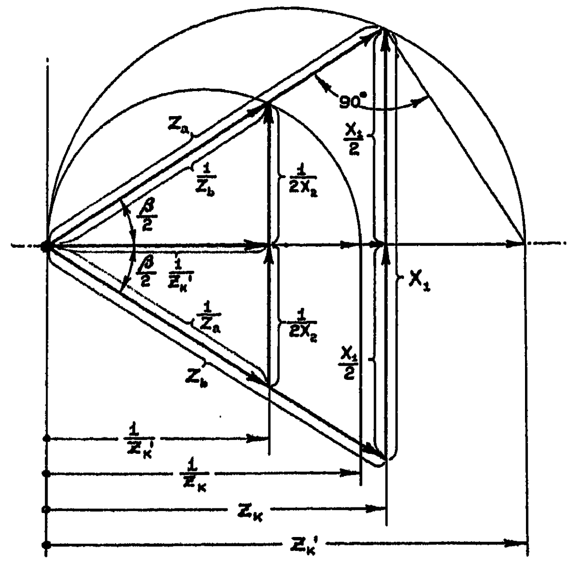



FIG. 15—VECTOR DIAGRAM OF A MID-SHUNT FILTER SECTION

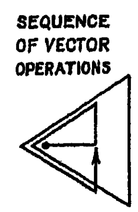



FIG. 15A—SEQUENCE OF VECTOR OPERATIONS

Solving for $\mathcal{Z}_k'$ in terms of $X_1$ and $X_2$ we get:

$$\boxed{\mathcal{Z}_k' = \sqrt{X_1 X_2}\,\frac{1}{\sqrt{1 + \dfrac{1}{4}\dfrac{X_1}{X_2}}}} \tag{24}$$

In Equation (24) $X_1$ and $X_2$ are functions of frequency. Plotting $\mathcal{Z}_k'$ (24) in terms of frequency for a band-pass filter (Fig. 38a) we see from Fig. 16

that $\mathfrak{Z}_k'$ has a value of zero at the cut-off $f_1$, and a value of infinite resistance and infinite capacitive reactance at the cut-off $f_2$.

*3.21 Iterative Impedance $\mathfrak{Z}_k'$ in Vector Diagram.* The variation of $\mathfrak{Z}_k'$ when the frequency is changed may also be shown vectorially.

If we decrease or increase the frequency, we vary $\mathfrak{Z}_k$ and $\mathfrak{Z}_k'$ as was shown in Figs. 7, 8, 9, 10, and 15. The variation of $Z_a$ and $Z_b$ with the frequency will be exactly the same for the mid-series as well as the mid-shunt filter. $\mathfrak{Z}_k$ and $\mathfrak{Z}_k'$, of course, vary differently. From Fig. 15 we see that $1/X_2$ is now the difference between the vectors $1/Z_b$ and $1/Z_a$ and we have:

$$\frac{1}{Z_b} - \frac{1}{Z_a} = \frac{1}{X_2} \tag{25}$$

$$\frac{1}{Z_a} = \frac{1}{\mathfrak{Z}_k'} - \frac{1}{2\,X_2} \tag{23}$$

$$\frac{1}{Z_b} = \frac{1}{\mathfrak{Z}_k'} + \frac{1}{2\,X_2} \tag{22}$$

Figs. 17 and 18 illustrate the vector positions outside the band.

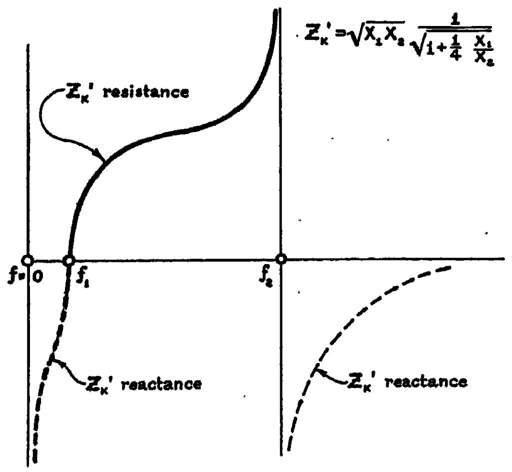



FIG. 16—ITERATIVE MID-SHUNT IMPEDANCE OF ONE TYPE OF BAND PASS FILTER

*3.3 Resistance and Reactance Limits of $\mathfrak{Z}_k'$.* Since the vector diagram (Fig. 5) of a mid-series filter and the vector diagram (Fig. 15) for a mid-shunt filter are identical, the frequency limits between which $\mathfrak{Z}_k$ and $\mathfrak{Z}_k'$ are pure resistances are also the same. That is to say, the cut-off points $f_1$ and $f_2$ are common for both.

Therefore, $\mathfrak{Z}_k'$ will be a pure resistance when

$$\boxed{0 > \frac{X_1}{X_2} > -4} \tag{15}$$

and a pure reactance when

$$\boxed{0 < \frac{X_1}{X_2} < -4} \tag{16}$$

Another interesting relationship may be shown at this point.

From Fig. 15 we see that

$$X_1 \div \frac{1}{X_2} = Z_a \div \frac{1}{Z_b} = \mathfrak{Z}_k \div \frac{1}{\mathfrak{Z}_k'}$$

$$\therefore \boxed{X_1 X_2 = Z_a Z_b = \mathfrak{Z}_k \mathfrak{Z}_k'} \tag{26}$$

This equation is very convenient for checking from mid-series to mid-shunt filters.

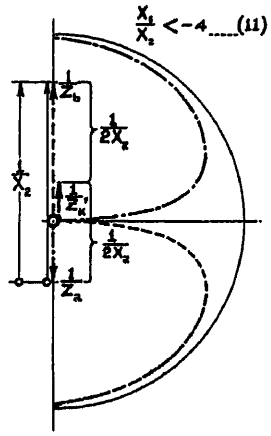



FIG. 17

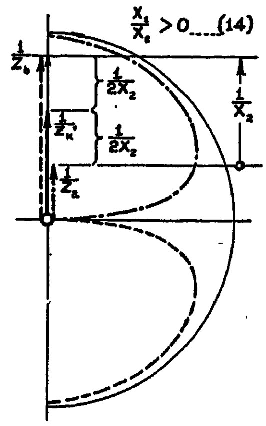



FIG. 18

FIGS. 17 & 18—POSITION OF THE CHARACTERISTIC VECTORS $Z_a$ AND $Z_b$ OUTSIDE THE FREQUENCY BAND

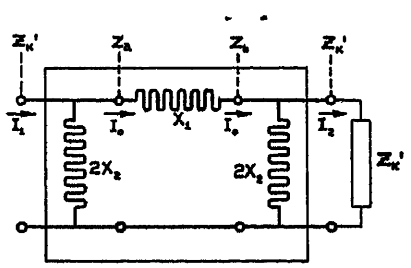



FIG. 19—CIRCUIT OF A MID-SHUNT FILTER SECTION INDICATING CURRENT ENTERING AND LEAVING THE SECTION

*3.4 Attenuation A and Phase Shift β.* It will be shown now that Equation (17) holds true for the mid-shunt as well as for the mid-series filter. (Fig. 19)

$$I_1 \mathfrak{Z}_k' = I_0 Z_a$$

$$I_2 \mathfrak{Z}_k' = I_0 Z_b$$

$$\therefore \frac{I_1}{I_2} = \frac{Z_a}{Z_b} \tag{17}$$

The phase shift and the attenuation are therefore identical for a mid-series and for a mid-shunt filter section of the same ladder type structure:

$$\boxed{\beta = \angle Z_a + \angle Z_b} \tag{21}$$

$$\boxed{A = ln \frac{|Z_a|}{|Z_b|}} \tag{20}$$

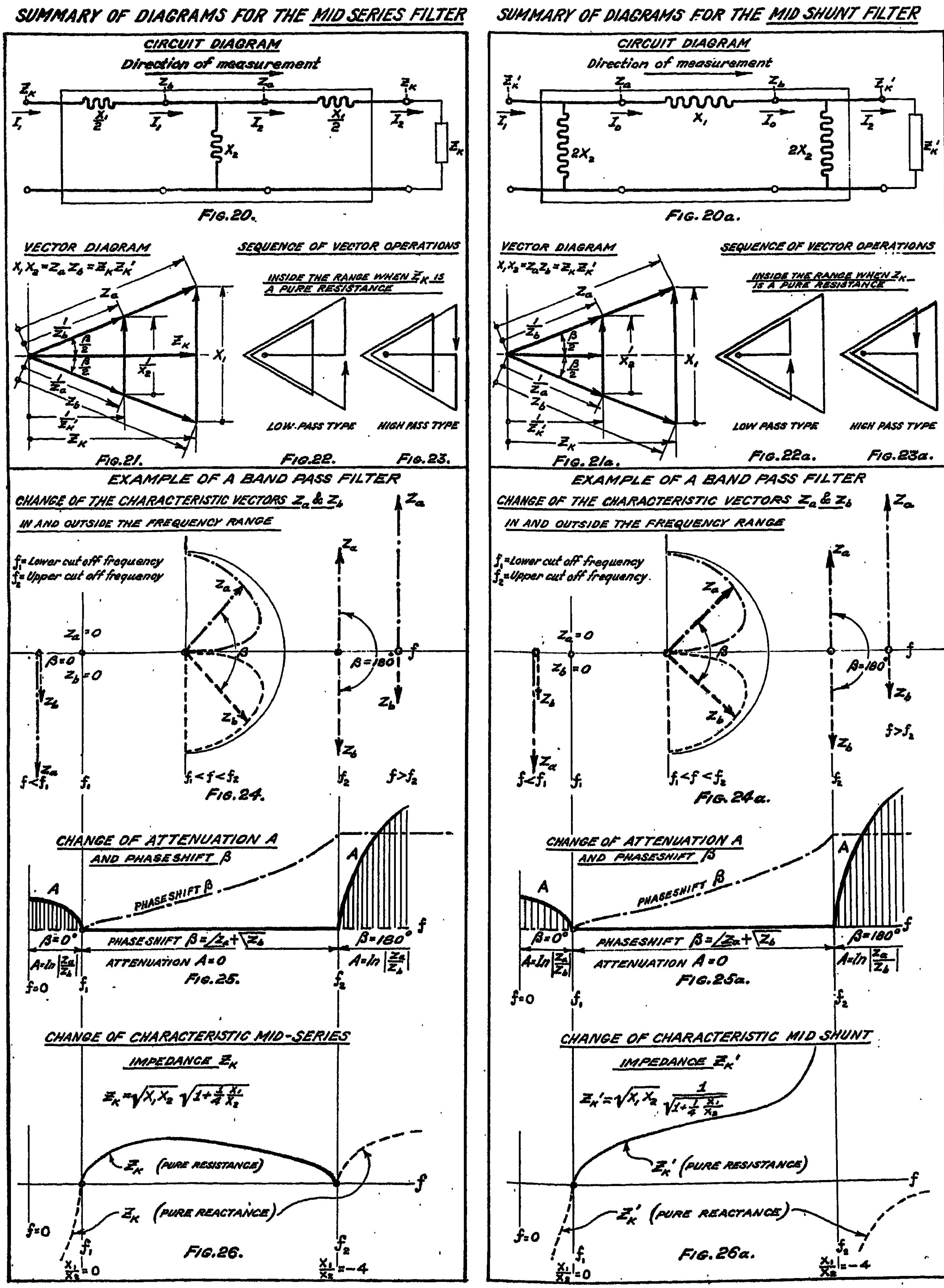



FIG. 26—COMPARISON OF CHARACTERISTICS OF MID-SERIES AND MID-SHUNT TYPES OF FILTERS

Summarizing the various steps in the treatment of the mid-series and mid-shunt filters, we can make a compact picture from which the action of a broad band iterative wave filter in and outside the frequency band is quite obvious. The chart refers to band pass filters of the low pass type. The difference between a filter of the low pass type and the high pass type is illustrated in Figs. 22 and 23.

### 4.0 The Low Pass Filter

In a low pass filter of the two element type (Fig. 27) the series reactance $X_1$ is a pure inductive reactance, and the shunt reactance $X_2$ is a pure capacitive reactance.

$$X_1 = j\,\omega L \tag{27}$$

$$X_2 = \frac{1}{j\,\omega C} \tag{28}$$

$$X_1 X_2 = \frac{\omega L}{\omega C} \tag{29}$$

With $\mathfrak{Z}_k$ properly chosen, we know from Equation (2) that

$$X_1 X_2 = Z_a Z_b \tag{2}$$

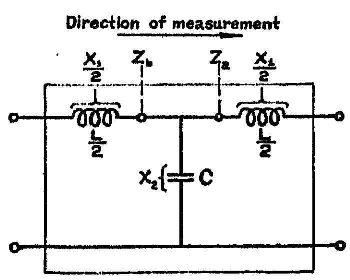



Fig. 27

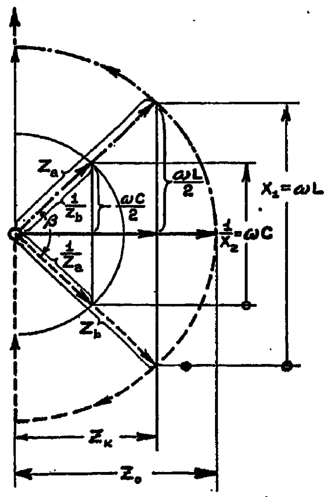



Fig. 28

Figs. 27 & 28—Circuit and Vector Diagrams of a Low Pass Filter Section of the Ladder Type

Substituting (29) in (2) we get:

$$Z_a Z_b = \frac{\omega\ L}{\omega\ C} = \frac{L}{C} \tag{30}$$

Within the band $|Z_a| = |Z_b|$

$$\boxed{|Z_a| = |Z_b| = \sqrt{\frac{L}{C}} = Z_0} \tag{31}$$

$Z_0$ is called the "nominal iterative impedance" or the iterative impedance at the frequency when the term $X_1/X_2$ in Equation (5) is equal to zero. In the case of the low pass filter this frequency is zero.

Since by Equation (31) the magnitudes of $Z_a$ and $Z_b$ are equal and independent of frequency, it is clear that if we vary $\omega$ the impedance vector $Z_a$ describes a section of a circle in the first quadrant, and $Z_b$ one in the fourth quadrant, (Fig. 28). When the angle $\beta$ has reached 180 deg. or when $\omega L/2 = Z_a$, then $Z_a$ and $Z_b$ are pure reactances. A further increase of $\omega$ causes an increase of $Z_a$ and a decrease of $Z_b$ in the direction of the $j$ axis. When $\omega$ increases, $Z_a$ therefore increases, while $Z_b$ decreases. The product, however, must always remain constant. (Equation (30).

*4.1 Cut-Off Frequency $f_1$ and $f_2$.* The lower cut-off frequency is determined by Equation (14)

$$\frac{X_1}{X_2} = 0 \tag{14}$$

Substituting (27) and (28) in (14) we get

$$-\ \omega^2 C L = -4\,\pi^2 f_1^2\, C L = 0$$

$$\therefore\ \boxed{f_1 = 0} \tag{32}$$

Equation (11) determines the upper cut-off $f_2$

$$\frac{X_1}{X_2} = -4 \tag{11}$$

or in the case of the low pass filter

$$\frac{X_1}{X_2} = -\ \omega^2 C L = -4\,\pi^2 f_2^2\, C L = -4$$

$$\therefore\ \boxed{f_2 = \frac{1}{\pi}\sqrt{\frac{1}{C L}}} \tag{33}$$

*4.2 The Iterative Impedance $\mathfrak{Z}_k$ and $\mathfrak{Z}_k'$.* Substituting (27) and (28) in (5) we obtain the equation for the iterative impedance of a mid-series filter.

$$\mathfrak{Z}_k = \sqrt{\frac{L}{C}}\sqrt{1 - \frac{1}{4}\,\omega^2 C L} \tag{34}$$

Substituting (31) and (33) in (34) we get

$$\boxed{\mathfrak{Z}_k = Z_0\sqrt{1 - \left(\frac{f}{f_2}\right)^2}} \tag{35}$$

Similarly we may obtain the equation for the iterative impedance of a mid-shunt filter.

$$\boxed{\mathfrak{Z}_k' = \frac{Z_0}{\sqrt{1 - \left(\frac{f}{f_2}\right)^2}}} \tag{36}$$

*4.3 Phase Shift $\beta$.* By Equation (21)

$$\boxed{\beta = \angle Z_a + \angle Z_b} \tag{21}$$

or in terms of $L$ and $C$, as shown in Fig. 28.

$$\sin\frac{\beta}{2} = \frac{\frac{\omega\ L}{2}}{Z_0} = \frac{\omega\ L}{2}\sqrt{\frac{C}{L}} = \frac{\omega}{2}\sqrt{LC} \tag{37}$$

*4.4 Attenuation A.* By Equation (20)

$$\boxed{A = l\,n \frac{|Z_a|}{|Z_b|}} \tag{20}$$

or in terms of $L$ and $C$ (Appendix C)

$$\text{Cosh}\,\frac{A}{2} = \frac{\dfrac{\omega\,L}{2}}{Z_0} = \frac{\omega\,L}{2}\sqrt{\frac{C}{L}} = \frac{\omega}{2}\sqrt{L\,C} \tag{38}$$

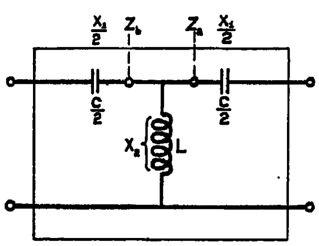


Fig. 30

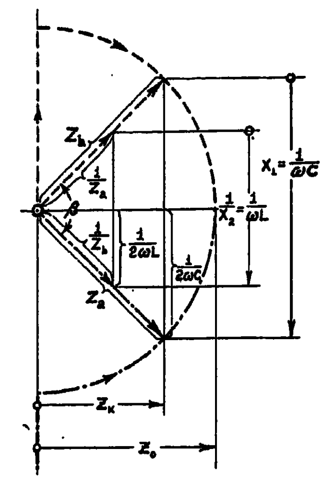


Fig. 31

Fig. 30 & 31—Circuit and Vector Diagram of a High Pass Filter of the Ladder Type

## 5.0 The High Pass Filter

In a high pass filter of the two-element type (Fig. 30) the series reactance $X_1$ is a pure capacitive reactance and the shunt reactance $X_2$ is a pure inductive reactance

$$X_1 = \frac{1}{j\,\omega\,C} \tag{39}$$

$$X_2 = j\,\omega\,L \tag{40}$$

The following formulas for the high pass filter may be readily derived by the same method as used for the low pass filter:

$$\boxed{f_1 = \frac{1}{4\,\pi}\sqrt{\frac{1}{L\,C}}} \tag{42}$$

$$\boxed{f_2 = \infty} \tag{41}$$

$$\boxed{Z_k = Z_0\sqrt{1 - \left(\frac{f_1}{f}\right)^2}} \tag{43}$$

$$\boxed{Z_k' = \frac{Z_0}{\sqrt{1 - \left(\dfrac{f_1}{f}\right)^2}}} \tag{44}$$

## 6.0 The Band Pass Filter

*6.1 Band Pass Filter of the Low Pass Type.* One of the most common filters, especially in mechanical structures, is a filter which has an inductance and a relatively small capacitance in series, and a capacitance in shunt.

$$X_1 = j\,\omega\,L_1 + \frac{1}{j\,\omega\,C_1} \tag{45}$$

$$X_2 = \frac{1}{j\,\omega\,C_2} \tag{46}$$

The circuit diagram and the impedance diagram of such a structure are shown in Figs. 32 and 33.

The cut off points are determined by Equations (11) and (14)

$$\frac{X_1}{X_2} = 0 \tag{14}$$

Substituting (45) and (46) in (14)

$$\frac{j\,\omega_1\,L_1 + \dfrac{1}{j\,\omega_1\,C_1}}{\dfrac{1}{j\,\omega_1\,C_2}} = 0$$

$$j\,\omega_1\,L_1 + \frac{1}{j\,\omega_1\,C_1} = 0$$

$$\omega_1 = \sqrt{\frac{1}{L_1\,C_1}}$$

$$\boxed{f_1 = \frac{1}{2\,\pi}\sqrt{\frac{1}{L_1\,C_1}}} \tag{47}$$

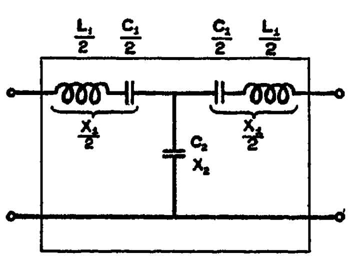


Fig. 32

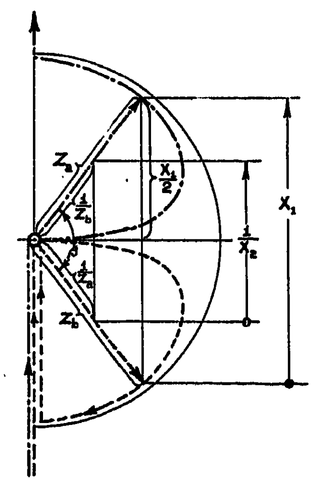


Fig. 33

Figs. 32 & 33—Circuit and Vector Diagram of a Band Pass Filter of the Ladder Type

Substituting the values for $X_1$ and $X_2$ in Equation (11) the other cut-off point is obtained

$$\frac{X_1}{X_2} = -4 \tag{11}$$

$$\frac{j\,\omega_2 L_1 + \dfrac{1}{j\,\omega_2 C_1}}{\dfrac{1}{j\,\omega_2 C_2}} = -4$$

$$\omega_2 = \sqrt{\frac{4 + \dfrac{C_2}{C_1}}{L_1 C_2}} \text{ and}$$

$$\boxed{f_2 = \frac{1}{2\,\pi}\sqrt{\frac{4 + \dfrac{C_2}{C_1}}{L_1 C_2}}} \tag{48}$$

## 7.0 Filters with Resistance

*7.1 Vector Diagram.* If the sections of a filter consist of reactances and resistances (Fig. 34) the vector

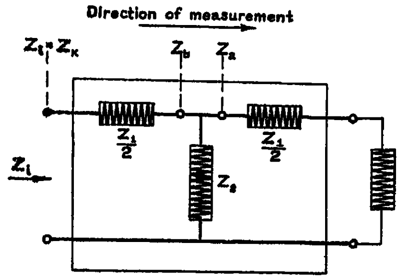



Fig. 34

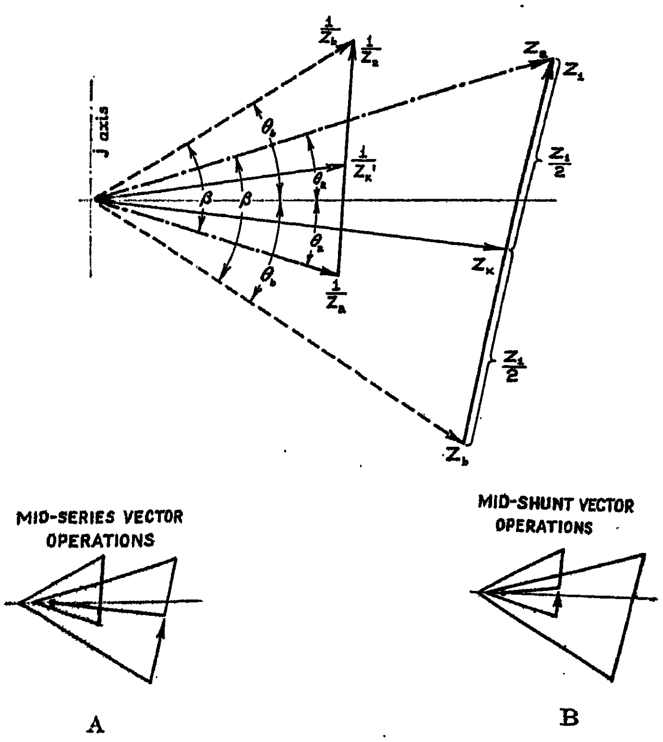



Fig. 35—Circuit and Vector Diagram of a Filter Section Having Small Resistance in the Shunt and Series Elements

diagram assumes the character as shown in Fig. 35. The vectors $Z_a$ and $1/Z_b$ as well as the vectors $Z_b$ and $1/Z_a$ are no longer in line.

The vectors $Z_1$ and $Z_2$ are not pure reactances and therefore they are not parallel to the $j$ axis. Nevertheless, the formulas (20) and (21) derived for filters with pure reactances hold true.

*7.2 Iterative Impedance $Z_k$.* It will be shown now vectorially that also in this type of filter the input impedance can be made equal to the iterative impedance and equal to the terminating impedance. This will be the case when the vector diagram (Fig. 35) is the same for every added filter section.

Starting with vector $Z_k$ and adding $Z_1/2$ the vector $Z_a$ is obtained. The angle of this vector is $\underline{/\theta_a}$ and the angle of $1/Z_a$ is $\overline{\backslash\theta_a}$. In other words, the two angles are equal, but opposite in sign. From circuit diagram (Fig. 34) we see that $Z_i - Z_1/2 = Z_b$ and since $Z_i = Z_k$ we may write

$$Z_k - \frac{Z_1}{2} = Z_b$$

This must also be the case in the vector diagram (Fig. 35). Having thus obtained vector $Z_b$ with angle $\overline{\backslash\theta_b}$ we get $1/Z_b$ with angle $\underline{/\theta_b}$.

In order to obtain a filter structure the vector $Z_2$ must be such that

$$\frac{1}{Z_2} = \frac{1}{Z_b} - \frac{1}{Z_a}$$

Since it is clear that

$$\frac{Z_a}{Z_b} = \frac{\dfrac{1}{Z_b}}{\dfrac{1}{Z_a}}$$

and since the angle $\beta$ of the triangle enclosed by $Z_a$ and $Z_b$ is equal to the angle $\beta$ of the triangle enclosed by $1/Z_a$ and $1/Z_b$ these triangles are similar

$$\frac{1}{Z_b} \div Z_a = \frac{1}{Z_2} \div Z_1$$

$$\therefore Z_a Z_b = Z_1 Z_2 \tag{49}$$

It is also clear from Fig. 35 that

$$\boxed{Z_a = Z_k + \frac{Z_1}{2}} \tag{50}$$

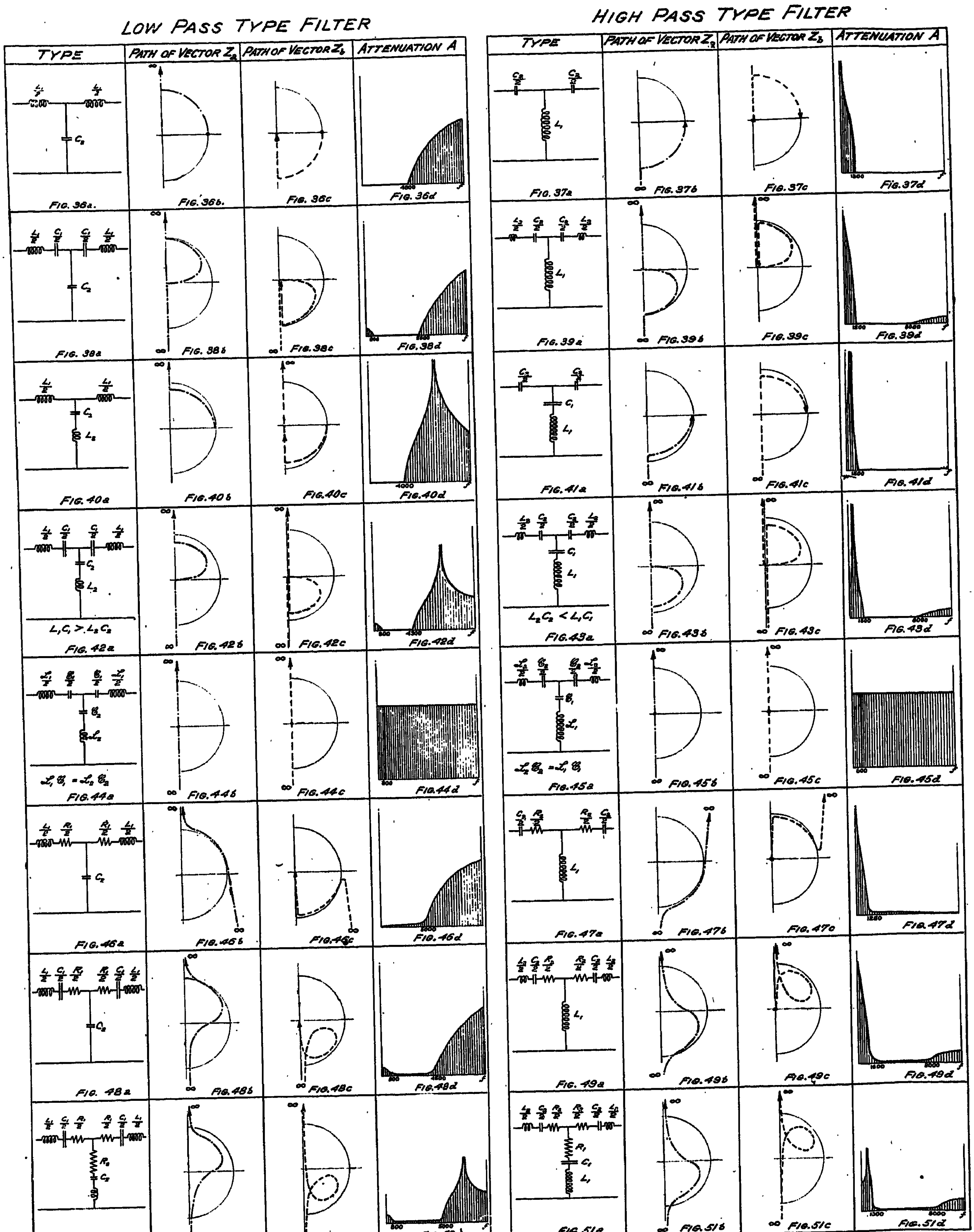
$$\boxed{Z_b = Z_k - \frac{Z_1}{2}} \tag{51}$$

Solving for $Z_k$ in terms of $Z_1$ and $Z_2$ we obtain for mid-series filter

$$\boxed{Z_k = \sqrt{Z_1 Z_2}\sqrt{1 + \frac{1}{4}\frac{Z_1}{Z_2}}} \tag{52}$$

and similarly for a mid-shunt filter

$$\boxed{Z_k' = \sqrt{Z_1 Z_2}\frac{1}{\sqrt{1 + \dfrac{1}{4}\dfrac{Z_1}{Z_2}}}} \tag{53}$$




$L_1$ = 0.277 HENRYS $C_1$ = 0.143 MICRO-FARADS $R_1$ = 100 OHMS
$L_2$ = 0.0277 HENRYS $C_2$ = 0.0143 MICRO-FARADS $R_2$ = 100 OHMS

$L_1$ = 0.277 HENRYS $C_1$ = 0.143 MICRO-FARADS $R_1$ = 100 OHMS
$L_2$ = 0.0277 HENRYS $C_2$ = 0.0143 MICRO-FARADS $R_2$ = 100 OHMS

FIGS. 36 TO 51—SUMMARY OF THE MOST COMMON TYPES OF BROAD BAND FILTERS OF THE ITERATIVE LADDER TYPE SHOWING PATHS OF THE CHARACTERISTIC VECTORS $Z_a$ AND $Z_b$, ALSO THE ATTENUATION

which two equations correspond with the pure reactance Equations (5) and (24)

*7.3 Phase Shift β and Attenuation A.* The same relations are true for phase shift and attenuation as with the reactance filter, therefore

$$\boxed{\beta = \underline{/Z_a} + \overline{\diagdown Z_b} = \underline{\Big/ \frac{1}{Z_b}} + \overline{\Big\diagdown \frac{1}{Z_a}}} \qquad (21)$$

$$\boxed{A = l\,n\,\frac{|Z_a|}{|Z_b|}} \qquad (20)$$

The difference between a filter with resistance and one without resistance is the fact that the frequency cut off points are not sharp and there will always be a certain amount of attenuation in the band, and phase shift variation outside the band.

### 8.0 Summary of Filter Structures

A summary of the most common filter structures is given in the accompanying chart (Figs. 36a-51d).

## Appendix A

### List of Symbols

| | |
|---|---|
| $A$ | Attenuation constant |
| $C$ | Capacity |
| $f$ | Frequency |
| $f_1$ | Lower cut-off frequency |
| $f_2$ | Upper cut-off frequency |
| $I_1$ | Current entering a filter section |
| $I_2$ | Current leaving a filter section |
| $j$ | Operator $\sqrt{-1}$ |
| $L$ | Inductance |
| $R$ | Resistance |
| $X$ | Reactance |
| $X_1$ | Iterative series reactance |
| $X_2$ | Iterative shunt reactance |
| $Z$ | Impedance |
| $Z_a$ | Characteristic impedance on the load side of the shunt member |
| $Z_b$ | Characteristic impedance on the source side of the shunt member |
| $Z_i$ | Input impedance for mid-series |
| $Z_i'$ | Input impedance for mid-shunt |
| $Z_k$ | Iterative impedance for mid-series |
| $Z_k'$ | Iterative impedance for mid-shunt |
| $Z_0$ | Nominal iterative impedance |
| $Z_1$ | Series impedance |
| $Z_2$ | Shunt impedance |
| $\beta$ | Phase shift or phase constant |
| $\epsilon$ | Natural base 2.718 |
| $\theta_a$ | Angle of vector $Z_a$ |
| $\theta_b$ | Angle of vector $Z_b$ |
| $\omega$ | $2\pi f$ |
| $\vert\ \vert$ | Absolute or scalar value of the complex quantity enclosed |
| $\vert Z_a \vert$ | Magnitude of vector $Z_a$ |
| $\equiv$ | Equal by definition |
| $l\,n$ | log to base $\epsilon$ |
| $\angle$ | Sign of a positive angle |
| $\diagdown$ | Sign of a negative angle |
| $<$ | Is less than |
| $>$ | Is greater than |

| | | |
|---|---|---|
| | $R$ | Resistance |
| | $X$ | Reactance |
| | $L$ | Inductance |
| | $C$ | Capacitance |
| | $\mathfrak{z}$ | Pure reactance or pure resistance |
| | $Z$ | Any kind of impedance |
| | $G$ | Generator or source |

## Appendix B

By Equation (17)

$$\frac{I_1}{I_2} = \frac{Z_a}{Z_b}$$

If $\alpha_1$ is the phase angle of the current vector $I_1$, and $\alpha_2$ the phase angle of the current vector $I_2$ the vectors $I_1$ and $I_2$ may be expressed in the following way:

$$I_1 = |I_1|\,\epsilon^{j\alpha_1}$$

$$I_2 = |I_2|\,\epsilon^{j\alpha_2}$$

$$\therefore \frac{I_1}{I_2} = \left|\frac{I_1}{I_2}\right| \frac{\epsilon^{j\alpha_1}}{\epsilon^{j\alpha_2}} = \left|\frac{I_1}{I_2}\right| \epsilon^{j(\alpha_1-\alpha_2)} = \left|\frac{I_1}{I_2}\right| \epsilon^{j\beta}$$

where $\beta$ is the phase angle between $I_1$ and $I_2$

Similarly

$$\frac{Z_a}{Z_b} = \frac{|Z_a|}{|Z_b|}\,\frac{\epsilon^{j\underline{/Z_a}}}{\epsilon^{j\underline{/Z_b}}} = \frac{|Z_a|}{|Z_b|}\,\epsilon^{j(\underline{/Z_a} - \underline{/Z_b})}$$

Equation (17) may therefore be written

$$\frac{|I_1|}{|I_2|}\,\epsilon^{j\beta} = \frac{|Z_a|}{|Z_b|}\,\epsilon^{j(\underline{/Z_a} - \underline{/Z_b})}$$

This formula is only true, if

$$\frac{|I_1|}{|I_2|} = \frac{|Z_a|}{|Z_b|} \qquad (18)$$

and $\beta = \underline{/Z_a} - \underline{/Z_b}$

or $\beta = \underline{/Z_a} + \overline{\diagdown Z_b}$ (21)

## Appendix C

The attenuation constant $A$ in terms of the characteristic vectors $Z_a$ and $Z_b$ is given by Equation (20).

$A$ may be readily expressed as a function of $X_1$ and $X_2$.

By Equation (19)

$$\epsilon^A = \frac{|Z_a|}{|Z_b|} \tag{19}$$

and by definition of a hyperbolic cosine

$$\text{Cosh}\,\frac{A}{2} = \frac{1}{2}\left(\epsilon^{\frac{A}{2}} + \epsilon^{-\frac{A}{2}}\right)$$

$$\therefore \text{Cosh}\,\frac{A}{2} = \frac{1}{2}\left(\sqrt{\frac{|Z_a|}{|Z_b|}} + \sqrt{\frac{|Z_b|}{|Z_a|}}\right)$$

$$= \frac{1}{2}\,\frac{|Z_a| + |Z_b|}{\sqrt{|Z_a|\,|Z_b|}}$$

By Equation (2)

$$|Z_a|\,|Z_b| = |X_1|\,|X_2| \tag{2}$$

$$\therefore \text{Cosh}\,\frac{A}{2} = \frac{1}{2}\,\frac{|Z_a| + |Z_b|}{\sqrt{|X_1|\,|X_2|}}$$

Similarly

$$\text{Sinh}\,\frac{A}{2} = \frac{1}{2}\left(\epsilon^{\frac{A}{2}} - \epsilon^{-\frac{A}{2}}\right) = \frac{1}{2}\,\frac{|Z_a| - |Z_b|}{\sqrt{|X_1|\,|X_2|}}$$

Outside the band when $X_1/X_2 > 0$ (Fig. 10) the vectors $Z_a$ and $Z_b$ are pure reactances and have the same direction. From Fig. 10 it is obvious that

$$|X_1| = |Z_a| - |Z_b|$$

$$\therefore \text{Sinh}\,\frac{A}{2} = \frac{1}{2}\,\frac{|X_1|}{\sqrt{|X_1|\,|X_2|}} = \frac{1}{2}\sqrt{\frac{|X_1|}{|X_2|}}$$

The vectors $X_1$ and $X_2$ having the same direction, we may substitute

$$\frac{X_1}{X_2} \text{ for } \frac{|X_1|}{|X_2|}$$

$$\therefore \text{Sinh}\,\frac{A}{2} = \frac{1}{2}\sqrt{\frac{X_1}{X_2}}$$

$$\boxed{A = 2\,\text{Sinh}^{-1}\left(\frac{1}{2}\sqrt{\frac{X_1}{X_2}}\right)} \tag{53}$$

Outside the band when $X_1/X_2 < -4$ (Fig. 9) the vectors $Z_a$ and $Z_b$ have opposite directions. Fig. 9 shows that

$$|X_1| = |Z_a| + |Z_b|$$

$$\therefore \text{Cosh}\,\frac{A}{2} = \frac{1}{2}\,\frac{|X_1|}{\sqrt{|X_1|\,|X_2|}} = \frac{1}{2}\sqrt{\frac{|X_1|}{|X_2|}}$$

The vectors $X_1$ and $X_2$ having opposite directions we may substitute $-X_1/X_2$ for $|X_1/X_2|$

$$\text{Cosh}\,\frac{A}{2} = \frac{1}{2}\sqrt{-\frac{X_1}{X_2}}$$

$$A = 2\,\text{Cosh}^{-1}\left(\frac{1}{2}\sqrt{-\frac{X_1}{X_2}}\right) \tag{54}$$

The relation between the phase angle $\beta$ and $X_1$ and $X_2$ may be read from Fig. 5.

$$\sin\frac{\beta}{2} = \frac{\dfrac{|X_1|}{|2|}}{|Z_a|}$$

$$|Z_a| \div \frac{1}{|Z_a|} = |X_1| \div \frac{|1|}{|X_2|}$$

$$|Z_a| = \sqrt{|X_1|\,|X_2|}$$

$$\therefore \sin\frac{\beta}{2} = \frac{1}{2}\,\frac{|X_1|}{\sqrt{|X_1|\,|X_2|}} = \frac{1}{2}\sqrt{\frac{|X_1|}{|X_2|}}$$

Since in the band the vectors $X_1$ and $X_2$ have opposite directions we may write.

$$-\frac{X_1}{X_2} \text{ instead of } \frac{|X_1|}{|X_2|}$$

$$\sin\frac{\beta}{2} = \frac{1}{2}\sqrt{-\frac{X_1}{X_2}} \text{ and}$$

$$\boxed{\beta = 2\sin^{-1}\left(\frac{1}{2}\sqrt{-\frac{X_1}{X_2}}\right)} \tag{55}$$

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## Discussion

**C. H. Dagnall:** An interesting vector diagram results when the output current is expressed in terms of a constant input current. The locus of the output current vector is a circle in

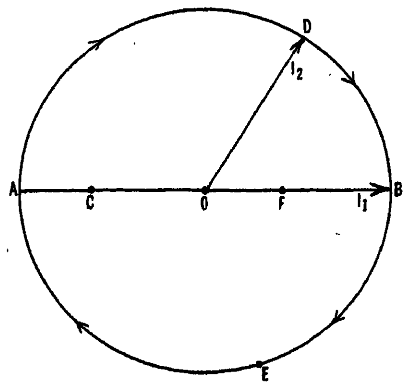


FIG. 1

the transmission bands and a diameter of the circle in the attenuation bands. A dissipationless structure terminated by its iterative impedance is assumed. In Fig. 1 herewith, let $I$ be held constant. The vector $I_2$ will terminate on the line $A\ B$ in the attenuation bands and on the circle in the transmission bands. For the structure of Fig. 2 herewith the locus will start at $O$, continue to $A$, then *clockwise*, around the circle past $D$, $B$, and $E$ to $A$, and finally along $A\ O$ to $O$. For the structure of Fig. 36 of the paper the locus would be $B$-$E$-$A$-$O$, the starting point $B$ corresponding to zero frequency.

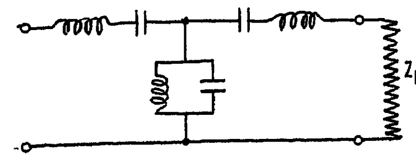


FIG. 2

When the filter possesses an infinite attenuation peak, such as that of Fig. 40, the terminus of $I_2$ goes to the origin at the peak and then moves out along $O\ A$ or $O\ B$. For the structure of Fig. 43 the locus would be $F$-$O$-$A$-$D$-$E$-$A$-$C$.

It will be noted that the diagram represents both the attenuation and phase characteristics. The phase constant $B$ is the angle by which $I_2$ *lags* $I_1$. In multisection filters the terminus of $I_2$ may go around the circle several times in one transmission band, since the phase constants of the several sections must be added.

# The Condenser Motor

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Fellow, A. I. E. E.

*Synopsis.—After a brief description of the construction and connections of the condenser motor the necessity of varying the capacitance is discussed, and the performance at start and under load is considered. Locus diagrams illustrating the operating performance in detail are given, followed by a more detailed discussion of starting torque. The Appendix gives the mathematical derivation of many of the formulas discussed.*

---

THE connections of a single-phase condenser motor are shown in Fig. 1. The motor itself is identical with a two-phase induction motor with the exception of the fact that the two windings are not necessarily alike. Winding 2 may have more or less turns than winding 1. The total weight of copper in the two, however, is approximately the same. The rotor is identical with that of any polyphase motor. It usually is of the squirrel-cage type although a wound rotor may of course be used.

To obtain the best results, the capacitance should be

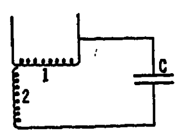


FIG. 1—SIMPLE CONDENSER MOTOR

large when the motor is being started and should be gradually reduced as the speed is increased. If it were practicable to adjust the capacitance to exactly the proper value corresponding to each value of load and speed, the motor itself would operate practically as a two-phase motor, the combination of the condenser and line operating as a phase changer to convert the single-phase supply into a two-phase supply. In practise a fixed value of capacitance may be satisfactory, providing a starting torque of about 50 per cent of full load running torque is sufficient. If more starting torque is necessary the motor may be connected as shown in Fig. 2. The switch $S_1$ is closed when the motor is at rest and is opened (usually automatically) when the speed is sufficiently high. Fig. 2 also illustrates a method by which the direction of rotation of the motor may be reversed, by throwing the switch $S_2$ to the right or left.

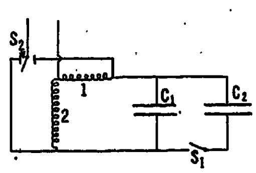


FIG. 2—CONDENSER MOTOR, REVERSIBLE AND WITH VARIABLE CAPACITANCE

Instead of using two condensers it is possible to supply the condensers through a variable ratio transformer as shown in Fig. 3. By applying a high voltage to the condenser at start and a smaller voltage for running, the same effect is produced as though the

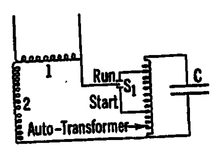


FIG. 3—CONDENSER MOTOR WITH VARIABLE TRANSFORMER

capacitance were changed. With this scheme the efficiency will necessarily be a little lower due to the losses in the transformer.

The vector diagram of a condenser motor is shown in Fig. 4. This was plotted from an actual test of a small motor under full load. In this case the motor was a standard two-phase motor.

The current $I_1$ in phase 1 lags, as usual, by a con-

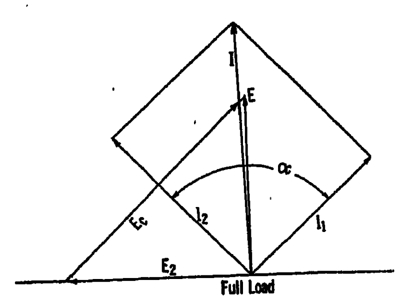


FIG. 4—VECTOR DIAGRAM OF CONDENSER MOTOR. (FULL LOAD)

siderable angle behind the line voltage $E$. Due to the introduction of the condenser, the current $I_2$ in phase 2 may be made to lead the line voltage. When the proper capacitance is used the two currents are approximately at right angles to one another and, if the two windings are alike, are nearly equal. Under these conditions the motor will operate just as though it were a two-phase motor and of course with the same efficiency.

From the above it will be apparent that we can build a single-phase motor having at full load practically the same efficiency as a two-phase motor and operating

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at or near 100 per cent power factor. It is self evident that its characteristics will be much better than those of a single-phase motor of the usual construction which necessarily operates at a lower efficiency and power factor than a two-phase motor.

In starting performance, the condenser motor is somewhat superior to the two-phase motor. Since one current leads and the other lags, the combined starting current is the vector sum of the two and is less than their arithmetical sum. For the same reason the power factor is excellent and is usually close to 100 per cent. The motor will develop even more torque than a two-phase motor and the current required is substantially less; in fact, the torque per ampere is nearly double that of a two-phase motor.

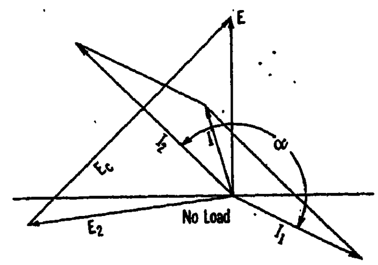


Fig. 5—Vector Diagram of Condenser Motor. No Load

When operating at or near synchronous speed the condenser motor like the usual single-phase motor has a nearly uniform rotating field, and consequently there is a nearly uniform induced voltage in the auxiliary winding, and the applied voltage across winding 2 will also be nearly constant. The induced voltage will be dependent upon the ratio of the number of turns in the two windings. When the windings are alike, the voltage across winding 2 will be nearly equal to the line voltage, the ratio being of course not exact, due to the resistance and reactance drops in the two windings. This voltage across the winding 2 will moreover be nearly 90 deg. out of phase with the voltage in winding 1. The voltage across the condenser, being the vector sum of the line voltage and the voltage of winding 2, will be approximately 1.41 times the line voltage with a one to one ratio. With any other ratio the voltage of winding 2 and the condenser voltage will be correspondingly changed.

The action of the motor under varying loads is shown in the vector diagrams of Figs. 4, 5, and 6. These diagrams have been plotted from tests upon a small two-phase motor connected as shown in Fig. 1. As previously explained, $E_2$, the voltage across phase 2, remains approximately constant both in magnitude and direction. The voltage across the condenser is marked $E_c$ and this likewise remains approximately constant. Since the current in the condenser (which is also the current in winding 2) must of course lead $E_c$ by 90 deg. and be proportional to it, it also varies but little in magnitude and direction.

When the motor is operating under no load (see Fig. 5) the power in phase 2 will obviously be more than necessary to operate the motor. It follows that the current in phase 1 must lag more than 90 deg. behind the line voltage so that it is returning power to the line. The vector sum of $I_1$ and $I_2$ is $I$, the line current. This must be of such a magnitude and phase as to give the necessary power to operate the motor. It will be obvious that at light loads both $I_1$ and $I_2$ are quite large and the motor will therefore not be so efficient under these conditions as a two-phase motor. The power factor is however much better.

The vector diagram of the same motor at 50 per cent over-load is shown in Fig. 6. The power in phase 2 remains nearly the same as at no load or full load and the current and power in phase 1 must therefore increase. The line current $I$ is now somewhat lagging although the power factor is still very high. Again it will be seen that since $I_1$ and $I_2$ are not equal the efficiency must be somewhat lower than that of the same motor operated two-phase. The power factor, however, is excellent and the efficiency better than that of a plain single-phase motor.

## Locus Diagrams

These relations can perhaps be more readily visualized from the locus diagram of Fig. 7. The curve marked $I_1$ represents the locus of the vector $I_1$ as the load is changed. The small circles represent the positions of the end of the vectors corresponding to no-load, full load, and 50 per cent over load and are taken from the same tests as those used in drawing Figs. 4, 5, and 6. Similarly the short curves marked $E_2$ and $I_2$ represent the loci of the vectors representing the voltage across phase 2 and the current in phase 2. The curve marked $I$ is the locus of the line current. In this particular case the current was leading at light load, in phase with the voltage at a little over full load, and slightly lagging for 50 per cent over-load. The power factor throughout this range of load was very close to 100 per cent.

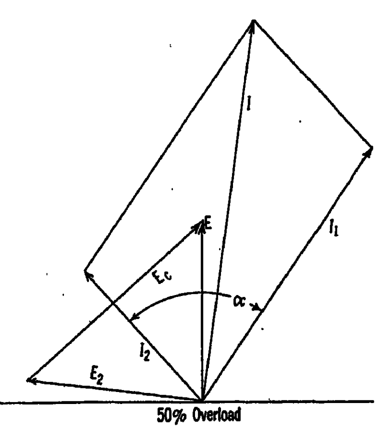


Fig. 6—Vector Diagram of Condenser Motor 50 per Cent Overload

Fig. 8 shows a locus diagram for the same motor but connected so that phase 2 has twice as many turns as phase 1. The voltage across the phase 2 is of course nearly twice as great as before. The condenser voltage obtained by drawing a line from any point on the curve $E_2$ to the end of the vector $E$ is more nearly at right angles to $E$ than before. Since the current $I_2$ must be at right angles to the condenser voltage it is brought

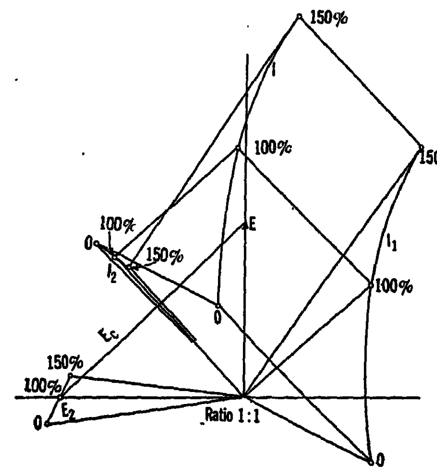


FIG. 7—LOCUS DIAGRAM OF CONDENSER MOTOR. WINDING RATIO 1 TO 1

more nearly into phase with $E$. Since the capacitance used was such as to give nearly 100 per cent power factor to the motor as a whole, it follows that the current $I_1$

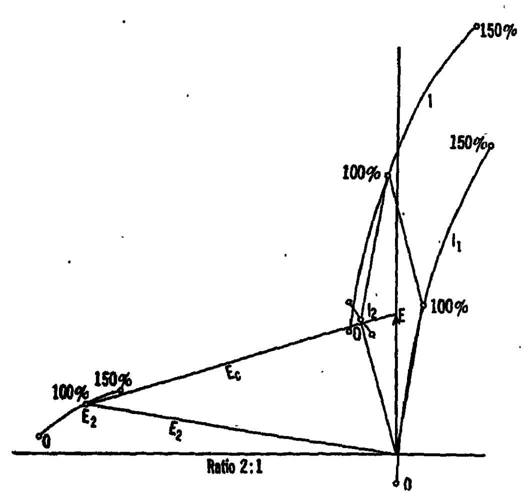


FIG. 8—LOCUS DIAGRAM OF CONDENSER MOTOR. WINDING RATIO 2 TO 1

was forced to be more nearly in phase with the line voltage. The vector sum of the two is represented by the current $I$ (the line current) and this is at nearly 100 per cent power factor throughout the range of the motor.

The advantage of having more turns in winding 2 than in winding 1 is that a higher voltage is applied to the condenser and consequently the capacitance can be greatly reduced; in fact the condenser used in making the tests represented in Fig. 8 was approximately one-half as large as that used in the tests of Fig. 7. It will of course be apparent that since the two currents $I_1$ and $I_2$ are no longer at right angles the motor is not operating so efficiently. In fact the conditions approach those of the ordinary single-phase motor,

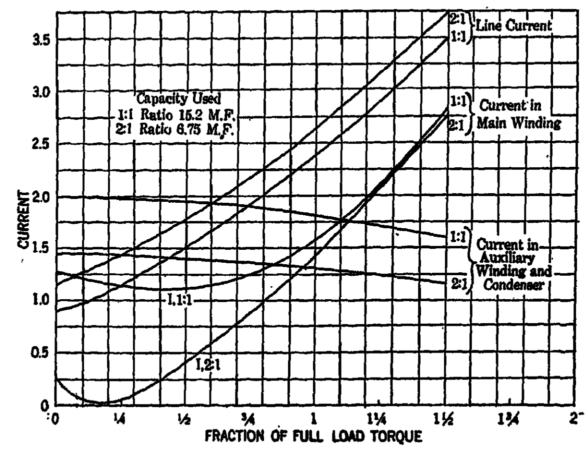


FIG. 9—TORQUES AND CURRENTS IN THE DIFFERENT WINDINGS OF CONDENSER MOTOR

since the current $I_1$ and $I_2$ do not differ very greatly in phase. The power factor is still excellent but the efficiency of the motor is somewhat reduced.

Fig. 9 has been plotted from the same data used in Figs. 4 to 8, and shows the variation of the various currents with the torque. Similarly, Fig. 10 shows the variation of the $I^2 R$ losses in the different windings.

## COMPARISON OF WATTS

In Fig. 11, the total watts input to the motor and also the watts in each of the windings have been plotted.

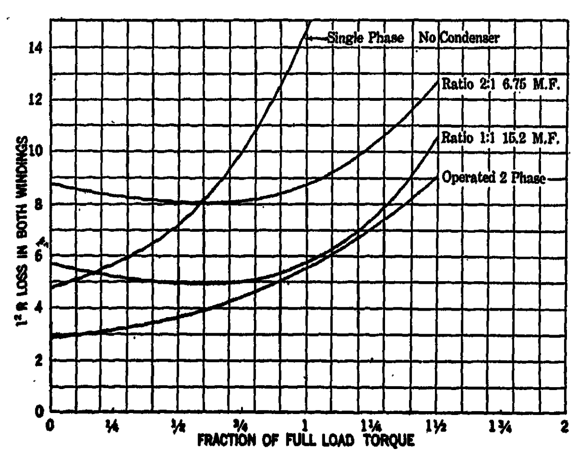


FIG. 10—TORQUE AND $I^2 R$ LOSSES IN CONDENSER MOTOR

The watts in the auxiliary winding are nearly the same with either connection and decrease only slightly as the load increases. The watts in the main winding are in both cases negative with light loads and of course increase as the load increases. The total power required with the two to one connection is greater than

with the one to one connection on account of the reduced efficiency.

### Effect of Changing Capacitance

It will be evident from the preceding discussion that the characteristics of a motor will be radically modified by any change in capacitance. The results of a test upon the same motor, previously referred to, have been embodied in Fig. 12, which shows the currents plotted in their proper phase relation, the output being held constant while the capacitance was varied.

It will be noted in both of these curves that the power component of the current, that is, the projection of the current upon the voltage axis, is less with the current somewhat lagging than it is with 100 per cent power factor. At full load the minimum power input and consequently the highest efficiency is obtained with a capacitance of about five microfarads and at half load with about three microfarads. The lowered efficiency, as previously explained, is due to the fact that the current does not divide between the two windings in

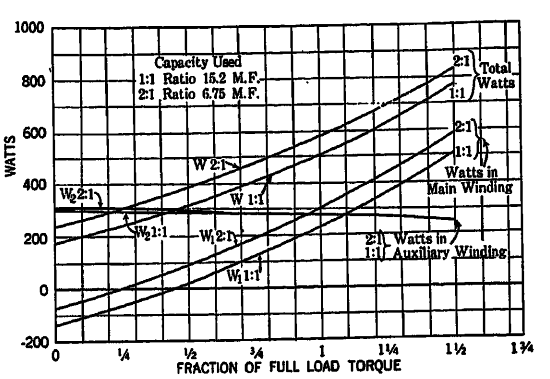


Fig. 11—Torques and Watts in Condenser Motor

the best ratio. In general, therefore, the capacitance which gives the best power factor will not necessarily be that which gives the highest efficiency. Particularly in the case of motors with a large number of turns in winding 2 compared with winding 1, it will be necessary to use a capacitance giving a somewhat lagging current if the best efficiency is to be obtained. With a one to one ratio, the points of best power factor and best efficiency will usually come more nearly together.

### Starting Torque

As shown in the appendix the starting torque is given by the equation

$$T = Q\, N_1\, N_2\, I_1\, I_2 \sin \alpha.$$

In other words, it is proportional to the ampere turns in the winding 1, the ampere turns in the winding 2, to the sine of the angle between the two currents, and to a constant which depends upon the construction of the motor.

The maximum current will exist in circuit 2 when the circuit is in resonance but under these circumstances the angle between the two currents will be unfavorable and consequently the greatest possible torque will not be developed.

It is shown in the appendix that the maximum starting torque is obtained when

$$X_3 = \frac{1}{C\ \omega} = -\,K^2\, Z_1$$

in which $Z_1$ is the impedance of circuit 1, $C$ is the capacitance of the condenser in farads, and $\omega = 2\,\pi\, f$. $K$ is the ratio of turns in winding 2 to the turns in winding 1.

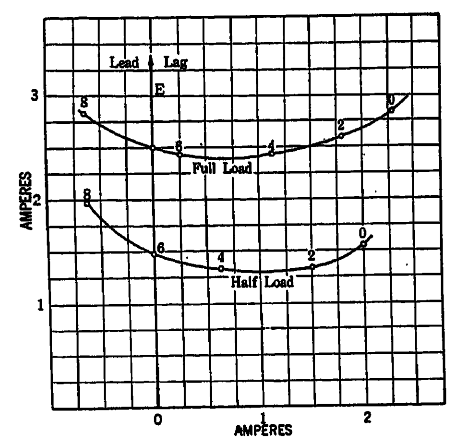


Fig. 12—Locus Diagram of Condenser Motor. Variable Capacitance and Constant Output. Winding Ratio 2 to 1

The negative sign indicates that the reactance $X_3$ is due to a condenser and not to a reactor. If we should use a reactor of the same value it would give us another but much smaller maximum value of torque. In the

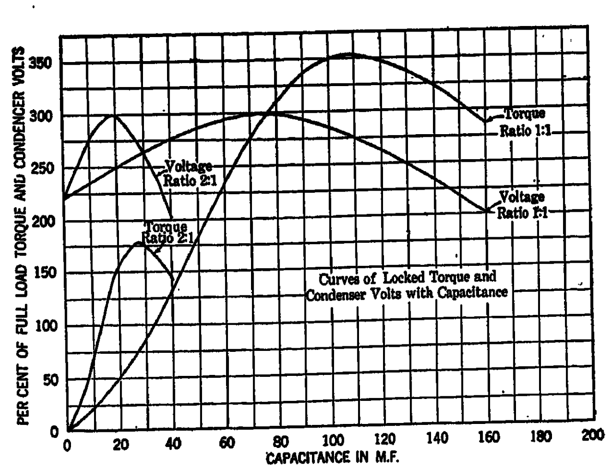


Fig. 13—Variation of Torque and Condenser Voltage with Capacitance

following the negative sign will be ignored when we are dealing with capacitance only. If the two windings are formed with coils of the same dimensions and if they contain the same weight of wire, the resistances, inductances, and impedances will all be in the ratio of $K^2$, $K$ being the ratio of the turns in winding 2 to those in winding 1.

In the above formula, then, $K^2\, Z_1$ is the impedance of

winding 2 and for maximum torque this impedance should be equal to the reactance of the condenser. The condition for resonance on the other hand would be that the reactance of winding 2 should equal the reactance of the condenser.

It is shown in the appendix that the locked torque is given by the following equation:

$$T = \frac{K\,N_1^2\,E^2}{Z_1^2} \cdot \frac{R_1\,C\,\omega}{K^4\,Z_1^2\,C^2\,\omega^2 - 2\,K^2\,X_1\,C\,\omega + 1}$$

in which the symbols have the same significance as before.

In Fig. 13 I have plotted the graph of this equation as computed for a particular motor with $K = 1$ and $K = 2$. The values of $Z_1$, $X_1$, and $R_1$ were obtained from readings with the rotor locked. Tables I and II show some comparisons between observed and computed torques.

TABLE I

For $K = 1$

| | | | | | | |
|---|---|---|---|---|---|---|
| $C$ in microfarads | 0 | 40 | 60 | 80 | 108 | 160 |
| $T$ calculated | 0 | 133 | 231 | 311 | 353 | 286 |
| $T$ observed | 0 | 130 | 210 | 285 | 355 | .... |

TABLE II

For $K = 2$

| | | | | | | |
|---|---|---|---|---|---|---|
| $C$ in microfarads | 0 | 10 | 15 | 20 | 27 | 40 |
| $T$ calculated | 0 | 66.5 | 115.5 | 155.5 | 176.5 | 143 |
| $T$ observed | 0 | 66.2 | 114 | 155 | 178 | 147 |

Using the capacitance to give maximum torque, namely $C = \frac{1}{K^2\,Z_1\,\omega}$ we obtain the following for the maximum torque with any given ratio of turns:

$$T_m = \frac{N_1^2\,E^2}{2\,K\,Z_1^2} \cdot \sqrt{\frac{Z_1 + X_1}{Z_1 - X_1}}$$

Inserting the known values of the constants we find that for $K = 1$, $C = 108$ microfarads and $T_m = 353$ per cent. The test values were 115 microfarads and 355 per cent.

For $K = 2$ the calculated capacitance for maximum torque was 27 microfarads and the test showed 28, the calculated maximum torque was 177 per cent and the tested value 178 per cent.

## Relation of Torque and Volt Amperes in the Condenser

As shown in the appendix the equation for torque may be written as follows:

$$T = \frac{K\,N_1^2\,R_1}{Z_1^2}\,E_c\,I_c = \frac{K\,N_1^2\,R_1}{Z_1^2} \cdot E_c\,I_2$$

In other words, the torque is directly proportional to the product of the volts across the condenser and the current flowing in it. This is important since the cost of the condenser (at least with voltages of 440 and above) is nearly directly proportional to the volt amperes in the condenser, rather than to its capacitance. That is, the cost of the condenser increases with the voltage it must withstand as well as with its capacitance.

If we insert the values of the constants of this particular motor we obtain the equation

$$T = 0.116\,K\,E_c\,I_2$$

In Fig. 14 I have drawn the curves corresponding to the above equation for $K = 1$ and $K = 2$. I have also plotted the test resulted on this motor and reasonably good correspondence is shown between the calculated and the test results.

The variation of the voltage across the condenser as we change the capacitance is also shown in the curves of Fig. 13. The voltage, for any ratio between the turns, is the same as the line voltage for $C = 0$, rises rapidly as $C$ is increased up to the point of resonance and then decreases. The maximum torque, as we should expect from the theory, is found with a value of $C$ somewhat greater than that which gives resonance.

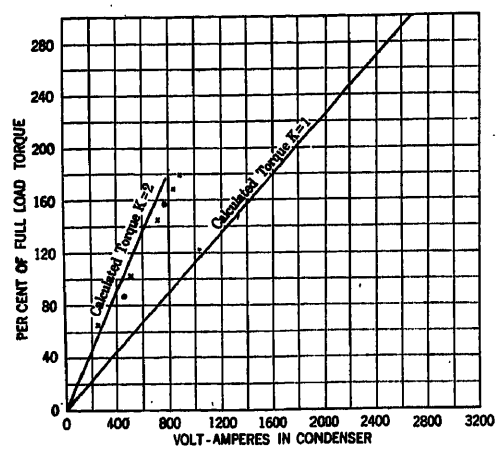


Fig. 14—Variation of Torque with Volt Amperes in Condenser

The condenser voltage at maximum torque is, however, usually decidedly above line voltage.

According to the theory as developed in the appendix the condenser voltage at the point of maximum torque is given by the following formula:

$$E_c = -\,E\,\sqrt{\frac{Z_1}{2\,(Z_1 - X_1)}}$$

It will be seen that this value is independent of the ratio $K$ and in the case of this particular motor the computed value is 274 volts. The actual values agreed fairly well with this, being 260 where $K = 1$ and 280 volts where $K = 2$.

## Summary

From the preceding it will be obvious that as we increase the number of turns in winding 2, the maximum locked torque decreases. However, the capacitance needed decreases even faster so that if locked torque only is to be considered the ratio $K$ of the turns

should be as great as possible and still permit the necessary torque to be developed.

It will also be apparent from the equation for maximum torque that this may be made as great as desired provided we make $K$ small enough. Excessive torques, however, are obtained at the expense of large currents and excessive cost of condensers.

### Starting Efficiency

The starting efficiency of any induction motor may be obtained by the following equation:

$$\eta = 0.142 \frac{\text{Lb. ft.} \times \text{Sync. Speed}}{\text{Volts} \times \text{Amperes}}$$

The voltage is that applied to the motor and the current is that in the supply line. For a motor of a given synchronous speed and voltage, it will be seen that this ratio is really the torque per ampere multiplied by a constant.

In Fig. 15 I have plotted the starting efficiency of this motor for the two ratios used. It will be seen that the efficiency increases with the torque and reaches

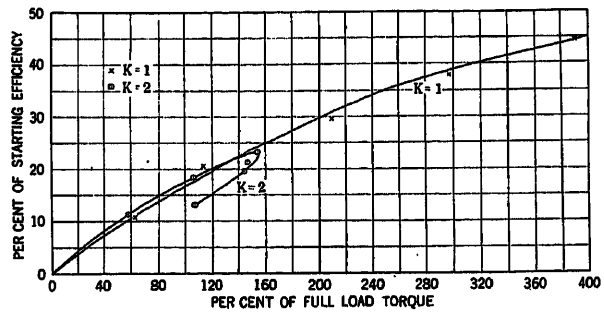


Fig. 15—Relation between Starting Torque and Starting Efficiency

higher values with the smaller values of the ratio $K$. The maximum efficiency obtained was 45 per cent. In comparison it may be mentioned that in split-phase motors the starting efficiency rarely exceeds 10 per cent and in polyphase squirrel-cage motors it is usually about 20 per cent. This particular motor was also tested as a two-phase motor and its starting efficiency was found to be 26.3 per cent. If the capacitance of the condenser used is increased indefinitely these curves of starting efficiency of course turn back and finally reach zero.

### Complete Torque Curves

In the actual application of the motor we need to know not only the locked starting torque but the torque throughout the complete range of operation. In Fig. 16 I have plotted speed-torque curves for another motor with a ratio of turns of 1 to 1 and with various values of capacitance. It will be seen that the torque with the larger values of capacitance is substantially maintained until the motor has reached normal operating speed. As the capacitance becomes smaller there is more of a tendency toward development of points of low torque at certain speeds. This will be particularly apparent in the curve for 20 microfarads. The point of lowest torque occurs at about $^1/_7$ of synchronous speed and would seem to indicate the presence of a backwardly rotating 7th harmonic.

I have also plotted the curve of current taken when 40 microfarad condenser was used. It will be seen

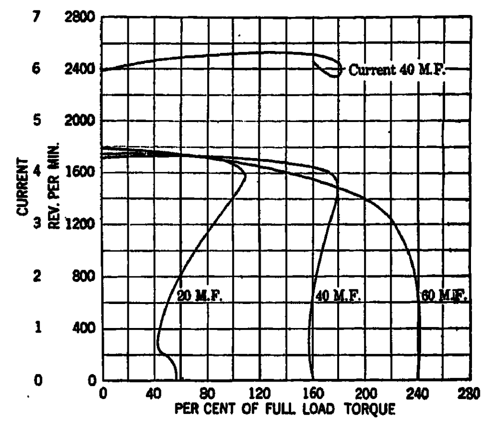


Fig. 16—Speed Torque Curves of Condenser Motor

that the current remains nearly constant throughout the entire starting period.

The torque developed when an attempt is made to reverse the motor at full speed is sometimes of im-

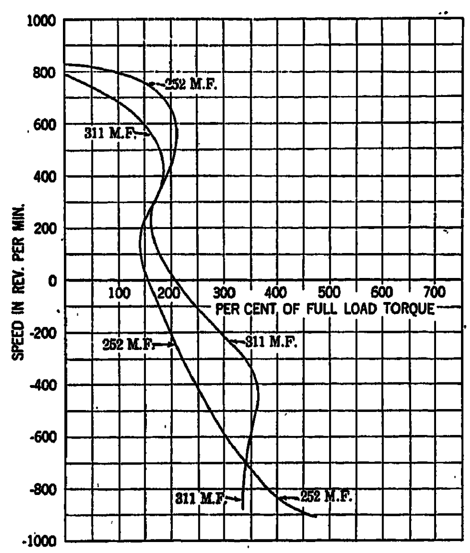


Fig. 17—Speed Torque Curves of Condenser Motor showing Reversing Torque

portance. The action under these conditions is shown in Fig. 17. These curves were taken on a different and much larger motor. Points below the line indicate the torque developed when the motor was running in opposition to the direction of the rotating magnetic field. In other words, they show what would happen if the motor were reversed at full speed. It will be seen that the torque developed at negative speeds is very great.

While this reverse torque is not of importance in the case of most motors, it is decidedly important in the case of motors used to operate elevators, hoists, or other similar devices. The ordinary split-phase motor with automatic switch for cutting out the starting winding and the repulsion start-induction run single-phase motors have no reserving torque. It is necessary to bring them to rest and reverse the connections in the case of the split-phase motor or shift the brushes in the case of the repulsion start-induction run motor and then start them in the new direction. The condenser motor, on the other hand, has excellent reversing torque and can be used in any service where so-called "plugging" is necessary.

## Appendix

### Theory of Locked Torque of Split Phase Motors

In Figs. 18 and 19 are shown the connections of a resistance split-phase motor and those of a condenser motor. Figs. 20 and 21 show the respective vector diagrams of the motor at the instant of starting. $I_1$ lags in both cases. $I_2$ lags less than $I_1$ in Fig. 20 and leads in Fig. 21. There is therefore an angle $\alpha$ between the two currents and this angle is much greater in the condenser motor. In place of a condenser or a resistor we might use a reactor in series with phase 2. This would give only a small angle between the currents and hence would be ineffective.

It can be rigorously proved (although it is nearly self-evident) that if the fluxes due to the two windings, differ 90 deg. in space phase and have sinusoidal varia-

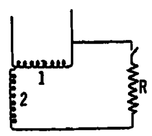


FIG. 18—RESISTANCE SPLIT-PHASE MOTOR

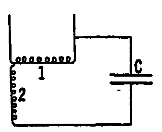


FIG. 19—CONDENSER SPLIT-PHASE MOTOR

tion in both time and space, the torque will be proportional to their product and to the component of one at right angles to the others. This can be expressed in the equation $T = Q_1 B_1 B_2 \sin \alpha$.

In which $B_1$ and $B_2$ are the fluxes, $\alpha$ is the time angle between $B_1$ and $B_2$, and $Q_1$ is a constant.

If we neglect saturation, $B_1$ and $B_2$ are proportional to the respective ampere turns $N_1 I_1$ and $N_2 I_2$ and

$$T = Q_2 N_1 I_1 N_2 I_2 \sin \alpha.$$

Here the purpose is to treat these three motors together. All that we do in these various connections is to change the resistance or reactance of phase 2. We shall assume that the constants of winding 1 are fixed. We can increase the resistance of circuit 2 as much as we please but we cannot reduce it below a certain value. Likewise, we can increase the reactance of circuit 2 as much as we wish. We can also reduce it if we wish, however, or even make it negative by using a condenser.

Let the locked impedance, resistance, and reactance of winding 1 be respectively $Z_1$, $R_1$, and $X_1$ and those of winding 2, be $Z_2$, $R_2$, and $X_2$. The constants of the *entire* circuit of which winding 2 is a part including any resistance, reactance, or condensive reactance used are designated by $Z$, $R$, and $X$. These constants are to be determined in an actual motor by applying a

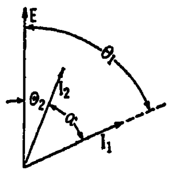


FIG. 20—VECTOR DIAGRAM OF RESISTANCE SPLIT-PHASE MOTOR AT START

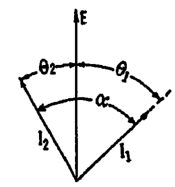


FIG. 21—VECTOR DIAGRAM OF CONDENSER MOTOR AT START

sinusoidal voltage of known frequency and measuring the volts, watts, and amperes. Then

$$Z_1 = \frac{E}{I_1}, R_1 = \frac{W_1}{I_1^2} \text{ and } X_1 = \sqrt{Z_1^2 - R_1^2}.$$

Similar equations hold for $Z_2$, $R_2$, and $X_2$ and for $Z$, $R$, and $X$. The latter three are of course taken with the resistors, reactors, or condensers used connected in the circuit.

It is important to note that the constants of circuit 2 are not affected by current in circuit 1 and vice versa. This is due to the fact that the windings are 90 electrical degrees from one another, and therefore current in circuit 1 does not induce voltage in circuit 2.

Adopting proper units for torque we may omit the constant and write

$$T = N_1 N_2 I_1 I_2 \sin \alpha.$$

$$\alpha = \theta_1 - \theta_2$$

$$\operatorname{Sin} \alpha = \frac{X_1 R - R_1 X}{Z_1 Z} \text{ and}$$

$$T = \frac{N_1 N_2 E^2}{Z_1^2} \cdot \frac{X_1 R - R_1 X}{R^2 + X^2}$$

In a given motor, $E_1$, $N_1$, $N_2$, $X_1$, $R_1$, and $Z_1$ are fixed. If the motor is to be of the resistance split-phase type,

$X = X_2$ and the only variable is $R$. On the other hand, in a condenser motor, $R = R_2$ and $X$ is the variable. A graph of this equation is shown in Fig. 22.

The condition for maximum torque is readily found. Assuming a resistance split-phase motor in which $R$ is the variable, we have for maximum torque,

$$\frac{d\,T}{d\,R} = 0 = \frac{X_1\,(R^2 + X^2) - 2\,R\,(X_1\,R - R_1\,X)}{(R^2 + X^2)^2}$$

and solving for $R$

$$R = \frac{X}{X_1}\,(R_1 \pm Z_1)$$

Since $Z_1$ is always greater than $R_1$ it is evident that one of the above values is negative. We have negative reactors, *i. e.*, condensers, but unfortunately we have no negative resistors. Hence only the positive value has any immediate importance for us.

Substituting the value of $R$ which will give us the maximum torque and calling the maximum torque as a resistance split-phase motor $T_{MR}$ we get

$$T_{MR} = \frac{N_1\,N_2\,E^2}{Z_1^2}\;\frac{X_1^2}{X_2}\cdot\frac{1}{2\,(Z_1 + R_1)}$$

In the above expression for the maximum torque

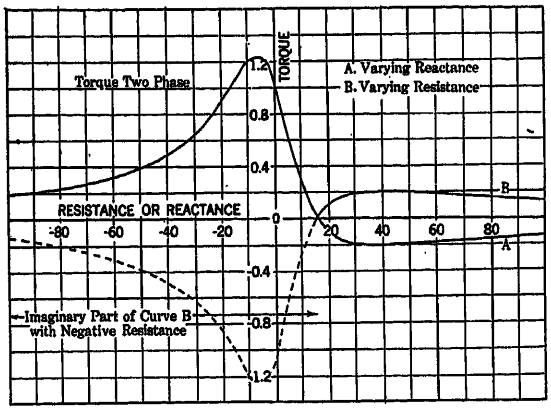


FIG. 22—CURVES OF TORQUE AND TOTAL RESISTANCE OR REACTANCE

the constants $Z_1$, $R_1$, $X_1$, and $N_1$ of the circuit 1 are rather rigidly fixed by the design of the motor. Since the circuit 2 is used only during the starting period, the number of turns and consequently its resistance and reactance can be varied within wide limits. Considering everything except $N_2$ and $X_2$ as constants and making use of the fact that $X_2$ is proportional to $N_2^2$ we may write

$$T_{MR} = \frac{\text{Constant}}{N_2}$$

This indicates that we can increase the torque of a resistance split-phase motor indefinitely by decreasing the number of turns in the starting phase and adjusting the resistance to correspond.

### THE CONDENSER MOTOR

We shall now consider the effect of changing the reactance of circuit 2, the resistance being kept constant. We proceed as before except that we now differentiate with respect to $X$ and have

$$\frac{d\,T}{d\,X} = 0 = \frac{-R_1\,(R^2 + X^2) - 2\,X\,(X_1\,R - R_1\,X)}{(R^2 + X^2)^2}$$

$$X = \frac{R}{R_1}\,(X_1 \pm Z_1)$$

Using this value of $X$ and subsituting in the expression for torque we have for the maximum torque

$$T_{MC} = \frac{N_1\,N_2\,E^2}{Z_1^2}\;\frac{R_1^2}{R_2}\;\frac{\pm 1}{2\,(Z_1 \pm X_1)}$$

It will be evident that the torque will be much greater when $X_1$ is negative in the above equation. This means of course that a condenser must be used. If we let $X_c$ equal the reactance used in addition to the inherent reactance $X_2$ of circuit 2, then

$$X = X_2 + X_c \text{ or } X_c = X - X_2 = \frac{10^6}{2\,\pi\,f\,C}$$

where $C$ is expressed in microfarads and $f$ is the frequency. From this equation we can readily calculate the capacitance necessary to give maximum torque.

It will be of interest to calculate some of the characteristics of an actual motor of typical design.

The motor chosen was a ½-hp., four-pole, two-phase, 60-cycle, 220-volt squirrel-cage induction motor. The phases were alike and had the following constants

$$Z_1 = Z_2 = 22.8,\ R_1 = R_2 = 15.9,\ X_1 = X_2 = 16.6$$

These were measured with the motor rotating slowly to average the constants for different positions of the rotor relative to the stator.

Our general expression for torque is

$$T = N_1\,N_2\,I_1\,I_2 \sin \alpha.$$

In a two-phase motor $N_1 = N_2$, $I_1 = I_2$ and $\sin \alpha = 1$, or

$$T_2 = N_1^2\,I_1^2 = \frac{N_1^2\,E^2}{Z_1^2}$$

Substituting this value in the expression for the torque of a split-phase motor, we have, since here the phases are alike

$$T = \frac{N_1\,N_2\,E^2}{Z_1^2}\;\frac{(X_1\,R - R_1\,X)}{R^2 + X^2} = T_2\;\frac{X_1\,R - R_1\,X}{R^2 + X^2}$$

If we consider our motor as a condenser motor, $R$ becomes constant and equals 15.9. For simplicity we may take $T_2 = 1$ and by varying $X$, compute the value of $T$. The result is shown in curve $A$ of Fig. 22. The reactance plotted is the total reactance of circuit 2 and is here equal to the external reactance plus the reactance of the winding 2.

With a reactance of 16.6 ohms, *i. e.*, no external

resistance or reactance, the torque is of course zero since the phases are exactly alike and the angle $\alpha$ is zero.

The reactance to give the greatest torque is given by the equation

$$X = \frac{R}{R_1}(X_1 \pm Z_1) = \frac{15.9}{15.9} \cdot (16.6 \pm 22.8)$$

$$= 39.4 \text{ or} - 6.2 \text{ ohms.}$$

The first value would of course be obtained by using a reactor of 22.8 ohms and the second by using a condenser of the same reactance.

Substituting these values in the equation for the torque we find $T_m = -0.201$ or $T_m = 1.245$.

It will be noted that these torques are in opposite directions and that the torque using a condenser is approximately six times that using a reactor.

Curve $B$ was obtained in a similar manner by assuming $X$ constant at 16.6 ohms and varying $R$. Calculation shows that the maximum torque will be obtained when the total resistance of phase 2 is 39.7 ohms. Using this value in the equation for torque we find $T = 0.213$. (Curve $A$ and Curve $B$ happen to be nearly alike since the resistance and reactance are nearly equal.)

The portion of Curve $B$ to the left of the point 15.9 cannot be used in practise since it corresponds to negative values of resistance.

### Comparison With Two-Phase Motor

In the expression used above we assumed that the factor $T_2$ or the torque as a two-phase motor was equal to unity. It will be evident, then, that the results are factors by which we may multiply the torque developed as a two-phase motor to find the torque as a split-phase motor. This particular motor with the proper resistance added to one phase develops single phase approximately one-fifth as much torque as when operated two-phase. With the proper value of capacitance added to one phase it develops single phase nearly 125 per cent of its two-phase torque.

### Locked Currents

*As a Two-Phase Motor.* Here we have two equal currents and to make a fair comparison we must of course consider the starting current as double that of one phase or

$$I_s \text{ (two-phase} = \frac{2 \times 220}{22.8} = 19.3 \text{ amperes.}$$

*As a Condenser Split-Phase.* The current in phase 1 is

$$I_1 = \frac{220}{22.8} = 9.65 \text{ amperes}$$

$$\text{Power component of} \quad I_1 = 9.65 \times \frac{R}{Z}$$

$$= 9.65 \times \frac{15.9}{22.8} = 6.72$$

$$\text{Wattless component of } I_1 = 9.65 \times \frac{X}{Z}$$

$$= 9.65 \times \frac{16.6}{22.8} = 7.02$$

In the circuit 2 the resistance is the same as in circuit 1. Using a condenser of 116 $\mu$ f. having a reactance of $-22.8\ \omega$ the total reactance of circuit 2 is $16.6 - 22.8 = -6.2$ ohms.

The impedance is

$$Z = \sqrt{15.9^2 + 6.2^2} = 17.16 \text{ and}$$

$$I^2 = \frac{220}{17.16} = 12.82 \text{ amperes.}$$

Power component of

$$I_2 = 12.82 \times \frac{15.9}{17.16} = 11.91 \text{ amperes}$$

Wattless component of

$$I_2 = 12.82 \times \frac{(-6.2)}{17.16} = -4.64 \text{ amperes}$$

Total power current

$$= 6.72 + 11.91 = 18.63 \text{ amperes}$$

Total wattless current $= 7.02 - 4.64 = 2.38$ amperes

Total line current $= \sqrt{18.63^2 + 2.38^2} = 18.8$ amperes

$$\text{Power factor} = \frac{18.63}{18.8} = 0.99$$

$$\frac{\text{Current condenser split-phase}}{\text{Current two-phase}} = \frac{18.8}{19.3} = 0.973$$

$$\frac{\text{Torque condenser split-phase}}{\text{Torque two-phase}} = \frac{1.245}{1} = 1.245$$

The condenser motor not only develops more torque but it does it with less current. The power factor is also better being 0.99 compared with 0.696 for the two phase. It is frequently assumed that no split-phase motor can compare with a polyphase motor in starting efficiency. Actually the condenser split phase is much better. If we had selected a condenser to give the greatest starting efficiency (instead of the greatest starting torque) the contrast with the two-phase motor would have been still greater.

In a similar manner we can compute the locked current as a resistance split-phase motor, and the power input with the two connections.

### Comparison of Various Connections

The results of the computations are embodied in Table III. This table applies of course to the par-

TABLE III

| | Locked torque | Locked current | Torque / Current | Power factor | Power | Torque / Power |
|---|---|---|---|---|---|---|
| Two phase........ | | 1 | 1 | 0.696 | 1 | 1 |
| Condenser split phase...... | 1.245 | 0.973 | 1.28 | 0.990 | 1.39 | 0.894 |
| Resistance split phase.......... | 0.213 | 0.760 | 0.280 | 0.795 | 0.855 | 0.249 |

ticular motor considered or to any motor having $R_1$ and $X_1$ in the same ratio and having the two windings alike. In general $R_1$ and $X_1$ are likely to be nearly equal so the table is fairly representative of average conditions.

It should of course be noted that if a motor were being designed as a resistance split-phase motor, the number of turns in the winding 2 would be made less than those in winding 1. This would increase the starting torque, but at the expense of a larger current.

Likewise in a condenser motor some economy could in general be effected by using more turns in winding 2 than in winding 1. This would reduce the starting torque, but since this is greater than in the polyphase motor, it could still be made great enough.

## Further Conclusions

As previously pointed out, condenser motors are frequently wound with the same weight of wire and the same distribution in both windings. In this case, if we let $K$ equal the ratio of turns in the two windings

$N_2 = K\,N_1,\ R_2 = K^2\,R_1,\ X_2 = K^2\,X_1,$ and $Z_2 = K^2\,Z_1$

If we confine our attention to the case of the condenser motor $R = R_2$ and $X = X_2 + X_3 = K^2\,X_1 + X_3$ where $X_3$ is the added reactance and if a condenser is used will of course be negative.

Our formula for torque

$$T = \frac{N_1\,N_2\,E^2}{Z_1^2}\cdot\frac{X_1\,R - R_1\,X}{R^2 + X^2}$$

becomes

$$T = -\frac{K\,N_1^2\,E^2}{X_1^2}\cdot\frac{R_1\,X_3}{K^4\,Z_1^2 + 2\,K^2\,X_1\,X_3 + X_3^2}$$

It is frequently convenient to express the torque in terms of the capacitance used. This is readily done by substituting

$$X_3 = -\frac{1}{C\ \omega}$$

Then

$$T = \frac{K\,N_1^2\,E^2}{Z_1^2}\cdot\frac{R_1\,C\,\omega}{K^4\,Z_1^2\,C^2\,\omega^2 - 2\,K^2\,X_1\,C\,\omega + 1}$$

The condition for maximum locked torque becomes

$$X = K^2\,X_1 + X_3 = \frac{R}{R_1}\,(X_1 \pm Z_1) = K^2\,(X_1 \pm Z_1)$$

and

$$X_3 = \pm\,K^2\,Z_1 \text{ or } C = \frac{1}{K^2\ Z_1\ \omega}$$

If we use the positive sign it means that $X_3$ is a reactor, if the negative sign it is a condenser. Since the latter gives the greater torque it alone will be considered.

Substituting, we have

$$T_m = \frac{N_1^2\,E^2}{2\,K\,Z_1^2}\ \sqrt{\frac{Z_1 + X_1}{Z_1 - X_1}}$$

From this equation it is evident that the maximum torque is inversely proportional to $K$ and may be made as large as desired by making $K$ small, *i. e.*, using a small number of turns in winding 2.

*Comparison with Two-Phase Motor.* In a two-phase motor we have

$$T_m = \frac{N_1^2\,E^2}{Z_1^2}.$$

$$\text{Then}\ \frac{\text{Torque condenser split-phase}}{\text{Torque two-phase}} = \frac{R_1}{2\,K\,(Z_1 - X_1)}$$

This shows that the torque of the condenser motor may be more or less than that of the two-phase motor, depending upon the value of $K$ and the relative value of $Z_1$, $R_1$, and $X_1$. If the motor has average characteristics, $R_1$ and $X_1$ will be nearly equal. If we assume that the phases are alike so that $K = 1$ and $R_1 = X_1$ then $Z_1 = \sqrt{2}\,X_1$ and the ratio becomes 1.21.

*The Condenser Voltage.* It is important that we know the voltage across the condenser since its cost is dependent upon the voltage it must stand as well as upon its capacitance. The condenser current is obviously the same as the current in circuit 2 and

$$I_c = I_2 = \frac{E}{Z} = \frac{E}{\sqrt{K^4\,Z_1^2 + 2\,K^2\,X_1\,X_3 + X_3^2}}$$

and the voltage across the condenser

$$E_c = I_c\,X_3 = \frac{E\,X_3}{\sqrt{K^4\,Z_1^2 + 2\,K^2\,X_1\,X_3 + X_3^2}}$$

If we use the value of $X_3$ which gives maximum torque *i. e.*, $X_3 = -\,K^2\,Z_1$. We have

$$E_c = \frac{-\,E\,K^2\,Z_1}{\sqrt{K^4\,Z_1^2 - 2\,K^4\,X_1\,Z_1 + K^4\,Z_1^2}}$$

$$= -\,E\,\sqrt{\frac{Z_1}{2\,(Z_1 - X_1)}}$$

This result is *independent of the ratio K*. It may be quite large if $X_1$ is nearly equal to $Z_1$. Assuming as before that $R_1 = X_1$ and $Z_1 = \sqrt{2}\,X_1$ we have

$$E_c = -\,1.31\,E$$

or in the average motor we may expect at start a condenser voltage about 30 per cent above the line voltage,

if maximum torque is developed. Since for maximum torque $X_3 = - Z_2$, it is obvious that the voltage across winding 2 will be equal to the condenser voltage.

*Relation of Starting Torque to Volt Amperes in Condenser.* The volt amperes in the condenser is given by

$$E_c\, I_c = \frac{E^2\, X_3}{K^4\, Z_1^2 + 2\, K^2\, X_1\, X_3 + X_3^2},$$

$$\text{also } T = - \frac{K\, N_1^2\, E^2}{Z_1^2} \cdot \frac{R_1\, X_3}{(K^4\, Z_1^2 + 2\, K^2\, X_1\, X_3 + X^2_3}$$

$$\text{therefore } T = - \frac{K\, N_1^2\, R_1}{Z_1^2} \cdot E_c\, I_c$$

Hence we arrive at the very important result that the torque is proportional to the volt amperes in the condenser and hence roughly proportional to the cost of the condenser.

$$\text{Also, torque per volt ampere} = \frac{T}{E_c\, I_c} = \frac{- K\, N_1^2\, R_1}{Z_1^2}$$

From this it is obvious that if we wish the torque per dollar to be high we should use a larger value of the ratio $K$. On the other hand as we increase $K$ the maximum torque we can obtain becomes less. We should therefore make the number of turns in winding 2 as great as possible and still obtain the necessary starting torque.

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## Discussion

For discussion of this paper see page 629.

# The Fundamental Theory of the Capacitor Motor

BY H. C. SPECHT*

Member, A. I. E. E.

*Synopsis.—A fundamental theory of the motor and capacitor is given partly by the algebraic method and partly by graphical method. The variables in the design of a complete capacitor motor unit for any desired performance are discussed. A few examples of unbalanced phases and performance are given. The suitability for various classes of service is discussed briefly.*

## INTRODUCTION

An ordinary two-phase motor may be used as a capacitor motor, one phase being connected directly, and the other in series with a condenser, to a single-phase circuit. The performance, however, may not be all that is desired. The hp. rating, especially, may have to be reduced from its normal two-phase value, in order to obtain sufficient relative pull-out torque.[1] By varying the capacitor continuously as the load changes, operating characteristics approximating those of the two-phase motor could be obtained. This, however, is not practical, and only one or two taps from the capacitor are permissible, generally one for the starting and one for the running load. Since the capacitor phase, with a constant capacity, will give almost constant torque, the torque for overloads has to be furnished by the main phase. For example, if the torque at full load is equally divided between the two phases and if the main phase is able to develop only twice its full load torque, then the combined pull-out torque will be only two and one-half times the full-load torque. On applications requiring motors with starting and pull-out torques of high values, this handicap will necessitate the designing of a motor having the phases unbalanced in order to use a smaller capacitor and thus keep the cost of the unit within reason.

The capacitor motor could be designed with a power factor of nearly 100 per cent and an efficiency nearly equal to a similarly rated two-phase motor. However, in order to obtain a smaller capacitor, a reasonable sacrifice in power factor and efficiency may be accepted.

In order to have a clear understanding of the various characteristics of the capacitor motor it may be well to deal first with the general theory of a capacitor motor, assuming the stator to be wound two phase and the windings spaced 90 electrical degrees apart.

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1. See I. Biermanns, *Archiv. für Elektrotechnik*, Vol. XVII, p. 519; Franklin Punga, *Archiv. für Elektrotechnik*, Vol. XVIII, p. 267; B. F. Bailey, *Elec. Wld.* 1928, pp. 597, 647.

*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

## GENERAL THEORY

(*A*) *Motor at Stand Still.*

The fundamental equation of starting torque for any kind of an electrical motor is as follows:

Starting torque = Rotor ampere turns × flux × cosine of the angle between their vectors × a constant.

$\Phi_1$ = Flux of main phase.
$\Phi_c$ = Flux of condenser phase.
$i_1$ = Stator amperes in main phase.
$i_{1c}$ = Stator amperes in condenser phase.
$i_2'$ = Current in the rotor of main phase and reduced to stator winding turns.
$i_{2c}'$ = Current in rotor of condenser phase and reduced to stator winding turns.
$e_1 = e_2'$ = Rotor voltage induced by $\Phi_1$, and reduced to stator winding turns.
$e_{1c} = e_{2c}'$ = Rotor voltage induced by $\Phi_c$ and reduced to stator winding turns.
$E$ = Line voltage.
$E_c$ = Terminal voltage of the capacitor phase.
$e_c$ = Capacitor voltage.
$t_1$ = Stator winding turns in main phase.
$t_{1c}$ = Stator winding turns in capacitor phase.
$k_1$ & $k_2$ = Main phase winding distribution factors.
$k_{1c}$ & $k_{2c}$ = Capacitor phase winding distribution factors.
$f$ = Line frequency.
$s$ = Slip.
$p$ = Number of poles.
$C$ = Capacity microfarads.
$r_1$ = Resistance in ohms of stator main phase.
$r_{1c}$ = Resistance in ohms of stator winding in capacitor phase.
$r_2'$ = Ohmic resistance of rotor reduced to main phase winding turns.
$r_{2c}'$ = Resistance in ohms of rotor reduced to stator condenser phase winding turns.
$x_1, x_{1c}, x_2'$ and $x_{2c}'$ = The corresponding leakage reactances.
$x_c = \frac{10^6}{2\pi f C}$ Capacitance in ohms.
$\psi_1$ = Angle between $i_2'$ and $\Phi_c$.
$\psi_2$ = Angle between $i_{2c}'$ and $\Phi_1$.
$\alpha$ = Angle between $\Phi_1$ and $\Phi_c$.

$T_1$ = Torque in $k\,g\,m$ developed by main phase.

$T_2$ = Torque in $k\,g\,m$ developed by condenser phase.

$$T_1 = 2.3 \times p \times \Phi_c \times i_2' \times t_1 \times \frac{k_1}{k_2} \times \cos \psi_1 \times 10^{-10}\, k\,g\,m \tag{1}$$

$$T_2 = 2.3 \times p \times \Phi_1 \times i_{2c}' \times t_{1c} \times \frac{k_{1c}}{k_{2c}} \times \cos \psi_2 \times 10^{-10}\, k\,g\,m \tag{2}$$

$$T_1 + T_2 = \text{Total starting torque.}$$

If the windings of the two phases have equal amounts of copper the formula for torque may be written as follows:

$$*T = \frac{e_2' \times e_{2c}'}{2\,\pi f} \times p \times \frac{r_2'}{(r_2')^2 + (X_2')^2} \times \sin \alpha \times 10^{-9} \times \text{Const.} \tag{3}$$

From this formula it follows that the maximum starting torque for different rotor resistances occurs when $r_2' = x_2'$ providing all other values remain the same. The induced voltage in the rotor depends on the stator impedance drop and in the capacitor phase also on the capacity. As the induced voltage varies the torque changes proportionately. Further, the torque depends on the angle $\alpha$ and this, for maximum starting torque, should be close to 90 deg. When figuring the torque it is convenient to use the graphical method as this gives a clear picture and helps in making changes necessary to obtain the best results. First of all the currents and their power factors are figured.

$$i_1 = \frac{E}{\sqrt{(r_1 + r_2')^2 + (x_1 + x_2')^2}} \tag{4}$$

$$\cos \varphi_1 = \frac{r_1 + r_2'}{\sqrt{(r_1 + r_2')^2 + (x_1 + x_2')^2}} \tag{5}$$

$$i_{1c} = \frac{E}{\sqrt{(r_{1c} + r_{2c}')^2 + (x_{1c} + x_{2c}' - x_c)^2}} \tag{6}$$

$$\cos \varphi_{1c} = \frac{r_{1c} + r_{2c}'}{\sqrt{(r_{1c} + r_{2c}')^2 + (x_{1c} + x_{2c}' - x_c)^2}} \tag{7}$$

The other values are obtained from the graphical method. (See Fig. 1.)

The locus of the vector $i_{1c}$ is a circle with the diameter of $\frac{E}{r_{1c} + r_{2c}'}$ and its center on the vertical line of $O\,E$.

Therefore the locus of the voltage vectors is also a circle, the center $M_{c1}$ of which is determined by the intersection of the perpendicular erected at the center of $O\,E$ and $A\,E$.

Also the locus of the vector of the induced voltage in the rotor is a circle and its center $M_{c2}$ is determined by the intersection of the perpendiculars through the center of $O\,B$ and $O\,C$. $O\,B$ represents the rotor voltage of the capacitor phase at a given capacity and $O\,C$ represents the rotor voltage at resonance. After the circles are determined it is an easy matter to pick from the diagram the voltages for any current because the angularity in regard to the individual vectors must be the same. The maximum starting torque for the various capacities occurs when $\sin \alpha \times e_{2c}' =$ maximum, which is at the point where the tangent at this rotor voltage circle of $e_{2c}'$ is parallel to vector $e_1$. This point is marked in Fig. 1 with $T_m$, the corresponding point is also indicated on the circle for the condenser voltage $e_c$ and the current $i_{1c}$. The point $T_m$ on the current circle is the tangent point to $i_1$ and $T_m$ on the voltage circle $E_c$ and $e_c$ is the tangent point to $O\,E$ or line voltage.

It will be noted from the diagram, that with an appreciably smaller value of current and capacity near the point $T_m$, the torque is not much smaller. Therefore, in order to keep the starting current as low as possible, it will be advisable to stay below the maximum point.

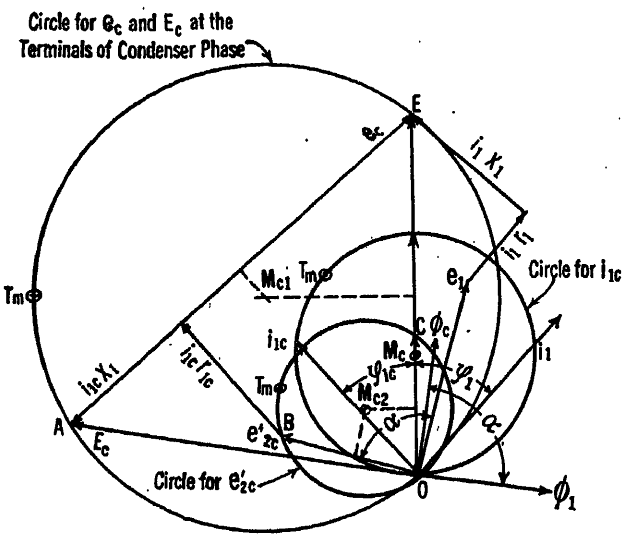


FIG. 1—VECTOR DIAGRAM OF MOTOR AT STANDSTILL

Since the capacity required to give a starting torque equal to or more than full load torque is so great, the capacitor may cost more than the motor, a series transformer should be used in connection with the condenser. The connection diagram most commonly used is shown in Fig. 2.

The advantage of this scheme is indicated very well by the fact that the capacity required decreases inversely with the square of the transformer voltage ratio and that at a certain voltage the cost of the condenser for the same volt-amperes is the lowest. The transformer also makes possible, by means of a transfer switch, the use of different effective capacities for both the starting and running conditions without breaking the condenser circuit. This is highly desir-

*See E. Arnold, *Wechselstrom Technik*, Volume 5.

able. This transfer switch may be of the centrifugal type or the magnetically operated type. The magnet coil of the latter type is connected, preferably, in the main winding circuit since the current of this winding varies through a wider range.

In designing the transformer it should be observed that the magnetizing reactance reduces the effective capacity, and that the watts loss also reduces the overall motor efficiency. Therefore, the transformer must be of ample size.

Considering starting torque only, it will be the cheapest proposition to work the main phase of the motor heavy and the capacitor phase light. How the value of the starting torque changes with an unbalanced winding system, is illustrated as follows.

*Example.* The amount of copper in both phases may be assumed as equal. The capacitor phase may, however, have twice as many turns as the main phase and therefore only half the cross section. The ohms resistance and the leakage reactance will be four times, and for simplicity the capacity may be only ¼. Therefore,

$i_{1c}$ = ¼ with its power factor remaining the same.

According to formulas (1) and (2) we find:

$T_1$ = ½ the value of the motor with balanced windings because the flux is half as great.

$T_2$ = ½ the value of the motor with balanced windings as the current is only ¼ as great and winding turns twice as many.

The resultant torque is therefore only decreased ½ while the condenser capacity has been reduced to ¼. The starting current in the line is decreased to 73 per cent of value with balanced windings, providing both current vectors remain 90 deg. apart.

Generally it can be stated that the starting torque decreases approximately in the inverse ratio of the winding turns in the capacitor phase providing the

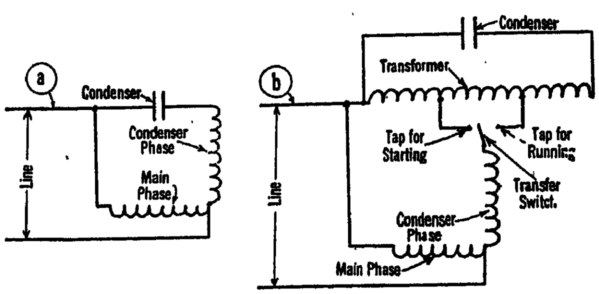


FIG. 2—(A) CONDENSER IN SERIES WITH AUXILIARY PHASE
(B) CONDENSER IN PARALLEL WITH TRANSFORMER

amount of copper in both phases is kept the same and the capacity decreased in the inverse ratio of winding turns squared. If the ratio of the amount of copper in the two phases is changed, the results are certainly different because the induced voltage in the rotor or the corresponding flux depends on the impedance drop in the stator winding. For commercial reasons, however, it is in some cases permissible to reduce the total copper section of the capacitor phase and still meet the required torques. However, in doing this the change in the performance of the motor under running load must also be given consideration.

(*B*) *Motor under Running Load.*

When the motor is running each phase will, in addition to its main flux, produce due to rotation a flux at

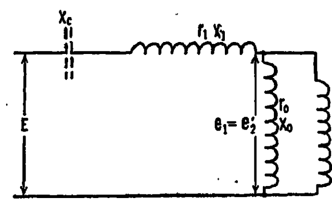


FIG. 3—CIRCUIT DIAGRAM OF MOTOR IMPEDANCE

right angles to the main flux. This field is a little smaller than the main field due to the rotor impedance drop. Therefore, for satisfactory operation it is necessary that the flux produced by the main and capacitor phase are at least approximately equal and displaced 90 deg. in time phase. If this is not the case, the rotational voltage produced by the main phase will not be equal to the transformer voltage of the capacitor phase and vice versa. This unbalanced voltage will cause a circulating current of such magnitude as to establish equilibrium. Such circulating currents result in a motor of lower performance and therefore in practise the capacitor motor will be designed with balanced flux condition without circulating current at normal operating load.

Although the method of calculation given in the following is limited in application to the balanced flux condition, it is a short and simple method for getting quick results which are sufficiently accurate for practical use. The impedance of an induction motor may be represented by the well known circuit diagram shown in Fig. 3.

In determining the complete vector diagram it is simplest to start out with the induced voltage in the rotor winding. According to the size of the motor the induced rotor voltage is generally from 4 to 8 per cent less than the line voltage. If at the end of the calculation this assumed voltage be found incorrect, the corrections can easily be made.

Since the magnetizing circuit is in parallel with the rotor circuit the corresponding conductance and susceptance must be used in our calculations.

$$i_1 = e_1 \sqrt{(g_0 + g_2)^2 + (b_0 + b_2)^2}$$

$$\cos \alpha_1 = \frac{g_0 + g_2}{\sqrt{(g_0 + g_2)^2 + (b_0 + b_2)^2}}$$

$$g_0 = \frac{r_0}{r_0^2 + x_0^2} \qquad g_2 = \frac{r_2'/s}{(r_2'/s)^2 + x_2'^2}$$

$$b_0 = \frac{x_0}{r_0^2 + x_0^2} \qquad b_2 = \frac{x_2'}{(r_2'/s)^2 + x_2'^2}$$

Having thus determined the value $i_1$, for both windings, lay off (Fig. 4) first the vectors $i_{1c}$ and $e_{1c}$ for the capacitor phase, then add the impedance drop of the stator winding. The vector $OA$ then represents the terminal voltage at the capacitor phase winding.

A line from $A$ at right angles to $i_{1c}$ and an arc with a radius equal to the line voltage around the point $O$ as a center will determine the voltage $e_c$ for the capacitor. Then the capacitance in ohms is $\frac{e_c}{i_{1c}}$ and in microfarads it is $\frac{i_{1c} \times 10^6}{2 \times \pi \times f \times e_c}$. If a capacitor with a transformer is used, the impedance drop of the transformer which is in series with the stator winding is also to be added.

The vector $e_1$ must be nearly equal to and at right angles to $e_{1c}$. By the angle $\alpha_1$ the vector $i_1$ is determined. The stator impedance drop added to $e_1$ should end again in the point $B$. If the two windings of the

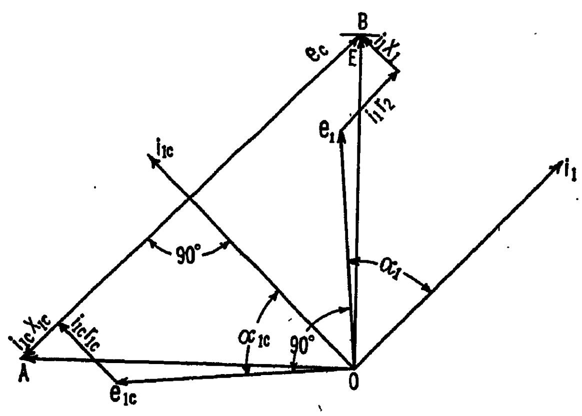


FIG. 4—VECTOR DIAGRAM OF MOTOR UNDER RUNNING LOAD

stator are not alike the induced voltage $e_{1c}$ will be $= e_1 \times \frac{t_{1c} \times f_{1c}}{t_1 \times f_1}$ where $t_1$ and $t_{1c}$ represent the corresponding winding turns and $f_1$ and $f_{1c}$ their winding distribution factors. If the capacity obtained should not give the desired pull-out torque, the windings or the value of microfarads of the capacitor may be changed. It will then be found that the induced voltage $e_{1c}$ in the rotor will change very little as long as the change stays within reasonable limits, because the voltage $e_{1c}$ is governed somewhat by the induced voltage $e_1$ of the main phase. Certainly the current in the capacitor phase will change almost in ratio with the change of capacity. If the capacity is made considerably greater than a balanced condition would require, the voltage in the capacitor phase will go up, and by its transformer action, will reduce the amperes and watts in the main phase considerably. The watts may even become negative. On the other hand, the capacitor phase will take more load in both current and watts. Naturally the line amperes and watts input will go up, resulting in lower efficiency and correspondingly increased heating of the motor. As an example, the following full load data were obtained by test on a ¼-hp. 110-volt, 60-cycle, 4-pole motor, having equal windings in the phases, and is given below in Table I.

It will be noted that at 33 $\mu f$. the phases are fairly well balanced and that the watts input from the line is at lowest value and the power factor is practically unity. With approximately twice the value of $\mu f$. the current and watts input are enormously increased and also the terminal voltage on the capacitor phase as well as the capacitor voltage is much higher. At light loads and particularly at no load, this increase in voltage on the capacitor and increase of watts input always exists on capacitor motors having approximately balanced phases at full load. This is another reason for designing the capacitor motor with a strong main phase and a weak capacitor phase.

As an example, Table II gives some test results of such a capacitor motor and for comparison the results of an ordinary split phase motor having the same starting torque. It will be noted that the capacitor motor has better performance. For the same starting torque the starting amperes are only half as great. The power factor and efficiency at full load are appreciably higher. The voltage on the capacitor phase is higher than the line voltage because the stator winding of the capacitor phase has approximately twice as many turns as the main winding.

As a matter of interest, in Table III and Figs. 5 and 6 are shown the great unbalance of the current and voltage vectors for various capacities at no load for a motor similar to the motor discussed in the preceding paragraph. It is interesting to see how by transformer action the current of the condenser phase drags the current vector of the main phase around the origin $O$ from lagging to leading value. Below the abscissa in Fig. 5 the current vectors of the main phase are negative and therefore give negative watts. It may also be noted in Table III how the speed decreases with the increase of capacity in microfarads. If in Fig. 6 the impedance drops were introduced it would be noted that the induced voltage in the rotor of both phases would be practically alike and 90 deg. apart. It is not the purpose of this paper to go into further details of the theory of unbalanced voltage conditions in split phase or polyphase motors. The theory given by Fortescue*, Slepian†, Biermanns‡, etc. may by some modifications, also be applied for unbalanced capacitor motors. However, these methods are rather complicated for daily use in design work and a more simple practical method would be highly welcomed by designing engineers.

Such extreme unbalance as shown in Figs. 5 and 6

*Fortescue, TRANSACTIONS A. I. E. E., Vol. 37, p. 1027.
†Slepian, *Elec. World*, 1920, p. 313.
‡Biermanns, *Archiv für Elektrotechnik*, Vol. XVII, p. 526.

TABLE I

| Capacity in $\mu$ f. | Line | | Main phase | | Capacitor phase | | | Cap. volts | Pull-out torque in oz. ft. |
|---|---|---|---|---|---|---|---|---|---|
| | Amps. | Watts | Amps. | Watts | Volts | Amps. | Watts | | |
| 22 | 2.6 | 269 | 2.37 | 179 | 98 | 1.25 | 90 | 150 | 24 |
| 33 | 2.45 | 265 | 1.73 | 110 | 115 | 1.94 | 155 | 158 | 26 |
| 44 | 2.7 | 283 | 1.22 | 53 | 126 | 2.85 | 230 | 172 | 29.5 |
| 64 | 4.2 | 398 | 0.55 | −25 | 154 | 4.55 | 423 | 189 | 34 |

TABLE II

| | Main phase | | | Aux. phase | | | Line | | | % | % | Cap. | | oz. ft. T | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) As cap. motor | V. | A. | W. | V. | A. | W. | V. | A. | W. | Eff. | PF | V. | Mfd. | FL | P O | Stg. |
| No load...... | 110 | 2.8 | 12 | 247 | 0.8 | 86 | 110 | 2.65 | 98 | .. | .. | 276 | 7.5 | 0 | .. | .. |
| Full load..... | 110 | 3.4 | 219 | 231 | 0.7 | 72 | 110 | 3.63 | 291 | 64.2 | 73 | 250 | 7.5 | 12 | 41 | .. |
| Locked...... | 110 | 19. | 1640 | 92 | 1.92 | 148 | 110 | 19.4 | 1715 | .. | .. | 118.5 | 45 | .. | .. | 26 |
| (2) As ord. split ph. motor | | | | | | | | | | | | | | | | |
| No load...... | 110 | 3.71 | 96 | .. | .. | .. | 110 | 3.71 | 96 | .. | .. | .. | .. | .. | .. | .. |
| Full load..... | 110 | 4.61 | 311 | .. | .. | .. | 110 | 4.61 | 311 | 60 | 61.2 | .. | .. | 12 | 39 | ... |
| Locked...... | 110 | 22.0 | 1805 | 110 | 20.2 | 1980 | 110 | 41.6 | 3785 | .. | .. | .. | .. | .. | .. | 26 |

TABLE III

| | Main phase | | | Capacitor phase | | | Line | | | Capac. volts | R. P. M. | Mfd. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | V. | A. | Watts | V. | A. | Watts | V. | A. | Watts | | | |
| 1 | 110 | 1.6 | 0 | 102 | 0.65 | 50 | 110 | 1.15 | 47 | 163 | 1798 | 10.6 |
| 2 | 110 | 1.475 | −27 | 112 | 1.05 | 81 | 110 | 0.775 | 52 | 171 | 1798 | 16.3 |
| 3 | 110 | 1.31 | −137.6 | 159 | 3.32 | 294 | 110 | 2.13 | 156.4 | 196 | 1795 | 45.3 |
| 4 | 110 | 1.7 | −168 | 179 | 5.02 | 493 | 110 | 4.12 | 318 | 204 | 1790 | 64.8 |
| 5 | 110 | 2.25 | −147 | 202.5 | 6.83 | 722 | 110 | 6.38 | 578 | 202 | 1775 | 88.5 |
| 6 | 110 | 3.28 | − 25 | 214.5 | 8.64 | 990 | 110 | 9.63 | 990 | 185 | 1760 | 126 |
| 7 | 110 | 5.1 | +390.5 | 201 | 9.26 | 950 | 110 | 12.15 | 1320 | 132 | 1730 | 188 |
| 8 | 110 | 6.0 | +571 | 183 | 9.0 | 840.5 | 110 | 12.6 | 1365 | 100 | 1690 | 239 |
| 9 | 110 | 6.35 | +681 | 160 | 8.8 | 645 | 110 | 12.4 | 1295 | 65 | 1610 | 315 |

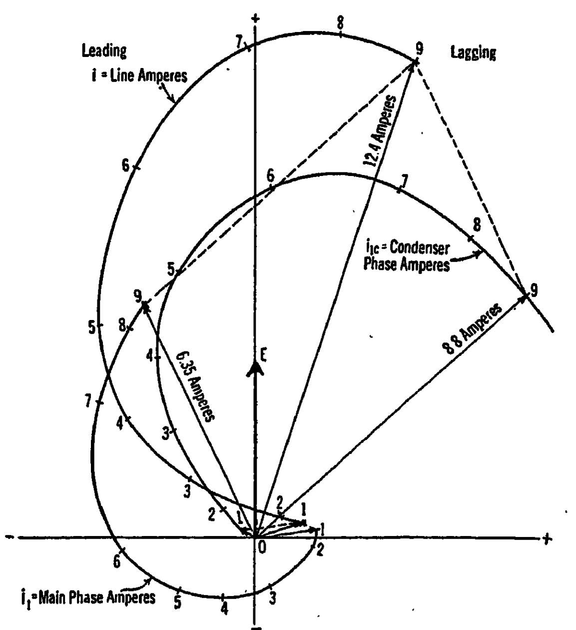


FIG. 5—CURRENT VECTORS FOR VARIOUS CAPACITIES AT NO LOAD

will not occur in practise. Generally it will be attempted to have the two phases at full load more nearly balanced and for such a case the performance can easily be determined. Due to the rather high effective ohms of the capacitor, the watts, as well as the current in the capacitor phase remain almost constant and all the variation in load is taken by the main phase. With the increase in load the watts and current in the capacitor phase will slightly decrease due to the higher impe-

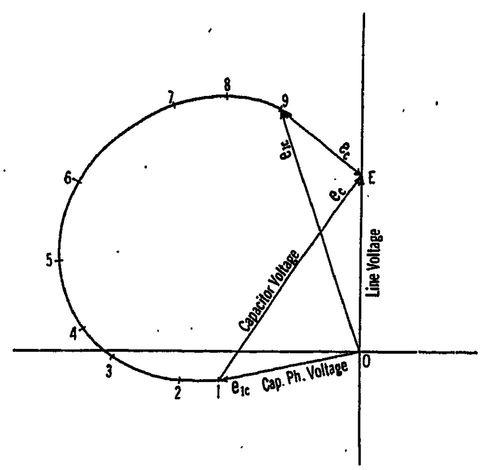


FIG. 6—VOLTAGE VECTORS FOR VARIOUS CAPACITIES AT NO LOAD

dance drop in the main phase and lower frequency of rotation. After the motor has pulled out, the current in the capacitor phase assumes the lowest value and stays practically constant to standstill. However, if the capacitor is much greater than required for balanced

condition at full load, *i. e.*, in the case of having the capacitor phase connected to the starting tap of the capacitor, then the current in the capacitor phase naturally will increase with the load because the capacitor phase is capable of carrying more load. After the motor has pulled out, the current stays practically unchanged since the total impedance in the capacitor phase has become almost constant. For example,

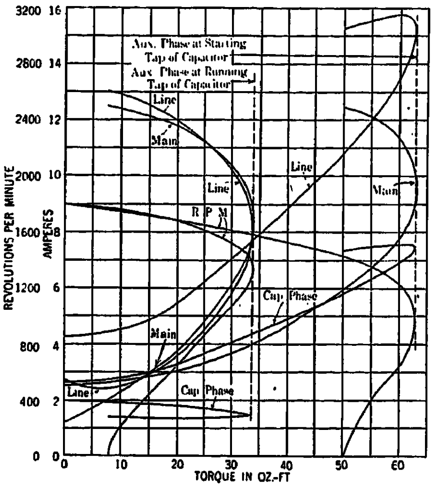


FIG. 7—SPEED-TORQUE-AMPERE CURVES OF A ¼-HP., 4-POLE, 60-CYCLE MOTOR

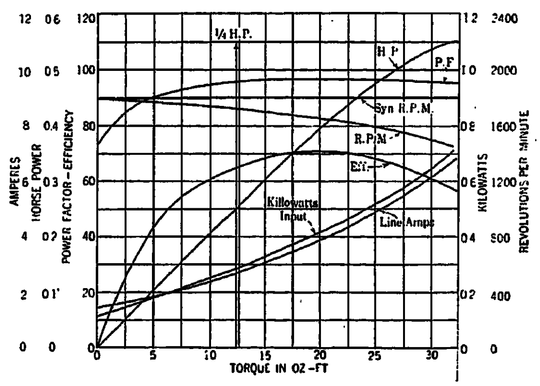


FIG. 8—PERFORMANCE CURVES OF ¼-HP., 4-POLE, 60-CYCLE MOTOR WITH CAPACITOR IN AUXILIARY PHASE

Fig. 7 shows the amperes and speed torque curves of a ¼-hp. motor 110 volts, 4 poles, 60 cycles, and in Fig. 8 the general performance of the same motor is shown.

*Capacitor.* As was mentioned before, the capacity required to insure a good capacitor motor is relatively high. The fact that the effective capacity varies with the square of the voltage, makes possible and desirable the use of a series auto transformer, as shown in Fig. 2. How far the voltage on the capacitor may be raised economically depends on the cost of the capacitor per kilovolt-ampere capacity and on the cost of the transformer. The cost of the transformer is quite an item. In cases where the line voltage is relatively high or where the starting torque required is low, the capacitor unit using a condenser only without a transformer may be more economical. By adding a transformer some of the capacitor effect is sacrificed, due to the magnetizing volt-amperes in the transformer.

The impedance diagram of the capacitor shown in Fig. 2B may be represented as shown in Fig. 9.

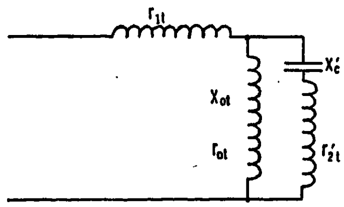


FIG. 9—IMPEDANCE DIAGRAM OF CAPACITOR

$r_{1t}$ = The ohmic resistance of the primary winding of the transformer.

$r_{2t}'$ = The ohmic resistance of the secondary winding reduced to primary turns.

$r_{0t}$ = The ohmic resistance due to iron loss.

$x_{0t}$ = The magnetizing reactance.

$x_c'$ = The inductive resistance of the condenser reduced to primary turns.

Since the leakage reactance of the transformer is very small, it is neglected. The admittance of the parallel circuit in Fig. 9 is:

$$Z = \sqrt{\left(\frac{r_{0t}}{r_{0t}^2 + x_{0t}^2} + \frac{r_{2t}'}{r_{2t}'^2 + x_c'^2}\right)^2 + \left(\frac{x_0}{r_{0t}^2 + x_{0t}^2} - \frac{x_c'}{r_{2t}'^2 + x_c'^2}\right)^2}$$

Since, in this equation, the reactive resistances are much greater than the ohmic resistances, it will be sufficiently accurate to add the primary resistance $r_{1t}$ in quadrature. Thus the total impedance of the capacitor is:

$$Y_t = \sqrt{r_{1t}^2 + \frac{1}{\sqrt{\left(\frac{r_{0t}}{r_{0t}^2 + x_{0t}^2} + \frac{r_{2t}'}{r_{2t}^{2\prime} + x'^2_c}\right)^2 + \left(\frac{x_{0t}}{r_{0t}^2 + x_{0t}^2} - \frac{x_c'}{r'^2_{2t} + x'^2_c}\right)^2}}}$$

For low watt losses in the capacitor resulting in not much reduction in over-all efficiency of the capacitor motor, the resistances $r_{0t}$ and $r_{2t}'$ will be comparatively so small that they may be neglected and the above equation may be simplified and condensed as follows:

$$Y_t = \sqrt{r_{1t}^2 + \frac{1}{\left(\frac{1}{x_{0t}} - \frac{1}{x_c'}\right)^2}}$$

The equation shows very clearly how much damage the magnetizing reactance $X_{0t}$ may do. As the saturation in the transformer iron increases, the reactance $X_{0t}$ decreases and the impedance $Y_t$ will increase. For a given voltage this results in a smaller current through the capacitor phase and hence less torque. The general rule that the starting torque of an induction motor increases as the square of the voltage holds true on a capacitor motor only as long as the iron in the transformer is not saturated. When the iron becomes highly saturated the torque may even decrease with increasing line voltage.

The saturation of the transformer iron exists only at the starting connection because at the running connection the primary winding turns are much greater. Generally the designing engineer is inclined to increase the flux density at starting to the highest possible limit; however in this case it is not wise to do so because the magnetizing volt-amperes directly reduce the capacity and with it the starting torque. It is somewhat different with the condenser itself as the voltage applied to it for such a short time may be greatly increased over its normal continuous rating.

It is somewhat different with the ohmic drop in the transformer winding as this gives only a small reduction in starting torque due to the fact that this acts at right angles to the inductive resistance. The heating of the transformer winding at start may limit the reduction in the size of wire providing the overall efficiency of the running conditions meets the guarantee. The size of the capacitor depends first of all on the design of the motor. For given values of torque and power factor the motor should be designed so as to require the least amount of capacity in order to keep the cost of the combined unit of motor and capacitor within reasonable limits. Ordinarily the motor will have a strong winding in the main phase and a weaker winding with a different number of turns in the capacitor phase.

The ratio of winding turns in the capacitor depends on the cost of the condenser. At a certain voltage the cost of the condenser per kv-a. will be least, which makes this the desirable voltage to figure on using, provided the winding cost of the transformer remains the same for the different ratio in turns. The lowest priced condenser may require a very high number of turns having a small wire section in the secondary winding, thus actually increasing the cost of the unit. In that case a larger condenser requiring a lower voltage will give a more economical design. The above statements show that the art of the design of a capacitor motor consists of properly balancing the design so as to obtain a satisfactory unit at a reasonable cost. This is far more difficult than in the design of an ordinary induction motor due to the large number of variables to be considered and especially so due to the additional cost of the capacitor.

## Application of Capacitor Motors

As far as the electrical performance is concerned, the capacitor motor may be used for almost any class of service where practically constant speed is desired. It has great flexibility of design. On applications where high power factor, high starting torque, and low starting current are required, the capacitor motor finds a good field. The possibility of obtaining various speeds by changing the capacity or, when wound slip-ring polyphase rotors are used, of changing the resistance in the rotor circuit, insures the motor a field for such applications as blowers and similar devices. Further, where reversing service or braking is required, the capacitor motor may be used. The possibilities of application of the capacitor motor are so numerous that many pages covering its field could be written.

## Summary

A summary of the above article on the capacitor motor is as follows:

1. The motor has good performance in respect to power factor, efficiency, starting current, and torques.
2. The motor is simple in construction and has no objectionable commutator or brushes. The rotor can be wound either squirrel cage or polyphase with slip rings.
3. The field of application for the capacitor motor is broader and less limited than for any other type of single phase motor.
4. The unit takes up more space on account of the capacitor.
5. Generally the total unit cost is higher than for any other type of single-phase motor. This is probably the only factor which may react against its present day use.

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## Discussion

For discussion of this paper see page 629.

# The Revolving Field Theory of the Capacitor Motor

BY WAYNE J. MORRILL*
Associate, A. I. E. E.

*Synopsis.—The paper presents an accurate theory of the split-phase motor, both as regards starting and running performance. The general equations for an unbalanced two-phase motor are first derived, and the results are then applied to the special case of the capacitor motor. Evidence of the validity of the theory is given in the form of curves, comparing test results with calculations.*

*The principal factors affecting practical capacitor motor design are discussed, and finally the performance characteristics of the motor are compared with those of repulsion-start induction motors. It is concluded that the capacitor motor has important advantages, which will justify its extensive use.*

*The complete derivation of the theory is given in an appendix.*

## INTRODUCTION

WITH the rapid increase which has recently taken place in the number of motors applied to such semi-continuously operated domestic loads as household refrigerators and oil burners, there has risen a demand for fractional horsepower motors of very high quality. Such motors operating on house lighting circuits as they do and running at all times of the night and day, must be quiet, have high operating characteristics, low starting current to prevent flicker of lights, and no radio interference. Of all the types of single-phase fractional horsepower motors available, the capacitor motor seems best suited for this sort of service. It not only has all of the characteristics necessary, but in addition is probably the simplest and most reliable of all high quality single-phase motors.

A number of manufacturers has appreciated the advantages of the capacitor motor and at the present time there are several different makes available. The renewed interest which has been shown in capacitor motors, by the actions of the motor manufacturers as well as by the attitudes of the power companies and the device manufacturers, makes it appropriate to discuss their theory and characteristics.

The capacitor motor as a type is not new. Some thirty years ago Doctor Steinmetz and his associates developed such a motor and secured a number of basic patents on it. Some of these motors were actually built and placed in service, but their manufacture was soon abandoned largely because of the difficulty, at that time, of securing capacitors of sufficiently reliable construction.

After this experience, there was a long period in which little interest in them was shown except for an occasional review which always brought the same answer; that is, until a cheap and reliable capacitor could be found, the capacitor motor would be commercially impractical.

In recent years great strides have been made in the manufacture of capacitors. The experience gained in their manufacture for the radio industry as well as for power-factor correction has been largely responsible for the improvements made. On the basis of this experience, there has been a definite effort made to develop a capacitor which would be satisfactory for use with motors, and the result is that at the present time the cheap and reliable capacitors for which the motor manufacturers have been waiting are available.

The purpose of this paper is to present the revolving field theory of the capacitor motor, and to show how, by the use of this theory, it is possible to explain and calculate the phenomena which appear in the operation of the motor. In addition to presenting the theory, an effort will be made to show that the possession of a sound theory is of great assistance in the design of a line of motors.

Of interest are the single-phase motor equations which can be obtained as the special case of a capacitor motor in which the auxiliary phase carries no current. It is believed that this method of treatment of a single-phase induction motor represents an advance in the art. By means of this treatment, the complete torque equation, including the alternating single-phase torque, is obtained.

A capacitor motor is really an unbalanced two-phase motor, in which both of the stator phases are connected directly across the same line. For this reason the equivalent circuit and general equations for an unbalanced two-phase motor will be first obtained, and after a brief discussion of the possible capacitor motor connections, these equations will be applied to the calculation of capacitor motor performance.

## THE GENERAL EQUATIONS OF AN UNBALANCED TWO-PHASE MOTOR

So far as the writer is aware, the general equations for the unbalanced two-phase motor were first derived

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*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

by Mr. P. L. Alger, early in 1924, as an extension of the A. I. E. E. article[1] he published at that time. The derivation of the general theory given here is largely founded on that work. In carrying on the further studies described in this paper, the writer has received encouragement and many helpful suggestions from Messrs. A. F. Welch, A. R. Stevenson, Jr., P. L. Alger, and C. J. Koch, to whom he wishes to express his obligation.

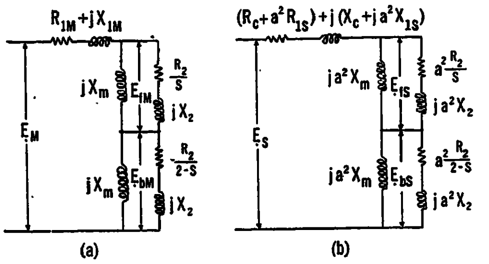


FIG. 1—SINGLE-PHASE MOTOR EQUIVALENT CIRCUITS

In the revolving field theory of single-phase motors the pulsating sinusoidal flux produced by the stator winding is resolved into two equal sinusoidal waves of flux gliding in opposite directions around the periphery of the air-gap at synchronous speed. The effect upon the stator produced by each of these revolving fluxes and the induced forward and backward revolving rotor fluxes can be represented by a parallel circuit of two branches. One branch is the magnetizing reactance and the other is composed of the secondary leakage reactance in series with the secondary resistance divided by the slip.

The equivalent circuit for a single-phase motor having a primary impedance of $R_{1M} + j\,X_{1M}$ and the primary winding of which is called the $M$ phase is represented in Fig. 1A.

In this equivalent circuit the impressed voltage equivalent to the voltage induced in phase $M$ by the forward revolving field of flux is $\bar{E}_{fM}$ and the impressed voltage equivalent to the voltage generated by the backward revolving field of flux is $\bar{E}_{bM}$.

If instead of the $M$ phase there is an $S$ phase displaced backward in position by 90 deg., electrical, from the position of the $M$ phase and having $a$ times as many turns as the $M$ phase, the single-phase equivalent circuit for the $S$ phase is shown in Fig. 1B. In the $S$ equivalent circuit the primary impedance is $a^2\,R_{1S} + j\,a^2\,X_{1S}$ and the external series impedance is $R_c + j\,X_c$.

If both the $M$ and the $S$ phases exist on the motor and are excited simultaneously, their fluxes superimpose without distortion and the equivalent circuits are the same as before except that in addition to the forward and backward voltages self induced in each phase there is a forward and backward voltage due to the fluxes of the other phase. Under this condition the equivalent circuit becomes as shown in Fig. 2 wherein the divided circuits have been replaced by series impedances of equal value.

Since the $M$ phase is displaced forward by 90 electrical degrees from the $S$ phase, the voltage generated in the $M$ phase by the $S$ forward flux must lag by 90 time degrees the voltage which the same flux produces in the $S$ phase and since the turns on the $M$ phase are to those on the $S$ phase as 1 is to $a$, the magnitude of the $M$ voltage must be $\frac{1}{a}$ times that of the $S$ phase.

From these considerations the equation for the impressed voltage equivalent to the voltage produced in the $M$ phase by the $S$ forward flux is:

$$\underset{Sf}{\bar{E}_{M}} = -\,j\,\frac{1}{a}\,\bar{E}_{fS} \qquad (1)$$

By a similar line of reasoning the impressed voltages equivalent to the other mutually generated voltages can be shown to be the values given on the equivalent circuit.

The equations for the voltages impressed on each of the primary phases are, from Fig. 2:

$$\bar{E}_M = \bar{I}_M\,[(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)] - j\,\bar{I}_S\,a\,[(R_f - R_b) + j\,(X_f - X_b)] \qquad (2)$$

$$\bar{E}_S = \bar{I}_S[R_c + a^2(R_{1S} + R_f + R_b) + j\,X_c + j\,a^2(X_{1S} + X_f + X_b)] + j\,\bar{I}_M\,a\,[(R_f - R_b) + j\,(X_f - X_b)] \qquad (3)$$

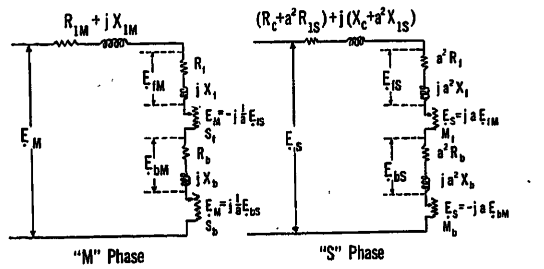


FIG. 2—UNBALANCED TWO-PHASE MOTOR EQUIVALENT CIRCUIT

## CURRENT EQUATION FOR AN UNBALANCED TWO-PHASE MOTOR

The simultaneous solution of (2) and (3) gives for the currents of an unbalanced two-phase motor:

$$\bar{I}_M = \frac{\bar{E}_M\,\{[R_c + a^2(R_{1S} + R_f + R_b)] + j\,[X_c + a^2\,(X_{1S} + X_f + X_b)]\} + j\,\bar{E}_S\,a\,[(R_f - R_b) + j\,(X_f - X_b)]}{\{[R_c + a^2\,(R_{1S} + R_f + R_b)] + j\,[X_c + a^2\,(X_{1S} + X_f + X_b)]\} \times \{[R_{1M} + R_f + R_b] + j\,[X_{1M} + X_f + X_b]\} - a^2\,[(R_f - R_b) + j\,(X_f - X_b)]^2} \qquad (4)$$

$$\bar{I}_S = \frac{\bar{E}_S\,[(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)] - j\,\bar{E}_M\,a\,(R_f - R_b) + j\,(X_f - X_b)]}{\{[R_c + a^2\,(R_{1S} + R_f + R_b)] + j\,[X_c + a^2\,(X_{1S} + X_f + X_b)]\} \times \{[R_{1M} + R_f + R_b] + j\,[X_{1M} + X_f + X_b]\} - a^2\,[(R_f - R_b) + j\,(X_f - X_b)]^2} \qquad (5)$$

1. See Bibliography.

### Equation for Average Torque of an Unbalanced Two-Phase Motor

Since only current and flux waves traveling at the same speed can act together to produce average torque, the average torque in synchronous watts produced by an unbalanced two-phase motor is equal to the sum of the watts consumed by the forward field in each phase minus the sum of the watts consumed by the backward field.

If the current in the $M$ phase be:

$$\bar{I}_M = A + j\,B \tag{6}$$

and that in the $S$ phase be:

$$\bar{I}_S = \mathbf{g} + j\,\mathbf{h} \tag{7}$$

the equation for the average torque can be shown to be:

$$T_{avg} = [I_M^2 + a^2 I_S^2][R_f - R_b] + 2\,a[A\,\mathbf{h} - B\,\mathbf{g}][R_f + R_b] \tag{8}$$

Since:

$$A\,\mathbf{h} - B\,\mathbf{g} = I_M\,I_S \sin\phi \tag{9}$$

Equation (**8**) may be reduced to (**110**) of Appendix I.

In addition to the average torque of Equation (**8**) there is an alternating torque produced through the action of the forward flux and backward current and vice-versa.

### Equation for Alternating Torque of an Unbalanced Two-Phase Motor

In Appendix I, Equation (**111**), it is shown that the maximum value of the alternating torque is:

$$\underset{Max}{T_{\sim}} = \sqrt{[I_M^4 + a^4 I_S^4 + 2\,I_M^2 I_S^2\,a^2 \cos 2\,\phi][(R_f - R_b)^2 + (X_f - X_b)^2]} \tag{10}$$

It will be noticed that the alternating torque ceases to exist when $I_S\,a$ is equal to $I_M$ and $\phi$ is 90 deg.

### The Capacitor Motor

A capacitor motor can be built in a number of forms as shown in Fig. 3.

The equations for each of the possible forms of a capacitor motor have been derived but careful investigations have shown that except in special cases the two forms in *d* and *c* of Fig. 3 are the most desirable from considerations of simplicity and economy.

The "shaded pole" and "permanent split" types are simpler than either *d* or *e* but their field of application is limited because their standstill torques are so low as to preclude their use on any loads which do not have a fan characteristic. Due to the unstable nature of the starting or capacitor phase, a peculiarity of capacitor motors which will be touched upon later, the starting phase takes an increasing amount of current as the motor speed approaches synchronism and as a result the normal and light load efficiencies are considerably reduced. A high-resistance rotor, due to the speed reduction which it produces, and the lesser amount of initial $S$ phase current needed to secure the starting torque, can be used in a "shaded pole" or "permanent split" motor without suffering loss in efficiency, and since a reduction in the size of the capacitor is obtained at the same time, high-resistance rotors are usually employed in such motors. The poor speed regulation obtained with a high resistance rotor permits of variable speed operation by variation of the impressed voltage.

Normally, there is nothing to be obtained from the "series" connection shown in Fig. 3c which cannot be got more economically from the simpler connections.

There is no difference between the motor shown in Fig. 3d and that shown in Fig. 3e except in the method of applying the capacitor. It has been found possible to impress a much higher voltage upon a capacitor for the short time required to start a motor than could be impressed continuously. Advantage has been taken of this fact in Fig. 3e by the use of an auto transformer which raises the voltage on the capacitor during the starting period and then by a change in taps reduces the voltage to a value safe for continuous operation. In addition to the change in voltage, the transformer

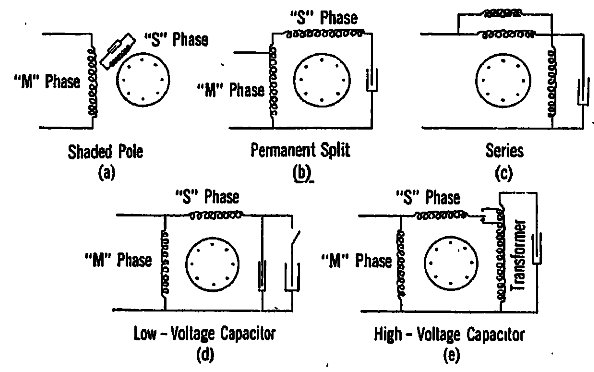


Fig. 3—Capacitor Motor Diagrams

makes it possible to impress upon the capacitor the exact voltage which is desired, regardless of the motor design.

### The Capacitor Motor Current and Torque Equations

For purposes of analysis it is possible to represent either the capacitor of Fig. 3d or the capacitor transformer unit of Fig. 3e by a resistance in series with a condensive reactance. If these values be substituted for the external impedance $R_c + j\,X_c$ in the unbalanced two-phase equations and the voltage $\bar{E}_M$ be substituted for $\bar{E}_S$ the unbalanced two-phase equations apply directly to a capacitor motor.

The unbalanced two-phase torque equations are not affected by the above substitutions and apply without change to a capacitor motor.

### Capacitor Motor Output

As shown in Equation (**112**) the equation for net output is:

$$W.\,O. = \{\,[I_M^2 + a^2 I_S^2]\,[R_f - R_b] + 2\,I_M\,I_S\,a\,[R_f + R_b] \sin\phi\,\}\,(1 - s) - W_f \tag{11}$$

wherein $W_f$ is the watts friction loss.

### Total or Line Current of a Capacitor Motor

The total current $\bar{I}$ of a capacitor motor is equal to the sum of $\bar{I}_M$ and $\bar{I}_S$ and if it be represented as follows:

$$\bar{I} = C + j\,F \tag{12}$$

the equation for the input may be written:

$$W.I. = C\,E_M + W_i \tag{13}$$

in which $W_i$ is the iron loss.

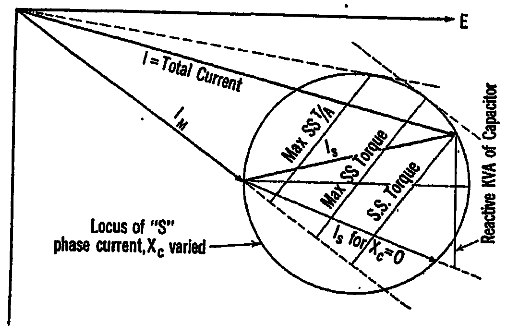


Fig. 4—Capacitor Motor Starting Diagram

### Standstill Torque Equation

At standstill the torque Equation (8) becomes:

$$T_{SS} = 2\,a\,[A\,\mathbf{h} - \mathbf{g}\,B]\,[R_f + R_b] \tag{14}$$

If at standstill the total impedance of the $S$ phase plus its external impedance be defined:

$$a^2 R_S + R_c = a^2 (R_{1S} + R_f + R_b) + R_c \tag{15}$$
$$a^2 X_S + X_c = a^2 (X_{1S} + X_f + X_b) + X_c$$

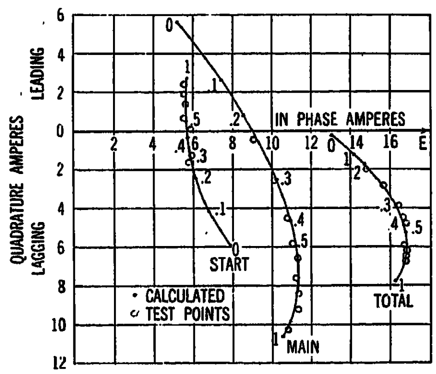


Fig. 5—Vector Current Relations for Starting Condition of a ¼-Hp. Capacitor Motor

Slip varied from "1" to "0"
. Calculated ⊙ Test points

CONSTANTS

| | | | | | |
|---|---|---|---|---|---|
| $R_{1M}$ | = 2.02 | $X_{1M}$ | = 2.79 | $X_M$ | = 33.4 |
| $R_2$ | = 2.06 | $X_2$ | = 1.06 | $a$ | = 1.18 |
| $R_{1S}$ | = 5.12 | $X_{1S}$ | = 2.31 | $Wf$ | = 13 |
| $Rc$ | = 3 | $Xc$ | = − 14.5 | $Wi$ | = 24 |

Equation (14) may be written in the following form:

$$T_{SS} = 2\,a\,E_M \left[\frac{-A(a^2 X_S + X_c) - B(a^2 R_S + R_c)}{(a^2 X_S + X_c)^2 + (a^2 R_S + R_c)^2}\right](R_f + R_b) \tag{16}$$

### Equation for Capacitor Reactance Required to Produce a Given Standstill Torque

If Equation (16) be solved for $X_c$ the equation for the capacitor reactance required to produce a given torque is obtained:

$$X_c = \frac{a E_M (R_f + R_b)}{T_{SS}} \left\{ -A \pm \sqrt{A^2 - \frac{4\,T_{SS}}{2\,aE_M(R_f + R_b)}\left[(a^2 R_S + R_c)\left\{B + \frac{T_{SS}\,(a^2 R_S + R_c)}{2\,a\,E_M\,(R_f + R_b)}\right\}\right]}\right\} - a^2 X_S \tag{17}$$

### Starting Diagram for Capacitor Motors

Equations (14) and (17) are useful but a starting diagram has been devised which furnishes graphically

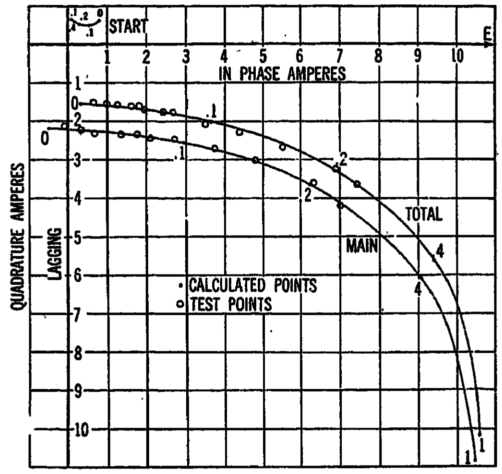


Fig. 6—Vector Current Relations for Running Condition of a ¼-Hp. Capacitor Motor

Slip varied from "1" to "0"
. Calculated ⊙ Test points
Constants are the same as Fig. 5 except $Rc$ = 9 and $Xc$ = − 172

the same information and at the same time gives a complete picture of all the quantities involved in the starting of a capacitor motor. This diagram is so simple and convenient to use that it is desirable to present it at this time.

If the locus of the total current $\bar{I}$ as $X_c$ is varied be plotted, the result is the circle shown in Fig. 4. From this circle the various starting quantities can be read in the manner indicated on the diagram.

It will be noticed that the maximum torque per ampere occurs when the total current is tangent to the circle and the maximum torque occurs at the point where a line parallel to $\bar{I}_M$ is tangent to the circle.

### Capacitor Motor Characteristics

The calculated current loci for a capacitor motor as its speed is varied are shown in Figs. 5 and 6. Loci

for both the starting and running connections are given and the torque and performance corresponding are given in Fig. 7.

On the preceding figures the circled points are test results. It will be noted that the calculations check the tests remarkably well.

On both "current loci" and "speed torque" curves the unstable nature of the $S$ phase current will be noticed. At high speeds the capacitor current and voltage increase rapidly making it desirable to change connections in order to secure economy and good operating characteristics.

Not only is the magnitude of the $S$ phase current unstable but its angular displacement also varies and with the starting connection the vectors actually cross over so that the second component of torque in Equation (8) becomes reversed in sign and acts to reduce the torque.

Investigation has shown that a high value for $a$ accentuates the instability of the $S$ phase current and for this reason it is often necessary to reduce $a$ in order that the torque on the starting connection at high speeds will be sufficient.

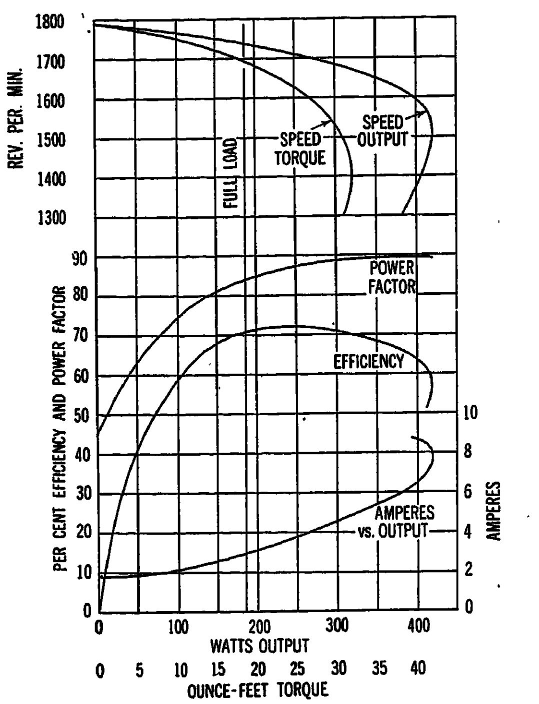


FIG. 7—PERFORMANCE CHARACTERISTICS OF ¼-HP. CAPACITOR MOTOR

(Same constants as Fig. 6)

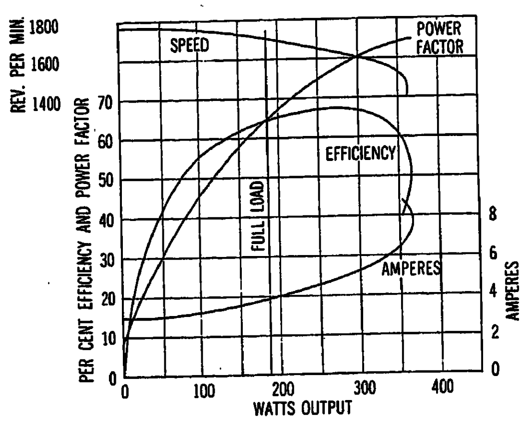


FIG. 8—SINGLE-PHASE PERFORMANCE OF ¼-HP. CAPACITOR MOTOR WITHOUT CAPACITOR

(Calculated curves)
"M" phase only, "S" phase disconnected
(Same constants as Fig. 6)

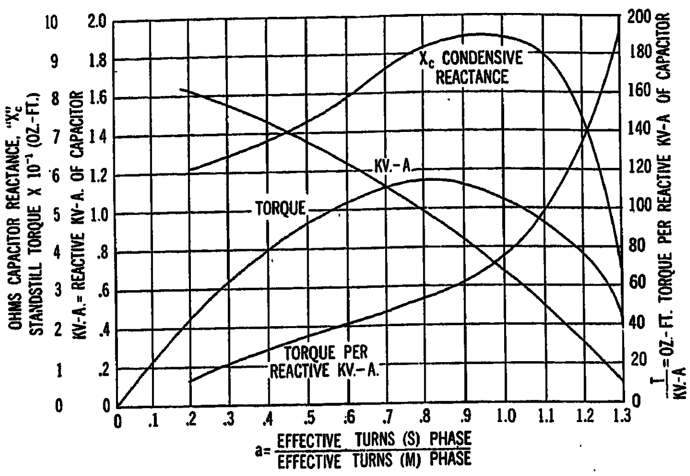


FIG. 9—CAPACITOR MOTOR STANDSTILL CHARACTERISTICS OF ¼-HP. MOTOR

Total current constant—Number of turns on starting winding varied
(Same constants as Fig. 5)

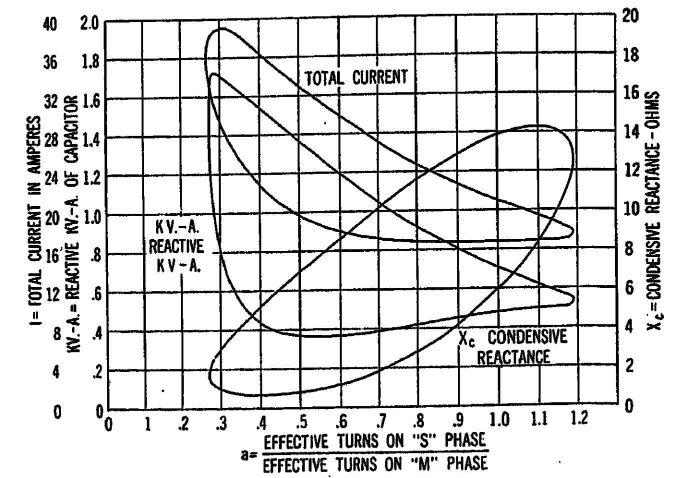


FIG. 10—CAPACITOR MOTOR STANDSTILL CHARACTERISTICS

(Calculated for ¼ hp.)
Constant torque, turns on "S" phase varied
(Same constants as Fig. 5)

For purposes of comparison Fig. 8 is inserted to show the calculated performance of the same motor with the $S$ phase disconnected and operated as a single-phase induction motor.

## DOUBLE FREQUENCY TORQUE

The alternating torque of a capacitor motor is given by Equation (10). While it is possible to approach the condition of zero alternating torque over a given range of load by properly proportioning the $S$

phase and the capacitor, this condition is seldom actually obtained. With a capacitor correctly specified for running operation, the alternating torque is always reduced, generally to less than half that of an equivalent single-phase motor. (See Fig. 12.)

## The Design of Capacitor Motors

(a) *Motor Proportions.* Experience shows approximately how much improvement is to be expected in the output and full load efficiency of a capacitor motor due to the action of the capacitor phase. Since both the efficiency and output are affected by a relatively small amount, it is possible to determine approximately the motor proportions as well as the electrical specifications of the $M$ phase and the rotor from single-phase considerations.

(b) *Calculation of the Best Capacitor Phase Proportions.* Since the primary purpose for using a capacitor motor instead of a resistance split phase motor is to secure high starting efficiency, the capacitor phase as well as the capacitor unit specifications should be determined from starting considerations.

The limiting condition on the torque specification varies and for this reason there is outlined a procedure which will apply to all cases with equal facility.

In general, it is found best to fill with copper the slots allotted to the $S$ phase and if this is done the only variable in the motor is the number of $S$ phase turns.

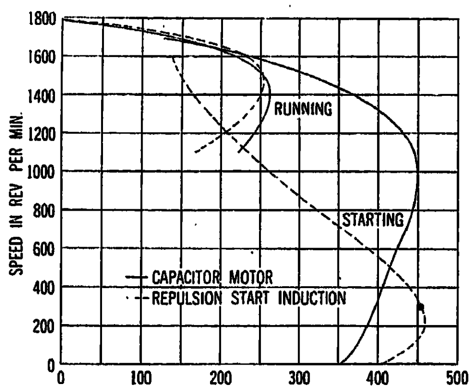


Fig. 11—Torque Curves of ¼-Hp. Capacitor Motor *vs.* ¼-hp. repulsion start induction motor (Average of several makes)
——— Capacitor motor
- - - - Repulsion start induction

Using the starting diagram it is possible to calculate the best capacitor specification for each of a number of values of $a$ and by plotting a curve of the results *vs.* $a$ the most desirable specification can be determined.

In order to show how the different quantities vary as $a$ is varied Figs. 9 and 10 are included.

c. *Calculation of Limiting Voltage on Capacitor.* The limiting voltage on the capacitor usually occurs at the cut-over speed and it is necessary to make a calculation of the starting connection for that speed in order to determine just what the highest voltage impressed on the capacitor will be.

d. *Verification of Design.* The design having been completed, its performance may be calculated and verified by means of the theory which has been presented.

The large number of variables involved in the design of a capacitor motor is very discouraging to a purely experimental investigation intended to secure the best motor and capacitor proportions. When it is remembered that in an experimental investigation such annoyances as heating, magnetic saturation, and observational errors are encountered, the advantages of a calculational investigation can be appreciated.

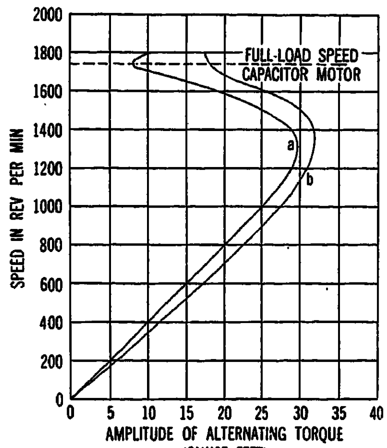


Fig. 12—Amplitude of Alternating Torque *vs.* Speed
(Same constants as Figs. 6 and 8)
(a) ¼-hp. capacitor motor
(b) Same motor operated single phase

As an illustration of the sort of results obtained in practical design Fig. 11 is presented showing a comparison of a ¼-hp. capacitor motor now in regular production with a composite repulsion start induction curve obtained by averaging the values for several of the leading makes of repulsion start induction motors.

## Conclusions

The foregoing theory and discussion of the capacitor motor have shown its possibilities as a very high quality motor. It has the following points of merit:

(a) It has excellent torque characteristics, being able to start and accelerate all it can carry;

(b) Its starting current is low, being only about 30 per cent to 50 per cent greater than that of a repulsion start induction motor of equal rating or about ½ that of a split phase motor of less torque;

(c) Its power factor is high, due to the use of the capacitor for power-factor correction after it has completed its duty in starting the motor;

(d) The motor noise is reduced, due to the reduction in double frequency torque and due to the fact that there are no brushes or other rubbing contacts;

(e) The motor is free from radio interference because it has no brushes.

ACKNOWLEDGMENT

The writer wishes to acknowledge his appreciation of the valuable assistance rendered by his associates in making calculations and plotting curves for this paper.

## Appendix

### THE UNBALANCED TWO-PHASE INDUCTION MOTOR

An unbalanced two-phase induction motor is a motor having two stator phases not identical electrically. The phases may be unbalanced with regard to windings, impressed voltage, or both.

It is well known that the action of a single-phase induction motor winding which is sinusoidally distributed in space and carries a current varying sinusoidally with the time, is to produce two sinusoidally distributed waves of m. m. f. of constant magnitude, gliding at uniform speed in opposite directions around the periphery of the air gap. It will now be shown that the revolving field theory, so admirable in its symmetry and simplicity when applied to a single-phase induction motor, readily lends itself to the calculation of an unbalanced two-phase motor.

The general construction of induction motors shows a number of characteristics peculiar to such machines. They usually have: small and approximately uniform air gaps, uniform slot spacing, symmetrical phase distribution, approximately fundamental sinusoidal distribution of the conductors in each phase, and unsaturated magnetic paths.

Advantage will be taken of these peculiarities to make some customary simplifying assumptions and these will now be listed.

*Assumptions.*

(*A*) *Perfect sinusoidal distribution of the conductors in each phase.* It can be shown that any induction motor winding can be resolved into an equivalent perfectly distributed fundamental winding plus an infinite number of perfectly distributed harmonic windings.

It can further be shown that except in very special cases all of the fluxes produced by the action of the harmonic windings link negligibly with any other windings except those on the member producing them and that in the case of a quadrature distributed two-phase motor the linkages of the harmonic fluxes are confined to the phase winding producing them. For these reasons, in the case of a uniformly distributed two-phase motor, the effect of the harmonic fluxes may be treated as that of self inductive reactance of the phase winding producing them.

(*B*) *Uniform or quadrature distribution of phases.* The methods of analysis employed in this paper can be used in the study of motors which do not have a quadrature distribution of phases, as, for example, the shaded pole motor; but such discussion does not come within the scope of the present paper.

(*C*) *Infinitesimally small slot openings.* This assumption is made in respect to the air gap only, and is for the purpose of securing a uniform gap and consequently a uniform permeance for the air gap. Correction for the slot openings can be made by the use of "average" gap permeance calculated allowing for the fringings.

(*D*) *Infinite permeability of the iron paths.* The iron paths of most induction motors are not worked above the "knee" of the saturation curve. Below the "knee" a small portion of the total m. m. f. is consumed in the iron paths and most of that which is so consumed is directly proportional to the flux density. For this reason it is possible to approximate with a high degree of accuracy to the conditions which actually prevail by altering the true reluctance of the air gap by an amount equivalent to the reluctance of the iron paths while introducing the assumption that the iron paths are of infinite permeability.

(*E*) *All the air gap flux crosses the gap radially.* With an infinitesimal gap and the preceding assumptions, this is exactly correct. As the gap becomes finite, the assumption is still very closely true for all but the very high harmonic fluxes. The high harmonics induce negligible voltages and, for this reason, with normal induction motors, the above assumption is sufficiently accurate.

(*F*) *Iron loss has no effect upon the motor fluxes.* This assumption introduces a negligible error in the calculation of motors of normal design. The actual loss will be allowed for as an additive effect after the other calculations are completed.

For special design motors and problems requiring extraordinary accuracy, more complicated methods of allowing for core loss can be employed with the theory herein presented; however such will not be considered in this paper.

*The Development of the Theory.*

If the primary of an induction motor consists of two phases displaced in space from each other by $\frac{\pi}{2}$ electrical radians, the equation for the conductor density of the first or $M$ phase is:

$$\Delta C_M = - \Delta C_{M\,max} \sin \frac{\pi}{\lambda} x \qquad (1)$$

and the equation for the conductor density of the second or $S$ phase may be written:

$$\Delta C_S = - a \, \Delta C_{M\,max} \sin \left[ \frac{\pi}{\lambda} x + \frac{\pi}{2} \right] \qquad (2)$$

wherein:

$x$ = the distance around the periphery of the air gap measured in inches from the reference point.

$\Delta C_M$ = the conductor density of the first phase per

inch of periphery at the point $x$ distant from the reference point.

$a$ = ratio $\dfrac{\text{effective conductors in second phase}}{\text{effective conductors in first phase}}$

$\lambda$ = pole pitch in inches.

If Equation (1) be integrated between limits of $x$ corresponding to a positive loop of the wave the relation between the maximum fundamental conductor density and the total fundamental conductors per pole is obtained. If $C_M$ be the fundamental conductors per pole the relation is:

$$\Delta C_{M\,max} = C_M \frac{\pi}{2\lambda} \tag{3}$$

Equations (1) and (2) may now be written

$$\Delta C_M = -\frac{\pi}{2\lambda} C_M \sin \frac{\pi}{\lambda} x \tag{4}$$

$$\Delta C_S = -\frac{\pi}{2\lambda} a\, C_M \sin\left[\frac{\pi}{\lambda} x + \frac{\pi}{2}\right] \tag{5}$$

The current in each conductor of phase $M$ will be taken as varying as the cosine of the time angle and the current in phase $S$ will be assumed to lead that of phase $M$ by $\phi$ time radians, the equations for the two primary currents are

$$i_M = I_{M\,max} \cos \omega t \tag{6}$$

$$i_S = I_{S\,max} \cos(\omega t + \phi) \tag{7}$$

The current per inch of periphery due to each phase may be obtained by multiplying the current per conductor in each phase by the conductor density of that phase.

$$\Delta i_M = -I_{M\,max} C_M \frac{\pi}{2\lambda} \sin \frac{\pi}{\lambda} x \cos \omega t \tag{8}$$

$$\Delta i_S = -I_{S\,max}\, a\, C_M \frac{\pi}{2\lambda} \sin\left[\frac{\pi}{\lambda} x + \frac{\pi}{2}\right] \cos[\omega t + \phi] \tag{9}$$

Equations (8) and (9) may also be written as shown in (10) and (11) as below.

$$\Delta i_M = -I_{M\,max} C_M \frac{\pi}{2\lambda} \left\{ \frac{1}{2} \sin\left[\frac{\pi}{\lambda} x - \omega t\right] + \frac{1}{2} \sin\left[\frac{\pi}{\lambda} x + \omega t\right] \right\} \tag{10}$$

$$\Delta i_S = -I_{S\,max}\, a\, C_M \frac{\pi}{2\lambda} \left\{ \frac{1}{2} \sin\left[\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right] + \frac{1}{2} \sin\left[\frac{\pi}{\lambda} x + \frac{\pi}{2} + \phi + \omega t\right] \right\} \tag{11}$$

In both Equation (10) and (11) the first term in the brackets represents a forward revolving field of current density and the second term a backward revolving component.

The m. m. f. at any point in the air-gap due to the forward revolving component of current of phase $M$ is equal to the integral of the current density at that point. The integration of the first term of Equation (10) with respect to $x$ produces

$$M\,M\,F_{Mf} = \frac{1}{4} I_{M\,max} C_M \cos\left[\frac{\pi}{\lambda} x - \omega t\right] + k \tag{12}$$

The constant of integration in Equation (12) is zero because the average potential of the stator is the same as the average potential of the rotor. In other words, there is no one-pole m. m. f.

The flux per inch of periphery due to the action of $M.M.F._{Mf}$ is equal to the product of (12) by the permeance per square inch of air-gap section and by the stacking length.

$$\underset{Mf}{\Delta} \Phi_M = \frac{1}{4} I_{M\,max} P\, l\, C_M \cos\left[\frac{\pi}{\lambda} x - \omega t\right] \tag{13}$$

The velocity of the flux is obtained by setting the cosine in Equation (13) equal to a constant and differentiating $x$ with respect to $t$. The velocity is:

$$V_{\Phi_f} = 2 f \lambda \text{ inches per second} \tag{14}$$

Since the velocity of the stator conductors is zero, the relative velocity of the flux with respect to the conductors is also (14).

The voltage per inch of periphery which is generated in the first stator phase by its forward flux is equal to minus* the product of (13) by (4) by the velocity of the flux with respect to the conductors, (14), and by $10^{-8}$ to convert to volts.

$$\underset{Mf}{\Delta} \epsilon_M = -[2 f \lambda]\, 10^{-8} \left\{ \frac{1}{4} I_{M\,max} P\, l\, C_M \cos\left[\frac{\pi}{\lambda} x - \omega t\right] \right\} \left\{ -C_M \frac{\pi}{2\lambda} \sin \frac{\pi}{\lambda} x \right\} \tag{15}$$

Equation (15) may also be written:

$$\underset{Mf}{\Delta} \epsilon_M = \frac{1}{4} \pi f P\, l\, C_M^2 I_{M\,max} 10^{-8} \left\{ \frac{1}{2} \sin\left[2 \frac{\pi}{\lambda} x - \omega t\right] + \frac{1}{2} \sin \omega t \right\} \tag{16}$$

If Equation (16) be integrated with respect to $x$ between limits of 0 and $\lambda$, the voltage per pole is obtained:

$$\underset{Mf}{\epsilon_M} = \frac{1}{4} \pi f P\, l\, C_M^2 I_{M\,max} 10^{-8} \frac{\lambda}{2} \sin \omega t \tag{17}$$

The impressed voltage equivalent to (17) may be written:

*The minus sign follows from Lenz's law.

$$E_{\underset{Mf}{M}} = -I_{M\,max}\,X_m \sin \omega t \tag{18}$$

wherein

$$X_m = 2\pi f C_M^2 \frac{1}{16} P l \lambda 10^{-8} \tag{19}$$

The quantity $X_m$ is called the magnetizing reactance of the $M$ phase to the forward or the backward field.

It can be shown that the magnetizing reactance for a balanced polyphase winding having $m$ phases is $m$ times as great as (19). Therefore, since the rotor phases are always balanced as far as either the forward or backward fields independently are concerned, and the paths for the rotor fluxes are identical with those for the stator, the magnetizing reactance per phase for a rotor having $m_2$ phases is,

$$X_{m\phi 2} = 2\pi f C_2^2 m_2 \frac{1}{16} P l \lambda 10^{-8} \tag{20}$$

If, as has been assumed, the rotor has $m_2$ phases the equation for the conductor density of the $n$th rotor phase may be expressed.

$$\Delta C_{2n} = -C_2 \frac{\pi}{2\lambda} \sin\left[\frac{\pi}{\lambda}x - \left(\frac{n-1}{m_2}\right)2\pi - \alpha - (1-s)\,\omega t\right] \tag{21}$$

In Equation (21) $\alpha$ is the angle by which the initial rotor phase is displaced forward from the initial stator phase at the time $t = 0$. $(1-s)$ is the speed of the rotor expressed as per unit of synchronous speed.

The voltage per inch of periphery which is generated in the $n$th rotor phase due to the forward flux of the first stator phase can be obtained in the same way as the phase $M$ voltage, Equation (15). In this case, however, the conductors as well as the flux are moving and it is necessary to solve for the velocity of the conductors in the same way that the velocity of the flux (14) was determined and to subtract the velocity of the conductors from the velocity of the flux in order to obtain the relative velocity.

The velocity of the rotor conductors is found to be:

$$V_R = (1-s)\,2 f \lambda \tag{22}$$

The relative velocity of the flux with respect to the rotor conductors is:

$$V_{\Phi f} - V_R = 2 f\lambda - (1-s)\,2f\lambda = 2 s f \lambda \tag{23}$$

The voltage per inch of periphery which is generated in the $n$th rotor phase at a point $x$ distant from the reference point is equal to minus* the product of (23) by (21) by (13) and by $10^{-8}$

$$\Delta \epsilon_{\underset{Mf}{2n}} = \frac{1}{4}\pi f s P l C_M C_2 I_{M\,max} 10^{-8}$$

$$\left\{\begin{aligned} &\frac{1}{2}\sin\left[2\frac{\pi}{\lambda}x - \left(\frac{n-1}{m_2}\right)2\pi - \alpha - (2-s)\,\omega t\right] \\ &+\frac{1}{2}\sin\left[-\left(\frac{n-1}{m_2}\right)2\pi - \alpha + s\omega t\right] \end{aligned}\right\} \tag{24}$$

The substitution of $X_m$ from (19) and the integration of (24) with respect to $x$ between limits of 0 and $\lambda$ produces:

$$\epsilon_{\underset{Mf}{2n}} = S\frac{C_2}{C_M} X_m I_{M\,max} \sin\left[-\left(\frac{n-1}{m_2}\right)2\pi - \alpha + s\omega t\right] \tag{25}$$

The differential equation for the current in the $n$th rotor phase is:

$$\epsilon_{\underset{Mf}{2n}} = R_{\phi 2}\, i_{\underset{Mf}{2n}} + L_{\phi 2}\frac{d}{dt} i_{\underset{Mf}{2n}} \tag{26}$$

wherein:

$R_{\phi 2}$ = resistance per rotor phase

$L_{\phi 2}$ = inductance per rotor phase

With a sinusoidal impressed voltage a sinusoidal current of the same frequency is produced. The equation for the current in (26) may be written:

$$i_{\underset{Mf}{2n}} = I_{\underset{Mf}{2n\,max}} \cos(\beta + s\omega t) \tag{27}$$

Employing (27) and its first derivative with respect to time, Equation (26) may be expressed

$$\epsilon_{\underset{Mf}{2n}} = I_{\underset{Mf}{2n\,max}}[R_{\phi 2}\cos(\beta + s\omega t) - L_{\phi 2}\, s\,\omega \sin(\beta + s\omega t)] \tag{28}$$

Assume the following

$$\sin\phi_f = \frac{R_{\phi 2}}{\sqrt{R_{\phi 2}^2 + (L_{\phi 2}s\omega)^2}} \tag{29}$$

$$\cos\phi_f = \frac{L_{\phi 2} S \omega}{\sqrt{R_{\phi 2}^2 + (L_{\phi 2}s\omega)^2}} \tag{30}$$

The substitution of (29) and (30) in (28) produces the following:

$$\epsilon_{\underset{Mf}{2n}} = \sqrt{R_{\phi 2}^2 + (L_{\phi 2}s\omega)^2}\, I_{\underset{Mf}{2n\,max}}[\sin\phi_f \cos(\beta + s\omega t) - \cos\phi_f \sin(\beta + s\omega t)] \tag{31}$$

Equation (31) may also be written:

$$\epsilon_{\underset{Mf}{2n}} = -\sqrt{R_{\phi 2}^2 + (L_{\phi 2}s\omega)^2}\, I_{\underset{Mf}{2n\,max}} \sin(\beta - \phi_f + s\omega t) \tag{32}$$

The product of the total inductance per rotor phase by $\omega$ is equal to the total reactance per rotor phase at fundamental frequency and since the total reactance is equal to the magnetizing reactance plus the leakage reactance.

$$L_{\phi 2}\,\omega = X_{m\phi 2} + X_{L\phi 2} \tag{33}$$

The substitution of (33) in (32) produces:

*See note Equation (15).

$$\underset{Mf}{\epsilon_{2n}} = -\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}\, \underset{Mf}{I_{2n\,max}} \sin (\beta - \phi_f + s\,\omega\,t) \qquad (34)$$

Equations (34) and (25) represent the impressed voltage on the $n$th rotor phase. Equating the two values for this voltage the following are obtained:

$$\underset{Mf}{I_{2n\,max}} = -\frac{s\dfrac{C_2}{C_M} X_m I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}} \qquad (35)$$

$$\beta = \phi_f - \left(\frac{n-1}{m_2}\right) 2\pi - \alpha \qquad (36)$$

The equation for the current in the $n$th rotor phase is obtained by the substitution of (35) and (36) in (27).

$$\underset{Mf}{i_{2n}} = -\frac{s\dfrac{C_2}{C_M} X_m I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}} \cos\left[-\left(\frac{n-1}{m_2}\right) 2\pi - \alpha + \phi_f + s\,\omega\,t\right] \qquad (37)$$

The current density in the $n$th rotor phase is equal to the product of (37) by (21)

$$\underset{Mf}{\Delta i_{2n}} = C_2 \frac{\pi}{2\lambda}\left[\frac{s\dfrac{C_2}{C_M} X_m I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}}\right] \sin\left\{\frac{\pi}{\lambda}x - \frac{n-1}{m_2} 2\pi - \alpha - (1-s)\,\omega\,t\right\} \cos\left\{-\frac{n-1}{m_2} 2\pi - \alpha + \phi_f + s\,\omega\,t\right\} \qquad (38)$$

Equation (38) may be written:

$$\underset{Mf}{\Delta i_{2n}} = C_2 \frac{\pi}{2\lambda}\left[\frac{s\dfrac{C_2}{C_M} X_m I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}}\right] \left[\frac{1}{2}\sin\left[\frac{\pi}{\lambda}x - \phi_f - \omega\,t\right] + \frac{1}{2}\sin\left[\frac{\pi}{\lambda}x - 2\frac{n-1}{m_2} 2\pi - 2\alpha + \phi_f - (1-2s)\,\omega\,t\right]\right] \qquad (39)$$

The total rotor current density is equal to the sum of the densities for all $m_2$ phases. For a polyphase rotor, which is always used, Equation (39) becomes:

$$\underset{Mf}{\Delta i_2} = \frac{m_2}{2} C_2 \frac{\pi}{2\lambda}\left[\frac{s\dfrac{C_2}{C_M} X_m I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}}\right] \sin\left[\frac{\pi}{\lambda}x - \phi_f - \omega\,t\right] \qquad (40)$$

The equation for the flux per inch of periphery due to (40) is obtained as was Equation (13)

$$\underset{Mf}{\Delta \Phi_2} = -\frac{1}{4} m_2 C_2 P\, l\left[\frac{s\dfrac{C_2}{C_M} X_m I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}}\right] \sin\left[\frac{\pi}{\lambda}x - \phi_f - \omega\,t\right] \qquad (41)$$

The voltage per inch of periphery generated in phase $M$ by the flux (41) is:

$$\underset{2Mf}{\Delta \epsilon_M} = -(2\lambda f)\, 10^{-8}\left[-C_M \frac{\pi}{2\lambda}\sin\frac{\pi}{\lambda}x\right] \left[\frac{1}{4} m_2 C_2 P\, l\right]\left[\frac{s\dfrac{C_2}{C_M} X_m I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2 s^2}}\right] \cos\left[\frac{\pi}{\lambda}x - \phi_f - \omega\,t\right] \qquad (42)$$

Equation (42) may be written:

$$\underset{2Mf}{\Delta \epsilon_M} = -\frac{1}{4}\pi f P\, l\, C_M C_2 m_2 10^{-8} \left[\frac{s\dfrac{C_2}{C_M} X_m I_{M\,max}}{\sqrt{R_{\phi 2}^2 + (X_{m\phi 2} + X_{L\phi 2})^2}}\right] \left[\begin{matrix}\dfrac{1}{2}\sin\left(2\dfrac{\pi}{\lambda}x - \phi_f - \omega\,t\right) \\ \dfrac{1}{2}\sin(\phi_f + \omega\,t)\end{matrix}\right] \qquad (43)$$

The voltage per pole is equal to the integral of (43) between limits for $x$ of 0 and $\lambda$. After performing the integration, substituting $X_m$ for its value and dividing the numerator and denominator of the first bracket by $s\left(\dfrac{C_2}{C_M}\right)^2 m_2$ the following may be written:

$$\underset{2Mf}{\epsilon_M} = I_{M\,max}\left[\frac{X_m^2}{\sqrt{\left[\dfrac{C_M}{C_2}\right]^4\left[\left(\dfrac{R_{\phi 2}}{s\, m_2}\right)^2 + \left(\dfrac{X_{m\phi 2} + X_{L\phi 2}}{m_2}\right)^2\right]}}\right] \sin[\phi_f + \omega\,t] \qquad (44)$$

At this point three new definitions will be made:

$R_2 = \left(\dfrac{C_M}{C_2}\right)^2 \dfrac{R_{\phi 2}}{m_2}$ Phase $M$ equivalent of rotor resistance. (45)

$X_2 = \left(\dfrac{C_M}{C_2}\right)^2 \dfrac{X_{L\phi 2}}{m_2}$ Phase $M$ equivalent of rotor leakage reactance. (46)

$$X_m = \left(\frac{C_M}{C_2}\right)^2 \frac{X_{m\phi 2}}{m_2} \qquad (47)$$

Equation (47) follows directly from (19) and (20).

Upon substitution of (45), (46), and (47) in (44) the following is obtained:

$$e_{\substack{M\\2Mf}} = -I_{M\,max}\left[\frac{X_m^2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2 + X_m)^2}}\right]\sin(\phi_f + \omega t) \quad (48)$$

The sine term of (48) may be resolved into two components and the equation for the impressed voltage equivalent to the generated voltage written:

$$E_{\substack{M\\2Mf}} = I_{M\,max}\left\{\frac{X_m^2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2 + X_m)^2}}\right\}$$

$$\{\sin \omega t \cos \phi_f + \cos \omega t \sin \phi_f\} \quad (49)$$

Equations (29) and (30) defined $\phi_f$ and it may be defined:

$$\sin \phi_f = \frac{R_{\phi 2}}{\sqrt{R_{\phi 2}^2 + (L_{\phi 2}\, s\, \omega)^2}}$$

$$= \frac{\frac{R_2}{s}}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2 + X_m)^2}} \quad (50)$$

$$\cos \phi_f = \frac{L_{\phi 2}\, s\, \omega}{\sqrt{R_{\phi 2}^2 + (L_{\phi 2}\, s\, \omega)^2}}$$

$$= \frac{X_2 + X_m}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2 + X_m)^2}} \quad (51)$$

The values in (50) and (51) will now be substituted in (49)

$$E_{\substack{M\\2Mf}} = I_{M\,max}\left[\frac{X_m^2 \frac{R_2}{s}}{\left(\frac{R_2}{s}\right)^2 + (X_2 + X_m)^2}\cos \omega t\right.$$

$$\left. + \frac{X_m^2 (X_2 + X_m)}{\left(\frac{R_2}{s}\right)^2 + (X_2 + X_m)^2}\sin \omega t\right] \quad (52)$$

The total primary voltage impressed upon the $M$ phase because of the forward field of flux set up directly by the $M$ phase and indirectly by the rotor current due to the $M$ phase is equal to the sum of (18) and (52).

$$E_{fM} = I_{M\,max}\left[\frac{X_m^2 \frac{R_2}{s}}{\left(\frac{R_2}{s}\right)^2 + (X_2 + X_m)^2}\cos \omega t\right.$$

$$\left. - \frac{X_m\left[\left(\frac{R_2}{s}\right)^2 + X_2 (X_2 + X_m)\right]}{\left(\frac{R_2}{s}\right)^2 + (X_2 + X_m)^2}\sin \omega t\right] \quad (53)$$

Equation (53) for the $M$ phase impressed voltage and (6) for the $M$ phase current are of the form of Equations (28) and (27). In Equation (53), by comparison with (28), the coefficient of the cosine term must represent a resistance and the coefficient of the sine term a $2\,\pi\, f\, L$ term or an inductive reactance. The coefficients of the two terms of Equation (53) may therefore be defined as the apparent resistance and reactance of the $M$ phase due to its own forward flux and the forward rotor flux which the $M$ phase induces.

$$R_f = \frac{X_m^2 \frac{R_2}{s}}{\left(\frac{R_2}{s}\right)^2 + (X_2 + X_m)^2}$$

= Apparent resistance due to the forward field. (54)

$$X_f = \frac{X_m\left[\left(\frac{R_2}{s}\right)^2 + X_2 (X_2 + X_m)\right]}{\left(\frac{R_2}{s}\right)^2 + (X_2 + X_m)^2}$$

= Apparent reactance due to the forward field. (55)

By a similar line of reasoning to that just presented the total primary voltage impressed upon the $M$ phase because of the backward field of flux set up directly by the $M$ phase and indirectly by the rotor current due to the $M$ phase is found to be:

$$E_{bM} = I_{M\,max}\left[\frac{X_m^2 \left(\frac{R_2}{2-s}\right)}{\left(\frac{R_2}{2-s}\right)^2 + (X_2 + X_m)^2}\cos \omega t\right.$$

$$\left. - \frac{X_m\left[\left(\frac{R_2}{2-s}\right)^2 + X_2 (X_2 + X_m)\right]}{\left(\frac{R_2}{2-s}\right)^2 + (X_2 + X_m)^2}\sin \omega t\right] \quad (56)$$

The apparent resistance and reactance of the $M$ phase due to its own backward flux and the rotor flux which the $M$ phase induces is therefore:

$$R_b = \frac{X_m^2 \frac{R_2}{2-s}}{\left(\frac{R_2}{2-s}\right)^2 + (X_2 + X_m)^2}$$

= Apparent resistance due to the backward field. (57)

$$X_b = \frac{X_m\left[\left(\frac{R_2}{2-s}\right)^2 + X_2 (X_2 + X_m)\right]}{\left(\frac{R_2}{2-s}\right)^2 + (X_2 + X_m)^2}$$

= Apparent reactance due to the backward field. (58)

The impressed voltages equivalent to the voltages induced in the $S$ phase by the $S$ produced forward and backward fluxes are:

$$E_{fS} = I_{S\,max}\, a^2 \left[R_f \cos(\omega t + \phi) - X_f \sin(\omega t + \phi)\right] \tag{59}$$

$$E_{bS} = I_{S\,max}\, a^2 \left[R_b \cos(\omega t + \phi) - X_b \sin(\omega t + \phi)\right] \tag{60}$$

The apparent impedances of the $S$ phase due to its forward and backward fluxes are, from Equations (59) and (60):

$$R_{fS} = a^2 R_f \tag{61}$$

$$X_{fS} = a^2 X_f \tag{62}$$

$$R_{bS} = a^2 R_b \tag{63}$$

$$X_{bS} = a^2 X_b \tag{64}$$

In addition to the forward and backward field voltages self induced in the $M$ and $S$ phases, there are two additional voltages generated in each due to the forward and backward fluxes set up by the other. Before deriving these mutually induced voltages, advantage will be taken of equations which have been developed to simplify the flux equations.

If Equations (13) and (41) be added and the symbols for apparent impedance substituted, the equation is obtained for the total forward flux per inch of periphery directly and indirectly produced by the $M$ phase.

$$\Delta \Phi_{fM} = \frac{\frac{1}{4} I_{M\,max}}{\frac{2\pi f \lambda 10^{-8}}{16} C_M} \left[X_f \cos\left(\frac{\pi}{\lambda} x - \omega t\right) - R_f \sin\left(\frac{\pi}{\lambda} x - \omega t\right)\right] \tag{65}$$

The corresponding backward flux is:

$$\Delta \Phi_{bM} = \frac{\frac{1}{4} I_{M\,max}}{\frac{2\pi f \lambda 10^{-8}}{16} C_M} \left[X_b \cos\left(\frac{\pi}{\lambda} x + \omega t\right) + R_b \sin\left(\frac{\pi}{\lambda} x + \omega t\right)\right] \tag{66}$$

For the $S$ phase the following flux equations may be written:

$$\Delta \Phi_{fS} = \frac{\frac{1}{4} a I_{S\,max}}{\frac{2\pi f \lambda 10^{-8}}{16} C_M} \left[X_f \cos\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right) - R_f \sin\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right)\right] \tag{67}$$

$$\Delta \Phi_{bS} = \frac{\frac{1}{4} a I_{S\,max}}{\frac{2\pi f \lambda 10^{-8}}{16} C_M} \left[X_b \cos\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} + \phi + \omega t\right) + R_b \sin\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} + \phi + \omega t\right)\right] \tag{68}$$

The voltage per inch of periphery generated in the $M$ phase due to the $S$ produced forward flux is equal to minus the product of (67) by (4) by (14) and by $10^{-8}$

$$\underset{Sf}{\Delta \epsilon_M} = \frac{2 a I_{S\,max}}{\lambda} \sin\frac{\pi}{\lambda} x \left[X_f \cos\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right) - R_f \sin\left(\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right)\right] \tag{69}$$

Equation (66) may be written:

$$\underset{Sf}{\Delta \epsilon_M} = \frac{2 a I_{S\,max}}{\lambda} \left[ X_f \left\{ \frac{1}{2}\sin\left(2\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right) - \frac{1}{2}\sin\left(\frac{\pi}{2} - \phi - \omega t\right)\right\} - R_f \left\{ -\frac{1}{2}\cos\left(2\frac{\pi}{\lambda} x + \frac{\pi}{2} - \phi - \omega t\right) + \frac{1}{2}\cos\left(\frac{\pi}{2} - \phi - \omega t\right)\right\}\right] \tag{70}$$

The integral of (67) with respect to $x$ between limits of 0 and $\lambda$ is the voltage per pole.

$$\underset{Sf}{\epsilon_M} = I_{S\,max}\, a \left[-R_f \cos\left(\phi + \omega t - \frac{\pi}{2}\right) + X_f \sin\left(\phi + \omega t - \frac{\pi}{2}\right)\right] \tag{71}$$

The impressed voltage equivalent to (71) is:

$$\underset{Sf}{E_M} = I_{S\,max}\, a \left[R_f \cos\left(\phi + \omega t - \frac{\pi}{2}\right) - X_f \sin\left(\phi + \omega t - \frac{\pi}{2}\right)\right] \tag{72}$$

The impressed voltage equivalent to the $M$ phase generated voltage due to the $S$ produced backward flux is found to be:

$$\underset{Sb}{E_M} = I_{S\,max}\, a \left[R_b \cos\left(\phi + \omega t + \frac{\pi}{2}\right)\right.$$

$$- X_b \sin\left(\phi + \omega t + \frac{\pi}{2}\right)\Big] \quad (73)$$

The impressed voltages equivalent to the voltages generated in the $S$ phase by $M$ produced fluxes are:

$$\underset{Mf}{E_S} = I_{M\,max}\, a \Big[ R_f \cos\left(\omega t + \frac{\pi}{2}\right)$$

$$- X_f \sin\left(\omega t + \frac{\pi}{2}\right)\Big] \quad (74)$$

$$\underset{Mb}{E_S} = I_{M\,max}\, a \Big[ R_b \cos\left(\omega t - \frac{\pi}{2}\right)$$

$$- X_b \sin\left(\omega t - \frac{\pi}{2}\right)\Big] \quad (75)$$

Thus far in the discussion no mention has been made of the primary impedance drop in either the $M$ or the $S$ phase. If the primary resistance and self inductive leakage reactance of the $M$ phase are:

$R_{1M}$ = primary resistance of the $M$ phase (76)

$X_{1M}$ = primary leakage reactance of the $M$ phase (77)

and if the primary resistance and leakage reactance of the $S$ phase are:

$a^2 R_{1S}$ = primary resistance of the $S$ phase (78)

$a^2 X_{1S}$ = primary leakage reactance of the $S$ phase (79)

wherein:

$R_{1S}$ = primary resistance of the $S$ phase if it had the same copper and distribution as it does and the same effective conductors as phase $M$ (80)

$X_{1S}$ = primary reactance of the $S$ phase if it has the same distribution as it does and the same number of effective conductors as phase $M$ (81)

the impressed voltages equivalent to the primary impedance drop in each of the two phases are:

$$E_{1M} = I_{M\,max}\,[R_{1M}\,T_{1M} \cos \omega t - X_{1M} \sin \omega t] \quad (82)$$

$$E_{1S} = I_{S\,max}\, a^2\,[R_{1S} \cos(\omega t + \phi) - X_{1S} \sin(\omega t + \phi)] \quad (83)$$

The total voltage impressed upon the primary of the $M$ phase is equal to the sum of the voltages impressed:

$$E_M = E_{1M} + E_{fM} + E_{bM} + \underset{Sb}{E_M} + \underset{Sf}{E_M} \quad (84)$$

The total voltage impressed on the $S$ phase is:

$$E_S = E_{1S} + E_{fS} + E_{bS} + \underset{Mf}{E_S} + \underset{Mb}{E_S} \quad (85)$$

The solution of Equations (84) and (85) for the currents results in the equations for the primary currents per conductor in each phase of an unbalanced two-phase motor.

Since the solution of (84) and (85) can be carried out more conveniently if the equations be expressed in vector form, steps will be taken so to express them.

If the impressed voltages and primary currents per conductor be defined in vector notation as follows:

$\bar{E}_M$ = vector voltage impressed upon the primary of phase $M$ (86)

$\bar{E}_S$ = vector voltage impressed upon the primary of phase $S$ (87)

$\bar{I}_M$ = vector current per conductor in the primary of phase $M$ (88)

$\bar{I}_S$ = vector current per conductor in the primary of phase $S$ (89)

the following vector equations may be written for the various components of (84) and (85)

$$\bar{E}_{1M} = \bar{I}_M\,(R_{1M} + j\,X_{1M}) \quad (90)$$
$$\bar{E}_{fM} = \bar{I}_M\,(R_f + j\,X_f) \quad (91)$$
$$\bar{E}_{bM} = \bar{I}_M\,(R_b + j\,X_b) \quad (92)$$
$$\underset{Sf}{\bar{E}_M} = \bar{I}_S\, a\,(X_f - j\,R_f) \quad (93)$$
$$\underset{Sb}{\bar{E}_M} = \bar{I}_S\, a\,(-X_b + j\,R_b) \quad (94)$$
$$\bar{E}_{1S} = \bar{I}_S\, a^2\,(R_{1S} + j\,X_{1S}) \quad (95)$$
$$\bar{E}_{fS} = \bar{I}_S\, a^2\,(R_f + j\,X_f) \quad (96)$$
$$\bar{E}_{bS} = \bar{I}_S\, a^2\,(R_b + j\,X_b) \quad (97)$$
$$\underset{Mf}{\bar{E}_S} = \bar{I}_M\, a\,(-X_f + j\,R_f) \quad (98)$$
$$\underset{Mb}{\bar{E}_S} = \bar{I}_M\, a\,(X_b - j\,R_b) \quad (99)$$

The vector expressions for (84) and (85) are, from (90) to (99) inclusive:

$$\bar{E}_M = \left\{ \begin{array}{l} \bar{I}_M\,[(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)] \\ + I_S\, a\,[(X_f - X_b) - j\,(R_f - R_b)] \end{array} \right\} \quad (100)$$

$$\bar{E}_S = \left\{ \begin{array}{l} \bar{I}_S\, a^2\,[(R_{1S} + R_f + R_b) + j\,(X_{1S} + X_f + X_b)] \\ - \bar{I}_M\, a\,[(X_f - X_b) - j\,(R_f - R_b)] \end{array} \right\} \quad (101)$$

The equations for $\bar{I}_M$ and $\bar{I}_S$ are:

$$\bar{I}_M = \frac{\bar{E}_M\, a^2\,[(R_{1S} + R_f + R_b) + j\,(X_{1S} + X_f + X_b)] + j\,\bar{E}_S\, a\,[(R_f - R_b) + j\,(X_f - X_b)]}{a^2[(R_{1S}+R_f+R_b)+j(X_{1S}+X_f+X_b)]\,[(R_{1M}+R_f+R_b) + j(X_{1M} + X_f + X_b)] - a^2\,[(R_f - R_b) + j\,(X_f - X_b)]^2} \quad (102)$$

$$\bar{I}_S = \frac{\bar{E}_S\,[(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)] - j\,\bar{E}_M\, a\,[(R_f - R_b) + j\,(X_f - X_b)]}{a^2[(R_{1S}+R_f+R_b)+j(X_{1S}+X_f+X_b)]\,[(R_{1M}+R_f+R_b) + j\,(X_{1M} + X_f + X_b)] - a^2\,[(R_f - R_b) + j\,(X_f - X_b)]^2} \quad (103)$$

*Development of Torque and Output Equations.*

The force exerted upon a conductor carrying a current across a magnetic field is in a direction perpendicular to both the current and the field and has a value expressed in absolute units of:

$$F_a = B_a\, l_a\, I_a \text{ dynes} \quad (104)$$

$B_a$ = flux density in lines per sq. cm.

$l_a$ = length of conductor perpendicular to the flux, in cm.

$I_a$ = current in the conductor in abamperes.

In the analysis of a-c. motors it is convenient to express the force or torque of a motor in terms of "Synchronous Watts."

*Definition of Synchronous Watts.*

A synchronous watt of force or torque is that force or torque which applied at synchronous speed would deliver mechanical energy at the rate of one watt.

The product of the force $F_a$, by the synchronous speed in cm. per second, $2 f \lambda_a$, is the power in dynes per cm. per second (ergs per second).

$$P_a = 2 B_a l_a I_a \lambda_a f \text{ ergs per second} \qquad \textbf{(105)}$$

In order to express the torque as the power in synchronous watts it is necessary to multiply (105) by $10^{-7}$. The performance of this operation together with the substitution of engineering units (inches and amperes) gives the equation:

$$T = 2 B l \lambda f i\, 10^{-8} \text{ synchronous watts} \qquad \textbf{(106)}$$

*General Equation for Torque.*

The general equation for the torque per inch of periphery may be expressed by substituting $\Delta \Phi$ for $B l$ and $\Delta i$ for $i$.

$$\Delta T = 2 \lambda f \Delta \Phi \Delta i\, 10^{-8} \text{ synchronous watts per in.} \qquad \textbf{(107)}$$

The total flux per inch of periphery is equal to the sum of the fluxes produced by both the $M$ and $S$ phase and the total primary current per inch of periphery is equal to the sum of the primary currents of each of the phases.

The equations for the torque per inch of periphery of an unbalanced two-phase motor is:

$$\Delta T = [\Delta \Phi_{fM} + \Delta \Phi_{bM} + \Delta \Phi_{fS} + \Delta \Phi_{bS}][\Delta i_M + \Delta i_S] 2 \lambda f 10^{-8}$$
$$= [(65) + (66) + (67) + (68)] [(10) + (11)] 2 \lambda f 10^{-8} \qquad \textbf{(108)}$$

The integral of (108) with respect to $x$ between limits of 0 and $\lambda$ produces the general equation for the torque per pole produced by an unbalanced two-phase motor.

*General Equation for the Torque of an Unbalanced Two-Phase Motor.*

$$T = \begin{bmatrix} I_M^2 [(R_f - R_b) + (R_f - R_b) \cos 2 \omega t \\ \quad - (X_f - X_b) \sin 2 \omega t] \\ + I_S^2 a^2 [(R_f - R_b) + (R_f - R_b) \cos 2 (\phi + \omega t) \\ \quad - (X_f - X_b) \sin 2 (\phi + \omega t)] \\ + 2 I_M I_S a [R_f + R_b] \sin \phi \end{bmatrix} \qquad \textbf{(109)}$$

$I_M$ = effective current per conductor in the $M$ phase.

$I_S$ = effective current per conductor in the $S$ phase.

It will be noticed that some of the torque components are constant while others vary sinusoidally with the time at twice line frequency. Over a period of time corresponding to any number of complete cycles of the line voltage the alternating torques integrate to zero and the average torque is:

$$T_{avg} = [I_M^2 + a^2 I_S^2] [R_f - R_b] + 2 I_M I_S a [R_f + R_b] \sin \phi \qquad \textbf{(110)}$$

The maximum value of the alternating torque is:

$$T_{\underset{Max}{\sim}} = \sqrt{[I_M^4 + a^4 I_S^4 + 2 I_M^2 I_S^2 a^2 \cos 2 \phi][(R_f - R_b)^2 + (X_f - X_b)^2]} \qquad \textbf{(111)}$$

*Output Equation for Unbalanced Two-Phase Motor.*

Since the torque is expressed in synchronous watts the output in watts is the product of the average torque by the speed in per unit of synchronous speed minus the friction loss.

$$W.O. = \{ [ I_M^2 + a^2 I_S^2] [R_f - R_b] + 2 I_M I_S a [R_f + R_b] \sin \phi \} (1 - s) - W_f \qquad \textbf{(112)}$$

*Input of Unbalanced Two-Phase Motor.*

The input can be calculated as the sum of the products of the voltage impressed on each phase by the in-phase current plus the iron loss.

*Single-Phase and Balanced Two-Phase Equations.*

It is of interest to notice how the current and torque equations behave under the special conditions of single phase and balanced two-phase operation.

When the following assumptions are made:

$$\begin{aligned} E_S &= + j E_M \\ a &= 1 \\ R_{1S} &= R_{1M} \\ X_{1S} &= X_{1M} \end{aligned} \qquad \textbf{(113)}$$

Equations (102) and (103) reduce to:

$$\bar{I}_M = \frac{\bar{E}_M}{(R_{1M} + 2 R_f) + j (X_{1M} + 2 X_f)} \qquad \textbf{(114)}$$

$$\bar{I}_S = \frac{\bar{E}_S}{(R_{1S} + 2 R_f) + j (X_{1S} + 2 X_f)} \qquad \textbf{(115)}$$

which are the current equations for a balanced two-phase motor.

If $R_{1S}$ be made very large (phase $S$ open circuited) the current $\bar{I}_S$ becomes zero and the current $\bar{I}_M$ becomes:

$$\bar{I}_M = \frac{\bar{E}_M}{(R_{1M} + R_f + R_b) + j (X_{1M} + X_f + X_b)} \qquad \textbf{(116)}$$

which is the current equation for a single-phase induction motor.

If the assumptions of (113) and currents obtained therefrom be applied to (109) the torque equation becomes:

$$T = 2 I_M^2 (2 R_f) \qquad \textbf{(117)}$$

which contains no alternating torque components and is the torque equation of a balanced two-phase motor.

If $I_S$ be made zero the torque equation becomes:

$$T = I_M^2 [(R_f - R_b) + (R_f - R_b) \cos 2 \omega t - (X_f - X_b) \sin 2 \omega t] \qquad \textbf{(118)}$$

which is the torque equation of a single-phase induction motor wherein the average torque is:

$$T_{avg} = I_M^2 (R_f - R_b) \tag{119}$$

and the maximum value of the alternating torque is:

$$\underset{Max}{T_\sim} = I_M^2 \sqrt{(R_f - R_b)^2 + (X_f - X_b)^2} \tag{120}$$

Equation (118) is particularly interesting in that it shows that one component of the alternating torque is proportional to the average or useful torque of the motor and, therefore, is not controllable by design. The other component is greatest at light loads and is subject to some control by design though an attempt at such control would probably be too costly for practical purposes.

*Capacitor Motor Equations.*

The equations for current and torque which have been developed for the unbalanced two-phase motor apply directly to a capacitor motor. Since the effect of the primary impedance is indistinguishable from external series impedance, the effect of a capacitor in series with the $S$ phase can be expressed by substituting for $a^2 R_{1S}$ and $a^2 X_{1S}$ the quantities $a^2 R_{1S} + R_c$ and $a^2 X_{1S} + X_c$ wherein:

$R_c + j X_c$ = vector impedance of the capacitor unit (121)

If the above substitution be made and $\bar{E}_M$ be substituted for $\bar{E}_S$, since both phases are connected across the same line, the current equations for a capacitor motor are:

$$\bar{I}_M = \bar{E}_M \left\{ \frac{[R_c + a^2 (R_{1S} + R_f + R_b)] + j\,[X_c + a^2 (X_{1S} + X_f + X_b)] + j\,a\,[(R_f - R_b) + j\,(X_f - X_b)]}{\{[R_c + a^2 (R_{1S} + R_f + R_b)] + j\,[X_c + a^2 (X_{1S} + X_f + X_b)]\}\,\{(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)\} - a^2 [(R_f - R_b) + j\,(X_f - X_b)]^2} \right\} \tag{122}$$

$$\bar{I}_S = \bar{E}_M \left\{ \frac{[(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)] - j\,a\,[(R_f - R_b) + j\,(X_f - X_b)]}{\{[R_c + a^2 (R_{1S} + R_f + R_b)] + j\,[X_c + a^2 (X_{1S} + X_f + X_b)]\}\,\{[R_{1M} + R_f + R_b] + j\,(X_{1M} + X_f + X_b)\} - a^2 [(R_f - R_b) + j\,(X_f - X_b)]^2} \right\} \tag{123}$$

$$\bar{I} = \bar{I}_M + \bar{I}_S$$

$$= \bar{E}_M \left\{ \frac{[R_c + a^2 (R_{1S} + R_f + R_b) + R_{1M} + R_f + R_b] + j\,[X_c + a^2 (X_{1S} + X_f + X_b) + X_{1M} + X_f + X_b]}{\{[R_c + a^2 (R_{1S} + R_f + R_b)] + j\,[X_c + a^2 (X_{1S} + X_f + X_b)]\}\,\{(R_{1M} + R_f + R_b) + j\,(X_{1M} + X_f + X_b)\} - a^2 [(R_f - R_b) + j\,(X_f - X_b)]^2} \right\} \tag{124}$$

The torque Equation, (109), is unaffected by the introduction of the capacitor and applies directly to a capacitor motor.

## NOMENCLATURE

The nomenclature has been systematically arranged by means of subscripts in such a way that the meaning of each symbol is apparent from its subscripts. For example, in Equation (17) of the appendix the symbol $\underset{Mf}{e_M}$ appears. The subscript $M$ indicates that the voltage is generated in the $M$ phase and the underscript $Mf$ indicates that it is due to the forward flux produced by the $M$ phase.

$A$ = "In phase" component of $M$ phase current

$a$ = Ratio $\dfrac{\text{Fundamental conductors on } S \text{ phase}}{\text{Fundamental conductors on } M \text{ phase}}$

$a^2 R_{1S}$ = Primary resistance of $S$ phase

$a^2 X_{1S}$ = Primary leakage reactance of $S$ phase

$B$ = "Reactive" component of $M$ phase current

$C$ = "In phase" component of total current

$\bar{E}_M$ = Vector voltage impressed on primary of $M$ phase

$\bar{E}_S$ = Vector voltage impressed on primary of $S$ phase

$F$ = "Reactive" component of total current

**h** = "Reactive" component of $S$ phase current

$\bar{I}$ = Vector total current

$I$ = Effective value of total current

$\bar{I}_M$ = Vector $M$ phase current

$I_M$ = Effective value of $M$ phase current

$\bar{I}_S$ = Vector $S$ phase current

$I_S$ = Effective value of $S$ phase current

**g** = In-phase component of $S$ phase current

$R_{1M}$ = Primary resistance of $M$ phase

$R_2$ = Secondary resistance reduced to $M$ phase

$R_c$ = External resistance in series with $S$ phase

$R_b$ = Apparent resistance to $M$ phase backward field

$R_f$ = Apparent resistance to $M$ phase forward field

$T_{avg}$ = Average value of developed torque

$T_{SS}$ = Standstill torque

$\underset{max}{T_\sim}$ = Maximum value of alternating torque

$W_f$ = Watts friction and windage loss

$W_i$ = Watts fundamental iron loss

$W.I.$ = Total watts input

$W.O.$ = Net watts output

$X_{1M}$ = Primary leakage reactance of $M$ phase

$X_2$ = Secondary leakage reactance reduced to $M$ phase

$X_c$ = External reactance in series with $S$ phase
$X_b$ = Apparent reactance to $M$ phase backward field
$X_f$ = Apparent reactance to $M$ phase forward field
$X_m$ = Magnetizing reactance of $M$ phase
$\phi$ = Angle by which $\bar{I}_S$ leads $\bar{I}_M$

The following Nomenclature refers to the Appendix:

$C_M$ = $M$ phase fundamental conductors per pole
$C_S$ = $S$ phase fundamental conductors per pole
$C_2$ = Fundamental conductors per rotor phase per pole
$\epsilon$ = Generated voltage
$E$ = Impressed voltage
$i_M$ = Instantaneous value of $M$ phase current
$i_S$ = Instantaneous value of $S$ phase current
$l$ = Axial length of air gap section
$L_{\phi 2}$ = Total inductance per rotor phase
$m_2$ = Number of rotor phases
$n$ = An arbitrary number denoting a particular phase
$P$ = Permeance per square inch of air gap section
$R_{\phi 2}$ = Resistance per rotor phase
$s$ = Slip in per unit of synchronism
$t$ = Time in seconds
$x$ = Distance in inches measured around periphery of gap
$\Phi$ = Flux
$_\phi$ = Subscript denoting "per phase" quantities
$\phi$ = Angle by which $\bar{I}_S$ leads $\bar{I}_M$ in time
$\phi_b$ = Compare with Equations (50) and (51)
$\phi_f$ = See Equations (50) and (51)
$\lambda$ = Pole pitch in inches
$\alpha$ = See Equation (21)
$\Delta$ = A prefix indicating per inch of periphery.

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## Discussion

### THE CONDENSER MOTOR
(BAILEY)
### THE FUNDAMENTAL THEORY OF THE CAPACITOR MOTOR
(SPECHT)
### THE REVOLVING FIELD THEORY OF THE CAPACITOR MOTOR
(MORRILL)

NEW YORK, N. Y., JANUARY 31, 1929

**C. R. Boothby:** Professor Bailey in his paper has discussed the starting and running performance of capacitor motors having an equal weight of wire in both phases. Such a design will require an expensive capacitor and the maximum torque will in general be low.

With these considerations in mind it will be found that if a greater percentage of the total copper is used in the main phase and a small percentage in the capacitor phase a more economical design will be obtained. I have in a manner similar to Professor Bailey's developed for such a winding the equations for starting torque, maximum starting torque, and capacitor required to give maximum starting torque. These equations reduce to the same form as given by Professor Bailey when the assumption is made that the winding resistance and reactance varies with the square of the number of turns. These equations were developed from the fundamental equations as used by Mr. Specht in his paper. The same notation is also used.

$$T = 2.66 \times P \times 10^{-8} \left[ \Phi_c I_1 T_1 \frac{K_1}{K_2} \cos \psi_1 + \Phi_1 I_{1c} T_{1c} \frac{K_{1e}}{K_2} \operatorname{Cos} \psi_2 \right] \text{oz. ft.}$$

For cage rotor $K_2 = 0.636$

$$\psi_1 = \varphi_2 + (\varphi_1 + \varphi_{1c} - 90) \qquad \psi_2 = \varphi_2 - (\varphi_1 + \varphi_{1c} - 90)$$

$$T = \frac{1.88\, P\, I_1\, I_{1c}\, K\, r_2 \,.\, \operatorname{Sin}(\varphi_1 + \varphi_{1c})}{f}$$

Where $r_2$ = Main phase rotor resistance in terms of the primary.
$K$ = Ratio of effective turns in capacitor phase to effective turns in main phase.

Making substitutions for $I_1$ and $I_{1c}$ and Sin. $(\varphi_1 + \varphi_{1c})$

$$T = \frac{1.88\, P\, E^2\, K\, r^2}{f} \times \frac{R\,c\,X - R\,(X\,c - C)}{[R^2 + X^2]\,[R\,c^2 + (X\,c - C)^2]}$$

Where $E$ = Line voltage
$R$ and $X$ = Total resistance and reactance of main winding
$Z$ = Impedance of main winding
$R\,c$ = Total resistance in capacitor phase including capacitor
$X\,c$ = Total reactance of the capacitor winding only
$C$ = Reactance of capacitor in ohms

Differentiating the above equation with respect to $C$ setting equal to zero and solving for $C$ we obtain the value of $C$ to give maximum torque *OR*

$$C_{max}\, T. = Xc - \frac{R\,c}{R}\,(X \pm Z)$$

For the capacitor motor the negative sign should be used and the equation becomes

$$C_{max}\, T. = X\,c + \frac{R\,c}{R}\,(Z - X)$$

Substituting the value of $C$ for maximum torque in the previous equation for torque we obtain the maximum starting torque.

$$T_{max} = \frac{0.944\, P\, E^2}{f} \times \frac{K\, r_2\,(Z + X)}{R\,c\,Z^2}$$

The maximum starting torque is found to be directly proportional to the ratio of the windings and the secondary resistance and inversely proportional to the total resistance in the capacitor phase circuit also a function of the main winding impedance.

The fact that the secondary resistance appears directly in the maximum starting torque equation suggests that this torque may possibly be increased provided the secondary resistance is a small part of the total winding impedance.

From this equation we see that the maximum torque is inversely proportional to the total resistance of the capacitor phase circuits. When a capacitor with a transformer is used it is therefore of importance to keep the total losses in the transformer low. In case of extremely high starting-torque requirements this may fix the size of the transformer required rather than the full load condition. In order to keep the effective resistance

of the capacitor low, it is an advantage to design the transformer with a small number of turns and high flux density as generally the copper loss will be reduced faster than the iron loss is increased. If at normal voltage the flux density is just below the knee of the saturation curve then we may have a condition that when the applied voltage on the motor is increased the starting torque will be decreased. This is due to transformer magnetizing reactance balancing part of the condenser reactance resulting in a reduction in effective microfarads in the capacitor.

In comparing the performance of a standard two-phase motor when operated from a two-phase supply with the performance of the same motor when operated with a capacitor from a single-phase supply it should be noted that only when the power factor of the two-phase motor is 71 per cent can we obtain the two-phase condition by means of a condenser. At other power factors we must be satisfied with either an unbalance in phase current or an unbalance in watts. This may become an important consideration with slow-speed motors having an inherently low power factor.

**P. L. Alger:** For some years past the utility companies have been more and more insistently demanding lower starting currents for single-phase motors on house-lighting circuits. Lately the manufacturers of domestic appliances, especially refrigerators, have been asking for higher starting torques on their motors. At the same time, the housewife has been insisting on quieter and quieter motors. And recently all of us have with one voice demanded that the motors shall not interfere with radio reception. Finally, the utility companies have come forward and set standards of performance that must be lived up to.

Under all this pressure of demands, the manufacturers have studied many different ways of improving their machines.

The simple resistance split-phase motor cannot possibly meet the torque per ampere now demanded of high-quality motors. The repulsion-start commutator motor is all right there, but it has brushes which make noise and give radio interference, and may require servicing. To make such a motor fully satisfactory, it must be spring-mounted to reduce its vibration, enclosed to reduce brush noise, and provided with a small capacitor to remove radio interference.

For all these difficulties, the capacitor motor appears the solution. By proper design, and by the use of a sufficient capacity, any reasonable starting characteristics, maximum output, and power factor can be secured. Also, the motor vibration is greatly reduced, radio interference is eliminated, and servicing, except of bearings, becomes unnecessary.

The three papers here presented show that, differ as we may in our points of view, the manufacturers now agree that all important characteristics of the capacitor motor can be predicted with reasonable accuracy, whatever the motor speed, and whatever the degree of unbalance.

Although the present high cost of capacitors will limit the application of the new motor for a time, it will undoubtedly find a field of usefulness wherever the value of superior performance is appreciated. I believe, therefore, that these papers mark the opening of a new era in single-phase motor development.

**H. C. Louis:** As an operating man, I wish to express appreciation of the development of the capacitor motor which has some very decided advantages. It is desired, however, to call attention to some of its limitations. It has many of the advantages claimed for it, but in most of these papers, and in many of the claims, it has usually been compared with split-phase motors.

For example, on the question of starting current, Mr. Morrill mentioned that it has only 30 to 50 per cent more starting current. This difference may be great enough in some cases to make it a bad application.

We had a particular case where a motor was claimed to be noisy by the customer, and we tried to replace it with a capacitor motor. Just this difference of 30 to 50 per cent made it impossible to use it in this particular location because it would have blown fuses.

I should like to ask if it is really an inherent or a necessary part of the design to have 30 or 50 per cent more starting current. In other words, if you compare this with a commutator motor where the brushes remain on the commutator, you will find it does not show up as well, as these have very good characteristics.

The question of condenser life is also raised. Trouble has been experienced with condenser failure. Can these be built to give satisfactory life without excessive cost?

**A. Nyman:** There is a large degree of misunderstanding on the rating of condensers. Particular condensers suitable for this work would be the paper or the electrolytic condenser.

The electrolytic condensers are of the liquid or the dry type, and the dry type is just beginning to be produced on a scientific basis. The electrolytic condenser has the advantage of very large capacity, but it has a disadvantage at the present time of very high losses. It is very suitable for filtering service on low voltages where there are only small a-c. components and their losses are important. For capacitor motors I do not believe the electrolytic is suitable.

The paper condenser resembles cables with regard to one feature, and that is the life. It is known from cable practise and it has been established with the condenser of the paper insulation type that the life is very much dependent on voltage. The life of a paper condenser varies as an exponential function of the voltage supplied to it.

If you rate the condenser conservatively, the life may be approximately eight years or ten years. If you double the voltage the life will be reduced considerably.

Now, on some capacitor motors the voltage on the condenser is doubled during starting. Therefore, in figuring out a condenser suitable for a capacitor motor you have to take into account the reduced life during the starting period and subtract that part of the reduction of life from the final life of a condenser as determined by the operating voltage.

The only consideration really necessary for a suitable condenser for capacitor motors is that its rating is conservative. A conservatively rated paper condenser will give long service for a capacitor motor.

**E. G. Michelsen and L. F. Hemphill:** The reversing characteristics of capacitor motors are discussed in Professor Bailey's paper and a test curve is included showing positive torques at slips greater than unity indicating the reversibility of the motor. This invites a discussion as to just what conditions are necessary to produce a positive torque curve.

It is self-evident that a capacitor motor is not necessarily reversible since the value of the condensive reactance used in series with the starting winding may be increased and as this value approaches infinity, the performance of the motor approaches that of a straight single-phase motor. Reversibility is therefore a function of the reactance employed in series with the starting winding.

Fig. 1 herewith shows two complete torque curves over the range of plus to minus synchronism. Curve No. 1 is for a motor using a relatively high condensive reactance wherein the torque characteristic similates that of a single-phase motor and is not reversible except at very low speeds, the effect of the load torque being neglected. Curve No. 2 is the result of employing a considerably lower value of condensive reactance in conjunction with the same motor and shows the motor to be completely reversible at all speeds independently of load torques. The trend of the curves for intermediate values of external reactance may be readily visualized.

The variation of the reversing torque with respect to the value of external reactance suggests a study of this effect at the speed at which it is desired to reverse the motor. The speed corresponding to a slip of 195 per cent has been chosen for this purpose, it being assumed that the motor is to be reversed at full

load speed. Fig. 2 herewith shows the torque curve as a function of the external reactance at this speed. The curve has a resonant characteristic, the maximum torque occurring at a value of condensive reactance somewhat higher than that required to render the starting circuit resonant. The range of reversibility extends from an inductive reactance of 22 ohms to a condensive reactance of 42 ohms for the motor considered. Values of

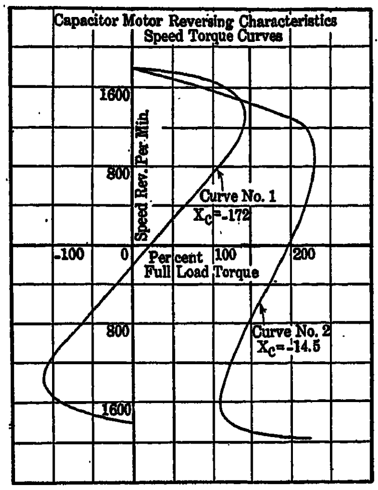


FIG. 1—CALCULATED FOR ¼-HP. CAPACITOR MOTOR

Slip varied from "0" to "2"
constants

| | | |
|---|---|---|
| $R_{1M} = 2.02$ | $X_{1M} = 2.79$ | $X_M = 33.4$ |
| $R_2 = 2.06$ | $X_2 = 1.06$ | $a = 1.18$ |
| $R_{1s} = 5.12$ | $X_{1s} = 2.31$ | |
| $R_c = 3$ | $X_c = 14.5, -172$ | |

reactance, either condensive or inductive, in excess of those mentioned result in negative torques and the motor will not reverse if the load torque and friction are neglected but will continue to operate in the same direction but with impaired characteristics. The addition of load and friction torques will, of course, increase the range of reversibility.

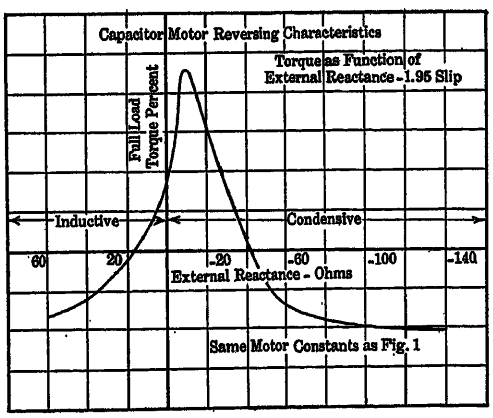


FIG. 2

Certain other factors will also enter to affect the reversing torque such as secondary resistance, the ratio of turns on the two quadrature windings, and the relative weights of copper in the two windings. The effects of these quantities are relatively unimportant compared to the effect of the external reactance and the scope of this discussion is therefore confined to this latter quantity.

The curves shown are based entirely on calculated values, the methods embodied in Mr. Morrill's paper having been employed. Past experience with the use of this method by the writers has led to very accurate results and we have every confidence in the correctness of the values herein given despite the fact that no confirming tests have been made.

We may conclude from the above that the capacitor motor is not inherently reversible but that its characteristics in this regard are dependent upon the design of the motor, the value of external reactance used during the reversing process being the predominant factor. The values of reactance normally used during the starting period with motors designed for heavy starting duty fall in the range of reversibility. The same is not true for motors designed for light starting duty as such motors are very likely to fall outside of the reversible range.

**A. G. Oehler:** Assuming that the demand for the capacitor motor is proportional to that of the more common types of single-phase motors now in service, what increase in cost can we reasonably expect for this type of motor? Also, to what range of sizes is such a motor applicable?

**W. I. Slichter:** The motor has had to wait for the development of a reliable capacitor of reasonable bulk and cost. Radio has given much material stimulation to the building of condensers and I have noticed that we are getting much better condensers for our laboratories than used to be the case. If the condenser can be relied upon this new motor will supersede the commutator single-phase motor which is now necessary when much starting torque is required of a single-phase motor, since the brush-lifting and commutator-short-circuiting mechanism is liable to mechanical troubles which cause it to fail occasionally, particularly in those cases where there is no experienced help in attendance.

I believe that the time has come when the capacitor is more reliable than the brush-lifting mechanism.

**H. C. Specht:** Professor Bailey certainly deserves much credit for presenting a paper that gives a picture of various performance by test with a capacitor motor. Unfortunately, however, all tests and conclusions are based on motors with equal amount of copper in both phases and the same distribution and winding turns of 1:1 and 2:1. This, however, is not the most economical design for the majority of applications.

In making a comparison of starting amperes with a two-phase motor the author apparently adds the currents of a two-phase motor numerically. This, however, does not seem to be fair. It would have been more correct to base such comparison on the amount of copper in the line which is required to give the same line drop. Also the comparison of power factor under running conditions with a two-phase motor is not fair for the same reason. Such a statement might give the impression that the capacitor motor is superior to a two-phase motor. It should be borne in mind, moreover, that the capacitor motor has less pull-out torque and is much higher in cost than a two-phase motor. Consequently there would be no excuse to recommend a capacitor motor in a case where a two-phase circuit is available. In Fig. 2 is shown a reversing switch. This simple arrangement is possible only if both stator windings are identical.

It is stated in the paper that the capacitor voltage reaches its maximum value at resonance. This happens sooner because with the increase in capacity the capacitance in ohms decreases in the inverse ratio and the current increases more slowly.

In Fig. 17 the speed curves of the motor with 311 and 252 $\mu$f. cross over at a certain speed and at higher speeds in both directions of rotation the smaller capacity gives more torque. This seems to be due to the great unbalance or the circulating current. I should appreciate it if the author will give his explanation of this peculiar characteristic.

**P. H. Rutherford:** The cost of the capacitor motor is rather hard to determine because of the fact that there is not a large production at the present time. If the public demands it and there is a large production, then there is a possibility

of making them at somewhere near the cost of the repulsion type of motor.

I don't believe anybody has spoken about the pull-up torque. Really the pull-up torque in refrigeration work is more important than the starting torque. The capacitor motor has a great deal better pull-up torque than the repulsion-induction motor.

**B. W. Jones:** Assuming that more capacity is used for the starting condition than is used for the running, it follows that a switching operation is necessary. The first question is, is it best to control this switching operation as a function of speed, or as a function of time? Also, assuming that this switching operation is made before the speed has reached the required value, what will be the shape of the speed-torque curve?

**P. S. Creager:** I have an impression that the field for this motor will be in the small sizes. Here the matter of cost will be important. It seems agreed that the cost of this type will be considerably higher for some time to come than the cost of competitive types. Hence, it would seem that the motor can be sold only on one or both of two bases, (a) freedom from radio interference, a very important matter in many communities, (b) quietness of operation.

Second, the question of starting demand has been mentioned several times. If, as stated above, the field for this motor is in the small sizes, is this question of starting current really important?

This leads me to ask another question somewhat apart from the subject of these papers but suggested by the discussion. Are not the limits of starting demand, etc., as specified by the utilities, too severe? The larger companies seem to be going to combined light and power primaries. In general, that means increased capacity per feeder. It would seem then that a larger starting demand could be handled. In that event, the increased starting demand of the capacitor motor would not appear to be a detriment to its use. Our laboratory is supplied from one of these feeders. So far 50-kv-a. swings have not, to my knowledge, produced complaints.

A more liberal policy in this matter would have large economic significance, to the power company, to the manufacturer, and to the user of electrical energy. This fact alone demands the most careful consideration of the general policy.

**B. F. Bailey:** The principal question raised in the discussion is that of the cost of the capacitor motor, compared with the cost of other types of single-phase motors. The motor itself is cheap; most of the added cost is in the condenser. The capacitance must be large if considerable starting torque is to be developed. The capacitance needed for running is comparatively small.

As pointed out by Mr. Nyman, electrolytic condensers are cheap, but there is a considerable loss of power in them. By suitable design, these losses have been greatly reduced. They are still too great to allow the use of an electrolytic condenser during normal running of the motor. They are, however, entirely suitable for use during starting. They are built completely sealed, so that they are as dry as a so-called dry cell. In actual service they give less trouble than paper condensers.

Reliability of paper condensers is a matter of manufacturing methods and the use of sufficient dielectric material. As the demand grows the cost will doubtless become less and their reliability will become greater. This is what has happened in the case of the condensers used in automobile ignition.

Another phase of the cost question should not be overlooked. If we consider the *entire* cost of a motor installation, the condenser motor is probably cheaper than the usual repulsion-induction motor. The entire cost includes the transmission lines, transformers, generators, switches, and local wiring. Each of these elements costs less for condenser motors. I have figures from a source in which I had great confidence, indicating that the central station can save from $10 to $20 per motor if condenser motors are used instead of the usual type. These figures are based upon refrigerator motors of the usual characteristics. This saving will more than make up the difference in price between the two motors. The difficulty is that this saving does not directly affect the customer although undoubtedly in the long run he will benefit by it, and yet he must pay the bill in the first instance. If there were some practicable way in which this saving could be passed over to the customer I believe the capacitor motor would immediately dominate the field in many applications, notably domestic refrigeration.

**H. C. Specht:** In closing the discussion I wish to say a few words to the points of radio interference and quietness in operation which Professor Creager has just brought out. I believe there is practically no radio interference either on the repulsion start induction motor or on the capacitor motor. Only at the start the repulsion induction motor may cause slight radio interference and so will the capacitor motor when the transfer switch operates.

As far as the magnetic noise of the capacitor motor is concerned it may be said that this motor is very slightly quieter than the repulsion induction motor due to the revolving field. However, this should be no argument in favor of the capacitor motor because the difference is too small and it is no more difficult to make the repulsion induction motor quiet enough for domestic service than it is to make the capacitor motor quiet.

The two real good features which speak most favorably for the capacitor motor are the good pull-up torque and the improved power factor and efficiency.

**W. J. Morrill:** One question has been asked concerning the starting current of the capacitor motor and another concerning the cost. These two questions are interrelated, because the starting-current limit determines the size and the cost of the capacitor. If the power companies decide that certain current limits must prevail, and the device manufacturer needs a certain amount of torque, then the motor manufacturer necessarily has to apply a capacitor which meets those two specifications.

It is difficult to say what will be the ultimate cost of the capacitor motor compared with the cost of its competitive motors, because, as Mr. Rutherford has stated, we have not had sufficient production experience on capacitor motors. There have been great strides made in the reduction of cost since the motor was first initiated, and we hope that the future will bring forth still further. Roughly, at the present time, it would appear the cost should be in the neighborhood of 25 or 30 per cent more than for a motor of similar characteristics of a competitive type.

I was interested in Mr. Specht's suggestion that perhaps a small motor should have a small formula. With small motors the most accurate formulas should be used because they are built in large quantitites, and it is necessary to secure the ultimate in the matter of reduction in cost.

As far as the sizes of capacitor motors are concerned, that is a matter of demand. Probably in the larger sizes we shall always have polyphase current available, and there is no need for a single-phase motor where a polyphase current is available.

The question of changing connections has been considered from a number of angles. It can be done by means of current variation and by means of speed, but it seems the most fundamental change is that by means of speed because the torque characteristic in reference to speed is always the same. Other means can be used for special reasons, but in the usual case, the speed method is the best method.

# Line-Start Induction Motors

BY C. J. KOCH*
Associate, A. I. E. E.

*Synopsis.—This paper considers the common types of polyphase induction motors suitable for starting at full voltage. The running and starting characteristics of these types are considered as well as the influence of the permissible starting current values upon these characteristics. No attempt is made to describe or consider special types designed to obtain low starting currents by the use of wound rotors, centrifugal switches, or other moving parts.*

## IMPORTANCE OF LINE START MOTORS

MANY manufacturers of polyphase squirrel-cage induction motors in the integral horsepower sizes have now added to their other products, a modified form of squirrel-cage motor in which the starting current is reduced below that of the standard squirrel-cage motor. The reduction in starting current is sufficient to warrant the power companies throughout this country to permit the starting of these motors directly across the line without the use of a starting compensator. This type of motor is now sold under a variety of trade names, such as line-start motors, auto-start motors, double-deck motors, double squirrel-cage motors, high-reactance motors, etc.

Because the use of this motor makes unnecessary the purchase and maintenance of a starting compensator, the popularity of the type is assured. The extent to which the standard squirrel-cage motor is already superseded indicates the popularity which the line-start motor will have in the future.

It will be the purpose of this paper to examine the running and starting characteristics of the line-start motor and to compare these characteristics with those of the standard squirrel-cage motor. Power factor and efficiency will be considered as well as the effect of the reduction in starting current on the starting torque and the maximum running torque which are available.

## HIGH-RESISTANCE AND HIGH-REACTANCE MOTORS RELATION OF MAXIMUM TORQUE TO STARTING CURRENT

Line-start motors are of two fundamental types, differing in the manner in which the starting current is reduced below that of the standard squirrel-cage motor. These types may properly be called the high-reactance type and the high-resistance type.

The high-resistance line-start motor may be considered as derived from the standard squirrel-cage motor by a reduction in the total amount of magnetic flux per pole, *i. e.*, by an increase in the number of turns per coil in the stator winding. The starting current will vary inversely as the square of the number of turns per stator coil. At the same time the starting torque and the maximum running torque will be reduced in the same proportion. The starting torque may be increased again to any desired value within limits by decreasing the size of the rotor bars but this leaves the amount of maximum torque unaffected since in an induction motor the maximum torque is independent of the rotor resistance, the latter serving only to determine the slip of the motor at which maximum torque occurs. In the high-resistance type of motor, therefore, the maximum torque and the starting current are quite closely proportional, any reduction in starting current resulting in a corresponding reduction in maximum torque.

In the design of a high-reactance line-start motor, the starting current is limited by increasing the leakage reactance of the motor. This is usually done by providing for each rotor bar a leakage path of comparatively low reluctance. If both the rotor and stator resistance were zero the starting current would be exactly inversely proportional to the leakage reactance. Actually the starting current will not decrease as rapidly as the leakage reactance is increased, due to the effect of resistance, but the departure from proportionality is not wide. In the same way, the maximum running torque which the motor will develop is approximately inversely proportional to the leakage reactance. In the high-reactance type of motor as well as in the high-resistance motor, therefore, the starting current and the maximum running torque are roughly proportional.

It follows that for both types of motors, the minimum starting current for which they can be designed depends upon the minimum value of the maximum torque which is accepted as a standard for general purpose motors. This is a matter for combined opinion to settle.

At the present time 200 per cent maximum torque is conceded to be high enough for general purpose applications. This amount of torque is sufficient to insure that at 90 per cent voltage the motor will be able to sustain satisfactorily an overload of 25 per cent for at least a short period of time and without stalling. Many motor manufacturers have stipulated in the past that standard squirrel-cage motors should be capable of this performance and it seems reasonable, therefore, to apply this same stipulation to any line-start motor designed to replace a standard squirrel-cage motor and compensator.

It is found by experience that high-reactance line-start motors built to have a starting current of 6 amperes per hp. on a 440-volt 60-cycle circuit will

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develop 200 per cent maximum torque with a slight margin and will have efficiencies comparing favorably with those of standard motors. This value of starting current represents therefore a minimum value for general purpose line-start motors.

Only motors for use in special applications where the torque requirements and voltage conditions are definitely known justify any lower value.

Such special applications exist in the driving of centrifugal fans and pumps. In these cases overloading the motor is not a likely occurrence and 200 per cent maximum torque is more than ample to take care of any low-voltage condition. For these cases line-start motors are built with starting currents as low as 4 amperes per hp. at 440 volts. Such motors will develop from 150 per cent to 160 per cent maximum torque and because their maximum torque is low they are usually specifically designated as "limited maximum torque" motors. The starting torque which they will develop is also limited, averaging from 75 per cent to 100 per cent of full-load torque and seldom exceeding the latter figure. Because their torque is low, these motors must be applied with care for a combination of low line voltage and overload would stall the motor.

The maximum torque obtainable with the high-resistance type motor is slightly higher than that of the high-reactance motor for the same starting torque and current. In spite of this it is not practicable to reduce the starting current of the high-resistance type motor below 6 amperes per hp. at 440 volts, owing to the sacrifice in efficiency involved and the resulting increase in heating of the motor.

### Limitations of Size of Line-Start Motors

The relation existing between amperes per hp. of starting current and maximum torque given in the preceding paragraphs is approximate only, but within the range of its approximation the relationship is independent of the motor size. If 6 amperes per hp. at 440 volts was an acceptable value of starting current for line-start motors irrespective of size, it would be possible to develop a line of general purpose line-start motors with 200 per cent maximum torque in any size requested. Actually this is not the case.

The most widely accepted rules regulating starting currents for 60-cycle circuits, the so-called "N. E. L. A. rules," are on a sliding scale, the permissible starting currents becoming more stringent for the higher hp. ratings. This imposes a definite limit to the size of a line-start motor which can develop 200 per cent maximum torque and still meet the starting current prescribed by these rules.

The N. E. L. A. rules comprise a set of recommendations drawn up by a subcommittee of the N. E. L. A. in 1923. Although never accepted officially by the N. E. L. A. these recommendations are recognized as good practise by many power companies. The N. E. L. A. rules in this way have grown to be an accepted standard recognized by the motor manufacturer and the power company alike.

Table I is an extract from the N. E. L. A. rules showing the prescribed starting current for three-phase motors on 440-volt 60-cycle circuits. For other voltages and for two-phase service, the rules prescribe starting currents which in each case amount approximately in kv-a. to the kv-a. represented by the currents in Table I.

TABLE I

| Horsepower | Starting current 3-phase—440 volts 60-cycles |
|---|---|
| 3 | 30 |
| 5 | 43.3 |
| 7½ | 58 |
| 10 | 70.5 |
| 15 | 98.5 |
| 20 | 125 |
| 30 | 180 |
| 40 | 190 |
| 50 | 200 |
| 100 | 400 |

The stringency of the rules at higher horsepowers will be noticed. Thus a 10-hp. motor is allowed a starting current of 7.06 amperes per hp., a 30-hp. motor is allowed 6 amperes per hp., and any motor above 50 hp. is allowed a starting current of only 4 amperes per hp.

What this actually represents may be better appreciated by referring to Table II, which shows the percentage of full-load current which these various starting currents represent. The power factors and efficiencies are taken from the recent N. E. M. A. recommendations for six-pole, 60-cycle, squirrel-cage motors.

TABLE II

| Hp. | Eff. | P. F. | Full-load current at 440 volts | N. E. L. A. | Per cent starting current |
|---|---|---|---|---|---|
| 10 | 85.5 | 85 | 14 | 70.5 | 503 |
| 30 | 88.0 | 88 | 40 | 180. | 450 |
| 50 | 89.0 | 89.5 | 62 | 200. | 322 |

It is evident from these figures that general purpose line-start motors designed to comply with the N. E. L. A. rules and to have 200 per cent maximum torque are limited to motors of 30 hp. and smaller.

Several manufacturers now list, in addition, line-start motors having ratings above 30 hp. and designed to have a starting current of 6 amperes per hp. at 440 volts. This value of starting current is above the N. E. L. A. recommendation but there are probably places in which such motors might be started across the line without objection, as for example, in central stations.

"Limited maximum torque" motors may be built to comply with the N. E. L. A. rules in any horsepower rating desired.

### Normal and High Starting Torque Motors

Line-start motors of two varieties are now built as standardized products in sizes from 5 hp. to 30 hp., differentiated from one another by the amount of starting torque developed. These are known as "normal-torque" motors and "high-torque" motors. Although differing in starting torque, both kinds of motors for the same rating are designed to have the same starting current, *i. e.*, the N. E. L. A. recommendation for that horsepower.

Normal-torque line-start motors may be of either the high-resistance or the high-reactance type but high-torque motors are always of the high-reactance type.

The normal-torque motor has a starting torque of about 150 per cent of full-load torque. This is slightly higher than that developed by a standard squirrel-cage motor when started on the 80 per cent tap of a starting compensator. The normal-torque motor is therefore applicable for general purpose work and is

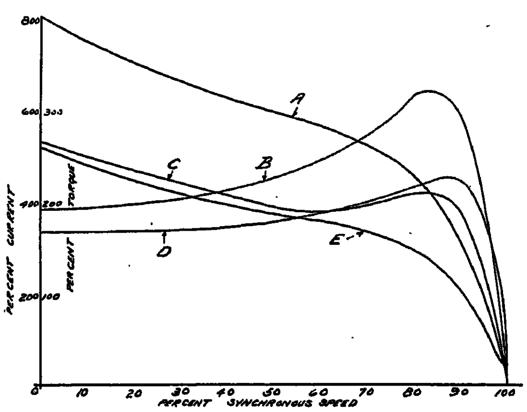


Fig. 1

*a.* **Current curve—Standard squirrel-cage motor**
*b.* **Torque curve—Standard squirrel-cage motor**
*c.* **Torque curve—High-torque high-reactance motor**
*d.* **Torque curve—Normal-torque high-reactance motor**
*e.* **Current curve—Normal-torque and high-torque high-reactance motor**

intended to be a substitute for the squirrel-cage motor and compensator.

The high-torque motor is designed to have from 225 per cent to 250 per cent starting torque and fills the special torque requirements of such loads as ammonia compressors, certain chain conveyers, and other applications where the torque requirements are severe at starting.

Comparative torque curves of a typical high-torque and normal-torque line-start motor are shown in Fig. 1. Both of the line-start motors are of the high-reactance type. A speed-torque curve of a standard squirrel-cage motor of the same rating is also added for comparison. Both the current and the torque curves are for full terminal voltage. The current curves for the normal-torque and high-torque motors are practically identical and for this reason a single curve has been drawn for the two motors.

Historically, the "high-torque" motor was first of the present line-start motors to find extensive industrial application. Designed as a double squirrel-cage or Boucherot motor with 225 per cent to 250 per cent starting torque, it was first applied industrially as a substitute for the slip-ring induction motor used until then for compressor drive.

At the same time it was found that the starting current of the double squirrel-cage motor, although not as low as that of the slip-ring motor with resistance, was appreciably lower than that of a standard squirrel-cage motor of the same rating. With the formulation of the N. E. L. A. recommendations previously mentioned, the small double squirrel-cage motor was endowed with the further advantage of not requiring a starting compensator. This economy led to the high-torque motor being installed in many places where the starting torque available from the motor was far in excess of the requirements of the load. In other words, in many installations, the attribute for which the high-torque motor was originally designed, that is, its high starting torque, was subordinated to the fact that the motor required no compensator. In these cases of misapplication considerable gains in efficiency and power factor could have been made by designing the motor for a lower starting torque.

There was an evident need for a motor having a starting current as low as that of the high-torque motor and with a starting torque equal to that of a squirrel-cage motor and compensator. Such motors are now available in the "normal-torque" motors.

That the development of the normal-torque motor has come about in this way is somewhat unfortunate. It seems destined eventually to supersede the standard squirrel-cage motor and compensator in many places. At the same time high-torque motors are still applied in many instances where normal-torque motors would do the work. This misapplication is delaying the time when the production of normal-torque motors will be swelled to the point where the manufacturers can invest in them the tool and die equipment needed to bring down their cost to that of the standard squirrel-cage motor of the same rating.

### The High-Reactance Type of Line-Start Motor

The majority of both normal- and high-torque line-start motors are of the high-reactance type. The reduction of the starting current below that of the standard squirrel-cage motor is effected by increasing in some way the leakage reactance of the motor. Usually the leakage reactance of the rotor bars is increased, although the same results may be obtained by increasing the reactance of the rotor end rings. For reasons which follow, it is not practicable to increase the reactance of the stator unless a very low starting torque is all that is necessary.

Consider, as an example, a standard squirrel-cage polyphase induction motor having a starting current

of 750 per cent of full load current and a starting torque of 200 per cent of full load torque at normal voltage. If it is assumed that for the particular motor in question the N. E. L. A. starting current is 450 per cent of full load current, the rotor reactance, if increased sufficiently to reduce the starting current to 450 per cent, would as a result reduce the starting torque to

$$\left(\frac{4.50}{7.50}\right)^2 \times 200\% = 72\%$$

assuming the rotor resistance to be unchanged. In order to obtain the 150 per cent starting torque which is considered necessary for a normal-torque motor it would be necessary to increase the rotor resistance by

$$\frac{1.50\%}{0.72\%} = 210\% \text{ or by } 350\% \text{ for a high-torque motor.}$$

If we further assume that the slip at full load was 2.5 per cent for the standard motor, the slip of the normal torque

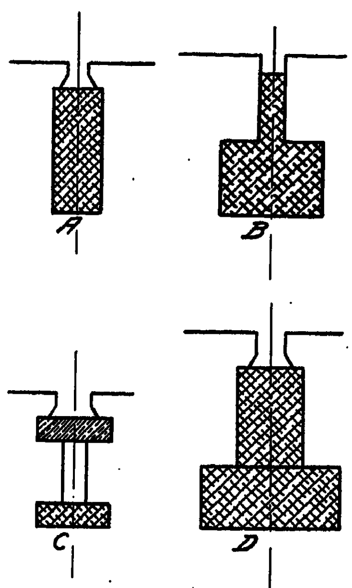


Fig. 2—Four Types of Rotor Slots

motor would be 2.1 $\times$ 2.5% = 5.3%, representing an objectional depreciation of about 2.5 per cent in the full load efficiency of the motor.

It is not necessary, however, to increase the running resistance of the rotor in order to obtain the needed starting torque. When the rotor is at standstill the frequency of the current in a rotor bar is the same as the line frequency. When, however, the rotor has accelerated to its full load speed the frequency of the rotor current is reduced to a very low value. If the rotor bars are so made that with the high frequency existing at starting eddy currents are induced, the equivalent increase in resistance due to these eddy currents appears as additional starting torque although the running resistance of the rotor has not been increased and the full load efficiency is but little affected.

The designer's problem in laying out either a normal-torque or a high-torque high-reactance motor therefore consists in designing a rotor bar or combination of bars which will have sufficient reactance to limit the starting current to the proper value, will give rise to enough eddy-current loss at starting to raise the starting torque to the desired value, will give good running efficiency, and will have as high a maximum torque and full load power factor as is consistent with the other conditions.

Choice of High-Reactance Rotor Slots

The methods for obtaining additional rotor resistance at starting are for the most part well known. Fig. 2 contains schematic drawings of four types of construction which will give the desired results. They are:

Fig. 2A This represents the well known deep bar in which the high reactance of the lower portion of the bar causes the current density to increase at the top of the bar when the frequency is increased, even though the total net load current in the bar is unchanged.

Fig. 2B This is a modification of the deep bar in which the lower portion of the deep bar is contracted to form the T-shaped bar in the figure. This may well be called a T-bar.

Fig. 2C This is the double squirrel-cage slot of the Boucherot motor. It may be considered as a development from the deep bar in which the reactance of the lower portion of the deep bar is purposely greatly increased by the introduction of the selective leakage path. The upper bar in the slot has a high resistance; the lower bar has a low resistance.

Fig. 2D The slot shown in Fig. 2D is similar to the slot for the T-bar but in this case the upper bar is idle, *i. e.*, it is not connected to the end ring and the net current which it carries is zero. At starting, however, the leakage flux crossing the idle bar due to the current in the active bar induces heavy circulating currents in the idle bar.

These four alternatives represent the principal choices open to the designer, although they by no means cover all of the possibilities. Among others, many ingenious arrangements have been devised for inducing eddy currents in the end rings rather than in the rotor bars. This method, although accomplishing the result in high-speed motors, is open to the objection that in low-speed motors the end rings contribute very little to the total resistance of the rotor. In addition, the introduction of any appreciable e. m. f. in the end ring makes it necessary to insulate the rotor bars in the slot in order to prevent short circuiting through the punchings.

Which of the alternatives in Fig. 2 represents the wisest choice? This depends upon the relative magnitude of what may be called the "incremental reactance" of each of these slots. When the motor in question has reached its full load speed and the rotor frequency is very low, the rotor current divides in inverse pro-

portion to the resistance of each individual axial filament of the rotor bar. At standstill, however, the current in the bar redistributes in a different manner and it is a fundamental fact that the leakage inductance of a bar of this kind is lower at fundamental frequency than it would be at the very low frequency obtained while running. Stated in other words, the leakage inductance of the rotor and hence the leakage reactance of the motor as a whole referred to the primary increases as the motor comes up to speed. This increase is the "incremental reactance" of the motor.

Since the maximum torque of the line start motor is decreased below that of the standard squirrel-cage motor by the additional rotor reactance required to limit the starting current, any further increase in the leakage reactance due to the "incremental reactance" means a further reduction in maximum torque. The designer's problem, therefore, consists in designing a slot which will have at standstill the proper leakage reactance, the proper a-c. to d-c. resistance ratio, and will at the same time have a minimum incremental reactance.

Some of the possibilities of Fig. 2 possess certain advantages over others. The deep bar is a poor choice as its incremental reactance is extremely high for very moderate values of a-c. to d-c. resistance ratio. This is due to the crowding of the current to the top of the slot where the reactance is lowest.

The idle bar is a much more "efficient" producer of eddy currents. This is readily appreciated since the idle bar permits the production of the same eddy currents as would exist in the top portion of a deep bar and at the same time restricts the load current to the lower bar where the reactance is high. This type of construction is particularly adaptable to high-speed motors of high horsepower rating where it is desired to increase the torque per ampere at starting as much as possible and at the same time avoid the mechanical complexity of a double-squirrel-cage with double end rings.

The T-bar has a low incremental reactance when it is desired to combine moderate values of resistance ratio with a considerable value of rotor reactance. It is particularly suited to the cast aluminum type of rotor construction and is often used for line-start motors for pump drive under 30 hp. in size and requiring not more than 100 per cent starting torque.

When it is necessary to get high values of resistance ratio with rather low values of rotor reactance the double squirrel-cage slot has the lowest incremental reactance of these four choices.

Besides having a low incremental reactance the double squirrel-cage slot possesses a very important advantage in that all of the bar is effective in providing the double squirrel-cage effect, the projection of the bar at the ends of the motor as well as the portion in the air ducts being just as effective as the portion in the slot. This follows from the fact that any division of current between the top and bottom bars which exists at one point of the rotor slot must exist throughout the bar length.

In the case of the other three alternatives, however, the resistance ratio of the portion of the bar embedded in the slot is higher than the resistance ratio of the rotor as a whole since the end rings and bar extensions contribute a constant resistance independent of frequency, no eddy current effect being present in them. A high resistance ratio is always associated with a high incremental reactance, so that for a given resistance ratio of the rotor as a whole, the advantage of the double squirrel-cage slot is evident. This advantage may be further increased by providing separate end rings for the top and bottom bars.

Fig. 3 shows rotor punchings from a variety of double squirrel-cage line-start motors. The double squirrel-cage in this case is not formed by a pair of bars driven

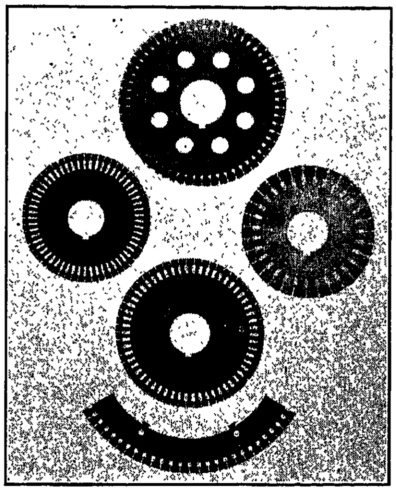

FIG. 3—ROTOR PUNCHINGS OF LINE START HIGH-REACTANCE MOTORS

into the slot as in Fig. 2c but in this case the whole slot is cast full of aluminum, the bars being cast integral with the end rings. The resistance ratio, incremental reactance, and leakage inductance at starting are controlled entirely by the shape of the slot.

Fig. 4 shows a double squirrel-cage rotor with cast aluminum bars, the top and bottom bars having separate end rings.

It is not the purpose of this paper to describe the methods by which the characteristics of these bars may be calculated. As a result of calculating a great many combinations, the author has found that the "vector method" of Professor W. V. Lyon[1] involves about the minimum of labor and gives results uniformly in agreement with test.

### EFFECT OF SATURATION UPON THE STARTING CURRENT

The starting current of line-start motors is usually measured with the rotor blocked at standstill and with

1. See Bibliography.

either full voltage or a reduced value of voltage applied at the terminals. In the latter case the starting current value at full voltage is found from the value measured at reduced voltage, assuming the starting current and the terminal voltage to be proportional. This proportionality does not actually hold true. The starting current at full voltage may be from 8 per cent to 15 per cent higher than the value extrapolated from reduced voltage readings. This is due to the fact that at full voltage the ampere conductors per slot in both the stator and rotor is higher than at reduced voltage and more saturation of the leakage flux paths occurs. This results in an appreciable decrease in leakage reactance at full voltage. A representative figure may be given for line-start motors, averaged from a large number of tests.

$$\frac{\text{Leakage reactance at full voltage}}{\text{Leakage reactance at full load current}} = 90 \text{ per cent}$$

It follows that the line-start motor which meets the N. E. L. A. rules at full voltage must have about 10 per cent more reactance running than a similar motor which meets the rules only at reduced voltage. The first motor is, therefore, under a certain handicap and

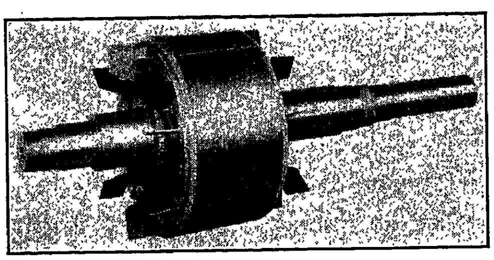

Fig. 4—Rotor from High Torque High-Reactance Motor

has a lower power factor and a lower maximum torque than the second motor.

It might be thought that the maximum torque would be unaffected by saturation since any saturation which occurs at starting would also occur at the maximum torque point of the torque curve, the current at these two points being not widely different. There are two reasons, however, why this is not true.

First: The maximum torque point of the torque curve is seldom approached in actual service due to overload but rather due to a combination of normal load and reduced voltage in which case the current at maximum torque is reduced also.

Secondly: The saturation effect is not an effect proportional to the current but after a certain point is reached it increases quite rapidly. Thus at normal voltage although the current at maximum torque is perhaps as much as 70 per cent of the starting current the actual saturation of the leakage paths at maximum torque may be small.

These two reasons indicate that the maximum torque should be based on a value of reactance measured at standstill at full load current and corrected for the incremental reactance rather than a value found by test at standstill and at full voltage.

The purchaser of a line-start motor is of course interested in knowing the actual starting current of the motor at full voltage, so that the better practise is to design line-start motors to meet the N. E. L. A. rules actually at full voltage. The N. E. L. A. recommendations, however, recognize and accept the measurement by extrapolation from reduced voltage.

## Operating Characteristics

### *Normal Torque High-Reactance Motors*

The normal torque high-reactance motor is the line-start motor of the high-reactance type designed for general purpose application and having 150 per cent starting torque.

At full load the leakage reactance of this motor is higher than that of a similar standard squirrel-cage motor. This difference results from the following three causes.

1. The reactance has been increased in order to limit the starting current.
2. Saturation of the leakage paths with full voltage starting.
3. The incremental reactance of the rotor bars.

The total additional reactance represented by these three causes constitutes an additional reactive kv-a. drawn from the line by the motor, the magnitude of which depends on the load. At full load this kv-a. will result in the normal torque high-reactance motor having a power factor from 2 per cent to 4 per cent lower than that of a standard motor of the same size. At reduced values of load, this difference in power factor decreases until at one-half load the power factor of the two motors is very nearly the same. Many motors in actual service operate only a portion of the time at full load and in cases of this kind, the total reactive kv-a. of the line-start motor over a period of time is but little more than that which a standard squirrel-cage motor would require.

This equality in power factor at light loads is only true if the two motors have identical stators and therefore have the same magnetizing current. This latter condition is almost always true, however, between 5 and 30 hp., for four- and six-pole, 60-cycle motors. Eight-pole and slower speed normal-torque motors have generally a higher magnetic flux per pole than the corresponding standard motors and in these cases the power factor of the line-start motor is below that of the standard motor even at the fractional load points.

The efficiency of the normal-torque high-reactance motor is the same as that of the standard squirrel-cage motor. Since its power factor is lower than that of the standard motor, the stator copper loss must be slightly higher, but this additional loss can be counterbalanced by decreasing by a small amount the running resistance of the rotor. This cannot be carried very far, however, without the incremental reactance becoming excessive

and resulting in a prohibitively low maximum torque. In the normal-torque motor it is possible to have the running resistance of the rotor lower than the resistance of a standard motor and still have a margin over 200 per cent maximum torque. The inter-relation which exists between maximum torque and rotor running resistance and therefore rotor copper loss is more important in the high-torque high-reactance motor than in the normal-torque motor and will therefore be considered again later.

*High-Torque High-Resistance Motors*

The "high-torque" high-reactance motor is the line-start motor of the high-reactance type designed for

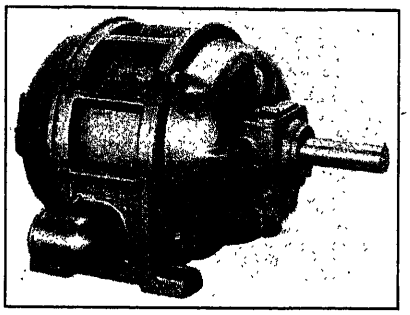

FIG. 5—LINE START GENERAL PURPOSE INDUCTION MOTOR
15 hp.—1200 rev. per min.

application to loads requiring a higher starting torque than that developed by the normal-torque line-start motor. It has 250 per cent starting torque as compared with 150 per cent for the normal-torque motor.

The rotor resistance at standstill of the high-torque motor must, therefore, be 167 per cent of that of the normal-torque motor. If the rotor running resistance were the same in the two motors, the high-torque motor would have a very high value of rotor resistance ratio, *i. e.*, $\left(\frac{\text{A-C. rotor resistance}}{\text{D-C. rotor resistance}}\right)$ and the maximum torque would be prohibitively low due to the attendant high incremental reactance. In order to keep the maximum torque above 200 per cent it is therefore necessary in the high-torque motor to have the slip at full load (*i. e.*, the rotor running resistance) higher than in the normal-torque motor. This distinction may be seen in Fig. 1.

It follows that the efficiency of the high-torque motor is lower than that of the normal-torque motor. This difference is from 1 per cent to 2 per cent at full load in sizes from 5 hp. to 30 hp.

The high-torque motor also has a lower power factor than the normal-torque motor. This is partly due to the fact that the incremental reactance is higher. The effect of this is shown in Fig. 1, the maximum torque of the high-torque motor being below that of the normal-torque motor. In many cases the total flux per pole of the high-torque motor must be higher than that of the corresponding normal-torque motor. This entails a further depreciation in power factor.

The running characteristics of the high-torque motor are consistently inferior to those of the normal-torque motor. It is therefore to the purchaser's advantage to use this motor only in cases which actually require the high torque developed.

*Normal-Torque High-Resistance Motors*

The high-resistance type of motor is distinguished from the high-reactance type by the fact that in the former there is no eddy current effect introduced in the rotor at standstill.

Limitation of starting current is obtained by either reducing the total magnetic flux per pole or increasing the horsepower rating of the frame in question.

The required starting torque is obtained by increasing the rotor resistance and since eddy-current effects are not used the rotor resistance is not altered in going from standstill to full speed. This type of motor has, therefore, an inherently high slip and low efficiency at full load. The speed torque curve of such a motor shown in Fig. 6 indicates a motor very similar to those special high slip motors designed for driving punch presses and other flywheel loads.

The high-resistance type of motor has an advantage in that the full load power factor is quite high but the full load efficiency is correspondingly low and becomes rapidly lower at overloads.

The low efficiency of the high resistance type of

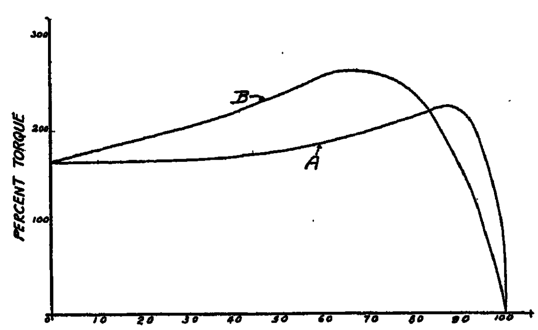


FIG. 6

*a.* Torque curve—Normal-torque high-reactance motor
*b* Torque curve—Normal-torque high-resistance motor

motor indicates that this motor will operate at a higher temperature rise than a motor of the high reactance type.

The average accelerating torque is higher than that of a high-reactance type motor having the same starting torque as shown in Fig. 6 where torque curves of the two types are superimposed.

High-torque line-start motors of the high-resistance type are beyond the range of practicability. The high value of rotor resistance which is required results in an objectionably low efficiency at full load. This class of motor is suitable only for application to a-c. elevators, skip hoists, or other installations where the duty is principally one of acceleration.

### Testing of Line-Start Motors

The testing of line-start induction motors and the subsequent use of these tests in preparing trustworthy guarantees necessitates precautions not necessary in the testing of standard squirrel-cage motors.

In preparing guarantees for standard squirrel-cage motors, it is common practise to compute the power factor and the conventional efficiency from data obtained from running light and blocked rotor tests. The computation may be done by the use of either the exact equivalent circuit or the circle diagram.

In preparing guarantees for line-start motors of the high-reactance type, however, the leakage reactance obtained from the blocked rotor test is not suitable for use in calculations at full load. It is too low by the amount of the incremental reactance, and power factor guarantees computed from it will be higher than the values actually realized in the motor.

Power factor guarantees of line-start motors should, therefore, be based primarily on carefully conducted wattmeter tests with the motor connected to a dynamometer.

An alternative method of arriving at the power factor consists in calculating the incremental reactance and adding it to the test value of standstill reactance in order to obtain the value of running reactance needed in the equivalent circuit or circle diagram. This method should be considered acceptable only when the individual designer has perfected the technique of calculating the incremental reactance to the point where wattmeter tests can be consistently checked by calculation.

Next to an interest in trustworthy power factor and efficiency guarantees the purchaser of a line-start motor should be particularly interested in how the starting current is measured. If the starting current is extrapolated from a measurement made at a reduced voltage the published value of the starting current may be from 10 per cent to 15 per cent lower than the actual starting current taken by the motor when connected to the lines. Any misunderstanding over this point could be avoided by a uniform practise among motor manufacturers of guaranteeing starting current values actually measured at full voltage.

### Conclusions

The principal points brought out in this paper may be summarized:

(1) The field for the line-start general purpose induction motor is large and is steadily expanding.

(2) The line-start motor has necessarily a lower maximum running torque than the standard squirrel-cage motor, the maximum torque being roughly inversely proportional to the starting current.

(3) Because of this lower maximum torque it is not advisable to build line-start motors for general purpose use with a starting current below the equivalent of six amperes per hp. on 440-volt 60-cycle circuits.

(4) Motors in sizes above 30 hp. built to meet the N. E. L. A. rules are not suitable for general purpose applications owing to the low maximum running torque which they develop.

(5) Two classes of line-start motors are available, normal-torque and high-torque motors. The normal-torque motor is for general purpose application. The high-torque motor is intended for a special type of service and its use involves a sacrifice in running characteristics.

(6) From the design point of view there are two types of line-start motors, high reactance and high resistance. As a whole the characteristics of the high-reactance type are superior.

(7) The running characteristics of normal-torque line-start motors compare favorably with those of standard squirrel-cage motors.

(8) Special care is needed in preparing guarantees for line-start high-reactance motors from no load test data due to the change in leakage reactance with speed.

(9) Starting current values should be measured at full voltage rather than increased proportionally from readings taken at reduced voltage.

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## Discussion

**F. E. Harrell:** Although I believe it was not stated bluntly, we can certainly infer from the statements made in this paper, that along with a great many others, Mr. Koch recognizes the fact that regardless of what type of N. E. L. A. motors we consider, the user is penalized in some respect in the performance of that machine due to the present low values of the N. E. L. A. permissible starting currents.

In this connection, I wonder if Mr. Koch can tell us whether the high-reactance type or the double-squirrel-cage motor with the minimum starting current which he mentions of six amperes per hp. for a 440-volt, 60-cycle supply, whether or not that design inherently prevents the speed-torque curve from having a drooping characteristic between the starting point and the maximum torque point.

It has been my general understanding that this one characteristic, namely, that of having some point between starting and maximum at which a torque lower than that developed at starting prevails, has been the thing which has prevented the more general use of this motor, both on the Continent and in America, although it was brought out over there twenty years ago.

It also occurs to me that the name "Resistance Type N. E. L. A. Motor" might cause it to be confused with the punch-press or flywheel load motors which have a high slip, up to 10 or perhaps as high as 15 or 18 per cent, which motors, of course, are practically always for intermittent duty and not for continuous duty. As a matter of fact, I believe that it is just as possible to design a good single-cage N. E. L. A. motor as to design a good double-cage N. E. L. A. motor. The thing which perhaps bears out Mr. Koch's conclusion that as a whole the characteristics of the double cage make it the most desirable type, undoubtedly involves the cost factor, or the manufacturing cost, because it is appreciated that it is entirely possible to meet a given starting current with given performance, in a single-cage motor by increasing the frame size, if not in overall dimensions, then by increasing the $D^2 L$.

There remains, of course, much to be said in favor of the single-cage or so-called high-resistance type of N. E. L. A. normal-torque motor which was not emphasized in Mr. Koch's paper, principally, the higher maximum torque and much higher power factor—in some cases as much as 5 per cent. Also, that existing types of single-cage N. E. L. A. motors, while they do have more slip than standard motors, do not have anything like the slip referred to as placing them in the class with punch-press motors for flywheel loads.

In conclusion, may I say that it is my opinion that Mr. Koch's conclusion that as a whole the characteristics of the high-reactance type are superior, is certainly a highly debatable one, with the balance of argument which might favor the double cage resting principally in the economy of manufacturing.

**J. C. Lincoln:** In the sixth page of the paper is the following statement: "It might be thought that the maximum torque would be unaffected by saturation since any saturation which occurs at starting would also occur at the maximum torque point of the torque curve, the current at these two points being not widely different. There are two reasons, however, why this is not true."

I can't reconcile that statement with Curve $C$ on the third page. The Curve $E$ gives the current of the normal torque, and the high-torque high-reactance motor. Curve $E$ has a maximum value at the start and it rather rapidly decreases to synchronism. At the start, therefore, we have a maximum current, and according to Curve $C$ showing the torque for the high-reactance high-resistance motor, the maximum torque occurs quite near synchronism when the current is quite small.

There is another question that I should like to ask Mr. Koch. The feature of a curve, as Curve $C$, showing a smaller torque between starting and maximum is rather general in double-cage motors, but what is the condition which produces a starting torque greater than the maximum torque?

**H. C. Specht:** In this paper I missed something and that is in regard to the temperature of the squirrel-cage winding. I believe if the temperature rises very rapidly, which would be the case in only the upper member where the section of the conductor is smaller, that would increase the resistance and drive some of the current down to the lower part of the squirrel-cage winding and thus give somewhat less torque. It seems from the curves that this is probably the case.

**P. L. Alger:** Mr. Koch's paper gives in a clear way the fundamental facts about the line start motor: that the per cent starting current is necessarily a little more than the per cent maximum torque (as Baffrey[1] has stated), $I_s$ equals approximately

$$\sqrt{4\,T_m^2 + T_s^2}$$

and that the full-load characteristics are necessarily inferior to the normal squirrel-cage motor. Briefly, if a standard squirrel-cage motor has a starting current $I$, a starting torque $T_s$, a maximum running torque $T_m$, occurring at 75 per cent of synchronous speed, and a maximum power factor, $P$, the most favorable results obtainable in a line start motor of the same efficiency and same dimensions, and having a starting current $k\,I$, are the same starting torque, a maximum torque $k^2\,T_m$, and a maximum power factor $1 - k^2(1 - P)$. If the standard motor has a per unit slip, $s$, at maximum torque less than $\frac{1}{4}$, the line-start motor can have a greater starting torque, without sacrifice of efficiency, equal to approximately

$$T_s\,[1 + k^2\,(1/2\,s - 2)\,]$$

These relations are only approximate, but they are useful in an appraisal of the relative merits of the motors.

Mr. Koch has thus given us full comparative data on standard and line-start motors. The question we should answer is "What is the proper field for each?" Present practise is, generally, to use standard squirrel-cage motors with full voltage starting in the smallest sizes, line-start motors in the next larger sizes, standard squirrel-cage motors with starting compensators in still larger, and slip-ring motors in the largest, sizes. The division points now come at perhaps 7½, 30, and 300 hp., respectively, in United States practise.

If starting-current limitations could be altogether removed, the line-start motor would probably disappear, while if these limitations are lightened, the median size of line-start motor will increase. It is quite certain that present practise is not ultimately the best, and that the line-start motor must either usurp more and more of the field, or that it will serve simply as a means to the end of introducing full-voltage starting of standard motors, and will ultimately disappear. My own opinion is that the starting-current restrictions can and will be lightened, and that full-voltage starting of standard motors will be used in continually larger, and line-start motors in still larger, sizes.

**C. W. Kincaid:** The design of these motors is dependent mainly on the locked current allowed by the operating company. Since these are not all the same, values as given by the N. E. L. A. are generally used as a basis.

From Table I, it can be seen that the amperes per horsepower allowed for the various ratings, decrease from 10 amperes at 3 hp. to 4 amperes at 50 hp.

Due to relatively large currents allowed on the small ratings, the standard design can be used. At 6 amperes per hp., a high-reactance motor is necessary, which can be of the deep-bar design if the required starting torque is low, but which must be double-deck if high starting torque is required. Between these limits all kinds will be used to suit requirements.

The maximum torque, which is directly dependent on the locked current, becomes too low for general-purpose motors when the current allowed is below 6 amperes per hp., and in consequence motors above 30 hp. are not made to meet N. E. L. A. current values. If the N. E. L. A. values for ratings above 30

1. *Elektrotech. u. Maschinenbau*, July 8, 1928, pp. 749-754.

hp. were made 6 amperes per hp. instead of 4 amperes per hp. above 40 hp., there would be no limit in the size of the motor that can be built as a line-start motor.

**A. M. MacCutcheon:** As has been said, in the case of the smaller motors it is relatively easy to meet the N. E. L. A. values with good performance and a suitable starting torque, but as you get up to the larger motors the double-cage motor seems to be at the present time the only answer. But is it correct that the industry should stop in the consideration of the current values as established by the N. E. L. A. and say that is a fixed thing?

Many plants that have their own power systems, or are tied in on the primary side of the distributing system and have their own secondary distribution system for the last five or ten years, have of their own initiative been throwing on to the line the standard induction motor. Mr. R. S. White, of Madawaska, Maine, told me he has been throwing motors as high as 2400 hp. on the line.

I have wondered whether the application of electric motors was not going to be furthered greatly if the power companies would again review this question. They have in the last few years reviewed it and revised it upward. I really believe it should be reviewed again with continued experience and taken upwards still further.

It has been said in the paper that the cost is reduced, but there has been no emphasis particularly placed on the fact that troubles are reduced, that is, provided the motor is so constructed that no trouble is experienced with the motor in so doing. From the users I hear that they have had far, far more trouble with compensators than they have had with squirrel-cage motors.

In the consideration by the power companies of this question of what should be done with motors when thrown on the line and what current should be allowed, I wonder if they have ever considered that they should classify in the plant of the user the type of motor used. If the motor is to be started every five or ten minutes, it seems to me that a very different current should be allowed on that motor from that on a motor which is started once a day or once a week. I think possibly that the power company should consider it from that light and make a classification as to how often a motor is to start, and if it is frequent, I think there should be a lower current value than for those motors which are started seldom. If the limits of current are extended upward, we will then have the simplicity of the straight squirrel-cage motor. I do not think that the double-cage motor is much more complicated, but it is somewhat, and I think it has a somewhat higher cost, but unless there are real good fundamental reasons why we should keep to these present N. E. L. A. values, I think the question should be opened up again.

**C. A. Adams:** I should like to support what Mr. MacCutcheon said about the revision of the N. E. L. A. allowance upwards.

It just happens that I have been recently operating a single-phase resistance welder connected to a three-phase system where we have loads lasting for a second or two as high as 700 or 800 kv-a. from one phase of the three-phase system, and there is no complaint of system disturbance.

The utility company permits us to take 500 kv-a. regularly. Under these circumstances, it seems to me absurd to retain the present low limits.

Referring now to the speed-torque curves, there are three points worthy of note. First, it is almost impossible, experimentally, to get the points on the unstable or undersides of these curves, since owing to the instability, the rotor is almost invariably either accelerating or decelerating. Second, the reason for the peculiar shape of the curve is fairly obvious. At very low speeds you have a very high resistance and low reactance in the secondary. As the speed increases, the reactance goes up rapidly and the resistance down until you fall on to the high reactance curve which carries you into the normal operating region. Third, what Mr. Specht said about the heating of the secondary is true if the two secondaries are separate. If in this case the speed is held for any length of time at a low point, there is bound to be a considerable increase in resistance, but the amount of this increase depends upon the nature of the material. If the material is copper, the increase in resistance might be considerable, but if it is made of some high-resistance alloy, the increase will be very much less. If, on the other hand, observations of these lower points are made rapidly, there will be little time for change of temperature and resistance. If, however, there is much change in the resistance due to slowness in making the observations, the starting torque, or torque at the low points of the curve, will obviously be affected thereby.

**C. W. Franklin:** With respect to starting currents, I feel most of the power companies are perfectly willing to allow starting currents up to any value that will not cause objection from customers. That value is variable depending largely on the type of system. We have in New York very large motors started across the line. We also have some 7½-hp. motors that cause appreciable lighting dips when they are fed from combined lighting and power secondaries. The large motor does not give trouble. The 7½-hp. does at times. That is the problem.

In the larger cities there is getting to be a general amount of combining of lighting and power secondaries and we are likely to get motors of many types and classes. Of course, most of them are rather small, but some get up as high as 300 or 400-hp., for cooling systems.

In order to prevent flicker trouble on lighting we have come to the conclusion that increment starting current is the only way. For instance, the lighting at the Roxy Theater requires 320 amperes per phase to start, requiring a final voltage drop of about 7 volts. These 7 volts if applied gradually do not cause any noticeable flicker in light. On the other hand, if we had a compensator slip motor on the line start which would take more amperes you would get a noticeable flicker.

So there are some cases where increment start is desired. That has meant resistance starters to date.

With respect to Mr. Koch's paper, we are highly in accord with the thought that the starting current should be measured with blocked rotor for a final proposition. Any other method may lead to argument. So we, and the N. E. L. A., I think, have set up blocked rotor values so far as the line-starting motor goes. I think it is an excellent way to determine the maximum starting current.

**L. L. Elden:** The paper on "line-start" motors will be most helpful in assisting those interested in power applications to a clearer understanding of the design and operating characteristics of this particular class of motors.

Written as it is to present the manufacturers' point of view, it gives only limited consideration to the commercial aspects of power supply which more intimately affect the utilities and their customers.

In late years the term "line-start" motors has been applied only to the newer types of motors described in the paper. These have been expected to effect a substantial reduction in the investment required for a given power installation, through the elimination of compensators or other starting devices.

The practise of starting motors across the line is not new. Standard general-purpose motors of all sizes have been so operated for many years in locations where conditions permit the use of liberal starting currents. As the term "line-start" is used, it is not clear why it should not generally apply to any motor which may be started across the line.

Up to within a very recent period, the manufacturers have emphasized the merits of standard squirrel-cage motors for general use, indicating that by concentrating large-scale production on a standardized product, substantial reductions in the cost of such motors could be effected. The introduction of numerous types of "line-start" motors appears to controvert

that theory since these motors have materially complicated the problems of power supply and the choice of motors for a given service. In the event that the production ratios of the two classes of motors are substantially modified, it will be logical to expect a revision of prices upward for squirrel-cage motors.

If on the other hand the relatively high cost of developing, producing, and marketing "line-start" motors in limited quantities was eliminated and an increase in the production of standard squirrel-cage motors thereby secured, it would be reasonable to expect a reduction in the cost of the latter which might even offset the anticipated savings to be secured by the omission of compensators.

Further, with no gain in efficiency or power factor and with an actual reduction in starting and running torque under given conditions, these new-type motors appear to offer only a possible reduction in costs and lower starting currents as a justification for their existence.

Without questioning the merits of the "line-start" motors for use in large power installations and in locations where limitations in starting current are unnecessary, the fact remains that such cases are far in the minority, when compared with the vast number of individual power applications where more or less strict limitations of starting currents must be enforced to conform to existing conditions in local supply systems. In such locations recourse must always be had to compensators or other starting devices, or to certain types of motors having low starting currents in order to preserve satisfactory service conditions.

Long experience in dealing with distribution facilities necessary to serve miscellaneous power applications is convincing to the end that the "line-start" motor has its true application only in large power installations where advantage may be taken of liberal starting currents for individual motors, none of which may be out of proportion to the maximum demands of the entire installation. Under such conditions it may well be that the standard squirrel-cage types of motors would and do perform equally well and with starting currents within permissible limits.

However, where advantage can be taken of such conditions the elimination of the compensator may effect a welcome reduction in costs although the added cost of the heavier wiring required must not be overlooked in weighing the final results.

The author has indicated that the motors described depend materially upon the maintenance of normal voltage and frequency for their best performance and even then their torque may be unfavorably affected. It would be helpful if definite statements could be added to describe the performance of these motors with their limited torque characteristics on network systems when the voltage is 10 per cent or more below rated motor voltage.

Reference is made to the N. E. L. A. motor rules in a number of places in the paper. The statements which appear on the second page in this connection appear to require some modification in order to conform with the facts in the case.

The so-called 1923 N. E. L. A. rules were never authorized and have never been approved by the Association. The record clearly shows that the members of the Sub-committee were unable to reach agreement upon any revision of the 1915 rules then in force, and that the rules prepared in 1923 were merely submitted for discussion. The 1923 rules were never acted upon, owing to lack of approval within the Association.

It having come to the notice of the N. E. L. A. that certain motor manufacturers were in some cases utilizing the unauthorized 1923 rules as one of the important factors controlling the design of new motors, official notice was sent informing the National Electric Manufacturer's Association of the true status of the rules in question. In addition individuals within the association have similarly advised various manufacturers to the same effect from time to time. It would appear that no notice has been taken of these advices.

Recent inquiry within the Association does not support the statement that "these 1923 recommendations are recognized as good practise by many power companies." The contrary appears to be the case since there is no evidence of any use of the 1923 rules or recommendations by any interest other than motor manufacturers.

The only rules that appear to be used in utility operations to any extent are those authorized in 1915. Even these rules have been used by only a limited number of companies, most companies preferring to enforce rules particularly suited to their own local conditions and in most cases limiting starting currents to even lower values than the 1915 rules.

Since the 1915 rules in most cases limit starting currents to values materially below those proposed in 1923, serious difficulties have arisen in many cases between utilities and their customers, when service has been refused for motors until their starting currents have been brought within acceptable limits. In these cases motor representatives invariably refer to the so-called 1923 rules as their understanding of N. E. L. A. requirements.

In view of the wide publicity now given to the unauthorized and unapproved N. E. L. A. recommendations of 1923 and the use which has been made of them in developing new motor designs, it would appear most desirable to modify the text of the paper where the rules are referred to, to convey a true picture of the situation first-hand in the final printing without depending upon a reading of the discussion to disclose the facts.

**W. A. Naudain:** (communicated after adjournment) The N. E. L. A. rules used by the power companies, which seem to govern the adaptation of this type of motor, do not fill the present needs.

The purpose of these rules is to eliminate fluxation of voltage, and the resultant complaints from other customers on the system. The line fluxation is governed more by the available power supply, and location of the equipment on the system than by the actual values of starting current drawn by connected apparatus. This being true, it appears that this type of motor, in rather large sizes, might be used in metropolitan areas where there is great concentration of load.

With these points in view, and the line-start current allowable being predicated on the specific location and application of the motor, it seems that these rules should be, to a large extent, nullified, allowing the motor designer to design motors having other desirable characteristics, and permitting the ultimate user to benefit.

**C. J. Koch:** Mr. Harrel and Mr. Lincoln have raised questions regarding the shape of the torque curve of line-start motors.

The shape of the speed-torque curve of the high-reactance type of line-start motor may be varied by changing the relative magnitudes of the resistances and reactances of the bars constituting the double squirrel cage. Thus curves *C* and *D*, Fig. 1, are from two motors having identical stators, the shape of the rotor slot, however, being different in the two cases.

If the double squirrel-cage slot is proportioned to give approximately 150 per cent of full-load torque at starting, for motors of about 50 hp. and below in size, the torque of the motor will increase as the motor comes up to speed as shown by Curve *D*, Fig. 1, reaching a maximum value somewhat above 200 per cent.

If, however, the proportions of the double squirrel cage are changed to give about 250 per cent starting torque as shown by Curve *C*, Fig. 1, the torque of the motor decreases at first as it comes up to speed since the starting torque is now higher than the torque at the maximum torque point. This droop which occurs in the torque curve of the high-torque high-reactance motor is not serious, however, since the motor is designed to have a high starting torque and it should not be forgotten that the speed-torque curve of the load must pass through 100 per cent at full-load speed. I do not recall any instances where a high-reactance motor failed to being its load up to speed after having

once started it. There have been cases where it may have failed to come up to speed due to harmonics but that is a different matter.

Regarding the question raised by Mr. Lincoln of the value of current taken by a motor from the line when passing through the maximum torque point of the torque curve: The current at maximum torque is theoretically about 70 per cent of the starting current. Actually it is less than this due to the increase of starting current due to saturation. Any amount by which it is less than 70 per cent of the starting-current value strengthens rather than vitiates the argument referred to in the paper. Throughout this paper by the term "maximum torque" is meant the highest torque which a motor will carry without suffering a great reduction in speed. Thus in Fig. 1 Curve *C* "maximum torque" occurs at about 85 per cent of full-load speed although the torque at this speed is actually less than the starting torque.

Regarding the effect of increase in temperature on the characteristics of the double-squirrel-cage motor, I do not believe that very large differences of temperature can exist between the top and bottom bars with the cast aluminum type of construction. It has been found, for example, that the starting torque of this type of motor is practically independent of the temperature of the rotor. Increased temperature certainly decreases the eddy-current effect but the increased resistivity normally means more starting torque, so that the one effect balances the other and the starting torque remains practically constant.

Mr. Elden's discussion is most timely and interesting. It was not the intention of this paper to create the impression that the "1923 recommendations" had ever found sanction by the N. E. L. A. or that there was any obligation on the part of the power companies to recognize these recommendations. However, the position which the motor manufacturers have taken in designing "line-start" motors to meet these recommendations is not unjustified. From this paper it will be appreciated that line-start motors designed to have starting currents any lower than the 1923 recommendations would not have desirable running characteristics, particularly overload capacity. The fact that there is a very considerable demand for motors having starting currents at the general level of the "1923 recommendations" indicates that there are many places in this country where the power company will permit the starting of such motors without a compensator. In order to meet this demand, motor manufacturers have designed line-start motors to meet the "1923 recommendations" not because there is any compulsion operating upon the power companies to accept such motors on their lines but because these recommendations comprise the best opinion we have of what starting currents, at a higher level than those prescribed in the 1915 rules, will be acceptable in some places if acceptable at all. The 1923 recommendations therefore have grown to be a standard on which the various motor manufacturers in this country can compete on the same basis in the design of line-start motors. Some standard of this kind is, of course, necessary. The production of line-start motors has burdened the motor manufacturer with an increase in the variety of his product but this increase would be very much more if manufacturers attempted to produce a number of individual groups of motors, each one representing the best balance of power factor, efficiency, and starting current for a particular locality.

# No-Load Induction Motor Core Losses

BY T. SPOONER[1] and C. W. KINCAID[1]

Member, A. I. E. E. Associate, A. I. E. E.

*Synopsis.—A detailed method of calculating the no-load core losses of induction motors is presented whereby each loss component is estimated separately. There is included a discussion of extra losses due to imperfection in workmanship. Calculated and test results on a few commercial machines are given as an indication of the reliability of this method.*

In general the simplest and most reliable method of predicting the core losses in a rotating machine of new design is to estimate them from the test results on machines of similar type. This is the method ordinarily used by experienced designers. This procedure is not adequate, however, in some cases. For instance, if the core losses in a certain type of machine are higher than anticipated, a more detailed analysis is necessary, or if a machine is proposed which has certain radically different features from any existing machine, a calculation of losses, based on fundamentals, is essential if these losses are to be properly estimated. It is our purpose here to consider all of the important factors which produce no-load core losses in induction motors and similar types of apparatus and to give a method of calculating these losses. This method has been used tentatively for several years for special cases and has been found to check the test results fairly well with, of course, occasional exceptions. These exceptions were due, it is felt, more often to errors of test or carelessness in shop construction than to inaccuracies in the method of calculation.

This method is presented primarily for the purpose of stimulating discussion and to obtain suggestions for improving and simplifying the calculations. There are certain weaknesses and elements of uncertainty in the method which we shall point out. Some details of the process have already been presented before the A. I. E. E. Only such portions of these previous papers will be repeated as are necessary to make the calculations and formulas complete.

## Types of Loss

This discussion will be confined to induction motors but it will be obvious that the methods can be applied to other types of machines in which both the stator and rotor are slotted and in which the air-gap flux has approximately a sine-wave distribution.

For the purpose of calculation, induction motor losses will be segregated into the following components:

1. Fundamental-Frequency Losses.
    a. Stator core
    b. Stator tooth
2. Pulsation Losses.
    a. Surface (stator and rotor)
    b. Tooth pulsation (stator and rotor)
    c. Copper eddy (stator and rotor)
3. Illegitimate Losses.

The fundamental-frequency losses are those due to the hysteresis and eddy-current losses in the core material corresponding to the applied frequency and are assumed to be the same as would occur under alternating flux, as in a transformer core, for the same maximum induction. This assumption may be open to question as far as the core flux is concerned since this flux has an elliptical rather than an alternating variation but the simpler assumption with corrections, if necessary, for the elliptical field seems to give satisfactory results. It is assumed that the frequency of the main flux in the rotor is so low that the losses are negligible.

The pulsation losses consist first, of the surface losses which are those hysteresis and eddy losses occurring just below the surface of the tooth tops due to the passage of the slots of the other member and are practically the same as the well known pole-face losses.

The tooth pulsation losses are those caused by the high-frequency pulsations of flux extending the whole length of the teeth and a little way into the core due to the reluctance changes in the air-gap as the slots of one member pass the teeth of the other. The distinction between surface and tooth-pulsation losses is somewhat arbitrary but seems to work fairly well.

The no-load copper eddy-current losses are, of course, not really core losses but appear as such by the ordinary methods of test. They are due to the high-frequency slot-leakage fluxes as the result of the momentary changes in the saturation of the teeth. They have both tangential and radial components.

The illegitimate losses are those caused by punching strains in the tooth iron, short-circuited laminations due to burrs and slot filing, bending strains in the iron sheets, and finally to leakage fluxes into the frames, end-bells, and other solid members.

## Calculation of Losses

The curves for the calculation of the iron losses will all be based on the specific losses occurring in a good quality of electrical sheet having a silicon content of approximately 1 per cent. For better grades of steel the calculated losses must be decreased accordingly. For instance, if a steel is used having a silicon content

1. Both of Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

*Presented at Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

of about 2½ per cent the iron losses, other things being equal, will be about 70 per cent of these for the sheet with the lower silicon content. This applies both to the fundamental-frequency and to the pulsation losses. For the same flux density, the copper eddy losses will be higher for the higher silicon material due to the fact that saturation occurs at a lower tooth induction and, therefore, the slot-leakage fluxes are greater.

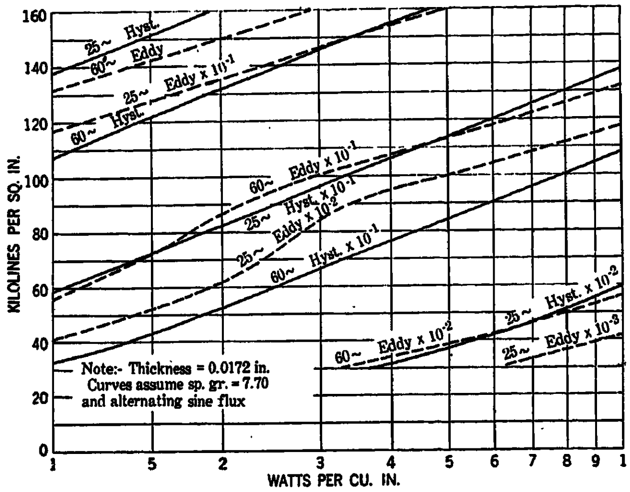


FIG 1—FUNDAMENTAL-FREQUENCY CORE LOSSES AT 25 AND 60 CYCLES. ONE PER CENT SILICON STEEL

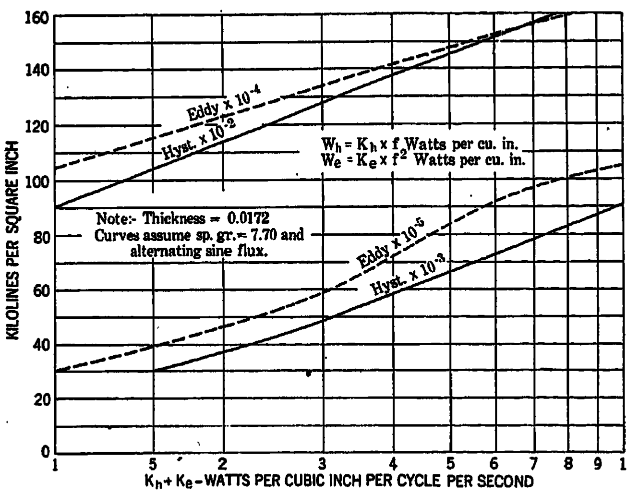


FIG. 2—FUNDAMENTAL-FREQUENCY CORE LOSSES FOR ANY COMMERCIAL FREQUENCY IN ONE PER CENT SILICON STEEL

## FUNDAMENTAL-FREQUENCY STATOR CORE LOSSES

The maximum induction in the core is to be calculated in the usual way by dividing one-half the flux per pole by the core cross-section corrected for space factor, namely: multiply the gross core cross-section by a factor of approximately 0.9 or greater, depending on the smoothness of the surface, thickness of insulation, and density of the particular core material which is used. The specific loss of the core material for the particular flux density as determined from standard core-loss curves for the sheet in question is multiplied by the net volume of the core (gross volume multiplied by space factor); this gives the total fundamental frequency iron losses for the core assuming no illegitimate losses and uniform flux density over the core section.

If the ratio of outside to inside radius of the core is not too large and if the number of poles is not too great, this procedure will give approximately correct losses except as there may be short-circuited laminations or leakage fluxes into the frames due to saturation of the laminated material or other causes to be discussed later.

Some years ago Alger and Eksergian[2] gave a method of calculating the increased iron losses due to non-uniform flux distribution in the core. In order to made these corrections, the hysteresis and eddy losses must be calculated separately. This may be done by the aid of Fig. 1 or 2, the former to be used for 25 or 60 cycles and the latter for other frequencies. In case Fig. 2 is used, multiply the value $K_h$ corresponding to the maximum induction by the frequency and for the eddy loss multiply the value of $K_e$ by $f^2$. Now refer to Figs. 3

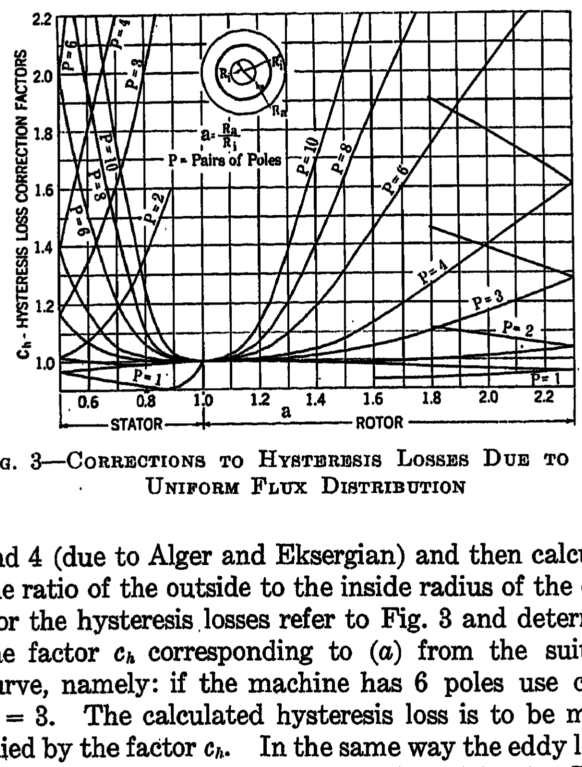


FIG. 3—CORRECTIONS TO HYSTERESIS LOSSES DUE TO NON-UNIFORM FLUX DISTRIBUTION

and 4 (due to Alger and Eksergian) and then calculate the ratio of the outside to the inside radius of the core. For the hysteresis losses refer to Fig. 3 and determine the factor $c_h$ corresponding to ($a$) from the suitable curve, namely: if the machine has 6 poles use curve $p = 3$. The calculated hysteresis loss is to be multiplied by the factor $c_h$. In the same way the eddy losses are to be multiplied by $c_e$ taken from Fig. 4. These curves were calculated on a basis of constant permeability. Due to the variations of $\mu$ with induction, the flux distribution will be different than assumed with a corresponding error in the corrections. The calculated eddy losses will be too small, and the calculated hysteresis losses too large.

## FUNDAMENTAL-FREQUENCY STATOR TOOTH LOSSES

The stator teeth of an induction motor are subjected to approximately a sine-wave variation of fundamental-

2. *Induction Motor Core Losses*, P. L. Alger and R. Eksergian, JL. A. I. E. E., October 1920, p. 906.

frequency flux on which generally is superposed a high-frequency ripple corresponding to the number of slots in the rotor. It has often been assumed that these high-frequency pulsations tend to reduce the fundamental-frequency hysteresis losses. It has been shown,[3] however, that this is not the case and that the fundamental-frequency hysteresis losses are proportional to the maximum flux density whether or not the high-frequency pulsations are superposed.

Referring to Fig. 5, $B$ is the maximum magnetic flux density in the teeth as ordinarily assumed and $B_m$ is the actual maximum flux density. It is necessary, therefore, to calculate the magnitude of these high-frequency flux pulsations in the teeth. This can be done by means of the formulas given in a previous A. I. E. E. paper.[4]

$P_s$ is the stator tooth pulsation in per cent of the maximum tooth flux

$P_{r1}$, $P_{r2}$, $P_{r3}$, and $P_{r4}$ are the rotor tooth pulsations for four cases

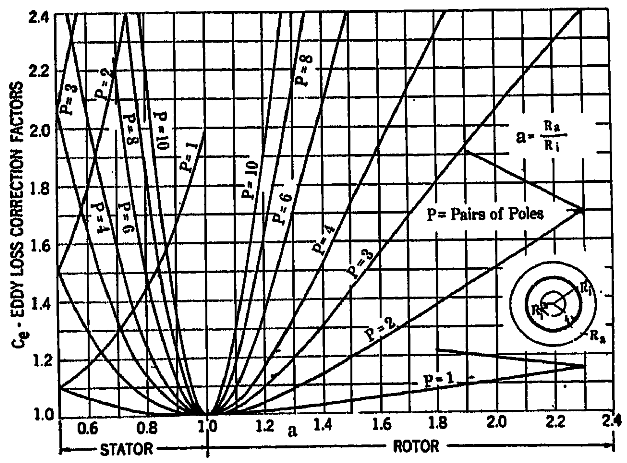


Fig. 4—Corrections to Eddy-Current Losses Due to Non-Uniform Flux Distribution

$\lambda_s$ is the stator slot pitch

$\lambda_r$ is the rotor slot pitch

$t_{es}$ is the effective stator tooth width

$t_{er}$ is the effective rotor tooth width.

Cases 1 and 2 are to be used when the rotor slot pitch is less than the stator slot pitch. Cases 3 and 4 are to be used when the rotor slot pitch is greater than the stator slot pitch.

$$P_s = \frac{\lambda_r - t_{er}}{t_{es}} 100 \qquad (1)$$

$$P_{r1} = \frac{\lambda_s - t_{es}}{t_{er}} 100 \quad \begin{cases} P_{r1} \text{ cannot exceed } 100 \\ t_{es} \text{ is greater than } t_{er} \\ \qquad (2) \qquad \text{Case (1)} \end{cases}$$

$$P_{r2} = \frac{\lambda_s - t_{er}}{t_{es}} 100 \quad \begin{cases} t_{es} \text{ is less than } t_{er} \\ P_{r2} \text{ has a positive sign} \\ \qquad (3) \qquad \text{Case (2)} \end{cases}$$

$$P_{r3} = \frac{t_{er} - \lambda_s}{t_{er} - \lambda_s + t_{es}} 100 \quad \begin{cases} t_{es} \text{ is less than } t_{er} - \lambda_s + t_{es} \\ t_{er} \text{ is less than } 2\,\lambda_s - t_{es} \\ \qquad (4) \qquad \text{Case (3)} \end{cases}$$

$$P_{r4} = \frac{\lambda_s - t_{es}}{t_{er} - \lambda_s + t_{es}} 100 \quad \begin{cases} t_{er} \text{ is greater than } 2\,\lambda_s - t_{es} \\ t_{er} \text{ is less than } \lambda_s + t_{es} \\ \qquad (5) \qquad \text{Case (4)} \end{cases}$$

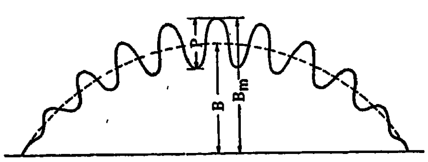


Fig. 5—Showing Tooth Flux Variations for Induction Motor

The effective tooth widths $t_e$ can be determined from Fig. 6. $B$ should be calculated on the basis of net section sine-wave flux distribution and for a tooth cross-section one-third of the distance from the smallest section. These formulas are based on the assumption that the teeth have zero reluctance. Above 70 to 80 kilolines saturation effects begin to be appreciable, namely: the tooth reluctance cannot be neglected. This tooth reluctance causes the percentage pulsations to decrease. Based on theoretical considerations, correction curves have been calculated to take care of the saturation effects. Assuming a pulsating wave having a maximum displacement $B$, and assuming an average reluctance for the induction-motor stator and

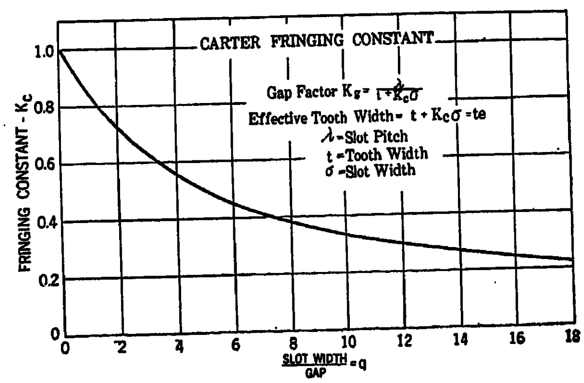


Fig. 6—Carter Fringing Constant

rotor teeth and an average length of air-gap, Fig. 7 has been calculated. This gives the percentage, as a result of saturation, of the calculated percentage pulsation as obtained by the above formulas plotted against $B$. For the case of the induction motor, $B$ is the maximum of the fundamental-frequency of the tooth induction. If there is a considerable departure from the assumed air-gap and tooth reluctances the curve is not very greatly shifted. A smaller relative tooth reluctance due to shorter teeth or a longer air-gap shifts the curve upward.

3. "Effect of Superposed Alternating Field on Apparent Magnetic Permeability and Hysteresis Loss," T. Spooner, *Phys. Rev.*, Vol. 25, Sect. 2, 1925, page 527.

4. *Tooth Pulsations in Rotating Machines*, T. Spooner, A. I. E. E. Trans., Vol. 43, 1924, p. 252.

To correct the tooth inductions for saturation, then, determine $B$ in the usual way assuming a sine-wave distribution of flux and a net tooth cross-section one-third of the distance from the narrowest part of the tooth. Calculate the percentage tooth pulsation $P$ from the suitable formula. Multiply by the factor $K_s$ determined from Fig. 7. Then the maximum tooth induction is:

$$B_m = B + \frac{B \times P \times K_s \times 10^{-4}}{2} \qquad (6)$$

From this value of $B_m$ and with Fig. 1 or 2 the fundamental frequency tooth losses may be calculated, assuming a suitable space factor in order to determine the net tooth volume.

### Surface Losses

Four variables are involved in the calculation of surface losses assuming that the member for which the losses are to be calculated has no slots. This assumption is approximately correct for rotors with nearly closed slots.

The variables are $B_a$ average air-gap induction in kiloline/sq. in. (flux per pole divided by air-gap area per pole).

$f_r$ or $f_s$ tooth frequency; $f_r$ is calculated from the rev. per min. and the number of rotor slots and $f_s$ from the rev. per min. and number of stator slots. It must be understood that for the calculation of rotor surface losses $f_s$ is to be used and for the calculation of stator surface losses $f_r$ is to be used.

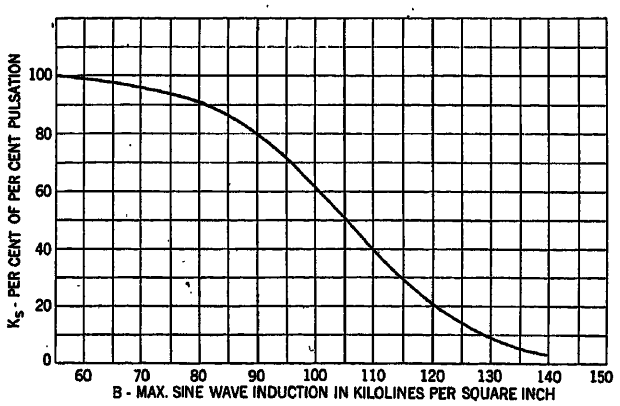


Fig. 7—Tooth-Saturation Correction Factors for Calculating Pulsations

$\sigma_r$ or $\sigma_s$ slot widths in inches for the rotor and stator respectively.

$q_r$ or $q_s$ equals $\frac{\sigma_r}{g}$ or $\frac{\sigma_s}{g}$ ratio of slot width at air-gap to single gap width.

The rotor surface losses per sq. in. of surface are then:

$$W_{sr} = K_{B_a} \times K_{f_s} \times K_{\sigma_s} \times K_{q_s} \qquad (7)$$

where the various $K$ factors are obtained from Fig. 8, the $K$ factors being read from the curves corresponding to the variables $B$, $f$, $\sigma$, and $q$.

The total rotor surface loss equals $W_{rs}$ multiplied by the air-gap area.

If the rotor has partly open slots multiply $B_a$ by the rotor-gap factor $K_g$ as determined from Fig. 6 and the rotor slot and tooth dimensions; also in this case, use the actual area of the rotor tooth tops.

If the rotor slot openings are small, the stator surface losses will be negligible. If the rotor slot openings are fairly wide use $B_a$ as corrected by the rotor gap factor and the formula

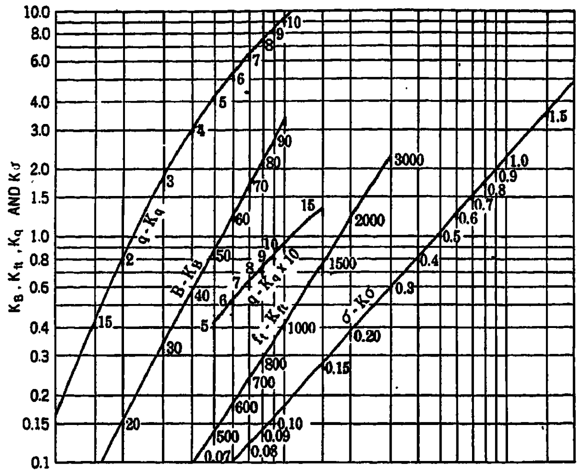


Fig. 8—Curves for Calculating Surface Losses

$W_s = K_B \times K_{ft} \times K_q \times K_\sigma$
$W_s$ = Watts per sq. in. of surface
$B$ = Average air gap induction in kilolines/sq. in.
$ft$ = Tooth frequency in cycles per sec.

$$q = \frac{\sigma}{g} = \frac{\text{Slot width}}{\text{Single gap}}$$

$\sigma$ = Slot width in inches

$$ft = \frac{\sim \times \text{no. of teeth}}{\text{pairs of poles}} = \frac{\text{rev. per min.} \times \text{no. of teeth}}{60}$$

For rotor use no. of stator teeth. For stator use no. of rotor teeth

$$W_{s_r} = K_{B_a} \times K_{fr} \times K_{\sigma r} \times K_{qr} \qquad (8)$$

$W_{s_r}$ is then to be multiplied by the gross area of the stator tooth tops.

The curves of Figs. 1 and 2 are for 1 per cent, 17-mil thick silicon steel. For a medium silicon steel (2½ per cent silicon) reduce the calculated losses by 25 or 30 per cent. For high silicon steel (4 to 5 per cent silicon) use a reduction factor of from 45 to 50 per cent.

### Tooth-Pulsation Losses

The method of calculating the magnitude of the tooth pulsations has already been given. Instead of correcting for saturation by Fig. 7, however, correction for the calculated pulsation $P$, as obtained from the formulas, may be made by Fig. 9 Assuming a straight line relation between induction and percentage pulsation $P$ as effected by saturation, the following formulas may be used instead of Fig. 9.

$$P_S = P\left(1.6 - \frac{B}{100}\right) \qquad (9)$$

Fig. 9 and formula (9) are based on theoretical considerations only. Experimental results on actual machines seem to indicate that the following formula more nearly fits the facts for large induction motors.

$$P_S = P\left(1.3 - \frac{B}{165}\right) \qquad (10)$$

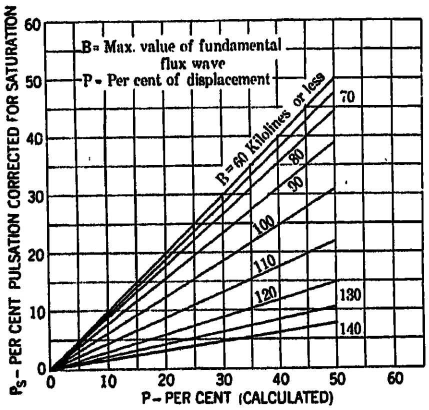


FIG. 9—TOOTH-SATURATION CORRECTION CURVES FOR CALCULATING PULSATION LOSSES

These corrections to $P$ are based on the assumption that the losses vary as the square of the pulsation amplitude. Of course, the effect of saturation increases as the pulsations follow up along the fundamental flux curve toward the maximum value. These saturation effects have been integrated for the whole fundamental wave. The $P_S$ values as determined from Fig. 7 and from Fig. 9 are not very different. If desired, therefore, either value of $P_S$ can be used for calculating fundamental-frequency and tooth-pulsation losses.

The variables for calculating pulsation losses are as follows: $f_{st}$ or $f_{rt}$ as above, where $f_{st}$, for instance, corresponds to the number of stator teeth but is used to calculate the rotor tooth-pulsation losses.

$B_m$, the maximum tooth induction, which for the stator teeth may be taken as the same as used in the calculation of the fundamental frequency tooth losses or more accurately should be recalculated using curve 9 or formula (9) or (10). In the latter case:

$$B_m = B\left(1 + \frac{P_S}{200}\right) \qquad (11)$$

where $B$ is the net tooth induction 1/3 from the narrowest section, $P_S$ is the percentage pulsation as corrected for saturation.

$P\,B_m$ the product of $B_m$ and $P_S$. The tooth-pulsation losses for a wound rotor per net, cu. in. of core material are:

$$W_{rtf} = \underset{\text{(Hysteresis)}}{(10^{-3} f_{st}\, K_{Bm}\, K_P)} + \underset{\text{(Eddy)}}{(K_{PBm}\, K_{fst})} \text{ watts} \qquad (12)$$

The $K$ factors are obtained from Fig. 10, $K_{Bm}$ corresponds to the rotor tooth $B_m$ as obtained from formula (11). $K_P$ is the rotor pulsation constant corresponding to the percentage pulsation as corrected for saturation.

The hysteresis component of this formula was obtained from a fundamental study of the losses of displaced hysteresis loops. The eddy-current component was obtained by many tests on an experimental machine supplied with various slot combinations.

The stator tooth-pulsation losses are calculated as follows when they are appreciable:

$$W_{stf} = (10^{-3} f_{rt}\, K_{Bm}\, K_P) + (K_{PBm}\, K_{frt}) \text{ watts} \qquad (13)$$

If the machine has a squirrel-cage rotor instead of a wound rotor the conditions are different. In this case due to the short-circuited turns around the teeth no very appreciable high-frequency flux can flow. Thus the tooth-frequency losses in the rotor teeth will be negligible. As pointed out by Alger and Weichsel[5] some years ago, however, the high-frequency currents which flow in the rotor bars produce a corresponding

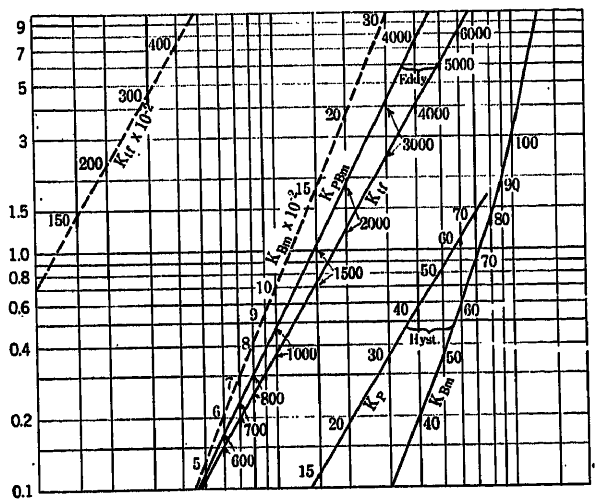


FIG. 10—CURVES FOR CALCULATING TOOTH PULSATION LOSSES

Note: All inductions and vol. are based on net section assuming a space factor of 90 per cent.

$W_{tf}$ = $(10^{-3} f_t\, K_{Bm}\, K_P) + (K_{PBm}\, K_{ft})$
$W_{tf}$ = Tooth frequency loss in watts per cu. in.
$f_t$ = Tooth frequency
$K_{Bm}$ = Max. tooth induction factor ($B_m$ is in kilolines per sq. in.)
$K_P$ = Tooth pulsation factor ($P$ is in per cent of $B_m$)
$K_{PBm}$ = Factor for % tooth pulsation and max. induction product
$K_{ft}$ = Tooth frequency factor

$$f_t = \frac{\sim \times \text{no. of teeth}}{\text{pairs of poles}} = \frac{\text{rev. per min.} \times \text{no. of teeth}}{60}$$

For rotor $f_t$ use no. of stator teeth
For stator $f_t$ use no of rotor teeth

m. m. f. which acts on the stator teeth to produce high-frequency pulsations in these teeth which may be responsible for very considerable losses.

Appendix I gives some experimental data showing this effect quantitatively. The magnitude of the stator-tooth pulsations as a result of the high-frequency rotor currents would be difficult to calculate accurately. Therefore, in the case of squirrel-cage motors the

5. A. I. E. E. TRANS., Vol. 44, 1925, p. 160.

expedient has been adopted of calculating the rotor-tooth pulsation percentages as if there were no squirrel-cage windings, then assuming that these are the percentage pulsations which actually occur in the stator teeth due to the rotor high-frequency currents. The rotor-tooth pulsations and therefore the pulsation losses are assumed negligible and the stator-tooth pulsation losses calculated from formula (**13**) assuming $P_s$ equal to the calculated rotor tooth pulsation percentage. When the number of stator and rotor teeth is not very different, this is a fairly reliable assumption.

### Copper Eddy Losses

The cause and nature of these losses have been discussed in a previous paper.[6] They become of appreciable magnitude only at quite high tooth flux densities. These losses are caused by high-frequency slot-leakage fluxes which occur due to the saturation of the teeth. In general the stator no-load copper eddy losses are negligible since the conductors are small and the teeth are less saturated but the rotor copper eddy losses are often appreciable both for squirrel-cage and wound-rotor induction motors.

In order to estimate their magnitude $B_m$ and $B_m - P_S$ (using Fig. 7) should be calculated and the difference in the m. m. f. corresponding to these two inductions $H_{max}$, $H_{min}$ determined from the normal induction curves of the class of sheet steel used in the rotor teeth. The copper eddy losses when the rotor conductors are of copper may be calculated from the following formula:

$$W_e = 96.8 \times 10^{-10} f_{st}^2\, W_{cu}^2\, H^2 \text{ per cu. in. when } m < 2. \qquad \textbf{(14)}$$

$$W_e = 10.9 \times 10^{-7} \frac{f_{st}^{1/2}\, H^2}{\left(\dfrac{t_{cu}}{1.5\,\sigma}\right)^{3/2} W_{cu}} \text{ per cu. in. when } m > 2. \qquad \textbf{(15)}$$

where $f_{st}$ is the tooth frequency as calculated from the number of stator slots and the speed of the rotor.

$W_{cu}$ is the radial depth of the individual bars in inches. $H$ is the difference in m. m. fs. between adjacent teeth (in gilberts per cm.)

$$m = 0.148 \sqrt{f_{st} \times \frac{t_{cu}}{\sigma}} \text{ for copper.}$$

$t_{cu}$ = thickness of the copper bars (tangential) multiplied by the number of bars side by side in the slot; $\sigma$ is the slot width.

In calculating $H$, of course, $H_{min}$ and $H_{max}$ do not occur simultaneoulsy in adjacent bars but a fair average is to assume that $H = (H_{max} - H_{min}) \times 2/3$.

### Illegitimate Losses

These losses have already been mentioned previously and are extra losses in addition to those which would occur in annealed unsaturated cores with perfect insulation between laminations. They are difficult to estimate due to their variability and should be eliminated as far as possible. They are one of the major causes of differences between calculated and test results.

One of the most common types of extra losses in rotating machines appears as the result of eddy currents caused by short-circuited laminations. Short-circuited laminations may be produced when types of construction are used such that the punchings are wedged onto dovetails or keyways or are forced on the shaft. This connects together electrically the back edges of the laminations. Now, if the teeth contain burrs or if the slots are filed, the laminations become short circuited and in this case there may result very considerably increased fundamental-frequency core losses. When the back edges of the laminations are not short circuited slot filing and tooth burrs do not cause much extra losses. Under conditions of badly short-circuited laminations, the core losses may be doubled or even tripled. Great care should be taken to eliminate burrs and slot filings, assuming stator punchings, so that the laminations will not be short circuited on the outside edges. Due to non-uniform flux as a result of these extra eddy currents, the hysteresis as well as the eddy losses may be increased.

For motors with narrow teeth, there may be considerably increased losses due to punching strains. At an induction of 65 kilolines per sq. in., the losses are increased by an amount which can be calculated approximately by increasing the standard loss by a factor equal to 14 divided by the mean tooth width in inches; namely:

$$W_{cor} = W_{std}\left(1 + \frac{0.14}{t}\right) \qquad \textbf{(16)}$$

At 100 kilolines the correction is perhaps not over one-half this value and at very high flux densities is very small. If the stampings are annealed after punching there is no such correction.

If the punchings are not flat, bending strains may appear increasing the losses at medium inductions. Above 100 kilolines there would be very little effect due to this cause.

If a motor is operated at very high core inductions leakage flux into the frames, end-bells, and other solid members may cause the losses to go up very rapidly with voltage. In a properly designed machine operated at normal voltage this should be a small factor.

One or all of these factors may be of appreciable magnitude in commercial machines. As a consequence, in calculating the fundamental-frequency losses it is advisable to substitute for the curves of Figs. 1 and 2 others which are determined from actual tests on commercial machines of various standard types. Small induction motors may require one set of curve and large motors another set. The losses,

6. *No-load Copper Eddy Current Losses*, T. Spooner, A. I. E. E. Trans., Vol. XLV, 1926, p. 231.

particularly at the higher inductions, will be appreciably greater for these empirical curves than for those given in Figs. 1 and 2. Such curves are not given here since they would have to be determined by every manufacturer for his apparatus and would be a function not only of design but of shop practise.

## Experimental Methods of Separating Losses

The several different kinds of loss which must be considered to obtain the total have been described and it would be very convenient if each one of these several losses could be tested alone or separated from the others. This ideal will hardly be realized, but tests which give a separation of losses with respect to the frequency which causes them can be made.

One method which is applicable only to wound rotor motors was described by R. Richter in the *Elektrotech. Zeitsch.*, January 6, 1921. This method separates the fundamental-frequency losses from the tooth-frequency losses and is carried out as follows: Three saturation curves are taken by varying the applied voltage and reading current and watts. One curve $a$ is taken with power applied to the stator and the rotor rotating with short-circuit armature. The second curve $b$ is taken with power applied to the rotor collector rings. The stator is short circuited and the rotor is rotating at full speed. The third curve $c$ is taken with power applied to either stator or rotor (preferably the stator) and with the other member open circuited. The rotor does not rotate in this case, but should be turned slowly to average the flux due to different positions of stator and rotor teeth.

All curves are corrected to eliminate the $I^2 R$ losses and the friction and windage in curves $a$ and $b$. Large rotors usually have a strap conductor and the resistance must be corrected for skin effect when calculating the $I^2 R$ loss in this member.

Curve $a$ (corrected) contains losses due to the line frequency in the stator and also all the high-frequency losses. Curve $b$ (corrected) contains losses due to the line frequency in the rotor and also all the high-frequency losses. Curve $c$ (corrected) contains losses due to line frequency in both rotor and stator and no losses due to tooth frequencies since the rotor is not rotating (practically).

Since curves $a$ and $b$ are taken from different members in which the voltage is very seldom exactly the same, the losses in the two curves are compared at voltages which give the same densities in the air-gap or any other part of the circuit.

If the curves $a$ and $b$ are added, the loss found contains the line-frequency losses in both stator and rotor and also twice the high or tooth-frequency losses. If the values from the $c$ curve, which contains only the line-frequency losses in both stator and rotor, are subtracted from the sum of curves $a$ and $b$, the result is twice the tooth-frequency losses.

In this way, the tooth-frequency losses are separated and if they are subtracted from curves $a$ and $b$, the line-frequency losses in the stator and in the rotor may be found separately.

This is the method that has been used in separating the losses in some of the motors shown in the curves.

Another method which is applicable to all types of machines is described by Messrs. Alger and Eksergian.[7] The fundamental idea is covered by their statement: "It is a general principle that whenever any cyclic variations in the permeance of a magnetic circuit are caused by the relative movement of parts of the circuit, all the losses caused by this variation are supplied by the mechanical agency causing the motion."

Applied to the induction motor, this means that the tooth-frequency losses are supplied by torque in the rotor. Now, the rotor can only receive power by running slower than the rotating field, *i. e.*, slipping, and by hysteresis loss in the rotor core and teeth. The slip can be measured very accurately by several means so that this portion of the rotor torque is known, but the hysteresis losses in the rotor core and teeth must be calculated. The hysteresis loss effect is as though it were at line frequency.

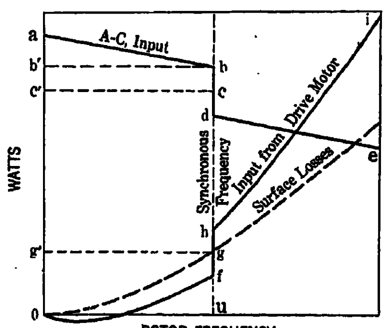


Fig. 11—Loss Diagram for Induction Motor

The no-load slip values decrease as the voltage is increased since the hysteresis loss is increasing and the induced torque due to slip does not need to be so much to overcome the friction and windage and the tooth-frequency losses. It might be of interest to note that in one test we made, the slip decreased to zero, indicating that the hysteresis loss was sufficient to drive the rotor in synchronism.

Another method, requiring a driving motor and also accurate meter readings, makes use of the step in the stator loss curve which occurs when the rotor is driven through synchronism. This has been described by Messrs. Alger and Eksergian[8] and by Messrs. Spooner and Kinnard.[9]

If a constant voltage and frequency are applied to the stator and the rotor is driven by another motor at different speeds, the power taken from the line drops

7. Alger and Eksergian, *loc. cit.*

8. Alger and Eksergian, *loc. cit.*

9. *Surface Iron Losses with Reference to Laminated Materials*, A. I. E. E. Trans., Vol. XLXXX, 1924, p. 252.

and the power taken by the driving motor increases abruptly when the rotor passes synchronous speed. This difference in watts before and after synchronous speed is equal to twice the rotor hysteresis loss.

This can be seen more clearly in Fig. 11.

Curve *a, b, c, d, e* is the stator input. Curve *o, f, g, h, i* is the driving motor output. $I^2 R$ losses and friction and windage losses have been taken out.

$b' c'$ is the rotor hysteresis loss and is one-half the step *b d* or *f, h*; $a\ b'$ is the rotor eddy current losses, which at normal speeds is practically zero. $O\ g'$ gives the tooth-pulsation losses and $o\ c'$ corresponds to the line frequency losses.

TABLE I
WOUND-ROTOR INDUCTION MOTORS
COMPARISON OF TESTED LINE FREQUENCY AND SLOT FREQUENCY LOSSES TO CALCULATED VALUES

| | Per cent normal voltage | Line-freq. loss | | High-freq. loss | | Total loss | |
|---|---|---|---|---|---|---|---|
| | | Test | Calc. | Test | Calc. | Test | Calc. |
| 60-cycle... | 56.8 | 1.00 | 1.36 | 2.20 | 2.33 | 3.20 | 3.69 |
| 12-pole.... | 79.5 | 2.10 | 2.32 | 3.70 | 3.55 | 5.80 | 5.87 |
| | 102.2 | 4.35 | 4.83 | 5.35 | 5.38 | 9.70 | 10.21 |
| | 125.0 | 9.80 | 9.90 | 7.40 | 7.20 | 17.20 | 17.10 |
| | | 0.50 | 0.50 | 1.05 | 0.86 | 1.55 | 1.36 |
| 60-cycle... | 56.8 | 0.96 | 1.06 | 1.94 | 1.98 | 2.90 | 3.04 |
| 16-pole.... | 79.5 | 1.98 | 1.97 | 2.82 | 3.20 | 4.80 | 5.17 |
| | 102.2 | 3.53 | 3.35 | 3.57 | 4.38 | 7.10 | 7.73 |
| | 125.0 | 6.03 | 6.41 | 4.17 | 5.83 | 10.20 | 12.24 |
| | | 2.06 | 2.11 | 1.65 | 3.60 | 3.71 | 5.71 |
| 60-cycle... | 56.8 | 4.67 | 4.55 | 6.13 | 8.35 | 10.90 | 12.90 |
| 24-pole.... | 79.5 | 8.67 | 8.70 | 11.63 | 13.18 | 20.30 | 21.89 |
| | 102.2 | 14.65 | 14.80 | 16.35 | 18.55 | 31.00 | 33.36 |
| | 113.7 | 19.55 | 20.15 | 19.85 | 21.59 | 39.40 | 41.74 |
| | 45.5 | 1.40 | 1.14 | 1.50 | 1.33 | 2.90 | 2.47 |
| 25-cycle... | 60.6 | 2.31 | 1.82 | 2.50 | 2.11 | 3.81 | 3.93 |
| 10-pole.... | 75.8 | 3.40 | 2.75 | 3.50 | 3.00 | 6.90 | 5.75 |
| | 90.9 | 4.80 | 4.05 | 4.60 | 4.06 | 9.40 | 8.11 |
| | 106.0 | 7.00 | 6.21 | 5.70 | 5.18 | 12.70 | 11.39 |
| | 121.1 | 11.10 | 10.50 | 6.60 | 6.89 | 17.70 | 17.39 |

Checks on other machines, which were not specially tested, show the same general agreement in the total losses.

This method may be used on wound-rotor motors but on squirrel-cage motors the line *a b* is so steep that it is very hard to get satisfactory readings unless the frequency and speed are very accurately controlled.

## Test Results

Several large wound-rotor motors have been tested according to the first method for loss separation and the losses have been compared with the losses calculated by the above described methods. The curve for the line-frequency losses was obtained from one of the tested machines and checks well with the loss curve for the grade of iron used. Table I shows the fundamental and the pulsation-frequency losses calculated separately. Table II gives data for other machines without separation of losses. Fig. 12 shows the results graphically for one of these machines.

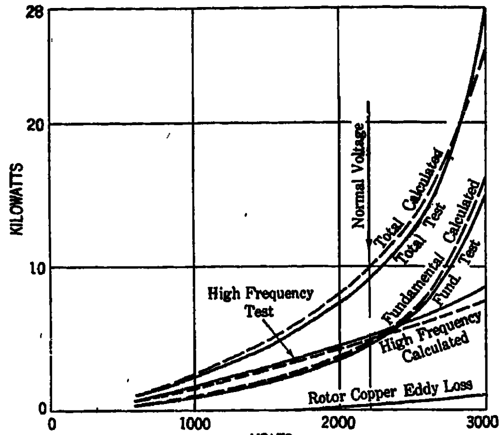


FIG. 12—WOUND ROTOR INDUCTION MOTOR

Comparison of calculated and test no-load core losses

In Table III are presented some data on squirrel-cage motors. These checks are not as good as for the wound-rotor machines partly because of the greater difficulties of test. These results were obtained about three years ago. The calculated pulsation losses are probably low because no account was taken of copper eddy-current losses.

TABLE III
SQUIRREL-CAGE INDUCTION MOTORS
COMPARISON OF TESTED AND CALCULATED CORE LOSSES AT NORMAL VOLTAGE

| Hp. | Freq. | Line freq. loss | | High freq. loss | | Total loss | |
|---|---|---|---|---|---|---|---|
| | | Test | Calc. | Test | Calc. | Test | Calc. |
| 250 | 60 | 1.98 | 1.87 | 1.72 | 1.22 | 3.70 | 3.09 |
| 500 | 60 | 6.04 | 6.02 | 3.93 | 3.14 | 9.97 | 9.16 |
| 175 | 40 | 1.08 | 1.07 | 0.95 | 0.64 | 2.03 | 1.71 |
| 125 | 40 | 0.82 | 1.15 | 1.30 | 1.27 | 2.12 | 2.42 |
| 225 | 40 | 1.83 | 1.96 | 2.09 | 1 53 | 3.92 | 3.49 |

TABLE II
WOUND-ROTOR INDUCTION MOTORS
COMPARISON OF TESTED AND CALCULATED TOTAL LOSS VALUES

| Per cent normal volts | 60 cycles 10 poles | | 25 cycles 4 poles | | 25 cycles 8 poles | | 25 cycles 10 poles | | 25 cycles 34 poles | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Test | Calc. | Test | Calc. | Test | Calc. | Test | Calc. | Test | Calc. |
| 34.1 | 2.87 | 3.50 | 1.03 | 12.2 | 4.8 | 5.6 | | | | |
| 45.5 | 4.89 | 5.46 | 1.68 | 20.2 | 11.8 | 9.9 | | | 7.4 | 6.7 |
| 56.8 | 6.69 | 7.22 | 2.37 | 30.8 | 16.8 | 16.1 | 68.0 | 50.4 | 15.1 | 13.4 |
| 68.2 | 9.55 | 10.36 | 4.16 | 45.3 | 28.4 | 26.0 | 88.6 | 86.2 | 24.6 | 21.7 |
| 79.5 | 12.70 | 13.36 | 6.22 | 61.4 | 38.0 | 38.1 | 137.0 | 141. | 34.9 | 31.9 |
| 91.0 | 16.58 | 16.64 | 8.00 | 80.6 | 49.5 | 49.7 | 175.0 | 185. | 46.3 | 44.4 |
| 100.0 | 20.26 | 19.85 | 10.66 | 10.60 | 63.8 | 64.2 | 204.0 | 222. | 53.8 | 52.8 |
| 113.7 | 25.40 | 26.70 | 12.35 | 12.33 | 81.2 | 83.9 | 258.0 | 272. | 57.9 | 59.6 |
| 125.0 | 32.00 | 32.46 | 14.10 | 14.67 | 99.0 | 98.2 | 345.0 | 342. | 68.1 | 75.3 |

## Conclusions

The above described method of calculating no-load induction-motor core losses appears to be rather tedious and complicated and as compared with the older empirical methods this is the case. However, by the use of a suitably prepared schedule the time may be greatly shortened. The time for such a calculation, provided all of the factors which have to be calculated for other purposes are available, is from 30 to 40 min. This is not excessive when dealing with a radically new design or with a type of machine which has given trouble due to high losses. It is hoped that this paper will stimulate discussion and will bring forth other methods for making these detailed calculations.

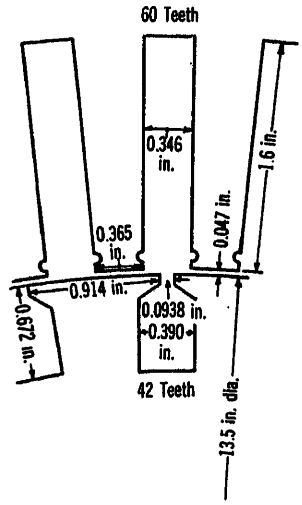


Fig. 13—Special Induction Motor Showing Tooth Slots

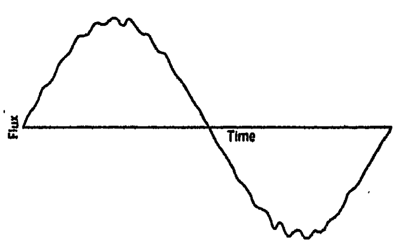


Fig. 14—Squirrel-Cage Induction Motor

Stator tooth flux
Normal voltage—no rotor bars

## Appendix

In a squirrel-cage induction motor having nearly closed rotor slots, open stator slots, and no rotor windings there would ordinarily be quite appreciable rotor-tooth flux pulsations but only very small stator-tooth pulsations. When, however, the squirrel-cage winding is applied, the rotor-tooth pulsations are damped out by the short-circuited copper turns. The high-frequency currents thus generated in the rotor bars produce a considerable m. m. f. which is approximately equal to the change in m. m. f. applied to the rotor teeth caused by the pulsating reluctance in the gap resulting from the stator slots. This high-frequency m. m. f. produced by the rotor windings causes pulsations of flux in the stator teeth which (for the ordinary induction motor having nearly the same number of stator and rotor teeth) are of the same order of magnitude as the pulsations which would have occurred in the rotor teeth had there been no rotor winding. The stator-tooth frequency will be that calculated from the number of rotor teeth and the peripheral speed.

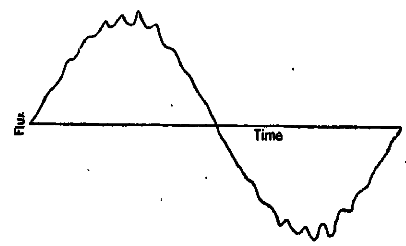


Fig. 15—Squirrel-Cage Induction Motor

Stator tooth flux
Normal voltage—rotor bars but no end-rings

In order to show this, test results were obtained on a 60-slot 4-pole stator fitted with a 42-slot rotor with a copper bar winding and brass end rings. The stator slots were open and the rotor slots nearly closed as shown by Fig. 13. This was a very special machine built for experimental purposes.

An exploring coil was placed around one of the stator teeth and oscillograms obtained of the induced voltage under conditions of normal induction and 20 cycles. This was done for 3 conditions:

1. No bars in the rotor slots.
2. Bars in the rotor slots but no end rings.
3. Bars in the slots and end rings connected.

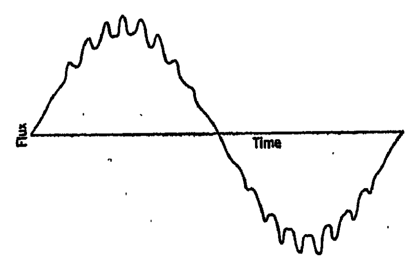


Fig. 16—Squirrel-Cage Induction Motor

Stator tooth flux
Normal voltage—rotor bars with end-rings

Oscillograms of the voltage in the stator-tooth exploring coils were obtained for the three conditions. These oscillograms, of course, show very large high-frequency voltage components.

These oscillograms were then enlarged and integrated in order to obtain the corresponding flux waves. These are shown by Figs. 14, 15, and 16.

These tooth-voltage waves may be integrated automatically if desired by inserting a large inductance in the circuit containing the exploring coil on the stator

tooth and the oscillograph element. This was actually done in this case. Due to the difficulty of obtaining a very large ratio of inductance to resistance the integration was not particularly satisfactory. Since the phase angle of the harmonics with respect to the fundamental was not correct, these oscillograms are not reproduced here.

It will be seen from the curves that, with no rotor bars, the stator-tooth pulsations were very small as would be expected from the fact that the rotor slots were nearly closed. When the bars were inserted in the slots, they made electrical contact with some of the laminations, thus producing somewhat the same effect as would result from high-resistance end rings. As a consequence high-frequency currents circulated around the rotor teeth through the bars and laminations producing increased pulsating fluxes in the stator teeth. (Compare Figs. 14 and 15.) With the end rings in position, these stator tooth pulsations were further increased due to larger high-frequency currents in the rotor bars. (Compare Figs. 15 and 16.)

The frequency of the stator-tooth pulsations should correspond to a 21st harmonic as calculated from the number of rotor teeth. Since, however, these pulsations were in turn proportional to the amplitude of the fundamental-frequency flux, a harmonic analysis should show that the pulsations consisted of a 20th and 22nd harmonic with no 21st. Due to the fact, however, that the variations in slot reluctance at the gap did not follow a sine law and to saturation effects, there were other harmonics present. A harmonic analysis of the flux waves was actually made and for condition one, with no rotor bars, the analysis gave the following results:

| Order of Harmonic | Resultant per cent |
|---|---|
| 20 | 1.8 |
| 21 | 0 |
| 22 | 1.7 |

For condition three, with rotor bars connected, the analysis is as follows:

| Order of Harmonic | Resultant |
|---|---|
| 20 | 3.3 |
| 21 | 0 |
| 22 | 4.9 |

The other harmonics have not been included since that would have added nothing of value to the discussion.

## Discussion

**P. L. Alger:** This paper is a logical sequel to Mr. Spooner's extremely well coordinated series of earlier papers. Many years ago Professor C. A. Adams suggested the rational analysis of induction-motor core losses, and his 1909 paper was the real starting point of the long series of investigations which Mr. Spooner has now brought to so successful a conclusion.

Of particular interest to me is the assumption made that the percentage flux pulsation in the stator teeth of a squirrel-cage induction motor is the same as that which would have occurred in the rotor teeth if a slip-ring winding had been employed. Ever since Chapman[1] showed the manner in which rotor flux pulsations set up circulating currents in a squirrel cage, it has been clear that these currents were a cause of extra pulsations in the stator, but the present proposal is the first practical one I know of for calculating them. While a more detailed analysis of these induced currents would be instructive, I think the assumption made is quite satisfactory.

There have been two especially important articles on core losses recently published abroad, a very complete treatise by Dreyfus[2] on core losses in slip-ring induction motors, and a paper by Coe and Taylor[3] on the theory of pole-face losses. The latter paper calls attention to the extremely rapid theoretical increase of pole-face loss with increase in the slot width over air-gap ratio, $q$, at small values of $q$.

Professor A. A. Bennett has recently made a careful analysis of this subject, with the results shown in the accompanying figure. The solid curves represent the relative theoretical pole-face losses as functions of $q$, for various ratios of slot-to-tooth width. These were derived by calculating the maximum, minimum, and average flux densities in the air gap, by elliptic function theory, for various values of $q$, deriving from these three quantities the zero, first, and second order harmonics, $A$, $A_1$, $A_2$, of a Fourier's series representing the flux distribution, and thence determining a relative pole-face loss factor equal to

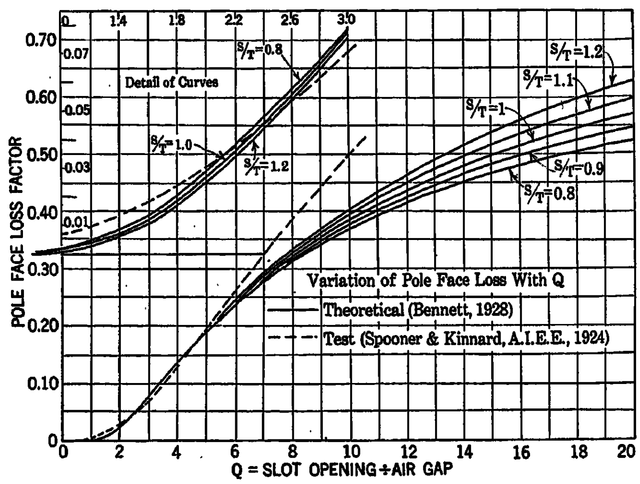


Fig. 1

$$\frac{A_1^2 + 2 A_2^2}{A_0^2}$$

The $A_2^2$ term was given double weight in determining the loss factor on the assumptions that the loss increases as the square root of the product of frequency and tooth pitch, and that the neglected higher-order harmonics would amount to about half as much as the $A_2$ term alone.

The dashed curve in the figure gives the average of the relative pole-face loss factors due to $q$ given by Spooner and Kinnard for four different kinds of laminated steel in their 1924 paper. The loss factors compared being merely relative, the points at $q = 5$ on the two curves were taken as the same. The theoretical curve shows that as the value of $q$ decreases below 1.5 the pole-face loss decreases with extreme rapidity, being zero for all practical purposes when $q$ falls a little below unity. The test

1. *London Electrician*, 1916.
2. *Archiv f. Elektrotech.*, May-July, 1928.
3. *Philosophical Mag.*, Vol. VI., July 1928.

data of Spooner give higher losses than the theoretical at both high and low values of $q$, but they agree very well in the middle range. The theoretical curve is of particular interest, as it explains how turbine generators, having very small values of $q$, can use solid steel rotors, while other machines with moderate values of $q$ must employ laminated rotors.

**C. A. Adams:** The point brought up by Mr. Alger seems to me of vital importance, namely, desirability of developing our rational knowledge of this subject to such a point that we can select slot ratios and other proportions to the end of reducing these losses to a minimum.

We are altogether too apt to hesitate to tackle theoretically a job which appears on the face of it to be very complicated, and I am sure that those of us who have ever persisted, have discovered that you can carry theoretical analysis much farther than was at first obvious, and to very great advantage. As long as we are dependent upon experimental data or upon the cut-and-try method, we are almost helpless when we attempt to solve many of the more intricate problems in this field.

**C. W. Kincaid:** I might say to Mr. Adams that before we had this method perfected we used one of his formulas. Of course, it was made up for d-c. machines and we made a few modifications. We have used that for years. In fact it is now difficult to convert some of the men to the new method. They still stick to his formula and say it is quicker than the one we are using now.

# Insulation Tests of Electrical Machinery
## Before and After Being Placed in Service

BY C. M. GILT[1] and B. L. BARNS[2]
Member, A. I. E. E. Member, A. I. E. E.

*Synopsis.—The A. I. E. E. Standards have provided high-potential test values as a means of establishing a standard basis of design and construction of insulation. Many purchasers test equipment after installation before placing it in service to be sure that the connections and equipment are in a safe condition for service. It is recommended that a Standard Test Voltage be established that may be used under these conditions.*

*It also seems to be advisable that suitable rules should be set up governing test voltages that should be used in a maintenance program. Periodic observations of the insulation resistance are suggested as substitute or supplemental tests. If there is a general desire that rules of this sort should be made, it is proper that the Institute should enter this new field of standardization.*

* * * * *

THE work of standardization performed by the American Institute of Electrical Engineers has been and is one of the most valuable undertakings that lie within its field of activities. This standardization has been primarily in the nature of establishing rating standards by which equipment of different manufacturers can be compared. While these standards have been based upon what is believed to be typical or average operating conditions they obviously are not standards for operation.

There is a rather decided opinion among many engineers that the Institute should undertake the development of standards for operation in fields in which they can be established.

One of the fields in which it is believed such standards can be developed is that of high-potential tests. These tests are not offered as standards of comparison of equipment nor as acceptance tests for purchasers from manufacturers, except as possibly determined by special contracts, but are intended as operating guides. They are an attempt to crystalize a variety of opinion as to what should be an adequate and safe test to determine the suitability of equipment for service, for those who wish to employ high-potential tests for this purpose.

This paper endeavors to present the subject in two parts, first, a test for new equipment before being placed in service, and second, a high-potential test for occasional check on equipment that is or has been in service. The first part has been prepared chiefly by the first author and the second part by the second author of the paper.

## Part I.

### INSULATION TESTS FOR INITIAL OPERATION

High potential testing may be roughly grouped in three general classes, each with a distinct purpose in view, but all with the fundamental object of determining the ability of the equipment to withstand the normal and abnormal potentials that may be reasonably expected under the operating conditions for which the device is designed.

The first group of tests is intended to determine the characteristics of the material or design in question. Such tests may be fundamental in character, such as determining the behavior of material under high-potential stresses, as well as of more immediate use, such as determining thickness of insulation or contours of creepage surfaces.

The tests of the second class are those carried on to discover factory errors in the manufacture of equipment or details of design. In general, the routine factory high-potential tests conducted on equipment at the completion of its manufacture fall into this class. While these high-potential tests are sometimes used as a check on the design when differing from the usual practise, the necessary thickness of insulation and clearance between live parts are so well established for ordinary service that there would be little need for the factory high-potential tests on normally designed equipment supplied by reputable manufacturers, when once the standard of design and over-potential stresses have been established, were it not for defects in material and workmanship that occasionally occur.

The third class of high-potential tests is made to determine the adequacy of equipment for service, to discover whether some accident has happened to the equipment since the factory test prior to its final connection to the lines ready for service or after once having been placed in service whether there has been sufficient deterioration of material or accidental damage to make the equipment unsafe for service. The tests after installation and prior to service commonly include, not only individual pieces of equipment such as motors or transformers, but assembled equipment in generator stations, substations, or customer's vaults and include cables, insulators, circuit breakers, transformers, instrument transformers, etc., as they are connected for placing in service.

The present rules of the Institute covering high poten-

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*Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.*

tial tests were primarily adopted as a means of establishing minimum standards of design for ordinary operating conditions and are essentially the result of experience that the equipment so designed will have a reasonable life under normal operating stresses including such overpotential surges as are commonly met with under ordinary operating conditions. The tests were further intended to insure that the insulation met the expectations of design. Over-potential surges of various magnitudes and duration occur repeatedly during the life of the equipment. Some have a higher and some a lower value than the test value used to determine the adequacy of the design and material but while there has been some agitation to increase the standards of insulation and insulation tests for given operating voltages, experience indicates that insulation that will withstand the standard factory test will withstand the ordinary repeated over-voltage surges during what has been considered a reasonable life.

While high-potential tests may have some deteriorating effect on certain classes of insulation, this deteriorating effect is so slight that it is accepted as a price that must be paid in the present development of the art as a means of reducing to a minimum the disastrous effects of failure of equipment in service as a result of defects or accidents that occasionally occur in manufacture and installation. Unfortunately the insulation processes are so different from that of working metals and alloys that a test sample of the material cannot always be taken as a reasonably conclusive proof of the strength and adequacy of the completed article.

Subsequent to the tests at the factory finishing touches are sometimes added to it. It is crated, shipped, and connected ready for service in the final installation and during this handling process accidents to the equipment occasionally occur, which if not discovered prior to placing in service, may result in serious damage to it, other equipment, and to the service rendered by the purchaser. These defects cannot always be discovered by observation. The placing in service of a new, large piece of equipment with its various connections is a nerve-racking undertaking and many of the operating companies believe it essential to take every precaution to be sure that no faults in equipment or connections exist that will result in expensive repairs and serious delays that occur when a fault develops with the tremendous capacity of modern systems feeding into it.

These tests are not ordinarily considered acceptance tests unless the equipment is delivered and erected by the manufacturer, as the equipment commonly passes out of control of the manufacturer when it leaves the factory, but are intended to cover the period and work between shipment and operation.

Obviously, equipment which is assembled on the premises of the purchaser without a complete factory test should receive the same test as though it had been previously assembled and tested at the factory.

As a generally accepted example of this practise, a test is placed on cable before it leaves the factory to discover defects in the manufacture of the cable and after the somewhat severe process of pulling into the ducts, a further test is placed upon it and the joints to be sure that the completed cable is ready for service. In certain respects this condition is somewhat different from other equipment as the test after installation is partly to determine the ability of the cable to withstand the process of pulling into ducts and as damage from exterior causes is somewhat easier to determine by inspection than in more complex pieces of equipment. It is recognized that the process of installation certainly reduces the strength of the insulation at various points and consequently the tests after installation are lower than those at the factory, or rather one might say, the factory tests are somewhat higher than the standards set after installation to make due allowance for the deterioration that results from handling.

Similarly, the various parts of assembled equipment are given individual high-potential tests before they are assembled in the completed machine and the value of the tests of the various parts is higher than that of the completed piece of equipment. The final tests of the completed machine are to insure that no damage has resulted to the individual parts and to place a test upon such connections as cannot be effectively tested before the final assembly. An allowance is made for the deterioration of portions of the equipment and the rules recognize a test on the completed assembly of 15 per cent lower than the tests on the part receiving the lowest individual test.

In spite of the very good reason for testing of equipment before energizing with service voltage, many careful purchasers of equipment forego this final check for one or both of two reasons. First, there is the general feeling that each high-potential test causes some permanent damage to the insulation, and second, that the insulation may not be in as good condition as before leaving the factory due to the accumulation of dust or the absorption of moisture and while safe for operating voltages the high-potential test might be damaging. It cannot be denied that many purchasers do not have the facilities for insuring that the insulation is thoroughly dry and in proper condition for a high potential test and the manufacturers quite naturally question the advisability of a high-potential test of a severe nature under these conditions. Experience has clearly indicated that there have been many pieces of equipment that have absorbed moisture to the point that a high potential test would be somewhat hazardous and yet have operated under load safely and have been dried out by that process.

The question of the voltage strain and damage to the insulation is one on which opinion is very widely separated. That there is probably some slight deterioration with each high-potential strain is commonly believed and one school of thought holds that in the

long run there are more failures and more damage as a result of the cumulative deterioration of equipment in reasonable condition as a result of the repeated strains imposed by tests than occur due to the normal deterioration of insulation and the consequent failures that are not anticipated by test. Certain insulating mediums such as oil and inorganic substances seem to be affected little or not at all by high potential stresses of any number of repetitions provided they are in good condition so that there is no appreciable leakage through them. The actual effect of high-potential stress on fibrous insulation is much more uncertain and whether overvoltage for short periods and within a reasonable value has any appreciable effect upon the insulation is not certain. When the insulation is stressed to a point near that of failure, the matter of time and duration is of great importance in the performance of the material and yet we know that insulation is subjected to brief overvoltage stresses of two or three times normal voltage many times during its life and whether the life of the insulation is reduced appreciably beyond the limit of the normal aging process in service, is not certain. It would seem that an additional application of overvoltage of short duration or of a value reasonably below the breakdown value of the insulation placed upon the equipment before going into service would not have any appreciable effect upon the life of the insulation. Unfortunately, the phenomenon of electrical breakdown of insulation is not clearly understood and it is hoped that out of the various studies that are now being conducted on what actually takes place when insulation fails, will come information that will point to the answer of this much discussed question of repeated tests. The tests reported by F. M. Clark on *Dielectric Properties of Fibrous Insulation,* (A. I. E. E. TRANS., Vol. XLV, 1925, p. 193) indicate that for a certain type of insulation the breakdown voltage is actually increased by a number of previous high-voltage tests, probably due to a drying out process, but that the insulation is permanently damaged by too large a number of overvoltage applications. The further conduct of such tests on various materials and with relative magnitudes of voltage and time will do much to settle this question.

It is obvious that equipment should not be subjected to high potential tests unless the insulation is in such a condition that it may be reasonably expected to withstand the test without damage or failure. The equipment must be carefully gone over to be sure that there are no apparent points of weakness or damaged parts of insulation, to be sure that there is no foreign material in the equipment that might cause a breakdown, that it is free from an accumulation of dust that might result in a failure, and above all one must be sure that the insulation is free from moisture. When equipment is shipped from a factory even though protected to some extent from atmospheric conditions it almost surely will absorb considerable moisture if at all hygroscopic. One would not think of operating or testing an oil insulated transformer that had been exposed to the air without oil for any but the briefest period, without a thorough drying out, and if shipped filled with oil a test of the oil should be made to insure freedom from moisture and high dielectric strength.

Similar precautions are and should be taken before equipment is placed in service even though no high-potential test is made. It should not be considered safe to operate equipment that is not in a condition to withstand safely an overvoltage test. This is particularly true of equipment connected to high-voltage lines where the equipment is quite commonly subjected to overvoltage stresses due to surges or various causes. Low-voltage equipment in which the insulating value of insulation is far beyond that required by the ordinary operating voltage may sometimes be operated with a fair degree of safety under conditions in which it would not withstand a high-potential test without serious danger of damage. Such conditions are rather the exception when considering equipment which a purchaser would ordinarily undertake to test.

Due to the fact that it is desirable not to cause any undue deterioration of the insulation by the test procedure and due to the fact that the equipment may have been subject to some slight deterioration as a result of handling and may have absorbed some slight amount of moisture, and that factory tests have already been made to insure against inherent defects in the insulation, it would seem reasonable that the test before operation should be somewhat less severe than at the factory. It should be searching enough to discover damage of the insulation that may later develop into a serious fault. The reduction of test may be in either time or magnitude of voltage or both and there is a variety of opinion as to what this reduction should be. A number of arguments has been advanced for a reduction of as much as 25 per cent of the original test voltage while others believe that the reduction should not be more than 15 per cent to correspond with the existing standards for the reduction in test voltage of assembled equipment while still other arguments are advanced for maintaining the magnitude of the original factory test voltage but reducing the time of test from one minute to momentary. The British standards recognize the establishment of such a test voltage for this purpose and have set it at 25 per cent below the factory test voltage.

In view of the differences in opinion as to what such a test voltage should be, it would seem desirable for the Institute to broaden its range of standards and include a standard of test voltage that may be used when desired by the purchaser when placing new equipment in service.

This standard is not intended to be mandatory but should be considered as establishing a standard test voltage that may be applied at the option of the purchaser, not as an acceptance test, but to assure him-

self as far as is possible that neither the equipment nor the service will be damaged by some accident that may have occurred to the apparatus.

It would appear that there is a field for such a standard and it is recommended for further consideration that the Institute establish a standard of test voltage that may be used for new equipment after installation of 85 per cent of the factory test voltage of the equipment connected at the time of the test receiving the lowest factory test and for a period of one minute.

## Part II

### Insulation Tests of Electrical Machinery in Service

For the same reason that it is desirable to set up a standard practise for testing the insulation of electrical apparatus at the time of initial installation, it is also desirable to establish a rule or recommendation as to what tests would be suitable for a maintenance program.

It is true that since the manufacturer is presumably desirous that apparatus of his manufacture should be properly maintained the user should be able to obtain recommendations of this sort from the manufacturer. This has been done to some extent but there is a desire that an agreement should be made regarding suitable tests that would be applicable to apparatus of any manufacture. Such a procedure would be of great assistance in formulating a suitable maintenance program where apparatus of more than one manufacture is installed in the same station on system.

The lack of any semblance of uniformity in practise in this respect has been drawn to the attention of the Committee on Electrical Machinery and after a careful consideration this committee has decided that a discussion of this subject should be presented to the membership of the Institute in the hope that it may lead to an agreement of some sort. Previous attempts have been made to formulate some standard practise or recommendation of this kind and probably the best efforts in this direction have been made by the Apparatus Committee of the National Electric Light Association by whom questionnaires were circulated among members for the purpose of determining their practises and the degree of satisfactory results. The reports of this committee indicate that the practises of the different members are so various that it is not practicable to choose any one as representing a general preference or giving better results than the others. Some pay no attention to making tests, some occasionally make tests to ground at rated potentials or at slightly higher values, while a small percentage regularly use test voltages approximating the values set out in the A. I. E. E. standards for new machinery.

In considering suitable means of checking the condition of insulation for use in a maintenance program it is but natural to turn to the present A. I. E. E. standards for guidance. But the insulation test voltages set up in these standards are designed to insure that the insulation has sufficient dielectric strength to allow for a certain amount of deterioration and still be sufficient to withstand the voltages that may be impressed upon it under operating conditions and in this sense is in the nature of an expected life test. It therefore appears that it should not be expected that after a period of operation in which undoubtedly some deterioration has taken place the insulation should withstand the same test as when new.

If the deterioration of insulation followed a known rate of reduction in dielectric strength under operation conditions it would be a simple matter to set up a schedule covering the expected strength over a given length of time and the corresponding test voltages. In that event tests would serve principally to search out possible mechanical injuries. But the life of insulation is an indefinite term. It contains, however, the element of time and is dependent upon the temperature to which it has been subjected as well as various other

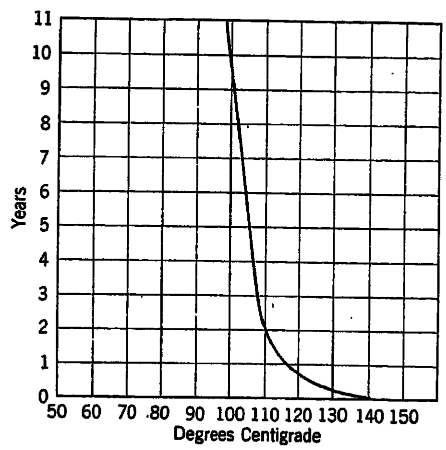


Fig. 1—Insulation Tests of Electrical Machines in Service

conditions. In their paper on *Temperature and Electrical Insulation*,[3] Steinmetz and Lamme offered curves showing the relationship between length of life and temperature. Fig. 1 is a reproduction of their curve for Class A insulation. The curve for Class B insulation is of the same type. In these curves the length of life does not refer to the time in which the insulation will entirely lose its dielectric strength but as explained by the authors it has to do with the time at which the fabric or varnish becomes brittle and loses its mechanical strength. The life of an insulation in this sense may be entirely spent and yet retain approximately its original dielectric strength provided it has not been mechanically injured. Therefore, the term life as used in connection with accepted practises in testing insulating materials is somewhat misleading for it does not have reference to its dielectric property. The useful life of an insulation is not dependent upon the effect of temperature and time alone but also upon various

3. A. I. E. E. Trans., Vol. XXXII, 1913, pp. 79-89.

other conditions which tend to destroy it by mechanical forces. It is reasonable then that the insulation should be expected to last an indefinitely long time if it were protected against mechanical injury barring, of course, the effect of excessively high temperatures which would change the chemical composition or the structure of the insulation. The common causes of mechanical injury are vibration and distortion due to temperature changes and electromagnetic forces. These causes are present in widely varying degrees due to differences in design and conditions of operations and it is, therefore, impractical to formulate a relationship between length of service and dielectric strength to indicate what may be expected of any particular type or class of insulation in a given kind of machine or to define a maximum test voltage that the insulation could be expected to withstand at given intervals.

But supposing that a relationship between time of operation and dielectric strength could be established for certain average conditions, would it be of much practical value in a maintenance program? Conceivably it might be used as a guide to indicate the time when the windings should be reinsulated or replaced and what test it would withstand at any given time. It does not seem reasonable that a maintenance program should call for a test at say double voltage at the end of one year of operation, 50 per cent overvoltage at the end of 3 years, and 25 per cent overvoltage at the end of 5 years if the 25 per cent overvoltage test gives sufficient insurance against breakdown during the ensuing period of operation and if uninterrupted service is of equal importance. It would appear, therefore, that the value of the test voltage should be chosen with respect to the maximum stresses that would occur in operation rather than the probable actual dielectric strength. If, then, operating conditions are defined it becomes possible to set up a value for the test voltage.

It is seldom that machines are required to operate at voltages above 10 per cent over normal. This is especially true of machines connected to low-voltage systems. Important a-c. generators are provided with overvoltage relays to limit abnormal rises of voltage to approximately 25 per cent overvoltage and the neutral point is grounded for relay protection of the winding. Under normal operating conditions the voltage between machine terminals and ground is 58 per cent of the rated terminal voltage and under an abnormal condition of 25 per cent overvoltage this would be increased to approximately 72½ per cent of the rated voltage. An insulation test between winding and ground at rated voltage would impose a stress approximately 38 per cent greater than would occur when the machine voltage is raised to 25 per cent above the rated voltages. Such a test would insure a sufficient margin in the dielectric strength for further service without risk of failure to ground. Such a test would not, however, be a check on the condition of the insulation between phases but by testing one phase at a time with the other two phases grounded the insulation between coils in different phases would be tested at a voltage higher than the operating voltage in proportion to their distance in the circuit from the terminals. The insulation between terminals would not be tested at a higher voltage than the normal operating voltage and there would not be any assurance of a margin in the condition of the insulation between phases.

By separating the tests and making one test to ground and another between phases, two different values of test voltage could be used, each suitable for its purpose without imposing an excessive voltage where a lower test is sufficient assuming that the rated voltage is sufficient for testing between winding and ground a higher voltage than could be chosen to test between phases which would produce the same relative overvoltage stress on the insulation between phases. This result would be obtained by testing between phases at 175 per cent of rated voltage with none of the phases grounded.

To those who are accustomed to associate all insulation test voltages with those that have been prescribed in the A. I. E. E. standards for new machinery, the proposed test voltage between winding and ground appears to be ridiculously low yet it fulfills the requirements. The proposed practise is not new since it has been used by the service departments of at least two of the large manufacturing companies when overhauling electrical apparatus for their clients. Their experience is that this program has produced satisfactory results. On the other hand the author has received many expressions of opinion from engineers who are responsible for the operation of electrical apparatus that the test voltage to ground should not be less than 150 per cent of the rated voltage. Nevertheless many instances have been cited of machines which have given years of service after the insulation has reached such a condition that it would unquestionably have failed at test voltages only slightly higher than normal. It is a commendable practise to maintain apparatus in a first class condition but the greater reliability thus obtained does not always justify the cost. It is to be expected that as the test voltage is increased so is the frequency of repairs increased.

It is generally agreed that high-potential tests of insulation do lasting damage to it and for that reason all unnecessary tests of that nature should be avoided. A maintenance program that would omit high potential tests and substitute some other test that would not damage the insulation is by far the better. While comparatively little experience has been obtained it is believed that the periodic measurement of the insulation resistance of windings would give warning of dangerous conditions that may develop. A program covering the determination of the insulation resistance at regular intervals during the period of operation and high potential tests at times of overhauling when it would be convenient to make repairs should afford a fair degree of

freedom from inopportune shut-downs. The adoption of such a scheme would call for the exercise of much good judgment to obtain satisfactory results and in the absence of sufficient practical experience upon which good judgment can be based it is advisable to set up a recommended practise in applying high potential tests.

In view of the divergence of opinion regarding the value of test voltage that should be used as between rated voltage to ground and 150 per cent rated voltage a compromise might be struck at 125 per cent of rated voltage. In order to use the same form of rule as now used for new machines a constant of 500 volts may be added to this value. Such a rule would then read as follows:

> It is recommended that except for distribution transformers which may offer hazards to life, the dielectric tests of windings of machines in service shall be at 125 per cent of the rated voltage plus 500 volts. When it is convenient to separate the phase windings a further test between phases shall be made at 175 per cent of rated voltage plus 500 volts.

## Conclusion

The theme of this joint paper, therefore, is a plea that the Institute enlarge its scope of operations to meet what appears to be a genuine need of the membership at large. This need is two-fold: first, for a basis of insulation testing prior to initial operation, and second, a similar basis for the operating period of this apparatus.

As a basis for consideration and possible acceptance by the Institute, these recommendations have been offered, which are in brief:

> For initial operation—a voltage 85 per cent of factory test for one minute, based on connected equipment receiving lowest factory test, and

> For maintenance—a test to ground of 125 per cent rated voltage plus 500; and between phases, 175 per cent plus 500 volts—supplemented by periodic measurements of insulation resistance.

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## Discussion

**J. S. Henderson:** The additional standards on insulation tests proposed by Messrs. Gilt and Barns are necessary. Even though the machinery has received insulation tests at the factory in agreement with the present Institute rules, an additional test should be applied after the apparatus is installed as insurance against damage since leaving the factory. In fact, most large pieces of apparatus are tested today after installation, and it is now only a question of what the voltage should be for this final test. I hope there will be a full expression on what is considered an adequate voltage.

The proposed standards for tests on machinery which has been in service are of a different type and are really an operating guide. The value of 125 per cent proposed agrees closely to the recommendation of 60 per cent of the original test voltage which the Westinghouse Company has made for some years.

One other point touched on by the authors is the question of drying out apparatus before making the insulation test. There has been a tendency to wish to omit this drying out, particularly on large turbine generators. It is undesirable to run these machines on short circuit because of possible damage to the steam turbine if it is operated at no-load for any length of time.

Generally, these units are handled with great care and any moisture on the winding is likely to be only on the surface of the end turns and not on the buried part of the coil. This surface moisture may cause failure because of reduced creepage resistance. In general, two courses are open. If the machinery has been shipped and erected in dry weather, the dry-out may be omitted. If, however, the shipment has been made in rainy weather, or if there has been damp air in the station during erection, the recourse can be made to using space heaters under the end windings and the insulation resistance watched until it becomes constant at constant temperatures.

The paper suggests as a test on new apparatus after erection, 85 per cent of the value specified in the A. I. E. E. rules. This is a test for one minute. The question of the length of the test should be carefully considered. Most high-voltage disturbances on a system are of short duration and last a second or less. The probability of failure while making the test, after the machine has once passed the standard high-voltage test, is likely to be due to decreased surface resistance caused by dust or moisture on the surface of the end turns. A high voltage for a second would not establish a carbon streak on the surface while a high voltage for a minute would. For this reason, a test for a minute is many times more severe than a high voltage for a second in service. It would seem, therefore, that the time could be reduced to a half minute and still leave a margin for insurance.

**J. R. Dunbar:** It is well recognized that new equipment that is assembled on the premises of the purchaser should receive the same tests as though it had been previously assembled and tested at the factory. However, when the purchaser decides to place the machine on commercial load without complete tests it does not seem reasonable to require it afterwards to stand up to the same tests as new apparatus, since there is bound to be a certain accumulation of dust and a consequent reduction of insulation resistance. It would, therefore, seem desirable that the Institute Standards include recommendations for voltage tests for the conditions described. These tests should be lower than the tests on new apparatus, yet higher than the maintenance tests described in the second part of the paper.

Second-hand and repaired equipment requires tests lower than the tests for new apparatus and possibly higher than the maintenance tests. In the case of repaired apparatus, when new coils are installed they should certainly be tested for the full voltage of new apparatus, but it is not to be expected that the complete winding containing old coils would stand up to the full tests. It is possible, in this case, that the maintenance tests would be sufficient, but it seems to me that this point is worthy of further consideration.

In the case of apparatus sold as second-hand, the purchaser naturally cannot expect the same life from the insulation as would be obtained from a new machine, but it would be reasonable to expect tests which would indicate, to a certain extent, the probable remaining life of the insulation. Such tests would have to be higher than the maintenance tests described by Messrs. Gilt and Barns in the latter part of the paper, but lower than the tests on new equipment.

A maintenance test between phases at 175 per cent of rated voltage with none of the phases grounded is mentioned in the paper. On high-voltage transformers with rated voltage of the order of 220 kv., 175 per cent of the test voltage is nearly 400 kv. The testing transformers for this voltage are usually designed with one terminal grounded, so that for high test voltages the proposed test would not be feasible with present testing equipment. Furthermore, it is undesirable during any test to apply voltages to a winding which is floating with respect to ground. In order to reduce the possible voltage stresses to ground where tests are applied between phases, with none of the phases grounded, it would be necessary to use a testing transformer so designed that

the center point of the winding could be grounded, or to use a special grounding transformer. If this is done the voltage stress from either phase to ground would be half the test voltage between phases. It is recommended that additional standards be established to govern all the foregoing conditions.

**F. E. H. Mowbray:** Mr. Gilt's suggestion that the prescribed test for insulation after assembly in the field be 85 per cent of the normal factory test is, I think, very good, but it seems to me it would apply only where the apparatus had been completely assembled and tested for insulation according to A. I. E. E. Rules, in the factory previous to shipment. I do not quite see how this insulation test could be reduced by 15 per cent, say in case of a large a-c. generator wound on the customer's premises. The customer would naturally expect the apparatus to fit the regular prescribed test.

However, most manufacturers have had cases where, as soon as the apparatus was assembled, it was put in service and then after several weeks or months, the customer would request that the final insulation tests be made. Under this or a similar condition, I should think that the maker would be entitled to some reduction and think that 85 per cent of normal is quite fair. In some cases that have come under my observation, a considerable amount of cement dust and other dust had collected on the windings and it was almost a physical impossibility to get it all out. Under this condition I do not think that the above reduction should cause objection.

Regarding the maintenance tests, I regret to say that I differ with the writer considerably. His proposed scheme means that each time a three-phase generator, for example, is tested, four insulation tests are to be made, *viz:*

(1) Ground test of complete winding
(2) Between phases $A$ and $B$
(3) Between phases $B$ and $C$
(4) Between phases $C$ and $A$.

In the first place a ground test on a large generator taking in the whole winding is likely to give unequal stresses in different parts of the winding due to capacity effect, and requires considerable care in its application and method of control. The average power company is not equipped with the proper apparatus for this test. Aside from this feature the proposed value of 125 per cent of normal rated voltage, plus 500, does not give the different voltage ratings the same per cent test above normal voltage. For example, a 2200-volt machine would get $2200 \times 1.25 + 500 = 3250$ volts or 47½ per cent above normal, while a generator rated at 13,200 volts would get $13{,}200 \times 1.25 + 500 = 17{,}000$ volts, or approximately 28½ per cent. Personally, I think that a flat rate of 30 per cent would be a more fair distribution and the time element should be not more than 15 sec.

With regard to the between-phase tests of 175 per cent of normal + 500 volts, this means that a 2200-volt generator would get $2200 \times 1.75 + 500 = 4350$ volts between phases. In the first case I consider this too high, and secondly it means that a special testing transformer with the middle point of the high-voltage winding grounded, or two testing transformers, would have to be employed. Again this test puts on another insulation test to ground of 2175 volts, repeating the ground test at roughly normal voltage, which in itself is approximately 70 per cent above the normal star voltage, assuming a grounded star. I also doubt very much if the average power company would wish to put a between phase test of from 79 per cent to nearly 100 per cent above normal terminal voltage, on their apparatus. In my opinion, the two tests should be combined and then the number of tests made would be three in number, with capacity and stress conditions identical, *i. e.*, ground two phases at both star and line terminals, and test the other phase to ground, thus getting a ground and between phase test in the one operation, and repeat this for the three phases.

The periodic checking of the insulation resistance while the apparatus is warm is a good check.

In conclusion, I should like to see the last paragraph of the article changed to read as follows:

For maintenance—a test to ground on one phase at a time with the other phases grounded of 130 per cent of normal line voltage for a period of 15 sec.

Generating apparatus which meets this requirement should be in a reasonably safe operating condition.

**D. A. McKenzie:** While the title of this paper is general, the impression upon reading it is that it applies chiefly to equipment which is expected to operate at moderately high voltages and above. This rather definitely confines its application to the main powerhouse generators as well as to the main station transformers (either step-up or step-down) and this is especially emphasized since the major part of the discussion is centered on fibrous insulation, its deterioration due to handling and due to the absorption of moisture, while still new, as well as its deterioration in service, due to heating from loads, natural aging, and voltage strains impressed during service, etc.

To an operating engineer, the suggestion that new equipment before being placed in service in the station is to be already considered as having deteriorated from factory conditions, is rather startling. While the later editions of the Standards of the A. I. E. E. specify that "Unless otherwise agreed upon, high-voltage tests shall be made at the factory," they also state that the "Commercial tests shall be made with the completely assembled machines," and it would seem that this last clause will have the effect of automatically transferring these tests to the power house of the customer, for the larger machines, which are assembled in place. However, the chief feature in which the purchaser or operator of electrical machinery is interested, is its ability to perform satisfactorily under operating conditions and the factory tests are of value or interest to them only as indicating its probable performance in operation. If this equipment is so designed that it will deteriorate in shipping or handling or installation so that after installation in the customer's powerhouse it will only withstand 85 per cent of the factory test, it would seem advisable to change either the design of the equipment or the method of care till ready for service, or to increase the value of the factory test, since it is believed that the safety factors intended by the Standards should be attainable when the equipment is ready for service. This is especially true since in most cases, these large pieces of equipment are under the care of the manufacturers until being actually placed in service. It would also seem that even while some large pieces of generating equipment might be assembled at the manufacturers shops, the facilities for properly drying it out are not available until it has been erected in its place in the powerhouse and connected up to its prime mover.

It would seem that the operating engineer is interested in having as high a safety factor as possible when the apparatus is to be placed in service, and is desirous of having acceptance tests on equipment made at that time unless there is good assurance that the equipment may be placed in service in equally as good condition as when tested at the factory.

With regard to the matter of testing the insulation of equipment in service, it will be found that most operating engineers are rather averse to running any risk of damaging machines or rendering them unfit for service for even a short time although information regarding the condition or progressive deterioration of machines would be valuable in arranging for reserve capacity.

Some years ago, the matter of making regular or periodic high-voltage tests on the generators of a rather large generating station in the Niagara district was considered. It was found that fairly expensive equipment would be required and moreover, owing to the high load factor on the plant it was necessary to keep all machines in service as continuously as possible and it was therefore not desired to run any risks of breaking down the insulation. Consequently the matter was dropped, without having decided on a ratio of test voltage to operating voltage.

It is, however, highly desirable to have some idea of the condition of the insulation of machines, especially of generating equipment, and more particularly with the large units now in general use.

We have in the Standards, specifications covering the minimum value of insulation resistance for the armatures of generators, together with the statement that while the insulation resistance may afford a useful indication as to whether the machine is in a suitable condition for the application of the high-voltage test, it shall not be considered a requirement of the Standards. They also state that these insulation resistance tests shall, if possible, be made with a d-c. voltage of 500.

This last suggestion seems to point rather definitely to a very convenient and useful instrument known as the "megger."

Now it would seem that in addition to information at a certain definite time, it is almost equally important that relative information on the same subject be secured throughout the complete operating history of the equipment to show the rate of change, if any. Therefore, if some definite measurement could be made on the machinery at monthly, or bimonthly periods, in exactly the same manner each time, it would seem that the rate of change would provide some criterion as to the condition of the equipment, even though any measurement by itself were not definitely accurate.

It is believed that many companies, by the use of meggers, have been able to observe the progressive deterioration of insulation on motors, and it would seem that similar measurements of the insulation resistance of generators would be of somewhat similar value. It might be necessary, however, to use a voltage more comparable with the generator terminal voltage than is obtainable from the megger.

With the advent of the kenetron it should be quite convenient for the manufacturers to supply compact equipment whereby the insulation resistance could be measured at a d-c. voltage approximately equal to, or if necessary greater than the terminal voltage of the machine. Similarly, it might be possible to obtain measurements or information regarding the performance of this same insulation under the application of alternating voltage using equipment which is readily portable. This will limit the voltage to a value which could not possibly result in any injury to the insulation of the machine, but might require special instruments.

Such a device should preferably be portable so that it could be readily moved directly to each machine with no appreciable length of high voltage testing leads or necessity for alterations in existing powerhouses. It should also be of small kv-a. capacity so that its supply voltage could be readily obtained at any part of the powerhouses. It would, of course, be self-contained as regards voltage regulation and complete with all necessary instruments. At present it would seem that a maximum voltage of about 15,000 direct current would be ample for this type of equipment for the normal voltage rating of most generating equipment in service. The opinion of the manufacturers as regards the value of such measurements and the possibility of obtaining the equipment would be of interest to many operating companies.

It is noted in the third paragraph on the fifth page that it is assumed that the neutral point of a-c. generators is grounded for relay protection of the winding. Many large generators, recently installed, are now operating with the neutral completely isolated and this tendency is apparently spreading.

In closing, the application of a high voltage to a winding does not give any definite information regarding the condition of that winding except that it was sufficiently strong at that instant to withstand that stress. It is conceivable that the immediate reapplication of this same voltage might result in breakdown. It would appear that if definite values relating to certain characteristics of the insulation were periodically obtained or measured during the history of the apparatus, and on a sufficiently large number of items of each class of equipment, it should be possible to evolve some law connecting the condition of the equipment with the measurements obtainable.

**F. D. Newbury:** I should like to discuss very briefly the first part of the paper covering the recommendation that the Institute adopt a standard for insulation test of generators before operation. I think that is an excellent suggestion and the Institute should take such action.

I am not quite so sure of the wisdom of the Institute's taking action regarding the second recommendation, that is, the establishment of standard tests during service. I think that part of the problem has some of the elements of insurance. The frequency of the test and the value of the voltage depend somewhat on the insurance value the operator wishes to take out to guard against breakdown at inopportune times. If he wishes to take out heavy insurance against breakdown, he will make a high-voltage test at frequent intervals. If he wishes to take greater risks, he will adopt a lower test or a longer interval. Judgment on the part of the operating company will, I think, be a controlling factor in the determination of the test program during service. For that reason I question whether the Institute could adopt any single value that would fit all situations and be satisfactory to all operating companies.

**P. L. Alger:** I believe the recommendations of Messrs. Gilt and Barns are reasonable in all respects. I consider, however, the complication of testing old machines, both phase to phase and phase to ground, undesirable, and I think it preferable to apply a single test of perhaps 1⅛ times normal voltage on each phase to ground separately, with the other phases grounded.

I should like to make some suggestions about the periodic tests on machines in service. The quality of insulation, after it has been in service for a long time, depends on three things: (1) on the amount of dust and dirt covering the end windings, and (2) on the internal condition of the insulation in respect to moisture and (3) in respect to air spaces. If the leakage current on direct current or the power factor on alternating current is measured periodically, it cannot be used as a guide to the changes in insulation quality, because of the variations in moisture present. If, however, in every case, measurements are made both cold and hot at definite temperatures, the differences between the measurements can be taken as a measure of the leakage current due to moisture, and the low temperature reading corrected for variations in the moisture can then be taken as a direct measure of the insulation quality. An increase in this current with time should then serve as a fairly reliable indication that the insulation is deteriorating. If an inspection of the windings showed the surfaces to be clean, there would then be a strong probability that the insulation was weak.

To separate by test the effects of air spaces in the insulation from other defects, readings can be taken at both high and low voltage at the same temperature. If the leakage currents here are not proportional to the voltages, air spaces are indicated, and hence periodic checks on these measurements will enable the increase of air spaces with time to be watched.

By thus increasing the number of measurements made periodically, I believe it is possible to develop a fairly reliable means of recording the deterioration of generator insulation.

**W. F. Dawson:** All of these discussions have mentioned the possibility of dirt and moisture in the windings when high-potential test is applied. Of course, everybody who knows anything about high-voltage insulation knows that those things are detrimental and I believe the existing A. I. E. E. rules state that before any high-potential test is made at least the surface condition of winding should be prepared by cleaning. I have seen, particularly in turbine alternators, great quantities of all sorts of dust. It may be dry dust that floats in from the roadway or is sucked in by the ventilation system, or it may be dust contaminated with oil. It may be good clean oil but probably it is not. It is loaded with coal dust and everything that it should not have. Of course, there is no sense in applying a high-

potential test to a winding in that condition. I have seen dust a quarter of an inch thick, not occasionally, but frequently. Any periodic testing ought to be preceded by a thorough cleaning out of the winding.

We have a prescription for cleaning,—first by blowing out, then by washing with gasoline or carbon tetrachloride if gasoline is considered an explosive danger. Clean thoroughly and dry and apply a coat or two of good air-drying varnish and know that the winding is in condition to receive a high-potential test before applying it.

**V. M. Montsinger:** There have been several discussions on rotating machinery. I should like to say a few words with reference to transformers.

In regard to the first part of the paper, there is no question but that some value or values should be set up; whether it should be 85 per cent or not is a question. We may have a condition where a transformer is shipped a long distance and perhaps collects a little moisture. If that transformer is put in service at normal voltage, the moisture may not do any harm. It may be driven out in two or three days and the transformer would be all right. On the other hand, if it is tested at too near the factory test values, it might break down. For this reason I do not favor setting the test value too near the factory test.

In reference to the second part, I think the conditions are somewhat different in transformers from those in rotating apparatus because we have oil which can be tested periodically. Practically all operating companies test their oil and if they find it is getting low in dielectric strength, measures are taken immediately to increase the dielectric strength and for that reason I doubt if there is as much need for rules for testing transformers periodically while in service as for other types of apparatus.

We have had this question before the committee for some time but have been handicapped because we do not know what the practises are and what the operating companies would like to have. So we were hoping that this paper would produce experiences of the operating companies. If we knew that, we would be in a much better position to judge whether such rules should be incorporated in the A. I. E. E. Standards.

As Mr. Newbury pointed out, there should be rules relating perhaps to the first part but whether there should be to the second part depends considerably on whether there is really any need for it.

**C. E. Skinner:** (communicated after adjournment) The manufacturers of electrical machinery and equipment have always, and rightly so, taken the responsibility for any damage which may occur to new machinery in the factory or at the time of installation due to the dielectric test, covered by the contract between the manufacturer and the purchaser. The responsibility for the repair of breakdowns which may occur as a result of the test after apparatus is ready for service as proposed by Messrs. Gilt and Barns may, in some cases, be open to question. The responsibility for breakdown after the machinery and equipment has been in service for some time should be distinctly that of the owners or operators. When machinery is returned to the manufacturer for repair there should be a definite understanding as to the dielectric test to be made and the owner of the machinery should be responsible for repairs made necessary by the application of the stated dielectric test. When the owner himself does over-hauling and repairing, he will, of course, assume the responsibility for repairs made necessary by breakdowns caused by the dielectric test applied.

It is suggested that the operators in consultation with the manufacturers should lead in taking the responsibility for the establishment of tests after machines are in service and for the test to be made on repaired apparatus. In the case of repaired apparatus, very much depends on the nature of the repairs, the urgency of the case, etc., as to whether the dielectric tests should be severe or not. A specific historical case will illustrate the point. A large armature was returned to the manufacturer for repair of broken leads. The armature was in bad condition with regard to dirt and after as thorough a cleaning as possible a dielectric test at normal operating voltage resulted in breakdown. Further difficulty was experienced in attempts to repair this breakdown with the final result that a complete new winding had to be put on this armature. The owner of the machine was extremely indignant, both at what he considered excessive time required to make the repairs and at the cost of the same, stating that he required nothing done except the repair of the broken leads. Definite rules or understandings before such repairs are begun would obviate similar difficulties.

With regard to the time of application of test, one minute for high voltage apparatus is entirely too long for tests after installation, and the time of application should be reduced as the voltage of the apparatus is increased. It is very doubtful if a test of more than two or three seconds should ever be applied to transformers of 100,000 volts and above after they have been in service for some time. Such tests have been known to cause damage to the insulation, resulting in breakdown later, and the test is applied to a part of the insulation which is rarely broken down as a result of service.

**C. M. Gilt:** There seems to be quite a consensus of opinion, on the first part of the paper, that it is advisable to establish some rule for testing and that the standard for tests of new equipment before placing in service should be somewhat less than the present Institute standard. Whether it should be, as some people think, the full voltage test for a minute or 85 per cent for one minute, or 85 per cent for a half minute, or, as I believe the British rule is, 75 per cent for one minute, is a question on which there is a difference of opinion.

It was my intent that this test was not to apply to equipment that was delivered and assembled without receiving the full factory test before shipment. I think most of the operating companies, and I believe most of the manufacturers as well, would prefer to see that equipment receive the full factory test once before placing in service. Where it occasionally happens that equipment is placed in service before it receives the full factory test and is operated for some time, it would seem obvious that the test then made should be at a somewhat lower value, probably at a value subject to special negotiation depending upon the time in which it had actually been in service. Perhaps the 85 per cent test is a reasonable compromise on that.

In regard to the second part of the paper, my opinion is that equipment that will not safely stand a 50 per cent overvoltage test is not actually safe for operation under modern conditions of heavy power supply. I would not recommend that as a regular monthly routine test. As far as our own company is concerned, we have established the practise that, when it is necessary to test equipment that is in service, but not as a regular routine test, we test it at 50 per cent above voltage because we would rather have the equipment fail on test than in service. This of course applies only to apparatus, not to cable tests.

**B. L. Barns:** The suggestion was made by Mr. Newbury that the test is in the nature of an insurance policy rather than just to satisfy curiosity, and therefore the conditions obtaining in the power system or the importance of uninterrupted operation will determine largely the amount of insurance you want to take out. This might be taken care of by raising the test value and shortening the time. Usually any disturbance that occurs on a system to impose an overvoltage on a machine winding is for a very short period of time. If we hold the voltage test for a whole minute, there is much more chance of doing damage and apparently the one-minute condition does not reproduce what might happen in actual operation.

Messrs. Dunbar and Skinner have mentioned dielectric tests on apparatus that may be returned to the factory for repairs. The establishment of standard test voltages for such occasions

was not in my mind when preparing this part of the paper. I was interested more in the possibility of establishing a recommended practise to be followed by the users of apparatus in a maintenance program and there was no thought of suggesting any responsible interest of the manufacturer in such tests. I agree fully with Mr. Skinner's opinion as to the responsibility for failures resulting from tests made after the apparatus has been in service. As to the establishment of rules for use when apparatus is repaired leaving all or part of the old windings in place, I believe, as has been remarked, that there are so many various conditions and circumstances that such a procedure would be impracticable.

I was interested in Mr. Dunbar's remarks about the desirability of some standards for second-hand apparatus. It was this idea that was first proposed to the Committee on Electrical Machinery. It was suggested that with some standard to govern tests on second-hand machines not only would the purchaser be benefited but perhaps the purchase of used apparatus would be stimulated and the industry as a whole would gain. I canvassed a number of established dealers in second-hand apparatus and was surprised and gratified that many make it a practise to test all machines according to the A. I. E. E. rules covering dielectric tests on new machines.

Messrs. Dunbar and Mowbray have pointed out some objections to the suggested tests between phases. These were suggested for the purpose of obtaining the same degree of assurance regarding the insulation between terminal coils as would be obtained by testing a phase winding to ground. Both tests would have the same margin over the corresponding operating voltage in the case of machines operating with grounded neutrals. The practical difficulties that would be met in applying such tests to high-voltage apparatus constitute a real objection to the proposed procedure. It was intended that this would apply only to apparatus having a rated voltage not exceeding, say, 15,000 volts. As Mr. Montsinger has pointed out, oil-insulated transformers should not be included in these tests since it is possible to determine the condition of the oil by testing a sample.

As for Mr. Mowbray's objection to adding an arbitrary 500 volts to 125 per cent of the rated voltage, it does not appear that this should be more objectionable than the additional 1000 volts that the A. I. E. E. rules specify for new apparatus.

The principal purpose of this part of the paper was not to establish definite values of test voltages for apparatus in service, but rather to present the question of whether it is desirable that the Institute should enter this new field by establishing recommended practises for the maintenance of apparatus in service. If an affirmative decision is reached the choice of test voltage values could then be discussed.

# Influence of Temperature on Large Commutator Operation

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*Synopsis.—The paper discusses the operating characteristics of large commutators. It is pointed out that many cases of blackened and burned bars are caused initially by a slight roughness which causes irregular commutation and a slight sparking which when once started gradually becomes worse. The requirements of the surface of a high-speed commutator are discussed, showing that radial variations of the order of one ten-thousandth of an inch between adjacent bars may give serious trouble. Some causes of roughness are discussed, showing that temperature is a major factor, but that redesigning the commutator to reduce the temperature does not necessarily cure the trouble, and may actually give poorer performance. Some methods of commutator testing are described with typical records showing the performance of various types of construction.*

*The question of safe temperature limits is then discussed. It is maintained that the question of the permissible temperature limits is entirely unlike that of insulated windings, since the materials used are not injured by temperatures considerably above the present limits. The permissible limit of a commutator is determined by its mechanical construction. Low temperature limits may lead to illogical designs, since large heat dissipating surfaces are required which may give higher stresses and larger expansion effects, which may be more harmful than higher temperatures. A temperature limit based on operating characteristics would be more satisfactory.*

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DURING the past five years, considerable experimental work on large commutator performance and characteristics has been under way and many different types of construction have been investigated and compared. Much information of value to designers has been accumulated and this information has led to a clearer understanding of the desirable characteristics of commutators and the influence of operating temperature on performance.

While this work was undertaken primarily to secure information for design, it has yielded information of a more general interest to operating engineers, and it is this latter phase of the subject that will be treated in this paper. Permission to publish the information in this paper was granted the authors on condition that it would be confined to information relating to commutator performance and operation, but even with this limitation, it is believed that the paper will make available to the profession information of considerable value. The paper, as indicated in its title, will deal with the subject from the standpoint of large commutators found, usually, only in power station and substation machinery.

A commutator is fundamentally a device for accomplishing a very rapid switching of the current in the armature coils. A rate of 5000 bars or switching operations per second is very common. The only practical device for this purpose has been the conventional form of stationary brushes, making contact with a rotating surface composed of copper bars electrically insulated from each other. Copper must be used because of its electrical, thermal, and chemical properties, but its mechanical properties are far from ideal, particularly its low elastic limit and tendency to "creep" or slowly yield at comparatively moderate stresses. Mica has exceptional insulating and heat resisting qualities, but its mechanical performance is erratic, and a large amount of research has been required to develop manufacturing methods which give the most desirable mechanical characteristics and uniform quality.

A large variety of constructions has been proposed for holding the bars, but in nearly all commercial designs, the bars are held by some form of shrink ring, giving a radial clamping, or by V rings acting at the ends of the bars. There are many other important features, such as whether the bar is held by clamping against a bush or drum, (drum bound), or by clamping against an arching pressure acting between the bars, (arch bound), or held by a wedging or V binding effect at the ends of the bars (V bound). There are also various means of providing for the thermal expansion.

Because of the nature of the materials and the extremely smooth surface required, the commutator problem is very unlike other problems with which engineers are familiar, so that the problem does not lend itself to theoretical mechanical analysis alone. The theoretical work must be supplemented by a large amount of research work and testing on both the materials and assembled commutators. From this, the best combination of factors for any given conditions can be determined, but this is a design question outside of the scope of this paper. In all types of construction, the heating of the commutator is inevitable because of the mechanical friction of the brushes and the electrical loss at the contact. In most electrical apparatus, the permissible temperature is determined by the temperature at which the insulation is permanently injured, but in large commutators, the materials are not injured by temperatures considerably above the limits commonly applied. There are other very important effects of temperature, but this is ordinarily not in the nature

1. Both of the Power Engineering Department, Westinghouse Electric and Manufacturing Co.

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of a definite limit fixed by the materials, but rather a limit based on the construction features of the commutator, and particularly its ability to withstand the change in dimensions caused by a change in temperature. For this reason, the authors believe that a different type of temperature limitation should be used for commutators from that used for apparatus where the temperature is limited by the point at which the material is injured. This paper will be devoted to a general discussion of the nature of the temperature limitation of commutators.

The primary function of the commutator is to make contact with the brush. A certain variation in the contact drop is permissible, but, as will be shown later, some commutators reach a condition where the brush cannot transmit direct current to the bars without vicious sparking, even when no commutation is involved. Under such conditions, satisfactory commutation is impossible. With the bars moving at a speed of a mile a minute or more, the commutator surface must be very smooth if the brushes are to maintain uniform contact with all bars.

Commutator bars deflect radially under centrifugal force and temperature. Since these bars are necessarily separated by insulating material, the bars may deflect unequally. If adjacent bars deflect unequally, an abrupt change or bump in the surface is produced. In considering the effect of commutator roughness, it is necessary to distinguish between eccentricity and other gradual irregularities, such as may occur either

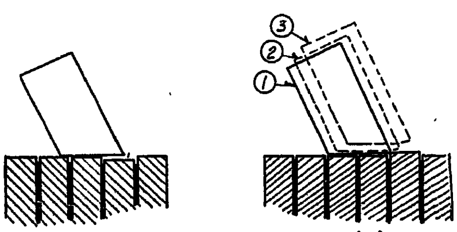



Fig. 1—Action of a Brush Passing Over an Irregular Surface

a—Shows the impossibility of making contact with a low bar

b—Illustrates the tendency of a brush to jump when it strikes a high bar which may be very serious at high peripheral speeds

in a slip-ring or commutator, and these abrupt changes from one bar to the next which are peculiar to commutators. Ordinarily, a brush will follow an eccentricity of several mils without losing the contact between bar and brush, but when the roughness occurs abruptly from one bar to the next, a very slight irregularity will give trouble. For instance, if one bar is lower than the adjacent bars, the brush usually covers several bars and therefore cannot make contact with a low bar. If a bar is higher than adjacent bars, the bar will strike the brush and cause it to bounce away from the commutator surface. Fig. 1 illustrates the action of a brush passing over a low and a high bar.

## Requirements of a Commutator Surface

The exact nature of the mechanism by which current is conducted between the bar and brush is somewhat of a controversial question, but at any rate there should be approximate mechanical contact between the bar and brush at least at certain points on the brush surface. Theoretically, there may be a separation even at these points, due to an air film and fine dust particles, but this is very minute. Whatever the exact nature of this contact, the commutator surface must be such that this contact can be maintained uniformly and without interruption. As has been pointed out, a commutator surface is composed of bars separated by insulation,

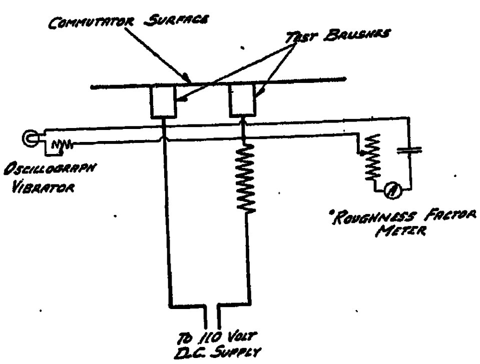



Fig. 2—Wiring Diagram for Brush Drop Test

Power circuits are shown by heavy lines and instrument circuits by light lines. The oscillograph and roughness-factor meter are both shown connected

so that there is the possibility of relative movement between adjacent bars giving an abrupt break in the contact surface. A very minute break in the surface can be absorbed in the surface deflections of the materials in contact, but with a greater variation a movement of the whole brush is required, and if the speed is high this will give an interrupted contact.

## Measurement of Commutator Surface Conditions

Any progress in science or engineering is dependent first on knowledge of the facts. Suitable measuring instruments and methods are fundamental to progress. This study of commutator design and performance made very slow progress until a satisfactory device for measuring commutator surface conditions was developed.

Since the condition of a commutator surface may change with speed and temperature, it must be measured at full speed, and operating temperature. The measurement of a variation of the order of one ten-thousandth of an inch between adjacent bars running at a surface speed of over a mile a minute is beyond the range of ordinary instruments. Because of the difficulty in measuring this mechanically, and since we are primarily interested in the effect of the surface condition on the brush contact drop, the measurement of this contact drop seems to be the most logical means of determining the surface condition. Fig. 2 shows the wiring diagram for a scheme of doing this which

will be called the "brush drop test." Fig. 4 shows a photograph of a small commutator set up for the test. In this test, two or more brushes are mounted in line so as to make contact with the same commutator bar. Current is passed from one brush to the bar and back through one or more brushes in series with sufficient resistance, so that practically constant current is maintained. The double-contact drop is recorded by an oscillograph. In this way, the surface condition of the commutator can be studied during heating and

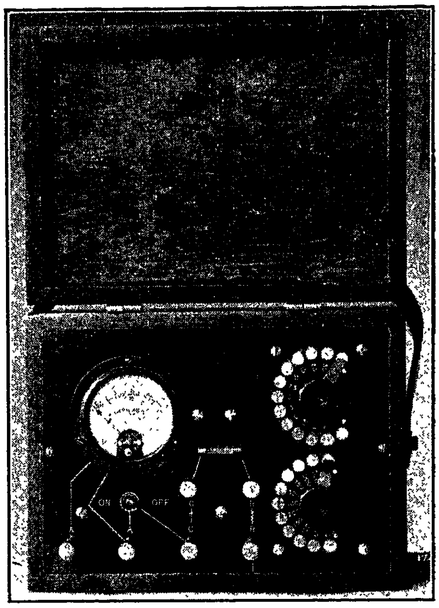


Fig. 3—"Roughness-Factor" Meter

cooling and for various speeds. If a timing contact is used as a second record on the oscillogram, the position of any roughness of the commutator can be located.

If it is not necessary to locate the bars giving trouble, but only to determine the average operating condition of the surface, then the "roughness factor meter" may be used instead of the oscillograph. Fig. 2 shows the wiring diagram for the brush-drop test with both the oscillograph and the "roughness-factor" meter connected to measure the brush drop. Fig. 3 shows a photograph of the "roughness-factor" meter. It consists of a large capacity condenser, variable resistance, and thermal milliammeter all in series. The condenser has sufficient capacity to offer only a negligible impedance to fluctuations in brush drop. For a perfectly uniform contact, the reading is zero, but in case the brush drop fluctuates, the current through the meter is limited by the resistance in series. The product of the resistance in ohms by the current in amperes is called the "roughness factor." It is the r. m. s. value of the fluctuations in brush contact drop in volts and is a measure of the surface condition of the commutator. With a good surface, it should be 0.15 or less, while with a bad surface, it may rise to several volts.

This method of measurement does not, of course, give bar movements in inches and in the case of eccentricity, a movement of several mils may give no indication, but since we are concerned with the uniformity of the contact drop, it does indicate the things in which we are interested. Because of its sensitivity, reliability, and simplicity, this test has been extensively used to study commutator surface conditions. Usually, the commutator is ground and the test brushes mounted. The commutator is then heated by narrow belts lined with maple blocks. These are so arranged that they do not strike the surface at points where the test brushes will bear. This method gives the surface heating and temperature distribution closely approximating service conditions. It also gives additional heat on any bar which tends to become high combined with considerable mechanical vibration, so that it is a very severe test which should disclose any trouble likely to develop in service.

This test shows the condition of certain parts of the surface. The roughness will almost never be the same along the full length of the bar. Since the test can be used to show the condition of any part of the surface, it can be used to study the nature of the bar deflections causing the roughness.

### Brush Action on a Rough Surface

The action of a brush in passing over an irregular surface is of importance mainly in designing the "brush rig" which is another major problem, but it must also be studied in connection with the commutator problem. For this purpose, a rotating copper disk was made with

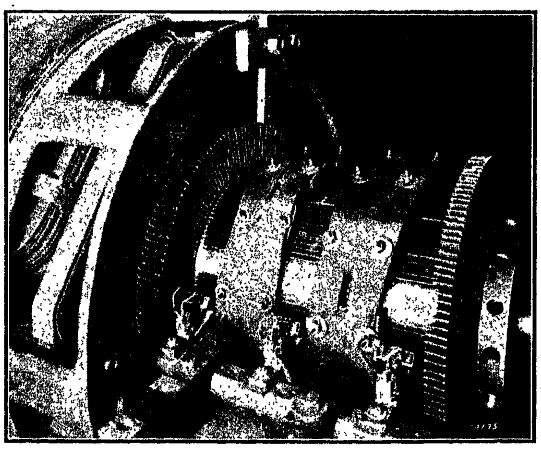


Fig. 4—Commutator Set up for Brush Drop Test

a surface slotted to represent an undercut commutator. In addition, four larger slots approximating the width of a commutator bar were cut in the surface. These four slots were then fitted with bolted-in fillers which could be adjusted by means of shims to give any desired surface roughness. By using gold leaf as a shim, an adjustment of a fraction of a ten-thousandth of an inch could be obtained. Brushes were mounted to run on this surface. The contact condition was determined by recording the contact drop with the oscillograph. Using an ordinary brush, the average contact condition

would be given and by dividing the brush into electrically insulated sections, the contact condition at any part of the brush could be studied. Fig. 5 shows the rotating disk, the brush mounting, and a device for recording brush movement.

These tests showed that for a given commutator surface condition, a wide variety of contact conditions could be obtained, depending on the brush material and surface condition, particularly for very slight irregu-

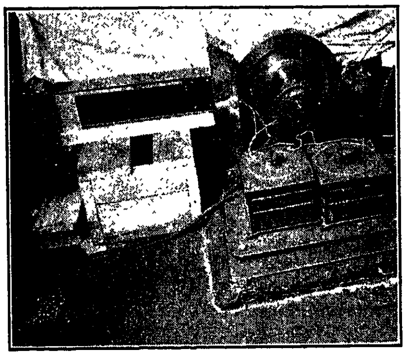


Fig. 5—Apparatus for Studying the Action of a Brush on a Rough Commutator Surface

larities in the surface. Fig. 6 shows three records of the brush voltage drop for brushes passing over exactly the same point in the surface, which, in this case, was a very slight roughness. This shows how with a very slight roughness, it is possible to get a brush condition which will pass over the roughness with almost no variation in voltage drop, while a slightly different brush condition will give a serious variation in the brush drop. With a somewhat greater roughness, the best brush condition would still give a serious fluctuation in brush drop and considerable sparking.

The surface speed is one of the most important factors

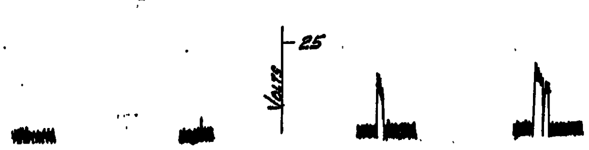



Fig. 6—Contact Drop Obtained with Four Different Brush and Brush Holder Arrangements

The cut was traced from oscillograms showing contact drop passing over the same surface roughness, only the brush and holder being changed

in determining the action of a brush. It is obvious that at very slow speed, a brush can pass over a considerable irregularity without losing contact with the surface. In Curve *A*, of Fig. 7, the fluctuation in brush contact drop is plotted against speed. The roughness or bump in the surface was made by raising the segment representing a bar 0.00015 in. above the rest of the cylindrical surface. The speed was changed without disturbing the brush so that the curve shows the effect of speed alone. This curve could be repeated for the given brush condition, but the result could be changed materially by a change in brush, brush holder, or by merely a different brush wearing surface as was illustrated by Fig. 6. Curve *B* of Fig. 7 is similar to Curve *A*, except that a different brush was used and the roughness or bump was 0.0007 in. high instead of 0.00015 in. These curves show how, for a slight roughness, the performance may be entirely satisfactory at one speed and give serious trouble at a slightly higher speed. Another effect of the higher speed not shown by the curve is the increased duration of the period of poor contact, as the peripheral speed is increased. This duration of poor contact tends to increase nearly with the square of the peripheral speed, which shows the importance of designing for the minimum peripheral speed.

These results show that the brush contact drop changes quite abruptly from the usual drop of about one volt, to a different type of current conduction giving a

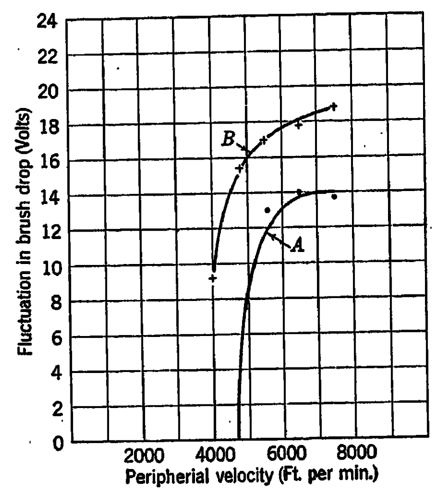



Fig. 7—Curves Showing Fluctuations in Brush Drop Plotted Against Speed

A—Roughness consisting of one bar raised 0.00015 in.
B—Roughness consisting of one bar raised 0.0007 in.

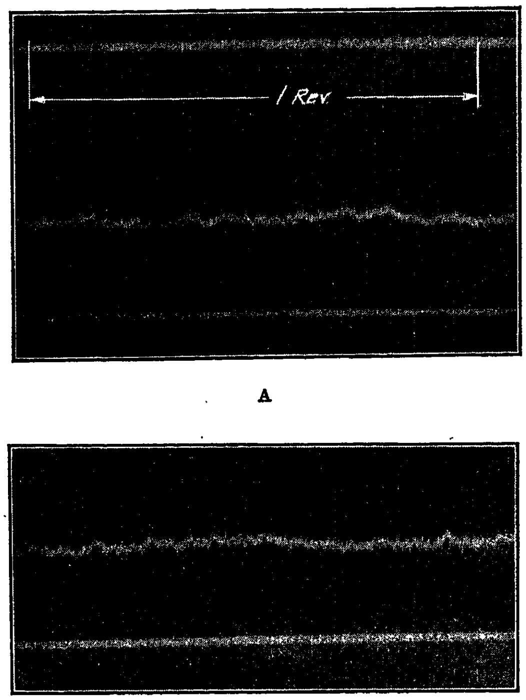
voltage drop of several volts, which is accompanied by a serious burning of the surface. The exact magnitude of the roughness which can be permitted before the brush drop is seriously disturbed is evidently a function of the surface speed and brush condition. However, for the purpose of discussing the accuracy required in a high-speed commutator, it can be considered to be of the order of one ten-thousandth of an inch for an abrupt variation from one bar to the next.

## Effect of Roughness on Operation

Commutation is a complex problem, involving the electrical characteristics of the machine, the brushes, and the commutator surface, so that it is difficult to tell just what is the cause of any given trouble. It is particularly difficult to analyze trouble caused by the surface condition, since the cause of the trouble does not necessarily produce a noticeable effect at once.

A commutator may develop a certain roughness at a high operating temperature. This roughness will

usually exist only over a part of a given bar. Some brushes will maintain contact with the given bar, and these brushes will suddenly be overloaded.[2] This may cause only a very slight sparking at first, but this gradually produces a burning or smutting of the bar which still further disturbs the brush action until a serious sparking results. In other cases, particularly with natural graphite brushes, a slight initial roughness may cause unequal wear of the copper, finally resulting




A

B

FIG. 8—EFFECT OF OVERSPEED

A—Brush drop record at normal speed
B—Brush drop record at 25 per cent overspeed

in a flat spot so that the commutator must be ground. In either of these cases, the effect may continue, even though the roughness which really caused the trouble has disappeared.

This paper is confined to the subject of the commutator surface. It should not be inferred that we consider this to be the only source of trouble, although it is particularly troublesome because it is one of the most difficult to control.

## CAUSES OF ROUGHNESS

Since the commutator bars must be supported through mica insulation, any load yielding of this mica would disturb the alinement of the bars, resulting in a rough surface. Because very little information has been available concerning the properties of mica, it has been blamed frequently for many troubles due to other causes. The requirements of commutator mica are very severe and its properties very different from most engineering materials, so that much research work has been required to develop a satisfactory material for the purpose.

Commercial mica must be built up using mica flakes cemented together by some form of bond. It must be possible to mold the material, and yet it must withstand high pressures in the commutator. In order to support all bars equally, it must be manufactured very accurately to the required thickness and at the same time have a high elasticity or "springiness" to absorb slight variations in the expansion of the metal parts. When the commutator is first assembled, even with V mica manufactured very accurately to thickness, a slightly plastic characteristic is desirable when first heated to allow the mica to conform to the machined surface, but it must quickly harden or set to a permanent condition, for if slippage or yielding of the mica takes place after the commutator is in service, a rough surface will result. For this reason, the compression of mica under the influence of temperature and pressure is very important. To determine these properties, a special mica testing machine was developed in which any temperature and

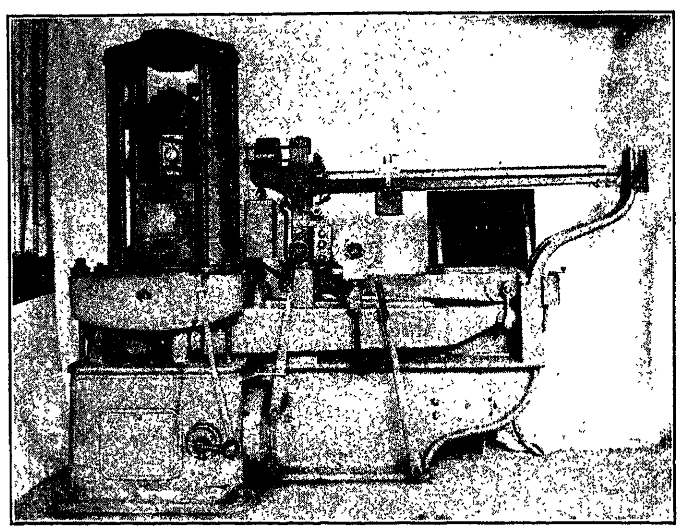


FIG. 9—MICA TESTING MACHINE

Used to study compression of mica under combined pressure and temperature

pressure can be automatically maintained and the compression of the mica accurately measured at any time during the test. Fig. 9 shows this testing machine, which consists of a standard 100,000-lb. capacity Riehle testing machine with automatic control added for maintaining a constant load in the test sample, and with a small electrically-heated oven for automatically maintaining the sample at any desired temperature. Special measuring devices were installed which indicate the compression of the sample at any time during the test.

2. For a brief discussion of the effect of a sudden change in load on a brush, see Arnold and la Cour, "Die Gleichstrommaschine," Par. 79.

These tests have made it possible to determine quickly if a given bond and grade of mica are suitable for the purpose. For instance, a new synthetic bond was recently developed which had desirable characteristics for some applications. It was considered for commutator use, but the compression tests showed that in its present form, it was not suitable for commutator V mica, mainly because if left soft enough to have any ability to conform to the machined surface, it would not season to a stable condition in a reasonable length of time.

On the basis of this kind of tests, it has been possible to develop a built-up mica plate which will quickly season to a stable condition and yet have satisfactory molding characteristics. Fig. 10 shows a compression curve for a sample of V mica. It shows a large initial compression cold and a large additional compression when first heated, but practically no additional yielding after the first 24 hr. of heating. The compression-pressure curve of the seasoned mica is also shown.

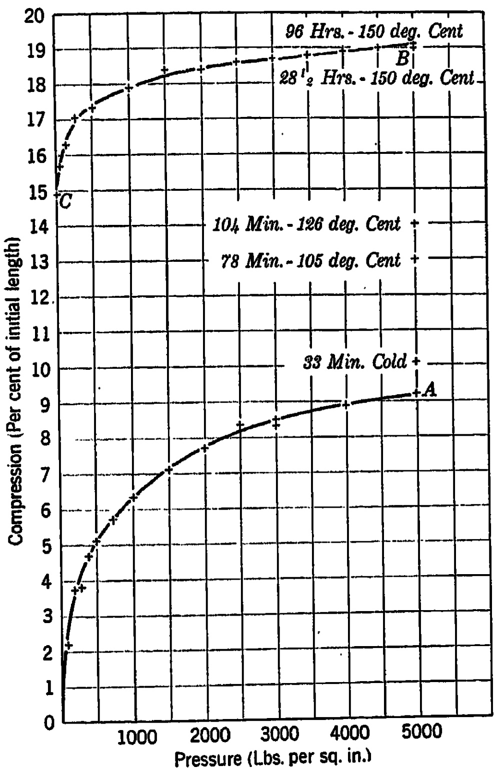



FIG. 10—COMPRESSION-PRESSURE CURVE FOR A TYPICAL SAMPLE OF MICA

O—A shows initial compression curve cold

Points A to B show compression at intervals of time during heating under 5000 lb. per sq. in. pressure

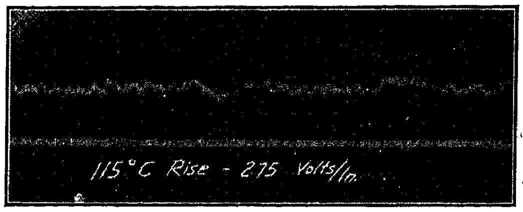
B-C shows compression curve of seasoned mica

Upon first thought, centrifugal force will usually be considered to be the most serious cause of commutator roughness. A long bar may deflect several mils under centrifugal force. Since this deflection varies with the fourth power of the distance between supports, a slight change in the support of adjacent bars such as might be introduced by irregular mica, would give a considerable variation in deflection, which would give a rough surface. Again, in a bad design, there is the possibility of a slow "creep" or yielding of the copper, which is not the same in all bars, leaving the surface rough. However, tests have shown that it is relatively easy to make a commutator that will remain perfectly smooth at all speeds up to a reasonable overspeed, *if tested cold*. Fig. 8 shows "brush drop" records for a typical commutator, first at normal speed and then at overspeed. In this case, the speed was increased 25 per cent, giving 56 per cent increase in centrifugal force and bar deflection without producing measurable roughness.

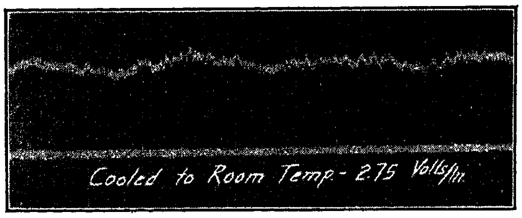
The effect of temperature is usually far more serious in commutator operation than centrifugal force. There are so many possible effects that it is particularly difficult to provide for all of them. One example of the effect of temperature is the difference between the expansion of the copper bar and the supporting parts. The steel parts have a lower coefficient of expansion, and in addition run cooler than the copper bars. This results in a differential expansion, which, if not properly provided for in the structure, may lead to serious results. As a quantitative example, if a copper bar at 50 deg. cent. rise is held between steel supports 15 in. apart and having a temperature rise of 20 deg. cent., the expansion of the copper is ($15 \times 50 \times 0.0000166$) = 0.0124 in. and of the steel parts is ($15 \times 20 \times 0.000011$) = 0.0033 in. or a difference in expansion of 0.0091 in., which, if not provided for in the structure, will result in serious stresses and distortions. For example, in the ordinary design in which the bars are held by V rings, this difference in expansion results in a large change in the bolt force holding the V rings together. This produces a distortion of the V rings and usually a bowing or bending of the copper bars. This same difference of expansion acts in a circumferential and in a radial direction. In some cases, this difference in expansion may result in a sliding action of the bar on the mica. It might seem that the solution of the problem is a low temperature rise. However, a low temperature rise requires a large surface for heat dissipation. For any given design, this usually requires a longer bar. The deflections causing roughness usually are of the nature of a bending or bowing of the bar which increases with the second to the fourth power of the bar length, depending on the type of support. Then, since the deflection increases as a power of the length while the temperature decreases only with the inverse of the first power of the length, for any given type of construction the roughness may be increased, rather than decreased by lowering the temperature rise.

Unequal temperatures between adjacent bars are particularly serious, since unequal expansion is a direct result and in almost every case, the temperature throughout the bar will not be uniform, giving an inherent warping of the bar. There are many possible causes of unequal heating. For example, a slight

smutting or burning of the bar will change both the contact drop and the brush friction. Any sparking will result in considerable energy lost as heat at that point. Any roughness disturbs the brush drop, giving unequal heating. The roughness produced by a given unequal heating can be reduced by certain mechanical constructions, but its complete elimination is practically impossible. Again a reduction in the temperature of the commutator as a whole will not necessarily reduce the roughness. Decreasing the temperature by increasing the cooling area has almost no effect on the temperature difference produced by a given unequal heating. The low thermal conductivity of the mica between bars retards temperature equalization, so that temperature difference for a given unequal heating can be reduced by increasing the area of the mica; that is, by increasing the bar length, but the deflection resulting will vary as a power of the length, so that for a given type of construction the resulting roughness will be increased rather than decreased by increasing the bar length.




A




B

Fig. 11—Ideal Performance

A—Brush drop record after heating commutator to 140 deg. cent.—115 deg. cent. rise)

B—Brush drop record after cooling to room temperature

In addition to the temperature effects mentioned, there are various mechanical causes of roughness, such as vibrations and other slight movements or shifting of structural parts. Mica slippage is decidedly influenced by temperature; but, as explained under "seasoning," in a properly designed commutator, the slippage and set of mica can be made to take place during the seasoning by heating the commutator somewhat above the highest operating temperature. The vibrations and other mechanical movements are usually independent of temperature and therefore not affected by the temperature limit.

## Types of Commutator Performance Under Influence of Temperature

*I. Ideal.* The ideal commutator should stay perfectly smooth under all operating conditions. Fig. 11 shows a brush drop test for a commutator of 23 in. active face length (26½ in. total bar length) and 5500 ft. per minute peripheral speed. It was heated to 140 deg. cent. (115 deg. cent. rise) using a belt lined with maple blocks and tested while cooling. Records at 115 deg. cent. rise and when cooled to room temperature show that a temperature rise far above present limits did not produce any measurable roughness in this commutator.

*II. Fair.* A commutator which becomes rough when heated to the overload temperature, but returns to a smooth condition when cooled is classed as fair. This might show some sparking at overload, but if this is not of sufficient duration to burn the bars, the performance should be satisfactory when the load is reduced. If, as has been pointed out, the overload is of long duration, the bars may be burned or smutted by the sparking, so that even though the load is reduced and the bars return to a smooth condition, the burned bars will still cause sparking. However, if the burning is not too severe, a period of very light load may polish the surface so that the performance will again be satisfactory.

Fig. 12 shows the brush drop records for a commutator in this class if the temperature limits were somewhat above the present limits. Fig. 12A shows the condition of the ground surface before heating; Fig. 12B after heating to 135 deg. cent., Fig. 12C after cooling to 105 deg. cent.; Fig. 12D after cooling to 70 deg. cent.

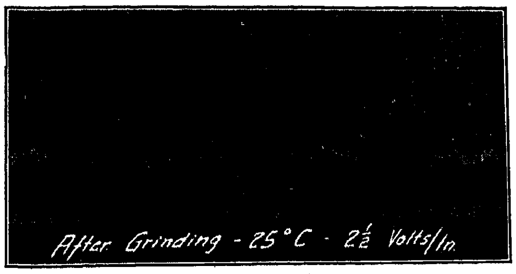
*III. Bad.* A commutator which when heated either to an operating or even a slight overload temperature, becomes rough and fails to return to a smooth condition on cooling is bad, because the surface must be re-ground to put it in a satisfactory condition. Fig. 13 shows a test on an old type construction which became seriously rough at 100 deg. cent. and this roughness only partially disappeared on cooling.

## Commutator Seasoning

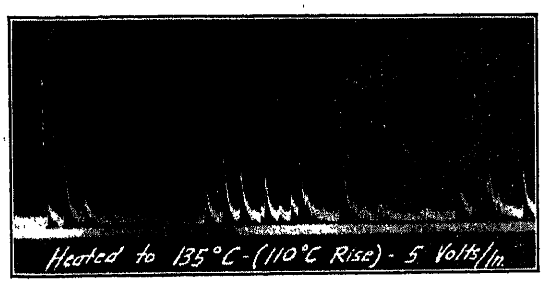
Practically all large commutators, if tested immediately after assembly and before any heating treatment, would show the performance just referred to as bad. This is due to the nature of the copper and mica, both of which tend to yield when first subjected to stress and temperature. The gradual yielding of the copper and mica under stress and temperature until finally a stable condition is reached is referred to as seasoning.

In a low-speed commutator, a sufficiently stable condition may be reached by merely repeated heating and tightening. In a commutator with higher rotational stresses these stresses must act on the commutator for a considerable time during heating and cooling before a stable condition may be reached. Originally,

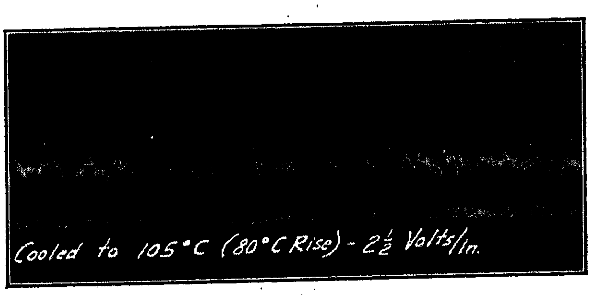
this was done by loading the machine with the brushes set off neutral to give excessive heating. Now, methods have been developed for seasoning the commutator, before it is even assembled on the machine. In some cases, this may be accomplished by merely rotating in an oven. In other cases, a more severe method known as "block seasoning" may be required. This consists in heating the commutator by the friction of belts lined with maple blocks. This gives the surface heating and temperature distribution approximating operating conditions, combined with mechanical vibrations.



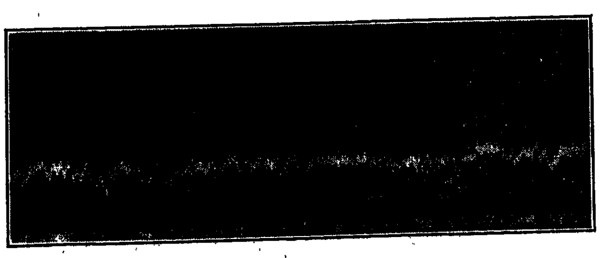

A




B




C



Cooled to 70 deg. cent. (45 deg. rise) 2½ volts/in.

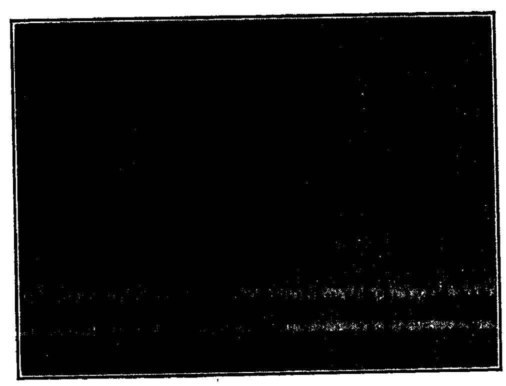
D

Fig. 12—Fair Performance

A—Brush drop record after grinding
B—Brush drop record after heating commutator to 135 deg cent. (110 deg. cent. rise)
C—Same except cooled to 105 deg. cent.
D—Same except cooled to 70 deg. cent.

The most obvious effect of seasoning is to eliminate the yielding of the mica, but in the best type of high-speed commutator, the copper must be permitted to yield slightly to allow it to conform to any irregular mica surface. Due to the peculiar nature of copper, this yielding will take place only partially during assembly. The rest will take place as a slow creep during the seasoning period. Tests have shown that the mica now used reaches stability in about 30 hours of seasoning at 150 deg. cent., while the copper may in some cases require 150 hours even when accelerated by overspeed and added building force.

A



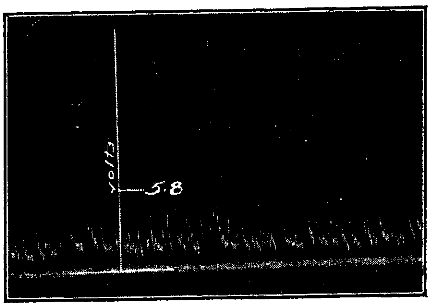
B

Fig. 13—Bad Performance

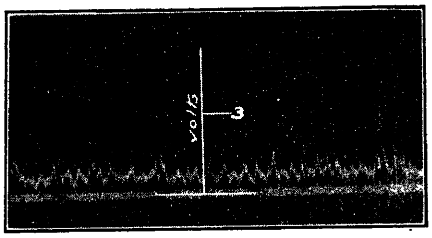
A—Brush drop record after heating to 100 deg. cent. (75 deg. cent. rise)
B—Brush drop record after cooling to room temperature

Fig. 14 shows the results obtained by block seasoning. Each record shows the roughness produced by a heating and cooling cycle. In the first test (Fig. 14A), taken after a short period of block seasoning, the surface was very rough so that only a small current could be used on the test brush. The roughness is shown by the irregularity of the contact drop. The second test (Fig. 14B) shows an intermediate condition. In the

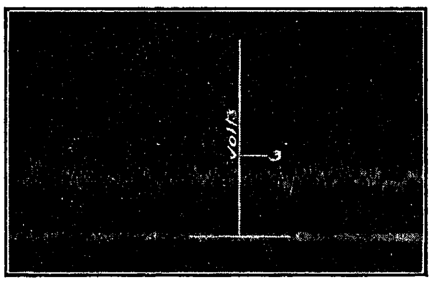
test after the final block seasoning the surface remained almost perfectly smooth so that full current was used on the test brush giving the higher normal contact drop evident in Fig. 14c. The roughness is indicated by the irregularity of the contact drop and this irregularity is almost independent of the test current used. The number and value of the "peaks" in the oscillogram are the significant factors, rather than the average value of the voltage drop. Even after all of these precautions, there may be some minute bar shifting during the first few months of operation, so that more maintenance is to be expected during this period than after the commutator has been in service for some time. A large commutator is one of the few mechanical devices which improve with use during early years of service.




A




B




C

Fig. 14—Effect of Seasoning

A—Commutator block seasoned for a short period and then ground smooth while cold, heated for 10 hr., and record taken after cooling to room temperature

B—Given additional period of block seasoning, ground smooth while cold, heated, and record taken when cold

C—Same as "b" except after an additional period of block seasoning

## Safe Temperature Limits

In the windings of electrical apparatus, fibrous insulation is used either exclusively (Class A), or as the mechanical support of mica insulation (Class B). In these cases there is a definite temperature, which if it be exceeded will rapidly and permanently injure the insulation, and the Institute practise of a definite temperature limit is logical. In commutators, however, mica is the only insulating material, so that the materials in the commutator would not be injured by temperature far in excess of the present limits. The insulation of adjacent windings would also be unaffected on account of the cooling effect and temperature drop introduced by commutator necks. There is, nevertheless, a definite temperature limit for any given construction, but this limit is based on the temperature at which the surface fails to remain smooth, rather than on the effect of temperature on insulation. Since this limit does not depend on the materials used, but rather on the type of construction employed, the present A. I. E. E. Standards should not logically apply to commutators.

A rational type of commutator design and construction may have a safe limiting temperature of 125 to 150 deg. cent without developing surface roughness, whereas other types of construction may be unsuited to limits as low as 75 deg. cent. The ability to withstand repeated temperature cycles without developing roughness determines a commutator's safe limiting temperature. The problems involved are entirely mechanical in nature, and the practise of the present rules in setting an arbitrary low-temperature limitation does not meet the requirements of the situation.

The present standard of a temperature-rise limit and load test is inconclusive. It is ordinarily made at a low-ambient temperature and does not show what trouble might develop from the additional total temperature obtained with the higher-ambient temperature permitted by the Standards, and it cannot usually be of sufficient duration to discover effects that may gradually accumulate due to very slight initial mechanical disturbances. The present practise of setting a temperature limit as a criterion for a mechanical problem is objectionable in that it results in design practise which actually encourages the development of operating trouble which it purports to prevent. Low-limiting temperature such as the present limit of 60 deg. cent. rise on converter commutators require large heat-dissipating surfaces on the commutator, which frequently require extra long commutators, one or more sets of ventilating vanes, and other characteristics which lead to high stresses, large expansion effects, etc., that may be more detrimental to operation than a higher limiting temperature. A physically large commutator having high mechanical stresses and high surface speed that will operate at a low temperature when newly ground and in good mechanical operating

condition, may well be a much less satisfactory commutator to the operating engineer than a physically smaller commutator that under the same conditions of loading, operates at a considerably higher temperature, but stays dead smooth under its maximum operating temperature range. Adherence to the present standards forces the designer to use a highly stressed commutator with high peripheral speed, that may be far more detrimental to good operation and low maintenance costs than would be the higher temperature rise resulting from a smaller lower stressed commutator.

A better measure than the load test, although scarcely practical in many cases, would be a repeated cycle test with the maximum ambient temperature. Satisfactory performance would be indicated by no appreciable increase in sparking with successive heat cycle tests. A less expensive, and therefore more feasible test for determining the permissible operating temperature, would be a "brush drop" or equivalent test, which could be made at the maximum temperature and would have the sensitivity to show any slight roughness which might in time give trouble.

## Discussion

**C. Richard Soderberg:** While there has always been a feeling that the majority of troubles in large commutators are of a mechanical nature, the general subject of commutation is too difficult and too imperfectly understood to enable the electrical designer to state with certainty whether or not the electrical performance of his machine is satisfactory. The brush-drop test, described in this paper, removed this difficulty, and I have no hesitation in stating that this method of analyzing mechanical commutator performance constitutes the greatest step forward in the art of building commutators, that has been made for a long time. It is only necessary to point to the fact that it is now possible to arrive at a definite conclusion as to whether a certain commutator is satisfactory or not, from a mechanical point of view, even before it has been assembled with its armature. If any criticism of the paper were warranted, it would be a criticism of their neglect to bring out this point with the emphasis it deserves.

In reading the paper one is struck by the complete absence of the usual controversy about different types of detail construction of commutators.

The results of this paper go beyond these minor differences, and any one can profit by making use of these results, regardless of the type of construction that he may favor.

**C. L. Dawes:** For a number of years I have been associated with manufacturers of mica segment plate such as the authors describe, and we already have performed a considerable amount of research work on built-up mica plate. There always has been great difficulty in finding any authoritative publications or information dealing in a scientific manner with the subject of segment plate, particularly with the conditions of its manufacture and the subsequent effect on the performance of the plate under operating conditions in commutators.

Hence, I am in perfect accord with the following statement of the authors: "Because very little information has been available concerning the properties of mica, it has been blamed frequently for many troubles due to other causes. The requirements of commutator mica are very severe and its properties very different from most engineering materials, so that much research work has been required to develop a satisfactory material for the purpose." Then the authors go on to say: "It must be possible to mold the material, and yet it must withstand high pressure in the commutator. In order to support all bars equally, it must be manufactured very accurately to the required thickness and at the same time have a high elasticity or 'springiness' to absorb slight variations in the expansion of the metal parts." It must also have the further property of becoming hard and unyielding after it has been baked in the commutators as otherwise the commutator will yield when in service. Also, in addition, many manufacturers require a high dielectric strength. Most of the foregoing properties of segment plate are controversial.

About ten years ago it was customary to use 10 to 12 per cent of binder which is ordinarily shellac. It was very difficult to reduce the amount of binder at that time because the plate was laid up by hand. It is very difficult to lay up the films by hand with sufficient uniformity and yet have the binder content below these percentages.

With the advent of laying machines the percentage of binder has been gradually reduced until at the present time it is somewhere between 3 and 5 per cent. Many users of plate, ignorant of the physical effects within commutators, began to think that low binder content was a criterion of good quality. Hence it developed in many cases that the manufacturer who could produce plate with the lowest binder content would obtain the business. If there is too little binder within segment plate, it becomes mechanically poor and brittle, and when punched the segments split apart. On the other hand, if there is too much binder, slipping develops in the commutator and the films slide out. With high binder content it is possible to have high dielectric strength and also to obtain the softness and springiness, which is required, and which the authors mention.

In order to obtain information which might assist us in manufacturing plate which would better meet conditions, we have written to a number of manufacturers and users of commutators. Among other questions we requested the temperatures at which they bake their commutators, so that we might adapt our plate to meet such conditions. A number did not reply, and it appeared to us that they did not know. I believe that the Westinghouse Company in one of the bulletins which they send to colleges state that they bake their commutators at 250 or 300 deg. fahr.

One of the manufacturers wrote to us stating that if anybody tells you the exact temperature at which they bake their commutators, they are not telling the truth because they don't know. This has been our experience.

In the smaller commutators such as are used for electric starters and vacuum cleaners, the commutators are sent through an electric oven usually maintained somewhere near 700 deg. fahr. The process is a continuous one and the commutators are sent through at such a rate that they never attain the temperature of the oven. Naturally under these conditions the maximum temperature which the commutators reach would not be known. The temperature and rate of passing through the oven are adjusted until satisfactory results have been obtained.

We find wide variations among the requirements of various commutator manufacturers. Some do not object to 6 per cent binder; others insist on having, say, less than 5 per cent. Sometimes plate is supplied with binder content as low as 3 per cent, but such plate is near the limit of good mechanical qualities. With the authors, I believe that it would be very helpful to all concerned if more exact scientific knowledge of commutator manufacture was available.

The curves in Fig. 10 showing the compression of molding plate are similar to curves which we have been obtaining for some time, not only for molding plate, but for segment plate as well. We have a much smaller and simpler apparatus than shown in Fig. 9. We use a very strong spring, carefully calibrated to obtain the compression, and by using small samples of plate we can subject it to the very high compression per unit

area and in that way measure the change of compression with time at various temperatures.

We have a large number of data giving the effect of the binder content on the compression, but they have not as yet been published since we are not certain as to how near our temperatures and pressure conform to actual practise. I believe that I am correct in saying there is less known scientifically both of the manufacture of commutators and of the actual operation of the commutators than of any one part of electric machines.

The authors, I believe, have accomplished much in the matter of increasing this knowledge by showing us in such a thorough and fundamental manner the reactions which are actually taking place in the commutator when it is under operation, and also in pointing out so specifically what the properties of mica plate should be to make a good commutator.

**W. I. Slichter:** Mr. Hague made a statement incidental to the paper which struck me as of very great importance at this particular time. That was that the A. I. E. E. rules of temperature limitation contributed to the design of a commutator which was not as good as it should be. As chairman of your committee on the Standards for d-c. rotating machinery delegated to confirm or improve these Standards, I am particularly interested in this because we have in our standards which are about to be submitted to the American Standards Association for final adoption, certain temperature limitations which are from 10 to 15 deg. higher than that adopted by European standards. As a member of the Advisory Board of the International Electrical Commission I am delegated to work to bring these two together and, representing the American engineers, to try to have the European temperatures brought to our higher levels. I am very much interested, therefore, in hearing whether that is a desirable thing to do or, as a result of the work of Messrs. Penney and Hague, it may be the European engineers have the better idea.

**E. S. Lee:** Professor Dawes' remarks are interesting concerning the available information relative to mica. There is much known about mica which may not have been published. It would take many more printed pages than we now have if everything were published.

I can subscribe to the effectiveness of the method described in the Hague-Penney paper of obtaining physical characteristics of mica at elevated temperatures, as we have used this method considerably during the past several years.

**F. D. Newbury:** Commutator construction has been and probably still is more of an art than a science. If the present paper has any value it is in placing this difficult branch of machine construction on a little more scientific and certain basis.

Mr. Soderberg in his remarks mentioned the difficulty of separating the mechanical and electrical performance characteristics of commutators. Those who have been responsible for the entire performance, whether it be mechanical or electrical, factory or design, know how serious the argument as to responsibility for poor performance may become, and the means described in this paper for testing a commutator solely with regard to its mechanical operating characteristics, I feel, is an achievement of prime importance.

I should like to say a word about the Standards aspect of this situation. In our present standards and, in fact, in the standards of the entire world, as far as I know, the only requirement from the standpoint of operation of commutators is the one of limiting temperature rise. The determination of that limit has been based wholly on the safe limiting temperature of insulation. While this limit is important, it is of only minor importance as compared with the limitation of smooth operating surface. The paper is timely from this aspect of the situation on account of the present discussion in Europe concerning temperatures of commutators. Our temperature limits are somewhat higher than European limits as Professor Slichter points out.

The point that I would emphasize is that our present Institute limit may be too high or it may be too low but whether it is one or the other is purely accidental because the limit has been based on the wrong criteria. It is a mechanical and not an insulation problem, and the test should be a mechanical test, not a temperature test. Temperature of commutators becomes important only because temperature may be related to smoothness. This paper presents a means of making a mechanical test and that, in turn, makes possible a rational standard. I don't mean that we are ready to include in our standards such a mechanical measure of commutator performance, but if others will use the means here described or some similar means, we will accumulate experience that will make a rational standard a possibility for the future.

**A. M. MacCutcheon:** I wish to emphasize the value of this paper. It is very interesting and helpful to all who are working with commutators.

I agree with Mr. Newbury in his feeling that the permissible temperature of commutators has not been established on a scientific basis.

I have often noted that good commutation accompanies a cool commutator. In particular, I recall one case where the current density was very high, the mechanical operation was not good, and yet the commutation was excellent. In this instance, the commutator was very cool, having less than 40 deg. rise.

**G. W. Penney:** Mr. Dawes raised the question of the temperature at which commutators are baked and quoted one manufacturer as saying that no one knows this temperature. This is probably true in the case referred to of small commutators passing through an oven in a continuous process, although even here the actual commutator temperature could be measured. However, a statement such as the one just referred to should not be taken to mean that the manufacturer of large commutators does not know the temperature at which his commutators are baked. In our practise large commutators are baked in a steam heated oven equipped with recording thermometers. Then in order to determine the difference between the actual commutator temperature and that of the recording thermometer, thermocouples have been embedded in commutators of various sizes and time temperature curves taken so that by comparing these results with the recorded thermometer temperatures for any other commutator the actual commutator temperature can be very closely approximated.

To answer Mr. Dawes' question, our practise is to bake large commutators at from 135 to 150 deg. cent., although this does not mean that all commutators should be baked at this temperature since the correct temperature for a given line of commutators depends on the binder used in the mica, on the kind of solder used, on the hardness of the copper, and other factors. I do not know where the figure of 250 to 300 deg. cent. came from. It might refer to an oven temperature in some process connected with small commutators but it obviously could not refer to the actual temperature of large commutators since pure tin solder is used and this would flow out at 230 deg. cent. and furthermore, a temperature of 250 to 300 deg. cent. would rapidly anneal the copper.

The remark is frequently made that experience shows that high operating temperatures give trouble and therefore there should be a low temperature limit. However, an investigation of such cases usually shows mechanical roughness which is a function of the temperature such as that described in the paper but where redesigning for lower temperature may not be as successful as redesigning to reduce the roughness at the same temperature. Brush trouble is another serious question but here again there apparently is no fundamental temperature limit.

One test of the temperature question is to use an external fan and blow a large quantity of air at high velocity against a commutator which is giving trouble so as to give a lower operating

temperature. In many types of trouble due to mechanical roughness this should improve the operation, but if the commutator is mechanically smooth this should not improve the surface. In order to determine if there was a temperature limitation of some other nature besides mechanical roughness a commutator which was apparently mechanically smooth but which still would not commutate well was cooled by an external fan but this gave no noticeable change in the performance. This is another indication that temperature of itself is not a limitation.

So long as the troubles are not a function of any given temperature and so long as there is apparently no fundamental temperature limit (within the operating range) the practise of fixing an arbitrary limit is hardly more logical than fixing an arbitrary limit to the speed for a machine of a given rating or an arbitrary limit to the current density that may be used in the winding. As Mr. Newbury has pointed out, this does not say whether the actual operating temperature should go up or down but merely that it should be determined on a more logical basis.

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